The quench action approach to out-of-equilibrium quantum integrable models

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Details of the large-Δ expansion

C.1 Large-Δ expansion of the saddle-point state.

In this appendix we would like to discuss briefly the derivation of the large-Δ expansion for the saddle-point state. In particular, we would like to discuss the derivation of the leading term of the expansion of η_n, which is the non-straightforward point of this calculation. As stated in Section 5.7, we need to expand the GTBA Eqs (5.40) and the BGT Eqs (2.66). We assume the following analytical ansatz for $\{\eta_n(\lambda)\}$

$$\eta_n(\lambda) \;=\; z^{\alpha_n}\eta_n^{(0)}(\lambda)\,\exp\left[\Phi_n(\lambda)\right], \qquad \Phi_n(\lambda) \;\equiv\; \sum_{j=1}^{\infty} z^j\,\eta_n^{(j)}(\lambda), \quad n \geq 1, \tag{C.1}$$

where $z = e^{-\eta}$, $\Delta = \cosh\eta$, and α_n are integer numbers. The functions $\eta_n^{(j)}(\lambda)$ with $j = 0, 1, 2, \ldots$ characterize the solution at order z^j in the expansion. From the leading behaviors of ρ_1 and of the exact solution (5.18) for $\rho_{1,h}$, we know that $\alpha_1 = 2$. This is the only information about $\rho_{1,h}$ we use in our expansion. The driving terms $\tilde{d}_n(\lambda)$ in Eqs (5.40) have a very simple expansion in z,

$$\tilde{d}_n(\lambda) = \begin{cases} 4\log z + \log\left(4\,\sin^2(2\lambda)\right) + 2\sum_{k=1}^{\infty}\frac{1}{k}\cos(4k\lambda)z^{4k}\,, & n \text{ odd}\,, \\ -\log\tan^2(\lambda) - 4\sum_{k=1}^{\infty}\frac{1}{2k-1}\cos[2(2k-1)\lambda]z^{2(2k-1)}\,, & n \text{ even}\,. \end{cases} \tag{C.2}$$

The leading order of the small-z expansion of Eqs (5.40) is a $\log(z)$-divergence. Since $\rho_{1,h}(\lambda)$ in Eq. (5.18) does not exhibit exponential behavior in λ, we expect (possible) divergencies in $\eta_n(\lambda)$ to be power law. This means that for the convolutions of the right-hand side of Eqs (5.40)

$$s * \log(1+\eta_n) = s * \log\left(1 + z^{\alpha_n}\eta_n^{(0)}\right) + O(z) = \Theta(-\alpha_n)\alpha_n + O(z^0)\,, \tag{C.3}$$

where $\Theta(x)$ is the Heaviside step function. This leads to a set of conditions on the parameters α_n,

$$\begin{aligned}
2\alpha_1 &= 4 + \Theta(-\alpha_2)\,\alpha_2\ , \\
2\alpha_n &= \Theta(-\alpha_{n-1})\,\alpha_{n-1} + \Theta(-\alpha_{n+1})\,\alpha_{n+1}\ , \qquad n \geq 2 \text{ even}\ , \\
2\alpha_n &= 4 + \Theta(-\alpha_{n-1})\,\alpha_{n-1} + \Theta(-\alpha_{n+1})\,\alpha_{n+1}\ , \quad n \geq 3 \text{ odd}\ .
\end{aligned} \tag{C.4}$$

Notice that $\alpha_n \leq 0$ for n even, and so from $\alpha_1 = 2$ we have $\alpha_2 = 0$. However, this set of equations does not have a unique solution. The general form of the solution for integers α_n is the following,

$$\begin{aligned}
&\{\alpha_1, \alpha_2, \alpha_3, \ldots\} \\
&= \{2, 0, 2, 0, \ldots, 2, 0, \alpha_{2k+1} < 2, \alpha_{2k+2}(\alpha_{2k+1}), \alpha_{2k+3}(\alpha_{2k+1}), \ldots\},
\end{aligned} \tag{C.5}$$

where k is a positive integer (or infinite), $\alpha_{2k+1} = 1, 0$ and $\alpha_{n>2k+1} < 0$ and they are unequivocally determined by α_{2k+1}. Our intuition is that this freedom in our ansatz is apparent and it disappears when we take into account the BGT Eqs (2.66). Indeed, we checked explicitly that the two $k = 1$ solutions are not consistent with Eqs (2.66). Therefore, the most natural choice is

$$\alpha_n = \begin{cases} 2 & \text{for } n \text{ odd}\ , \\ 0 & \text{for } n \text{ even}\ . \end{cases} \tag{C.6}$$

