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Published in:
IMF Economic Review

DOI:
10.1057/imfer.2014.8

IMF Economic Review
Vol. 62, No. 2

Sovereign Default and the Stability of Inflation-Targeting Regimes

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We analyze the impact of interactions between monetary and fiscal policy on macroeconomic stability. We find that in the presence of sovereign default, macroeconomic stability requires monetary policy to be passive if the feedback from debt surprises back to the primary surplus is too weak. An active monetary policy can however only contribute to the stabilization inflation and output, if the primary surplus is increasing in debt with a slope that increases with the default probability. The results are relevant for the design of fiscal and monetary policy in emerging markets where sovereign credibility is not well established. Recent debt developments in Western Europe and in the United States suggest these results may become relevant for more mature financial markets too once the current low inflation period is over. [JEL E52, E63, F41]
IMF Economic Review (2014) **62,** 261–287. doi:10.1057/imfer.2014.8;
published online 1 July 2014

Can aggressive inflation targeting lead to macroeconomic instability when high public debt levels trigger fears of sovereign default? In the last decades inflation targeting has become the preferred modus operandi of central bankers across the world, with wide support from the academic community. The practice has rapidly gained ground not only in developed countries but also among emerging market economies (International Monetary Fund (IMF), 2005; OECD, 2008).

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Yet some have argued that it might be an unsuitable strategy for countries with high sovereign debt[1] and as yet shaky reputations as inflation fighters. And some 10 years after the initial warnings of Blanchard (2005), the unfortunate coincidence of high public debt and deficits and a concurrent need to restrain inflation is emerging once again in major emerging market countries like India and Brazil.

These concerns have mostly been expressed with emerging market countries in mind, but recent postcredit-crisis developments have shown that the issue may become relevant in more mature financial markets too, as rescue and stimulus packages have led to rapid increases in deficits and debt levels in Western Europe and the United States. Of course the ongoing recession has pushed inflation fears to the background in Western Europe, Japan and to a lesser degree the United States, but there is an enormous mass of liquidity hanging over the market after successive rounds of unconventional monetary transactions. What a future world will look like is as yet unknown, but it will be a high debt world and may once again be a higher inflation world as liquidity is absorbed. And then our questions will become relevant for OECD countries too.

The implementation of inflation targeting is premised on the assumption that high real rates slow down inflation; in that case a mean reversion to the inflation target is likely to be a stable process. But if for any reason high real rates do not slow down inflation, macroeconomic stability cannot be guaranteed. For example if high real rates and the ensuing increase in debt service burden lead to higher default fears, capital outflows, and pressure on the exchange rate, a perverse impact on inflation may well arise. If then an active interest rate policy would be maintained anyhow, such a perverse effect can clearly become an element of instability, as suggested for example in Blanchard's (2005) discussion of Brazil. An analysis of the potentially destabilizing impact of such interactions thus is of particular relevance for economies where the reputation of fiscal solidity is not well established, be they emerging market economies or more mature countries in the aftermath of the credit crisis.

The literature on inflation targeting is too large to survey even in summary; an overview is given by Svensson (2005). At the heart of its theoretical foundation is the idea that the central bank should minimize fluctuations in inflation and the output-gap, which are costly because of the existence of price rigidities (see Svensson and Woodford, 2005). Although the central idea is in principle not related to any particular policy instrument, inflation targeting is commonly associated with the use of interest rate feedback rules. The idea is that a central bank should adjust interest rates in response to an increase in expected inflation in a way that reduces aggregate demand enough to stabilize inflation around its target value. The consensus view from that literature is that macroeconomic stability will be assured as long as interest rates are set according to the Taylor principle, that is, respond to inflation by more than one for one (see Woodford, 2003). Feedback effects of debt service costs on the

[1]See Blanchard (2005) or Sims (2011).

default probability, and the possibility of emerging stability problems, have not been considered in this literature.[2]

These feedback effects (from debt service costs to default risk premiums and from there back to debt service costs) are at the core of this paper. We set up a mostly standard model of a small open economy with a floating exchange rate and perfect international capital markets, where a rigidity in domestic producer prices is the main macroeconomic distortion. This implies that the central bank should stabilize domestic producer prices instead of the CPI (Gali and Monacelli, 2005). The government follows a tax rule like in Bohn (1998), with a feedback from higher debt levels on taxation. Such a rule guarantees intertemporal solvency in the sense of Bohn (1998), but may imply rates of taxation that are perceived as politically infeasible (a so-called fiscal limit, in the language of Davig and Leeper, 2010, and Davig and others, 2011). We use a simple model of the default process, but one that shares a characteristic with many more ambitious models of strategic default decisions that the probability of default rises after debt levels are pushed up. This feature, that is, a probability of default that is increasing in real government debt, commands overwhelming empirical support (see Edwards, 1984; Eichengreen and Mody, 2000; Aizenman and others, 2013).

We assume an independent monetary authority that follows a simple inflation-targeting policy. Within this environment, we analyze the stability implications of a standard interest rate rule by which the nominal interest rate is increased in response to changes in (domestic producer price) inflation. We show that in the absence of a sufficient feedback from debt surprises on the primary surplus, an active interest rate policy will render equilibrium determinacy, that is, the existence and the uniqueness of equilibrium sequences that converge to a steady state, impossible. Only when there is a sufficiently strong feedback from higher debt levels on higher primary surpluses (in our context higher taxes) equilibrium determinacy prevails. The more crisis-prone the country is, that is, the higher the default probability, the stronger that feedback needs to be. If, however, the debt feedback is not strong enough, equilibrium determinacy requires a passive interest rate policy.

This result largely resembles Leeper's (1991) conditions for stable local equilibria with stationary public debt.[3] As tax rates respond positively to increases in public debt, so-called non-Ricardian fiscal policies are ruled out (see Bohn, 1998). However, Ricardian equivalence does not apply, as changes in real debt alter default expectations and thereby the effective rate of return on government bonds. Hence, a stable equilibrium can only exist if the debt sequence converges to a long-run value, due to its nonneutrality. Our analysis shows that if real debt affects default expectations, macroeconomic stability (in the sense of equilibrium determinacy) under a tax policy which responds too weakly to public debt requires a passive monetary policy.

[2]Bi and others (2010), who examine interactions between monetary and fiscal policy under sovereign default risk, provide a local determinacy analysis for the case of a fixed default rate.

[3]Leith and Wren-Lewis (2000) derive similar conditions for an overlapping generations model where fiscal policy also matters for equilibrium determination.

We find that the conditions for macroeconomic stability do not depend on the openness of the economy. When higher interest rates raise public debt and the perceived default probability, the fall in the effective real rate of return does not only affect the exchange rate but can also reduce domestic savings. Hence, a perverse response of inflation to an increase in interest rates is also possible in an economy which is less open and where public debt is mainly held by domestic households. Our analysis further suggests that the destabilizing effect of active interest rate policies is also relevant in the case where the government issues debt that is denominated in foreign currency.[4]

It should be noted that the analysis in this paper does not imply that inflation targeting is a source of macroeconomic instability under a weak fiscal policy and fears of sovereign default. Instead, the results described above only apply to the case where the central bank aims at implementing an inflation-targeting policy by setting the interest rate. If however an inflation-targeting policy is implemented via contingent money supply adjustments, the fiscal policy stance is less crucial.[5]

In the final part of the paper, we demonstrate that monetary and fiscal policy interactions are not only relevant for equilibrium determinacy, but affect macroeconomic volatilities as well. In particular, we find that higher feedback from debt on the primary surplus can improve the inflation-to-output trade-off faced by the central bank. Our results therefore provide formal backing for the claim often heard from central bankers that loose fiscal policy reduces the leeway a central bank has in pursuing its anti-inflation goals.

The remainder of this paper is organized as follows. Section I develops the model. In Section II we analyze conditions for stable local equilibria in the presence of endogenous default premiums. In Section III we examine the impact of fiscal policy on the central bank's inflation-to-output trade-off. Section IV concludes.

I. A Small Open Economy Model

In this section we present a model of a small open economy that is mostly standard[6] except for the treatment of sovereign default. Domestic and foreign households have access to a complete set of contingent claims on foreign currency and to domestic currency-denominated public debt. For simplicity, we neglect holdings of money and assume that the economy is cashless,[7] without loss of generality. Nominal (real) variables are denoted by large (small) letters.

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[4]Details on the conditions for macroeconomic stability for the indexed debt case are available upon request from the authors.

[5]Schabert (2010) provides a related argument in favor of money supply policies in a flexible price framework, where sovereign default is modeled according to Uribe's "fiscal theory of sovereign risk" (2006).

[6]Cf, for example, Gali and Monacelli (2005).

[7]See Woodford (2003) for a discussion of this approach.

The Public Sector

The public sector consists of two parts, the government and an independent central bank. The government levies lump-sum taxes $P_t \tau_t$ on domestic households (P_t denotes the price level of the aggregate consumption good), purchases goods g_t, which are exogenously given, and issues one-period discount bonds B_t. Domestic government debt is internationally traded and either held by domestic households $B_{H,t}$ or by foreign households $B_{F,t}$:$B_t = B_{H,t} + B_{F,t}$. At the beginning of each period t the government issues new bonds B_t to finance purchases of goods and outstanding debt obligations. Government bonds are traded at the domestic currency price $1/R_t$ and each unit of debt B_{t-1} issued in $t-1$ leads to a promised payoff of one unit of the domestic currency in period t.

Following Bohn (1998), we assume that the government follows a simple fiscal rule for its core tax policy $\tilde{\tau}_i$. These taxes are raised in a lump-sum way up to a fraction $\kappa > 0$ of the outstanding stock of debt in excess of a target level b^*defined in real terms:

$$\tilde{\tau}_t = \kappa\left(\left(\frac{B_{t-1}}{P_t}\right) - b^*\right), \quad \text{where} \quad \kappa \in (0, 1]. \tag{1}$$

We account for the possibility of sovereign default and its role for macroeconomic stability using a deliberately simple model of the default process. We assume that the default is not a strategic "premeditated" decision of the government but may occur nevertheless once an unanticipated shock: If the shock is such that the core tax policy $\tilde{\tau}_t$ would require a level of taxation in excess of a level deemed politically inacceptable by the government, as is unintentionally possible in this stochastic setup, it defaults on its debt obligations for that period rather than seeing taxes rise to politically unacceptable levels. This notion is similar to the existence of a "fiscal limit" $\overline{T}$ in the language of Davig and Leeper (2010) and Davig and others (2011). Define a default indicator Δ_t: if Δ_t equals 1, there is a sovereign default (that is, B_{t-1} goes unpaid), if $\Delta_t = 0$, debt is serviced as scheduled: thus $\Delta_t = 1(0)$ when core tax policy $\tilde{\tau}_t$ exceeds (falls short of) $\overline{T}$.

We assume that the fiscal limit itself (that is, the maximum tax level that is politically acceptable) is not known with certainty to investors, and that their beliefs on $\overline{T}$ are represented by the probability density function $f(\overline{T})$. An alternative interpretation of the same mathematical structure would be that the government decides on where its fiscal limits are using a random draw mechanism following the pdf $f(\overline{T})$.[8] Then assuming rational expectations on the part of investors implies that their beliefs about $\overline{T}$ can once again be summarized by the probability density function $f(\overline{T})$. The density function $f(\overline{T})$ implies a distribution for Δ_t, with the decision rule for Δ_t as stated above (default whenever core tax policy would imply $\tilde{\tau}_t > \overline{T}$). We do not impose any restriction on f other than that it

[8]There are policy game set ups where such Bayesian strategies emerge as optimal (see for example Pastine (2002) who shows this for the exchange rate crisis model outlined in Cumby and van Wijnbergen (1989)). Note also that this assumption convexifies the problem which considerably simplifies solving the model.

is a proper pdf. We furthermore assume that the gains due to default are handed out in lump sum fashion, specifically not proportional to the holdings of B_{t-1}. The latter restriction implies that, first, while defaults occur they do not have any effects since these are offset by the lump sum transfers, but, second, their anticipation does have real effects since the individual compensation is not tied to individual holdings of debt. Core tax policy $\tilde{\tau}_t$ equals the level of taxes that would have resulted if no default would have occurred. So the actual level of taxes τ_t equals:

$$P_t\tau_t = P_t\tilde{\tau}_t - \Delta_t B_{t-1}. \tag{2}$$

Thus, the ex-post public sector budget constraint is:

$$B_t R_t^{-1} + P_t\tau_t = P_t g_t + (1-\Delta_t)B_{t-1}, \quad \text{where} \quad B_t = B_{H,t} + B_{F,t}. \tag{3}$$

Suppose, without loss of generality, that $b^* = 0$. With Equation (3), and Equation (1), public debt evolves according to $B_t/R_t = P_t g_t + (1-\kappa)B_{t-1}$, so nominal debt grows at a rate that is smaller than the nominal interest rate. This obviously guarantees intertemporal government solvency, that is, $\lim_{k\to\infty}(B_k/R_k)\Pi_{i-1}^{k}R_{i-1}^{-1} = 0$, for any finite initial value B_{-1} (see also Bohn (1998)). Hence, fiscal policy is consistent with the households' unwillingness to support a government Ponzi-game (that is, satisfies the transversality condition).[9] Alternatively, this condition can be seen as a capital market participation constraint without which the government could not place its debt. However, public debt will be nonneutral for the equilibrium allocation given that the investors' rationally perceived default probability depends on the stock of outstanding debt (see below). Government expenditures are exogenous and assumed to be constant over time, for convenience.

The central bank targets the price of government bonds when it conducts its (unmodeled) liquidity providing facilities, for example by trading reserves against treasuries in open market operations. It thereby takes the investors' willingness to hold risky government bonds fully into account and aims at adjusting the target rate with changes in macroeconomic indicators. Specifically, we assume that the nominal interest rate on government bonds is adjusted contingent on changes in domestic producer price inflation $\pi_{H,t}$:

$$R_t - R = R(\pi_{H,t} - \bar{\pi}_H), \qquad R' \geq 0,\ R_t > 1, \tag{4}$$

where the central bank sets the target inflation rate $\bar{\pi}_H$ and considers an average interest rate R. Gali and Monacelli (2005) show that for the special case of unit intra- and intertemporal substitution elasticities, when imperfectly set domestic producer prices are the main distortion, monetary policy should aim at stabilizing the domestic price inflation rate, not the CPI inflation rate.[10] This principle also applies here, given that the main difference to their model, that is, sovereign default, is modeled in a way that does not distort the allocation of resources.

[9] Hence, "non-Ricardian" policy regimes are ruled out (see Kocherlakota and Phelan, 1999).

[10] Moreover, stabilizing the CPI raises the likelihood of equilibrium multiplicity (see De Fiore and Liu, 2004).

The Private Sector

Investors' Expectations

Defaults occur when servicing the debt would require the politically infeasible level of taxation $\overline{T}$ (the so-called fiscal limit in the language of Davig and others (2011) and Davig and Leeper (2010)). As discussed in the previous section (see text following equation (1)), lenders/investors do not know the exact value of $\overline{T}$, but have rational expectations on the rule the government actually follows, or, in a simpler interpretation, are uncertain about the exact value of $\overline{T}$. Either way their beliefs about $\overline{T}$ are summarized by the prior pdf $f(\overline{T})$. Given that tax revenues $\tilde{\tau}_t$ are set according to Equation (2), the probability of default then equals the probability that the tax rule implies a level of $\tilde{\tau}_t$ exceeding $\overline{T}$:

$$\delta_t = \int_0^{\tilde{\tau}_t} f(\overline{T})d\overline{T} = \int_0^{\kappa(B_{t-1}P_t^{-1} - b^*)} f(\overline{T})d\overline{T}. \tag{5}$$

Clearly δ_t equals the investors' expectation of the value of Δ_t. The impact of real debt on the default rate can be found by simply differentiating Equation (5) which gives:

$$\frac{\partial\delta_t}{\partial B_{t-1}/P_t} = \kappa \cdot f\left(\kappa\left(B_{t-1}P_t^{-1} - b^*\right)\right) > 0. \tag{6}$$

Thus, the perceived default probability is strictly increasing in the real value of beginning of period debt. equation (6) is the key relation in our debt default model. Although our simple framework for the default process is more of a reduced form approach rather than resulting from a fully specified optimizing decision framework, it shares this feature (higher levels of debt leads to higher expectations of default) with many explicitly optimizing models of sovereign default (cf Aguiar and Amador, 2013, for a recent survey). For the local analysis of the model we will repeatedly use the product of the default elasticity with respect to the real value of public debt B_{t-1}/P_t at the steady state with the ratio $\delta'/(1-\delta)$: $\Phi = (b/\pi)\cdot\delta'/(1-\delta)$, where b denotes steady-state real debt and π the steady-state CPI inflation rate. For simplicity we will refer to Φ as the default elasticity.

Domestic Households

Assume a continuum of infinitely lived domestic households, with identical asset endowments, time endowments, and preferences. Their consumption basket c_t is an aggregate of domestically produced goods c_H and foreign goods c_F; $c_t = \gamma c_{H,t}^{1-\vartheta} c_{F,t}^{\vartheta}$, where $0 \leq \vartheta \leq 1$ and $\gamma = [\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}]^{-1}$. This leads to the standard share equations $c_{H,t} = (1-\vartheta)(P_{H,t}/P_t)^{-1}c_t$ and $c_{F,t} = \vartheta(P_{F,t}/P_t)^{-1}c_t$, where $P_{H,t}$ and $P_{F,t}$ are the price indices of the domestically produced and foreign consumption goods, respectively, and ϑ denotes the import share. The price index of the aggregate consumption good (CPI) is: $P_t = P_{H,t}^{1-\vartheta}P_{F,t}^{\vartheta}$. Contemporaneous utility u_t of a representative domestic household rises with aggregate consumption and with leisure l_t, where $l_t \in [0, 1]$ and

$n_t = 1 - l_t$ is the working time. Its objective is to maximize expected utility of consumption and leisure over time:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \zeta \frac{(1-n_t)^{1-\sigma_l}}{1-\sigma_l} \right) \right], \qquad \sigma>0, \sigma_l \geq 0, \tag{7}$$

with $\beta \in (0, 1)$ the time preference discount factor. The household earns labor income $P_t w_t n_t$, pays taxes $P_t \tau_t$, and receives profits from monopolistically competitive firms indexed with $i \in [0, 1]$.

Domestic and foreign households can further borrow and lend in terms of nominally state contingent claims, which are internationally traded.[11] Let $\Gamma_{t,t+1}$ denote the stochastic discount factor for a one-period ahead nominal payoff, that is, the period t price in foreign currency of one unit of foreign currency in a particular state of period t+1 normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the time t domestic currency price of a random payoff D_{t+1} in period t+1 is given by $E_t[S_t \Gamma_{t,t+1} D_{t+1}]$, where S_t is the nominal exchange rate (measured as units of domestic currency per unit of foreign currency). The household maximizes lifetime utility (equation (7)) subject to the budget constraint, which takes into account default beliefs ($\delta_t \geq 0$),

$$E_t\left[S_t \Gamma_{t,t+1} D_{t+1}\right] + \left(\frac{B_{H,t}}{R_t}\right) \leq S_t D_t + (1-\delta_t) B_{H,t-1} + P_t w_t n_t - P_t c_t - P_t \tau_t + \Sigma_t, \tag{8}$$

and a no-Ponzi-game condition, taking prices, taxes, dividends, the default probability, and the initial wealth endowment D_0 and $B_{H,-1}$ as given. $\sum_t$ collects firms' profits. The first-order conditions corresponding to the solution of the constrained maximization problem are:

$$\zeta(1-n_t)^{-\sigma_l} = w_t c_t^{-\sigma}, \ \beta E_t\left\{(1-\delta_{t+1}) c_{t+1}^{-\sigma} \pi_{t+1}^{-1}\right\} = \frac{c_t^{-\sigma}}{R_t}, \beta S_{t+1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1}$$
$$= S_t c_t^{-\sigma} \Gamma_{t,t+1}, \tag{9}$$

where $\pi_t = P_t / P_{t-1}$. The first equation equates the marginal disutility of work to the marginal utility of the consumption it permits; the other two equations equate the intertemporal terms of trade using both available assets to the trade off between (marginal utility of) consumption today and consumption tomorrow. Further, the budget constraint holds with equality and the transversality condition is satisfied:

$$\lim_{k \to \infty} E_t \left(\frac{S_{t+k} \Gamma_{t+k,t+1+k} D_{t+1+k} + B_{H,t+k}}{R_{t+k}} \right) \left(\frac{S_t}{S_{t+k+1}} \right) \Gamma_{t,t+1+k} = 0. \tag{10}$$

[11] This assumption is made only for convenience, that is, to facilitate deriving analytical results. Market incompleteness, for example, by assuming that only risk-free bonds instead of a complete set of contingent claims are available, does not change the main properties of the model.