This means that the leading scaling exponent of $\eta_n(\lambda)$ is only due to the $\log(z)$ part of the driving term (C.2). At order z^0, the convolutions on the right-hand side of Eqs (5.40) are independent of λ, and therefore the functional behavior of $\eta_n^{(0)}$ is determined by the driving terms only, *i.e.*,

$$\eta_n^{(0)}(\lambda) = \begin{cases} c_n \sin^2(2\lambda)\ , & \text{for } n \text{ odd}\ , \\ c_n \tan^{-2}(\lambda)\ , & \text{for } n \text{ even}\ , \end{cases} \tag{C.7}$$

where $c_n \geq 0$ on physical grounds (densities cannot be negative). The convolutions $s * \log(1 + \eta_n)$ at order z^0 are zero if n is odd and $2\log\left(1 + \sqrt{c_n}\right) + O(z)$ if n is even. Substituting this into Eqs (5.40), we have

$$c_n = \begin{cases} 4\left(1 + \sqrt{a_{n-1}}\right)\left(1 + \sqrt{a_{n+1}}\right)\ , & \text{for } n \text{ odd}\ , \\ 1\ , & \text{for } n \text{ even}\ , \end{cases} \tag{C.8}$$

where by convention $a_0 = 0$. We find that

$$\eta_n^{(0)}(\lambda) = \begin{cases} 8\sin^2(2\lambda)\ , & \text{for } n = 1\ , \\ 16\sin^2(2\lambda)\ , & \text{for } n \geq 3 \text{ odd}\ , \\ \tan^{-2}(\lambda)\ , & \text{for } n \text{ even}\ , \end{cases} \tag{C.9}$$

The functions $\eta_n^{(j)}$ for $j > 0$ can then be computed. Up to $j = 3$ we have

$$\begin{aligned}
\Phi_1(\lambda) &= 2z\cos(2\lambda) + z^2\left[\cos(4\lambda) + \tfrac{1}{2}\right] \\
&\quad + z^3\left[\tfrac{2}{3}\cos(6\lambda) - 3\cos(2\lambda)\right] + O(z^4)\,, && \text{(C.10a)}\\
\Phi_2(\lambda) &= z^2\left[-8\cos(2\lambda) + 6\right] + \mathcal{O}(z^4)\,, && \text{(C.10b)}\\
\Phi_3(\lambda) &= 4z\cos(2\lambda) + z^2\left[2\cos(4\lambda) + \tfrac{3}{2}\right] \\
&\quad + z^3\left[\tfrac{4}{3}\cos(6\lambda) - 5\cos(2\lambda)\right] + O(z^4)\,, && \text{(C.10c)}\\
\Phi_n(\lambda) &= z^2\left[-8\cos(2\lambda) + 8\right] + \mathcal{O}(z^4)\,, \qquad n \geq 4 \text{ even}\,, && \text{(C.10d)}\\
\Phi_n(\lambda) &= 4z\cos(2\lambda) + z^2\left[2\cos(4\lambda) + 2\right] \\
&\quad + z^3\left[\tfrac{4}{3}\cos(6\lambda) - 4\cos(2\lambda)\right] + O(z^4)\,, \qquad n \geq 3 \text{ odd}\,. && \text{(C.10e)}
\end{aligned}$$

Using this expansion and the BGT Eqs (2.66), the expansion for the densities [Eqs (5.80) and (5.81)] can then be computed as well.

C.2 Large-Δ expansion of the GGE state

In this appendix we would like to discuss briefly the derivation of the large-Δ expansion for the GGE. In particular, we derive the leading terms of the expansion, making the computation of the next-leading terms straightforward.

As stated in Section 5.7, we need to expand the GTBA Eqs (5.16) for $n \geq 2$ and the BGT Eqs (2.66) for $n \geq 1$, and use the exact formula (5.18) for $\rho_{1,h}$. All information about the expectation values of the local charges is thus encoded in $\rho_{1,h}$, and we do not need to to compute the chemical potentials that appear only in the driving term of the $n = 1$ GTBA Eq. (5.16). Two useful sum rules to check the correctness of our assumptions are

$$2\sum_{m=1}^{\infty}\int_{-\pi/2}^{\pi/2} \mathrm{d}\lambda\, \rho_m(\lambda) = 1 - \int_{-\pi/2}^{\pi/2} \mathrm{d}\lambda\, \rho_{1,h}(\lambda)\,, \tag{C.11a}$$

$$2\sum_{m=1}^{\infty} m \int_{-\pi/2}^{\pi/2} \mathrm{d}\lambda\, \rho_m(\lambda) = 1\,. \tag{C.11b}$$