Of course the possibility of trading over time using two different assets gives rise to a no-arbitrage condition between the returns of those two assets:

$$\frac{1}{R_t} = E_t\left\{(1-\delta_{t+1})\left(\frac{S_t}{S_{t+1}}\right)\Gamma_{t,t+1}\right\}. \tag{11}$$

Hence, higher expected default probabilities lead investors to demand a higher interest rate on government bonds.

We assume that preferences of foreign households exhibit the same qualitative structure as domestic households. Hence, their demand for domestically produced consumption goods $c^*_{H,t}$ and foreign consumption goods $c^*_{F,t}$ satisfies $c^*_{H,t} = \vartheta^*(P^*_t/P^*_{H,t})c^*_t$ and $c^*_{F,t} = (1-\vartheta^*)(P^*_t/P^*_{F,t})c^*_t$, where $\vartheta^* \in (0, 1)$ and c^*_t is aggregate foreign consumption. We assume there is a strong home country bias in consumption: $\vartheta^* << 1-\vartheta$ or the Laursen-Metzler condition $1-\vartheta-\vartheta^*>0$ such that the import shares add up to less than one.

Foreign households also have access to a complete set of contingent claims and they can hold domestic public debt $B_{F,t}$, which is denominated in domestic currency. We assume that the instantaneous utility function of foreign households is similar to the one of domestic households and that they have the same discount factor β. Their first-order conditions for investments in both assets are given by

$$\begin{aligned} \beta E_t\left\{[1-\delta_{t+1}]\left(c^*_{t+1}\right)^{-\sigma*}\left(\pi^*_{t+1}S_{t+1}\right)^{-1}\right\} &= \left(c^*_t\right)^{-\sigma*}(R_tS_t)^{-1}, \\ \beta\left(c^*_{t+1}\right)^{-\sigma*}\left(\pi^*_{t+1}\right)^{-1} &= \left(c^*_t\right)^{-\sigma*}\Gamma_{t,t+1}, \end{aligned} \tag{12}$$

where $\pi^*_t = P^*_t/P^*_{t-1}$ and σ^* is the inverse of the foreign households' intertemporal elasticity of substitution. Note that the price $1/R^*_t$ of a portfolio of state contingent claims that mimic a risk-free one-period bond, which pays one unit of foreign currency in period t+1, has to satisfy $1/R^*_t = E_t[\Gamma_{t,t+1}]$. Thus, Equations (9) and (12) imply for a risk-free foreign interest rate R^*_t:

$$\beta E_t\left\{\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{q_{t+1}}{q_t}\frac{1}{\pi^*_{t+1}}\right\} = \frac{1}{R^*_t}, \quad \beta E_t\left\{\left(\frac{c^*_t}{c^*_{t+1}}\right)^{\sigma}\frac{1}{\pi^*_{t+1}}\right\} = \frac{1}{R^*_t}, \tag{13}$$

where q_t is the real exchange rate defined as $q_t = S_tP^*_t/P_t$ and P^*_t is the foreign consumption price index.

Firms and Domestic Production

The production sector consists of two parts. Firstly, intermediate production is conducted by a continuum of monopolistically competitive firms, each producing a differentiated good being indexed on $i \in [0, 1]$. Their technology is linear in labor, $y_{H,it} = n_{it}$, where $n_t = \int_0^1 n_{it}di$. Secondly, there are perfectly competitive firms producing the domestic consumption good $y_{H,t}$ by combining the differentiated intermediate goods as inputs: $y_{H,t}^{(\varepsilon_t-1)/\varepsilon_t} = \int_0^1 y_{H,it}^{(\varepsilon_t-1)/\varepsilon_t}di$, where $\varepsilon_t>1$ can vary stochastically, giving rise to a standard cost push shock to aggregate supply. Firm i sets the price for the intermediate good $y_{H,it}$ in home currency $P_{H,it}$. The final good producer's cost minimizing demand is $y_{H,it} = \left(P_{H,it}/P_{H,t}\right)^{-\varepsilon_t} y_{H,t}$, implying $P_{H,t}^{1-\varepsilon_t} =_0^1 P_{H,it}^{1-\varepsilon_t} di$ for the price index of home-produced goods.

The price setting decision of an intermediate domestic producer is based on Calvo (1983) and Yun (1996). A fraction $\phi \in (0, 1)$ of firms is assumed to adjust their prices with the steady-state rate of domestic producer price inflation π_H, where $\pi_{H,t} = P_{H,t}/P_{H,t-1}$, such that $P_{H,it} = \pi_H P_{H,it-1}$ and there is price dispersion in the long run. In each period a fraction $1-\phi$ of randomly selected firms sets new prices $\overline{P}_{H,it}$ in order to maximize the expected sum of discounted future profits:

$$\max_{\overline{P}_{H,it}} E_t \sum_{s=0}^{\infty} \phi^s q_{t,t+s} \left(\overline{P}_{H,it} y_{H,it+s} - P_{H,t+s} mc_{H,t+s} y_{H,it+s}\right), \tag{14}$$

s.t. $y_{H,it+s} = (\pi_H^s \overline{P}_{H,it})^{-\varepsilon_t} P_{H,t+s}^{\varepsilon_t} y_{H,t+s}$, where $q_{t,t+s}$ is the stochastic discount factor and $mc_{H,t}$ denotes real marginal costs. The first-order condition for price $\overline{P}_{H,it}$ can be written $Z_t = \frac{\varepsilon_t}{\varepsilon_t - 1} Z_{1,t}/Z_{2,t}$, where $Z_t = \overline{P}_{H,it}/P_{H,t}$, $Z_{1,t} = c_t^{-\sigma} y_{H,t} mc_{H,t} + \phi\beta E_t (\pi_{H,t+1}/\pi_H)^{\varepsilon_t} Z_{1,t+1}$, and $Z_{2,t} = c_t^{-\sigma} y_{H,t} + \phi\beta E_t (\pi_{H,t+1}/\pi_H)^{\varepsilon_t - 1} Z_{2,t+1}$. Using the demand constraint, we obtain $1 = (1-\phi) Z_t^{1-\varepsilon} + \phi(\pi_t^H/\pi^H)^{\varepsilon_t - 1}$. Given that aggregate labor input is $n_t = \int_0^1 n_{i,t} dj$ and $n_{i,t} = (P_{H,it}/P_{H,t})^{-\varepsilon_t} y_t$, aggregate domestic output depends on the price dispersion, $y_t = a_t n_t / s_t$, where $s_t \equiv \int_0^1 (P_{H,it}/P_{H,t})^{-\varepsilon_t} dj$ and $s_t = (1-\phi) Z_t^{-\varepsilon_t} + \phi s_{t-1} (\pi_{H,t}/\pi_H)^{\varepsilon_t}$ given s_{-1}. Suppose there exists a steady state where home prices grow at the rate π_H, while all real variables are constant, for example, $mc_H = MC_H/P_H = (\varepsilon - 1)/\varepsilon$. Then, one can derive the following marginal-cost-based Phillips curve in terms of percent deviations from steady-state values (for example, $\hat{\pi}_{H,t} = \log \pi_{H,t}/\pi_H$):

$$\hat{\pi}_{H,t} = \beta E_t \hat{\pi}_{H,t+1} + \chi \hat{m}c_t + \hat{z}_t, \tag{15}$$

where $\chi = (1-\phi)(1-\beta\phi)\phi^{-1} > 0$ and $\hat{z}_t$ is a function of the stochastic elasticity ε_t. Finally, labor demand in a symmetric equilibrium satisfies

$$w_t = \left(\frac{P_{H,t}}{P_t}\right) mc_{H,t}. \tag{16}$$

Market Clearing

The home country is assumed to be small in the sense that its exports are negligible in the foreign price indices. The foreign producer price level $P_{F,t}^*$ is then identical to the foreign consumption price index P_t^*, $P_t^* = P_{F,t}^*$. The law of one price holds (separately) for each good such that $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$, where $P_{H,t}^*$ is the price of home-produced goods expressed in foreign currency. Thus, we get the following relation between the real exchange rate and the relative price ratio $P_{H,t}/P_t$: $q_t = (P_{H,t}/P_t)^{1-1/\vartheta}$ implying for CPI inflation

$$\pi_t = \pi_{H,t} \left(\frac{q_t}{q_{t-1}}\right)^{\frac{\vartheta}{1-\vartheta}} \qquad \forall t \geq 1. \tag{17}$$

In equilibrium, the market for domestically produced final goods clears, $y_{H,t} = c_{H,t} + c^*_{H,t} + g_t$. Substituting in demand functions and using $q_t = (P_{H,t}/P_t)^{1-1/\vartheta}$ yields:

$$y_{H,t} = (1-\vartheta) q_t^{\frac{\vartheta}{1-\vartheta}} c_t + \vartheta^* q_t^{\frac{1}{1-\vartheta}} c_t^* + g_t. \tag{18}$$

Given that state contingent claims and domestic public debt is internationally traded, the noninterest current account surplus satisfies:

$$P_{H,t} y_{H,t} - P_t c_t - P_t g_t = S_t \left\{ E_t \left[\Gamma_{t,t+1} D_{t+1} \right] - D_t \right\} - \left(\frac{B_{F,t}}{R_t} \right) + [1 - \delta(t)] B_{F,t-1}. \tag{19}$$

Perfect International Risk Sharing

Recall that domestic and foreign households are assumed to have access to a complete set of contingent claims, so risk is fully shared internationally. The domestic and foreign first-order conditions (equation (9) and (12)) imply that the consumption growth rates are related:

$$\left(\frac{c_{t+1}}{c_t} \right)^{\sigma} = \left(\frac{c^*_{t+1}}{c^*_t} \right)^{\sigma *} \left(\frac{q_{t+1}}{q_t} \right) \qquad \forall t \geq 0.$$

This equilibrium condition on the growth rates of c_t, c_t^*, and q_t determines the relation between their levels up to a constant ξ,

$$c_t^{\sigma} = \xi q_t \left(c_t^* \right)^{\sigma *}, \tag{20}$$

where the constant $\xi > 0$ can be pinned down by initial asset endowments and intertemporal solvency (that is, the intertemporal budget constraint).

Equilibrium

Throughout the analysis, foreign macroeconomic variables (starred variables) are independent from domestic variables, that is, they are exogenously determined. To simplify the analysis we assume that aggregate foreign consumption is constant, $c_t^* = c^*$, which implies that foreign monetary policy is conducted in a way that is consistent with a constant real interest rate $R_t^*/E_t \pi_{t+1}^* = 1/\beta$ (see the second equation in equation (12)). The real exchange rate then acts as a shock absorber, maintaining consistency with Equation (20).

Households are fully rationally expecting the government to occasionally default following the mechanism outlined in the section "The Public Sector." Their expectations are also summarized in the pdf $f(\overline{T})$, which in turn gives rise to positive perceived default probabilities.

In equilibrium, market clear and the first-order conditions of domestic and foreign households and firms are satisfied for given domestic monetary and fiscal. In equilibrium, domestic households are indifferent between holding internationally traded risk-free private securities and domestic public debt. As the distribution of public debt between domestic and foreign households is indetermined, we assume that domestic public debt is solely held by foreign investors,

$$B_{H,t} = 0 \Leftrightarrow b_t = b_{F,t}.$$

The equilibrium is described in more detail in Appendix I, “Equilibrium.” If there would be no risk of default ($\delta_t = 0$) or if the risk premium would be independent of the level of public debt ($\delta' = 0$), then the sequence of foreign holdings of government bonds $b_{F,t}$ would be irrelevant for the equilibrium allocation. We briefly come back to this case below.

To derive conditions for a stable equilibrium under different stances of fiscal and monetary policy, the equilibrium conditions are log-linearized at the steady state (see Appendix I, “Equilibrium”). Thus all the results are locally valid around the long-run equilibrium. Below we characterize that steady-state long-run equilibrium.

Steady State

In steady state q is constant; this implies $\pi = \pi_H$ (cf equation (17)), where long-run domestic price inflation equals the central bank’s target $\pi_H = \bar{\pi}_H$. Using aggregate production and the first of the households’ first-order conditions (equation (9)) to substitute out domestic output and working time from the commodity market clearing equation (18), we get an equilibrium relation between domestic consumption c, foreign consumption c^*, and the real exchange rate q, given that there is no long-run price dispersion ($s = 1$) due to price indexation with π_H. Combined with Equation (20), the steady-state levels of c and q can be determined as a function of ξ, preference parameters, the mark-up, and an exogenous level c^*. Domestic output y_H and hours worked n then follows from the commodity market equilibrium condition. Hence, neither changes in monetary and fiscal policy nor in the perceived default probability will affect the long-run real allocation and inflation.

Given that there is no long-run growth, a steady state requires a constant real value of public debt in terms of the aggregate consumption good.[12] Since we focus on the case where the domestic government is indebted, we only consider cases where the steady state satisfies $b = g/[(1/R)-(1-\kappa)/\pi]>0$, implying—together with Equation (9) and $b^* = 0$—a sufficiently large feedback coefficient $\kappa>\delta$. Investors’ expectations are also consistent with a long-run equilibrium and satisfy $\pi = R\beta(1-\delta)$. Thus, changes in δ affect the long-run equilibrium interest rate R for a given inflation target.

II. Debt, Deficits, and Macroeconomic Stability

In this section we examine the conditions for macroeconomic stability, while restricting our attention to the case of positive steady-state debt levels. In the main part of this section, we analyze the case of a small open economy. We then consider the cases where the economy is closed and where debt is neutral.

[12]Note that without growth a constant level of debt also implies a constant steady-state debt-output ratio.

The Blanchard Effect

To assess the stability implications of fiscal-monetary policy regimes, we reduce the model to a set of conditions for $b_{F,\,t}$, q_t and R_t and $\pi_{H,\,t}$. In a neighborhood of the steady state the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions (see Appendix I, "Equilibrium"). An equilibrium for constant government expenditures is then defined as follows: An equilibrium is a set of sequences $\{\hat{b}_{F,t}, \hat{\pi}_{H,t}, \hat{R}_t, \hat{q}_t\}_{t=0}^{\infty}$for $\delta \in (0, 1)$ and $\Phi \in (0, 1)$ that converge to the steady state (b_F, π_H, R, q) and satisfy

$$
\begin{aligned}
&(a)\ \frac{1-\Phi\vartheta}{1-\vartheta}E_t(\hat{q}_{t+1}-\hat{q}_t) = \hat{R}_t-(1-\Phi)E_t\hat{\pi}_{H,t+1}-\Phi\hat{b}_{F,t},\\
&(b)\ \hat{\pi}_{H,t} = \beta E_t\hat{\pi}_{H,t+1}+\psi\hat{q}_t+z_t, \qquad \psi=\chi\left[\frac{\sigma_n}{n}\left(\frac{(1-\vartheta)c}{\sigma}+\frac{(1-\vartheta)c\vartheta+\vartheta^{*}c^{*}}{1-\vartheta}\right)+\frac{1}{1-\vartheta}\right]>0,\\
&(c)\ \hat{b}_{F,t} = \hat{R}_t+\Lambda\hat{b}_{F,t-1}-\Lambda(1-\vartheta)\hat{\pi}_{H,t}-\Lambda\vartheta\hat{R}_{t-1}, \qquad \Lambda=\frac{1-\kappa}{\beta(1-\delta)}\frac{1}{1-\Phi\vartheta}>0,\\
&(d)\ \hat{R}_t = \rho_\pi\hat{\pi}_{H,t}, \qquad\qquad (21)
\end{aligned}
$$

(where $\sigma_n = \sigma_l n = /(1-n)$ and $\chi = (1-\phi)(1-\beta\phi)\phi^{-1}>0$) given $b_{F,-1}>0$ and $R_{-1}>1$.

Relation (equation (21d)) is the central bank's reaction function. The equilibrium relation (equation (21a)), which originates in the asset pricing condition for public debt, relates the real interest rate to the change in the real exchange rate in an almost conventional way. A higher (home) real interest rate requires a future real depreciation to be consistent with asset market equilibrium, at least for sufficiently small values for Φ. In standard overshooting fashion, a future real depreciation requires an instantaneous real appreciation up front. The implied real appreciation ($\widehat{q}_t \downarrow$) leads to a decline in aggregate (domestic and foreign) demand for domestically produced goods (see equation (18)). As a consequence, domestic producers tend to lower their prices, as can be seen from the aggregate supply relation (equation (21b)). At the same time a rise in the nominal interest rate tends to raise real public debt $\widehat{b}_{F,t}$ (see Equation (21c)).

A rise in real debt $\hat{b}_{F,t}$, however, tends to lower its expected total return, since it raises default expectations. This can be seen from the RHS of (equation (21a)), which decreases with $\hat{b}_{F,t}$. How the rise in public debt affects the previously described chain of events crucially depends on monetary policy, because that determines the initial interest rate rise, and on fiscal policy, which determines the issuance of new debt.

As suggested by Blanchard (2005) the negative feedback from public debt to its return, which originates in sovereign default expectations, may render macroeconomic stabilization impossible. To get an intuition for this, suppose that inflation exceeds its steady-state value due to a cost push shock. The central bank, which aims to stabilize inflation ($\rho_\pi>0$), will then raise the nominal interest rate. The rise in the nominal interest rate can then cause an increase in real debt $\hat{b}_{F,t}$, if the fiscal feedback coefficient κ is small (and Λ is large). The perceived default probability will then rise, which reduces the foreign households' willingness to invest in public debt.

The associated real depreciation ($\hat{q}_t \uparrow$, see LHS of equation (21a)) then exerts an upward pressure on domestic prices through different channels. A rise in the real exchange rate q_t directly raises aggregate consumption $\hat{c}_t$, as implied by Equation (20), which increases the demand for home goods. In addition, expenditure switching of domestic and foreign households in response to the exchange rate change further increases the demand for domestically produced goods. This adds to the price pressure as producers incur higher marginal costs at higher output levels. Moreover, households will demand a higher nominal wage, since the price level of aggregate consumption will rise due to higher prices of imported goods. Hence, domestic producers will unambiguously raise their prices in response to the real depreciation (see equation (21b)), which reinforces the initial rise in inflation. Owing to these channels, a rise in the nominal interest rate can actually lead to an upward pressure on inflation if κ is small and ρ_π is high.

The interaction of monetary and fiscal policy is decisive for equilibrium determinacy, that is, for the existence and the uniqueness of locally stable equilibrium sequences. The system (equation (21)) features two predetermined variables, such that equilibrium determinacy requires exactly two stable eigenvalues. The conditions for equilibrium determinacy are given in the following proposition.

Proposition 1 *Suppose that taxes are set according to* (1) *for* $\kappa \in (\delta, 1)$*, that monetary policy satisfies* (equation (21d))*, and that* $\delta \in (0, 1)$ *and* $\Phi \in (0, 1/(1+\psi/\beta))$.

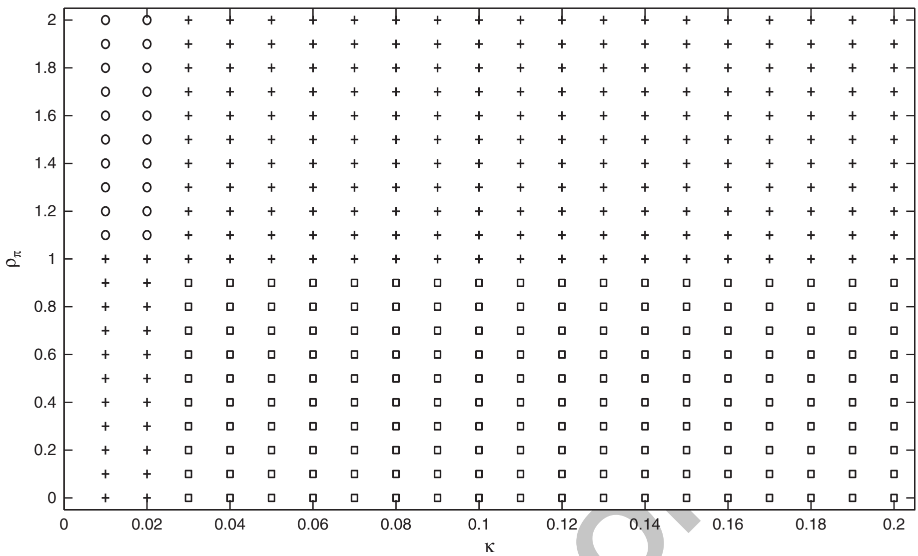
(1) *For* $\rho_\pi < 1$*, there exists a uniquely determined equilibrium if and only if* $\kappa < \kappa_1$.
(2) *For* $\rho_\pi > 1$*, there exists a uniquely determined equilibrium if but not only if* $\kappa \in (\kappa_1, \kappa_2)$*, where* $\kappa_1 = 1 - \beta(1-\delta)(1-\Phi)$ *and* $\kappa_2 = 1 + (1-\delta)[(1-\vartheta)(1-\Phi)\psi - (2\beta - 1)(1-\Phi\vartheta)] > \kappa_1$.

See Appendix I for the proof. Proposition 1 shows that existence and uniqueness of a locally stable equilibrium depends on the particular monetary and fiscal policy stance, measured by the feedback parameters ρ_π and κ.[13] Although the conditions presented in part (1) of Proposition 1 are necessary and sufficient for equilibrium determinacy, the conditions in part (2) are sufficient but not necessary. Specifically, a numerical analysis of the characteristic polynomial shows that the condition $\kappa < \kappa_2$, which guarantees the existence of at least one stable eigenvalue, is in fact hardly ever binding (see also Figure 1).

The main result summarized in Proposition 1 is that a monetary policy that aims to stabilize inflation through an active interest rate policy ($\rho_\pi > 1$) will rule out equilibrium determinacy if the feedback from debt surprises back to the primary surplus is too weak. In particular if $\kappa < \kappa_1$, an active interest rate policy would tend to destabilize the sequence of public debt, which is inconsistent with a locally stable equilibrium in the presence of default risk. This property is clearly at odds with the main principle (the Taylor-principle) known from many models of closed and open economies, which demands monetary policy to react actively, that is, by

[13] It should be noted that the parameter restriction $\Phi < 1/(1+\psi/\beta)$, which will be satisfied throughout the subsequent analysis, ensures $\kappa_1 < \kappa_2$. For example, the parameter values introduced in Section III lead to $\Phi = 0.01$, $1/(1+\psi/\beta) = 0.183$, $\kappa_1 = 0.022$, and $\kappa_2 = 0.134$.

Figure 1. Regions of Equilibrium Determinacy +, Indeterminacy □, and No Stable Equilibrium ○



more than one for one, to changes in (domestic producer price) inflation ρ_π>1 in order to ensure equilibrium determinacy (see Gali and Monacelli, 2005, for example). Thus when public debt is associated with a default risk premium, which in turn is influenced by the level of public debt, a feedback smaller than one-for-one is required from inflation to the nominal interest rate, if the feedback from debt to taxes is small, $\kappa<\kappa_1$.

Notably, these conditions closely relate to the stability conditions in Leeper (1991), where sovereign default is not considered while equilibria are nevertheless restricted to exhibit stationary debt sequences. The difference between his conditions and ours are the default rate and its elasticity, which both tend to increase the threshold for the fiscal feedback. The parameter values applied in Section III, for example, imply an increase of the threshold from 0.008 to $\kappa_1 = 0.023$. Thus, fiscal policy has to be more responsive to changes in real debt in order to allow the central bank to stabilize inflation and output via an active interest rate policy. The modification of the fiscal policy threshold relates to Leeper and Walker's (2013) requirement for a stable debt process (which mainly differs from κ_1 by the default elasticity Φ), for the case where public debt is characterized by stochastic default decisions. (In-)stability of real debt is relevant in their flexible price closed economy as fiscal policy may interact with monetary policy when it comes to the determination of the price level.[14]

[14]The role of fiscal policy then relates to the Fiscal Theory of the Price Level (Kocherlakota and Phelan, 1999).

To see the intuition for the conditions in Proposition 1, consider the case where a temporary shock leads to a rise in public debt. as expected default then rises, investors would be less willing to hold domestic public debt. The associated depreciation (see equation (21a)) would lead to a rise in the demand for domestic goods and thus to an upward pressure on inflation (see equation (21b)). If the central bank aggressively raises the interest rate in response to higher expected inflation, $\rho_\pi>1$, debt service costs would rise strongly, and would for a small value for κ lead to an even further increase in real debt and thus to divergent debt dynamics, which is inconsistent with equilibrium determinacy. If, however, the monetary policy is passive, that is, $\rho_\pi<1$, the real value of debt can decrease due to the revaluation via a higher domestic price level, which leads to a uniquely determined locally stable equilibrium.

For a high feedback coefficient $\kappa>\kappa_1$, active monetary policy $\rho_\pi>1$ typically leads to equilibrium determinacy. A temporary rise in real debt will again tend to raise expected default and inflation, but with the higher feedback coefficient κ tax revenues will be sufficiently high to eventually lower future real debt. Forward-looking price setters and investors realize the fiscal stance, and will therefore not raise prices and will not demand a higher default premium. The feedback from debt to the rate of return will then not blur the logic underlying the Taylor-principle. Precisely, with a feedback from debt increases to the primary surplus satisfying $\kappa\in(\kappa_1,\kappa_2)$ active monetary policy leads to a uniquely determined locally stable equilibrium (see also upper right corner of Figure 1).

Figure 1 illustrates conditions for equilibrium determinacy in terms of the policy parameters κ and ρ_π. Specifically, it presents regions of equilibrium determinacy (marked with black crosses), of indeterminacy (marked with red cubees), and regions of no stable equilibrium (marked with blue circles). For the computation of the eigenvalues, we applied the parameter values that are introduced in detail in Section III (implying $\kappa_1=0.023$). In the upper left corner, both fiscal and monetary policy are active ($\kappa<\kappa_1$ and $\rho_\pi>1$) in the terminology of Leeper (1991), with inconsistency as a result: in that region no stable equilibria exist. Persistence with inconsistent policy rules is obviously not possible for more than a fixed time period, so there is going to be a regime shift one way or another. What actually happens depends on which rule will change into a passive stance; see Davig and Leeper (2010) for an analysis of possible outcomes of anticipated regime switches.[15]

When both policy stances would switch to passive ($\kappa>\kappa_1$ and $\rho_\pi<1$), we enter into a region of indeterminacy (in the lower right corner), opening up the possibility of arbitrary "sunspot" equilibria. With a passive monetary policy, higher expected inflation will tend to reduce the real rate of return on domestic bonds. The associated real depreciation can then lead to an increase in inflation according to the channels laid out above, such that multiple equilibria with self-fulfilling inflation expectations are possible. For this, fiscal policy has to be passive

[15]A very early example of such a consistency restoring regime switch is of course discussed in the classic paper by Sargent and Wallace (1981).

implying that its tendency to self-stabilize the debt sequence is consistent with a multiplicity of inflation sequences. If, however, fiscal policy is active, $\kappa<\kappa_1$, arbitrary inflation sequences are in general inconsistent with a locally stable debt sequence, such that Equation (21c) uniquely selects an equilibrium inflation sequence that stabilizes real debt, ruling out sunspot equilibria (see lower left corner).

Two Extreme Cases: A Closed Economy and Debt Neutrality

Let us first consider the closed economy version of the model, $\vartheta = 0$, where public debt is held by domestic households, $b_t = b_{H,t}$, and CPI inflation equals PPI inflation, $\pi_t = \pi_{H,t}$. In this case, the model can be reduced to a set of three equilibrium conditions, that is, an aggregate demand condition, an aggregate supply condition, and the government budget constraint (see Appendix II). The stock of public debt is again nonneutral due to its impact on the perceived default probability. Under a weak feedback from debt to taxes, $\kappa<\kappa_1$, an increase in the real interest rate tends to increase public debt and thereby default expectations, like in the open economy case. The associated decline in the effective rate of return on domestic debt leads to a depreciation in the open economy case, which reduces the relative price against foreign goods today and against domestic goods in the future. This decline in the relative price then leads to a higher aggregate demand for domestically produced goods through different channels (see previous section). In the closed economy case the decline in the rate of return just reduces the intertemporal price of current consumption goods, which suffices to reduce domestic households' willingness to save. As a consequence, aggregate demand for consumption and prices tend to increase, which implies that interest rates set according to an active feedback rule can render equilibrium determinacy impossible. This extends qualitatively to the open economy case, where the consumption response to changes in real rates is more pronounced due to international price changes. Hence, for the case where the economy is closed and public debt is held by domestic households equilibrium determinacy can be guaranteed under conditions that correspond to those presented in Proposition 1 (see Appendix II).[16] We can therefore conclude that the Blanchard (2005) effect is not only relevant for a small open economy, but should also be taken into account for large developed economies like the United States or the Eurozone, which may more credibly be modeled as a closed economy.

To relate our findings to existing results in the literature, we consider the second "extreme" case, where public debt is not perceived to be risky, such that $\Phi = 0$. Notably, this case cannot be assessed by analyzing the limiting case $\Phi \to 0$, since the model exhibits a discontinuity at $\Phi = 0$: When public debt is not risky $\Phi = 0$, it is neutral and there exists infinitely many debt sequences that are consistent with one set of sequences for the equilibrium allocation, which is an implication of the applicability of the Ricardian equivalence theorem in this case.

[16]It should be noted that the macroeconomic dynamics, for example, impulse reponses to aggregate shocks, are nevertheless different in both versions (see Figure A1 in Appendix III for impulse responses to cost push shocks).

Hence, convergence of the public debt sequence (and thus the existence of a stable eigenvalue that can be assigned to it) is neither necessary nor sufficient for the existence of locally stable equilibrium. As the level of public debt does not affect its rate of return, consumption growth depends solely on the interest rate $\hat{R}_t$, which is set by the central bank, and on inflation $\hat{\pi}_{H,t+1}$: $\hat{c}_{t+1} - \hat{c}_t = (1-\vartheta)\sigma^{-1}(\hat{R}_t - \hat{\pi}_{H,t+1})$. Given that condition (equation (21b)) can be written as $\hat{\pi}_{H,t} = \psi\sigma^{-1}\hat{c}_t + \beta\hat{\pi}_{H,t+1}$, the equilibrium allocation can be determined independently from fiscal policy and, therefore, in an entirely forward-looking way, like in Gali and Monacelli (2005). Equilibrium determinacy then requires interest rate policy to be active $\rho_\pi>1$, as shown in Gali and Monacelli (2005).[17] Hence the stark contrast between the determinacy conditions in Proposition 1 and the traditional principles of stabilizing interest rate policies in models with risk-free debt is due to the existence (and not the size) of default expectations. When the tax feedback coefficient is sufficiently large, the central bank can apply an active interest rate policy to stabilize inflation and output via the conventional Fischer effect: the depressing effect of high real rates on aggregate demand for domestic goods then is high enough to slow inflation down. Without such a feedback, or with too weak a feedback, the interaction between default fears and exchange rate depreciation would trigger an upward shift in inflation and would render equilibrium determinacy impossible if active interest rate rules are implemented nevertheless.

Successful Inflation Stabilization and Sovereign Default Risk

We demonstrated that an active interest rate policy is less desirable in the presence of sovereign default risk if not supported by a sufficiently strong fiscal policy response to debt. Of course this does not imply that inflation stabilization is infeasible or unwanted in those circumstances, but that the central bank should not use an interest rate on debt that is associated with a default risk premium as its instrument. But there are alternative monetary policy instruments that sidestep the problems caused by the endogeneity of default premiums on debt (see Schabert, 2010). Taylor (2002) already conjectured this for environments with high and variable risk premiums: “Thus, policy makers in emerging market economies might want to give greater consideration to policy rules with monetary aggregates, even if rules with the interest rate become the preferred choice” (Taylor, 2002, p. 445). In line with Taylor’s suggestion, we show in Schabert and Van Wijnbergen (2006) that the central bank can safely control inflation through a money rule independent of interest rates or actual inflation. In this way it implements a uniquely determined locally stable equilibrium. Such an analysis of course neglects problems like those stemming from for example money demand instability. But it shows that money-supply-based inflation stabilization policy is feasible even with risky public debt, without the problem plaguing interest rate rules in such circumstances.

[17]Figure A1 in Appendix III presents impulse responses to cost push shocks for neutral public debt.

III. Debt Stabilization and Macroeconomic Fluctuations: A Numerical Example

We have shown that at least some degree of debt stabilization is necessary for the existence of a stable equilibrium under an interest rate rule to exist. The government has to raise taxes to a sufficiently large extent in response to debt surprises, that is, κ has to be sufficiently high, to allow successful stabilization of inflation and aggregate demand by using an active interest rate policy $\rho_\pi>1$. This result seems to suggest that, as long as $\kappa>\kappa_1$, fiscal policy is irrelevant for the stabilization of macroeconomic aggregates. Yet, this would overlook the impact of (the time path of) public debt on the effectiveness of interest rate adjustments through its impact on default probabilities.

In this section we demonstrate that fiscal policy matters for monetary stabilization policy even if condition (2) in Proposition 1 is satisfied. For this, we use a numerical example that is intended to show how the public debt dynamics alter the central bank's ability to reduce macroeconomic fluctuations under cost push shocks, which wouldn't be accommodated from a welfare maximizing perspective.[18] The parameter values are therefore chosen in the first place to clarify the role of debt stabilization and to isolate the effects from changes in the policy parameter κ and of the aggressiveness of interest rate policy as measured by ρ_π. For this exercise we set nonpolicy parameter and steady-state values equal to standard values (with periods interpreted as quarters).

The discount rate is set equal to 0.9923 to match a reasonable risk-free long-run interest rate (see below), the elasticity of intertemporal substitution to 0.5 ($\sigma=\sigma_n=2$), the domestic import share $\vartheta=0.5$, and the foreign import share $\vartheta^*=0.01$. We set the relative size of the foreign country to $c^*/c=20$ to get close to the small country assumption. The government share equals $(g/y)=0.3$, and the fraction of nonprice-adjusting firms $\phi=0.8$, while the preference parameter ζ and initial endowments (and thus ξ) are chosen to get working time $n=0.5$ and the real exchange rate $q=1$ in steady state. To examine whether the average size and the elasticity of the perceived default probability matter even at relatively small values, we set them equal to $\delta=0.005$, implying a plausible annualized premium of about 2 percent, and $\Phi=0.01$, for simplicity. We further assume that the central bank aims at zero inflation in the long run ($\pi=1$). The long-run nominal interest rate on public debt then equals $R=1.03$ (implying an annual interest rate of 5.3 percent). We vary the policy parameters $\kappa>\kappa_1$ and ρ_π within a reasonable range around the benchmark values, $\rho_\pi=1.5$ and $\kappa=0.1$, implying equilibrium determinacy and a steady-state debt-to-output ratio of 86 percent. The cost push shock $\hat{z}_t$, which is a function of the stochastic substitution elasticity ε_t only, is assumed to satisfy $\hat{z}_t=\rho\hat{z}_{t-1}+\varepsilon_{z,t}$ with $\rho=0.9$ and $\varepsilon_{z,t}$ is i.i.d. with $E_{t-1}\varepsilon_{z,t}=0$ and $\mathrm{var}(\varepsilon_{z,t})=0.01$.

Table 1 shows unconditional variances of producer price inflation $\hat{\pi}_{H,t}$, domestic output $\hat{y}_{H,t}$ and real debt $\hat{b}_t$ for several values ρ_π and κ. The key result

[18] This possibility has been shown by Linnemann and Schabert (2010), in an environment where public debt is nonneutral because it provides transaction services.

Table 1. Unconditional Variances (Benchmark Values $\rho_\pi = 1.5$ and $\kappa = 0.1$)

	$\kappa = 0.05$	$\kappa = 0.1$	$\kappa = 0.2$	$\rho_\pi = 1.25$	$\rho_\pi = 1.5$	$\rho_\pi = 1.75$
$\Phi = 0.01$						
$\mathrm{var}(\hat{\pi}_{H,t})$	0.103	0.100	0.097	0.243	0.100	0.054
$\mathrm{var}(\hat{y}_{H,t})$	1.116	1.111	1.107	0.914	1.111	1.210
$\mathrm{var}(\hat{b}_t)$	36.10	2.60	0.30	2.48	2.60	2.65
$\Phi = 0.02$						
$\mathrm{var}(\hat{\pi}_{H,t})$	0.1117	0.108	0.102	0.265	0.108	0.059
$\mathrm{var}(\hat{y}_{H,t})$	1.126	1.115	1.105	0.917	1.115	1.213
$\mathrm{var}(\hat{b}_t)$	58.42	3.05	0.319	2.873	3.05	3.134

is that a higher fiscal feedback coefficient tends to lower all three variances, which should ideally equal zero. The debt variance is most strongly affected by higher κ's, but inflation and output variances are also reduced, be it to a much smaller extent. Of interest is the fact that a higher κ lowers both the variance of inflation *and* output, thereby improving the trade-off between inflation and the output-gap.

In contrast, higher values for the inflation feedback ρ_π of interest rate policy lower the inflation variance, but at the expense of higher output variance. At the same time, the debt sequences become more volatile, since more pronounced interest rate adjustments tend to increase variations in debt servicing costs. Overall, debt variations have a relatively minor impact on inflation and output fluctuations due to the small value for the default elasticity Φ, while a higher fiscal feedback coefficient facilitates macroeconomic stabilization by lowering both output and inflation variance.

These effects are more accentuated at higher values of Φ as the lower half of Table 1 shows: the reduction in output variance is slightly over 2 percent, more than twice as large as in the upper half of Table 1 when κ goes from 0.05 to 0.20. Inflation variance is reduced by 6 percent for $\Phi = 0.01$ but by almost 10 percent when $\Phi = 0.02$. The impact of varying ρ_π is similar for both the lower and the higher value of Φ, as can be seen by comparing the lower and upper blocks of Table 1.

The true value of Φ depends on investors' beliefs, which are of course an empirical matter. But it is clearly possible, by judiciously choosing a positive κ in combination with an active interest rate policy $\rho_\pi > 1$, to lower inflation variance substantially without having to accept higher output variance in return.

IV. Conclusion

Inflation targeting based on interest rate control has become the preferred modus operandi of Central Banks around the world. Yet concerns have emerged about the wisdom of applying this framework in an environment where doubts about the willingness to pay out on debt service obligations are persistent and increasing in measures of indebtness of the government involved. Taylor (2002) expresses similar concerns: "nominal interest rates are a less appropriate instrument in cases where risk premia can be high and variable" (Taylor, 2002, p. 444). Yet the formal literature on inflation targeting has not addressed this issue.

Nevertheless the issues are real. If fears of debt default are positively correlated with the debt service burden, macroeconomic stability cannot be ensured as we show in this paper. In that case, higher interest rates lead to an increased debt service burden, hence to higher fears of debt default, in an open economy to real exchange rate depreciation and higher domestic goods prices, which in turn call for higher interest rates under a Taylor rule.

We examine this mechanism in a dynamic general equilibrium model of a small open economy where price rigidity provides a rationale for inflation stabilization. The model used is mostly standard, except for the introduction of default probabilities on public debt. We introduce default probabilities and an associated default premium in interest rates based on full rationality of debt holders in capital markets.

We show that a uniquely determined locally stable equilibrium cannot be guaranteed when interest rates are raised aggressively in response to higher inflation. In fact we obtain a very strong result: Unless there is a sufficiently strong feedback from higher debt to higher (primary) surpluses on fiscal account, active interest rate policy would destabilize debt dynamics that rule out the equilibrium determinacy. This provides formal support for the view often expressed by central bankers, that their leeway on monetary policy is much reduced when there is insufficient back up from fiscal policy.

If no such fiscal support is forthcoming, central banks are not powerless: the central bank is still able to stabilize inflation if it does not use the interest rate as its instrument. An inflation-targeting policy based on controlling money supply can safely be implemented even in the presence of endogenous default fears. And it is clearly possible, by choosing a sufficiently positive feedback of rising debt levels to higher primary surpluses, in combination with an active interest rate policy, to lower inflation variance substantially without having to accept higher output variance in exchange. But active, interest rate rule-based, inflation targeting without such fiscal stringency equally clearly is not recommendable.

There are many questions inviting future research. Are countries with heavily indexed debt structures more vulnerable to this problem than countries without indexed debt? What about nominal deficit targets, do they imply a sufficiently strong feedback from debt levels (through interest payments) to implicit primary surplus targets to put to rest instability fears? Does the choice of exchange rate regime matter for this debate? Whatever the answer to these questions, it is clear that in crisis-prone environments, monetary policy cannot be seen in separation from fiscal policy. There is much to be said in favor of recommending feedback rules calling for higher primary surpluses when debt levels are increasing, to complement inflation targeting through active Taylor rules.

APPENDIX I

Equilibrium

A rational expectations equilibrium is a set of sequences $\{mc_{H,t}, w_t, \pi_t, \pi_{Ht}, c_t, n_t, q_t, y_{H,t}, R_t, R_t^*, b_{H,t}+b_{F,t}, Z_t, Z_{1,t}, Z_{2,t}, s_t\}_{t+0}^{\infty}$ satisfying

$$
\begin{aligned}
\zeta(1-n_t)^{-\sigma_l} &= w_t c_t^{-\sigma},\\
1/R_t^* &= \beta E_t\left\{(c_t/c_{t+1})^{\sigma}(q_{t+1}/q_t)/\pi_{t+1}^*\right\},\\
1/R_t &= \beta E_t\{(1-\delta_{t+1})\,(c_t/c_{t+1})^{\sigma}/\pi_{t+1}\},\\
c_t^{\sigma} &= \xi q_t (c_t^*)^{\sigma^*},\\
w_t &= q_t^{-\vartheta/(1-\vartheta)}\, mc_{H,t},\\
y_{H,t} &= n_t/s_t,\\
\left(b_{H,t}+b_{F,t}\right)/R_t &= g-\tau_t+\left(b_{H,t-1}+b_{F,t-1}\right)\pi_t^{-1},\\
\tau_t &= \kappa\left[\left(b_{H,t-1}+b_{F,t-1}/\pi_t\right)-(b^*/\pi)\right],\\
y_{H,t} &= (1-\vartheta)q_t^{\vartheta/(1-\vartheta)}c_t+\vartheta^* q_t^{1/(1-\vartheta)}c_t^*+g_t,\\
\pi_t &= \pi_{H,t}(q_t/q_{t-1})^{\frac{\vartheta}{1-\vartheta}},\\
1 &= (1-\phi)\,Z_t^{1-\varepsilon}+\phi(\pi_t^H/\pi^H)^{\varepsilon_t-1},\\
Z_t &= \tfrac{\varepsilon_t}{\varepsilon_t-1}Z_{1,t}/Z_{2,t},\\
Z_{1,t} &= c_t^{-\sigma}y_{H,t}mc_{H,t}+\phi\beta E_t(\pi_{H,t+1}/\pi_H)^{\varepsilon_t}Z_{1,t+1},\\
Z_{2,t} &= c_t^{-\sigma}y_{H,t}+\phi\beta E_t\left(\pi_{H,t+1}/\pi_H\right)^{\varepsilon_t-1}Z_{2,t+1},\\
s_t &= (1-\phi)Z_t^{-\varepsilon_t}+\phi s_{t-1}\left(\pi_{H,t}/\pi_H\right)^{\varepsilon_t},
\end{aligned}
$$

the transversality condition, and a monetary policy for given sequences $\{\varepsilon_t, g_t\}_{t+0}^{\infty}$ and $\{c_t^*, \pi_t^*\}_{t+0}^{\infty}$ satisfying $\beta E_t\{(c_t^*/c_{t+1}^*)^{\sigma}/\pi_{t+1}^*\} = 1/R_t^*$, initial asset endowments and an initial price level.

In a neighborhood of the steady state the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions. Note that total public debt can be determined, while its distribution between domestic and foreign household is indetermined. We therefore assume that domestic households' holdings of public debt equals zero, $B_{H,t}=0$. The equilibrium can be defined as follows (where $\widehat{x}_t$ denotes the percent deviation of a generic variable x_t from its steady-state value $x : \widehat{x}_t = \log x_t - \log x$):

Definition A rational expectations equilibrium for $B_{H,t}=0$ and $g_t = g$ is a set of sequences $\{\widehat{w}_t, \widehat{\pi}_t, \widehat{\pi}_{H,t}, \widehat{c}_t, \widehat{q}_t, \widehat{y}_{H,t}, \widehat{R}_t, \widehat{b}_{F,t}\}_{t=0}^{\infty}$ satisfying

$$\hat{w}_t - \sigma\hat{c}_t = \sigma_n \hat{y}_{H,t}, \tag{i}$$

$$\Phi\hat{b}_{F,t} - \Phi E_t\hat{\pi}_{t+1} = \hat{R}_t - E_t\hat{\pi}_{t+1} - \sigma(E_t\hat{c}_{t+1} - \hat{c}_t), \tag{ii}$$

$$\sigma\hat{c}_t = \hat{q}_t, \tag{iii}$$

$$\hat{\pi}_{H,t} = \beta E_t \hat{\pi}_{H,t+1} + \chi\left(\hat{w}_t + \left[\frac{\vartheta}{(1-\vartheta)}\right]\hat{q}_t\right) + \hat{z}_t, \tag{iv}$$

$$\hat{y}_{H,t} = \left[\frac{\left(\frac{c}{n}\right)\vartheta + \vartheta^*\left(\frac{c^*}{n}\right)}{(1-\vartheta)}\right]\hat{q}_t + (1-\vartheta)\left(\frac{c}{n}\right)\hat{c}_t, \tag{v}$$

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \left[\frac{\vartheta}{(1-\vartheta)}\right](\hat{q}_t - \hat{q}_{t-1}), \tag{vi}$$

(where $\sigma_n = \sigma_l n_{/(1-n)}$, $\Phi = (b/\pi)\delta'/(1-\delta)>0$, and $\hat{z}_t = z(\varepsilon_t)$), the transversality condition, and monetary and fiscal policy characterized by

$$\begin{aligned} \widehat{b}_{F,t} &= \widehat{R}_t + \frac{1-\kappa}{\beta(1-\delta)}\widehat{b}_{F,t-1} - \frac{1-\kappa}{\beta(1-\delta)}\widehat{\pi}_t, \\ \widehat{R}_t &= \rho_\pi E_t \widehat{\pi}_{H,t}, \end{aligned}$$

where $\rho_\pi = R'\pi_H/R \geq 0$ and $\eta = (1-\kappa)/(1+\kappa(R-1)) \in (0, 1)$ for $\{\varepsilon_t\}_{t+0}^{\infty}$ and initial values $\widehat{b}_{F,-t}$ and $\widehat{q}_{-1}$.Eliminating $\widehat{w}_t$ and $\widehat{y}_{H,t}$ with (i) and (v), the aggregate supply constraint (iv) can be rewritten as $\hat{\pi}_{H,t} = \beta E_t \hat{\pi}_{H,t+1} + \chi[(\sigma_n(1-\vartheta)cn^{-1} + \sigma)\hat{c}_t + (\sigma_n\alpha + \vartheta/(1-\vartheta)\hat{q}_t] + \hat{z}_t$, where $\alpha = (1-\vartheta)c\vartheta + \vartheta^* c^*/n(1-\vartheta)$. Further, eliminating $\widehat{\pi}_t$ and $\widehat{c}_t$ with (iii) and (vi), the set of equilibrium conditions can be reduced to the following system in $\{\widehat{\pi}_{H,t}, \widehat{q}_t, \widehat{R}_t, \widehat{b}_{F,t}\}_{t=0}^{\infty}$:

$$-\Phi\hat{b}_{F,t} = -\hat{R}_t + (1-\Phi)E_t\hat{\pi}_{H,t+1} + \frac{1-\Phi\vartheta}{1-\vartheta}(E_t\hat{q}_{t+1} - \hat{q}_t), \tag{vii}$$

$$\hat{b}_{F,t} = \hat{R}_t + \frac{1-\kappa}{\beta(1-\delta)}\frac{1}{1-\Phi\vartheta}\hat{b}_{F,t-1} - \frac{1-\kappa}{\beta(1-\delta)}\frac{1-\vartheta}{1-\Phi\vartheta}\hat{\pi}_{H,t} - \frac{1-\kappa}{\beta(1-\delta)}\frac{\vartheta}{1-\Phi\vartheta}\hat{R}_{t-1}, \tag{viii}$$

$$\hat{\pi}_{H,t} = \beta E_t \hat{\pi}_{H,t+1} + \chi\left[\frac{\sigma_n}{\sigma}\frac{(1-\vartheta)c}{n} + 1 + \sigma_n\alpha + \frac{\vartheta}{1-\vartheta}\right]\hat{q}_t + \hat{z}_t, \tag{ix}$$

$$\hat{R}_t = \rho_\pi E_t \hat{\pi}_{H,t}, \tag{x}$$

Under certainty, we thus end up with the system (equation (21)).

Proof of Proposition 1

In order to prove the claims made in the proposition, the interest rate is eliminated in Equation (21a–d) by substituting in the policy rule $\widehat{R}_t = \rho_\pi \widehat{\pi}_{H,t}$, leading to the following 4×4 system in $\widehat{b}_{F,t}\, \widehat{\pi}_{H,t}, \widehat{q}_t$, and the auxiliary variable $\widehat{x}_t$:

$$\begin{aligned} \Phi\widehat{b}_{F,t} = \frac{1-\Phi\vartheta}{1-\vartheta}\widehat{q}_t - \frac{1-\Phi\vartheta}{1-\vartheta}\widehat{q}_{t+1} + \rho_\pi\widehat{\pi}_{H,t} - (1-\Phi)\widehat{\pi}_{H,t+1}\ \widehat{b}_{F,t} = \Lambda\widehat{b}_{F,t-1} \\ + (\rho_\pi - \Lambda(1-\vartheta))\widehat{\pi}_{H,t} - \Lambda\vartheta\rho_\pi\widehat{x}_{t-1}\beta\widehat{\pi}_{H,t+1} = -\psi\widehat{q}_t + \widehat{\pi}_{H,t}\ \widehat{x}_t = \widehat{\pi}_{H,t} \end{aligned}$$

(where $\Lambda = (1-\kappa)/(\beta(1-\delta))(1)/(1-\Phi\vartheta)$), which can be rewritten as

$$\begin{pmatrix} \widehat{b}_{F,t} \\ \widehat{q}_{t+1} \\ \widehat{\pi}_{H,t+1} \\ \widehat{x}_{t} \end{pmatrix} = A \begin{pmatrix} \widehat{b}_{F,t-1} \\ \widehat{q}_{t} \\ \widehat{\pi}_{H,t} \\ \widehat{x}_{t-1} \end{pmatrix}$$

$$A = \begin{pmatrix} \Phi & \frac{1-\Phi\vartheta}{1-\vartheta} & 1-\vartheta & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 & \frac{1-\Phi\vartheta}{1-\vartheta} & \rho_\pi & 0 \\ \Lambda & 0 & \rho_\pi - \Lambda(1-\vartheta) & -\Lambda\vartheta\rho_\pi \\ 0 & -\psi & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial of A is given by

$$\begin{aligned} H(X) = XG(X),\ G(X) = X^3 &+ X^2 \frac{\psi(\vartheta-1)(\Phi-1) - (\Phi\vartheta-1)(\beta+\Lambda\beta+1)}{\beta(\Phi\vartheta-1)} \\ &+ \Lambda \frac{\psi\rho_\pi(\vartheta-1)-1}{\beta} \\ &+ X \frac{-\psi(\vartheta-1)(\Phi-1)\rho_\pi + (\Phi\vartheta-1)(\Lambda+\Lambda\beta+\Lambda\psi-\Lambda\psi\vartheta+1)}{\beta(\Phi\vartheta-1)}. \end{aligned}$$

One eigenvalue equals zero and can be assigned to $\widehat{x}_t$. Since there remains one further predetermined variable $\widehat{b}_{F,t-1}$, a uniquely determined convergent equilibrium requires $G(X)$ to exhibit exactly two unstable and one stable eigenvalue. To identify the conditions for this, we first examine $G(0)$,

$$G(0) = -(1-\kappa)\frac{\psi\rho_\pi(1-\vartheta)+1}{\beta^2(1-\delta)(1-\Phi\vartheta)} < 0,$$

which is strictly negative, for δ and Φ not exceeding one. One or three negative stable roots are further ruled out, since $G(-1)$ is strictly negative (given that $\kappa \leq 1$):

$$G(-1) = -\frac{\begin{pmatrix} (\rho_\pi+1)\psi(1-\vartheta)(-\kappa+1+\beta(1-\delta)(1-\Phi)) \\ +2(\beta+1)(-\kappa+1+\beta(1-\delta)(1-\Phi\vartheta)) \end{pmatrix}}{\beta^2(1-\delta)(1-\Phi\vartheta)} < 0.$$

Two or zero positive stable roots are ruled out, if $G(1)$, given by

$$G(1) = -\psi(1-\rho_\pi)(1-\vartheta)\frac{\kappa-1+\beta(1-\delta)(1-\Phi)}{\beta^2(1-\delta)(1-\Phi\vartheta)}$$

is strictly positive. There are two sets of conditions that lead to $G(1)>0$: For $\kappa>\kappa_1$, where $\kappa_1 = 1-\beta(1-\delta)(1-\Phi)$, monetary policy has to be active, $\rho_\pi>1$, while for $\kappa>\kappa_1$ monetary policy has to be passive, $\rho_\pi<1$. Then, there exist either one or three stable eigenvalues, from which at least one is positive. We further examine if the sum of all eigenvalues exceeds 3, which delivers a sufficient (but not necessary condition) for the existence of one unstable eigenvalue and is equivalent to check if $G''(1)<0$, where

$$G''(1) = 2\frac{\kappa-(1+(1-\delta)\,[(1-\vartheta)\,(1-\Phi)\,\psi-(2\beta-1)\,(1-\Phi\vartheta)])}{\beta(1-\delta)\,(1-\Phi\vartheta)}.$$

For $\kappa>\kappa_2 \Rightarrow G''(1)<0$, where $\kappa_2=1+(1-\delta)[(1-\vartheta)(1-\Phi)\psi-(2\beta-1)(1-\Phi\vartheta)]$, implying that there exists at least one unstable eigenvalue, while $\kappa_1<\kappa_2$ for $\Phi<1/(1+\psi/\beta)$. Then, for $\rho_\pi<1$ there exists a uniquely determined equilibrium if and only if $\kappa>\kappa_1$, and for $\rho_\pi>1$ there exists a uniquely determined equilibrium if but not only if $\kappa\in(\kappa_1,\kappa_2)$. This establishes the claims made in the proposition.

APPENDIX II

Consider a closed economy version of the model, $\vartheta=0$, where public debt is held by domestic households, $b_t=b_{H,t}$, and PPI inflation equals CPI inflation, $\pi_t=\pi_{H,t}$. The set of linearized equilibrium conditions can then be reduced to the following conditions in real debt, consumption, inflation, and the nominal interest rate

$$\Phi\widehat{b}_{H,t}-\Phi E_t\widehat{\pi}_{t+1} = \widehat{R}_t-E_t\widehat{\pi}_{t+1}-\sigma\left(E_t\widehat{c}_{t+1}-\widehat{c}_t\right),\ \widehat{\pi}_t = \chi(\sigma+\sigma_n)\,\widehat{c}_t+\beta E_t\widehat{\pi}_{t+1},$$
$$\widehat{b}_{H,t} = \widehat{R}_t+\frac{1-\kappa}{\beta(1-\delta)}\widehat{b}_{H,t-1}-\frac{1-\kappa}{\beta(1-\delta)}\widehat{\pi}_t,$$

and $\widehat{R}_t = \rho_\pi\,\widehat{\pi}_t$. Eliminating the interest rate, the system can be written as

$$\begin{pmatrix}\widehat{b}_{H,t}\\ \widehat{c}_{t+1}\\ \widehat{\pi}_{t+1}\end{pmatrix} = A\begin{pmatrix}\widehat{b}_{H,t-1}\\ \widehat{c}_t\\ \widehat{\pi}_t\end{pmatrix},\quad A=\begin{pmatrix}\Phi & \sigma & 1-\Phi\\ 1 & 0 & 0\\ 0 & 0 & \beta\end{pmatrix}^{-1}\begin{pmatrix}0 & \sigma & \rho_\pi\\ \Lambda & 0 & \rho_\pi-\Lambda\\ 0 & -\Psi & 1\end{pmatrix}$$

where $\Lambda=(1-\kappa)/(\beta(1-\delta))>0$ and defining $\Psi=\chi(\sigma+\sigma_n)>0$. The characteristic polynomial of A is

$$F(X) = X^3+\frac{-\Psi-\beta+\Phi\Psi-\Lambda\beta-1}{\beta}X^2+\frac{\Lambda+\Psi\Lambda+\Lambda\beta+\Psi\rho-\Phi\Psi\rho+1}{\beta}X-\Lambda\frac{\Psi\rho+1}{\beta},$$

where $F(0) = -\frac{\Lambda}{\beta}(\Psi\rho_\pi+1)<0,\ F(1) = -\Psi(1-\rho_\pi)\left[\Phi-1+\frac{1-\kappa}{\beta(1-\delta)}\right]/\beta.$

$$F(-1) = -[\Psi(1+\rho)\,(1+\Lambda-\Phi)+2(1+\beta)\,(1+\Lambda)]/\beta<0.$$

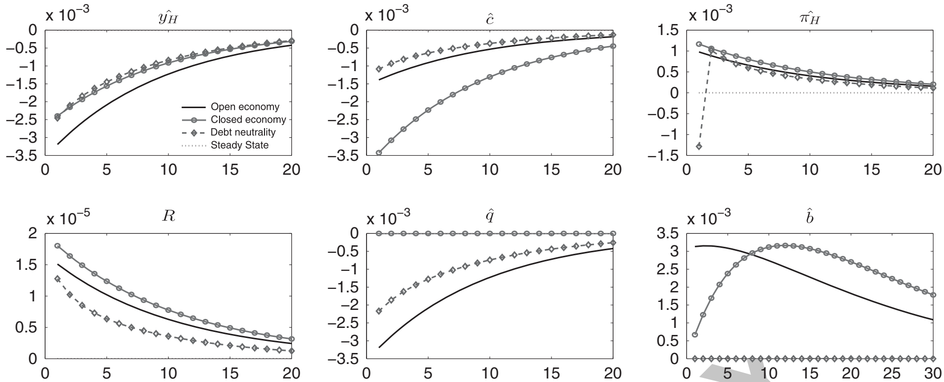
There are two sets of conditions that lead to $F(1)>0$: For $\kappa>\kappa_1$, where $\kappa_1=1-\beta(1-\delta)(1-\Phi)$, monetary policy has to be active, $\rho_\pi>1$, while for $\kappa<\kappa_1$ monetary policy has to be passive, $\rho_\pi<1$. Then, there exist either one or three stable eigenvalues, from which at least one is positive. We further examine if the sum of all eigenvalues exceeds 3, which delivers a sufficient (but not necessary condition) for the existence of one unstable eigenvalue and is equivalent to check if $F''(1)<0$, where

$$F''(1) = 2\{\kappa-(1+(1-\delta)\,[\,(1-\Phi)\,\Psi-(2\beta-1)])\}/\beta(1-\delta).$$

For $\kappa<\kappa_2 \Rightarrow F''(1)<0$, where $\kappa_2=1+(1-\delta)[(1-\Phi)\Psi-(2\beta-1)]$, implying that there exists at least one unstable eigenvalue, while $\kappa_1<\kappa_2$ for $\Phi<1/(1+\Psi/\beta)$. Then, for $\rho_\pi<1$ there exists a uniquely determined equilibrium if and only if $\kappa<\kappa_1$, and for $\rho_\pi>1$ there exists a uniquely determined equilibrium if but not only if $\kappa\in(\kappa_1,\kappa_2)$. These conditions for equilibrium determinacy correspond to those presented in Proposition 1.

APPENDIX III

Figure A1. Impulse Responses to Cost Push Shocks



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