The first one is a consequence of the BGT Eqs (2.66), while the second one expresses the conservation of the total magnetization. Our analytical ansatz is

$$\eta_n(\lambda) = z^{\alpha_n}\eta_n^{(0)}(\lambda)e^{\Phi_n(\lambda)}\,, \qquad \Phi_n(\lambda) = \sum_{l=1}^{\infty} z^l \eta_n^{(l)}(\lambda)\,, \tag{C.12a}$$

$$\rho_{n,h}(\lambda) = z^{\gamma_n}\rho_{n,h}^{(\gamma_n)}(\lambda)\left[1 + \sum\nolimits_{l=1}^{\infty} z^l \rho_{n,h}^{(l+\gamma_n)}(\lambda)\right]\,, \tag{C.12b}$$

where $\gamma_n \in \mathbb{N}$. Since $z = 0$ corresponds to the quenchless point, we have $\rho_1(\lambda) = 1/(2\pi) + O(z)$. Since $\rho_{1,h}(\lambda) = 4z^2\sin^2(2\lambda)/\pi + O(z^3)$ [Eq. (5.18)], we have

$\gamma_1 = \alpha_1 = 2$. Inserting the ansatz (C.12a) into the GTBA Eqs (5.16) for $n \geq 2$ and isolating the terms proportional to $\log(z)$, we obtain

$$2\alpha_n = \theta(-\alpha_{n-1})\alpha_{n-1} + \theta(-\alpha_{n+1})\alpha_{n+1}\,, \qquad n \geq 2\,. \tag{C.13}$$

From here it follows that, for $n \geq 2$, $\alpha_n \leq 0$ and hence $\alpha_n = (n-1)\alpha_2$. Let us now expand the BGT Eqs (2.66) for $n \geq 2$. The leading term of the l.h.s. is proportional to $z^{\gamma_n} + z^{\gamma_n - \alpha_n} \sim z^{\gamma_n}$, while the r.h.s is proportional to $z^{\gamma_{n-1}} + z^{\gamma_{n+1}}$. Notice that the term proportional to z^{γ_n} in $s * \rho_{n,h}$ is always strictly positive as $\rho_{n,h}$ is always positive while $s(\lambda) = 1/(2\pi) + O(z)$. Therefore, we can conclude that $\gamma_n = \gamma_2 \leq 2$ for $n \geq 2$. Because of our analyticity hypothesis $\gamma_n \in \mathbb{N}$, there are three possible values for γ_2: 0, 1 and 2. Let us now expand the $n = 1$ BGT Eq. (2.66) up to the second order. The case $\gamma_2 = 0$ can be excluded because $\rho_1(\lambda) = 1/(2\pi) + \mathcal{O}(z)$. Similarly, $\gamma_2 \neq 1$ because if $\gamma_2 = 1$ we would have that $\int_{-\pi/2}^{\pi/2} \mathrm{d}\lambda\, \rho^{(1)}(\lambda) > 0$, in contradiction with the sum rules (C.11). Therefore, we conclude that $\gamma_n = \gamma_2 = 2$. Moreover, we can conclude that $\alpha_{n\geq 2} = \alpha_2 = 0$, because otherwise $\rho_n \to +\infty$ for $z \to 0$ and n sufficiently large.

We are now in the position to compute all $\eta_n^{(0)}$. As we can see by expanding Eq. (5.16), they are actually constant and obey the recursive relations

$$\log(\eta_2^{(0)}) = \frac{1}{2}\log(1+\eta_3^{(0)})\,, \tag{C.14a}$$

$$\log(\eta_n^{(0)}) = \frac{1}{2}\left[\log(1+\eta_{n-1}^{(0)}) + \log(1+\eta_{n+1}^{(0)})\right]\,. \tag{C.14b}$$

The solution

$$\eta_{n\geq 2}^{(0)} = n^2 - 1 \tag{C.15}$$

is the only one consistent with the sum rules (C.11). Expanding now the BGT Eqs (2.66) for $n \geq 2$ up to the second order, we have

$$\rho_{2,h}^{(0)}\left(1 + (\eta_2^{(0)})^{-1}\right) = \frac{1}{\pi} + \frac{1}{2}\rho_{3,h}^{(0)}\,, \tag{C.16a}$$

$$\rho_{n,h}^{(0)}\left(1 + (\eta_n^{(0)})^{-1}\right) = \frac{1}{2}\left[\rho_{n-1,h}^{(0)} + \rho_{n+1,h}^{(0)}\right]\,. \tag{C.16b}$$

The only solution to this system of recursion relations is $\rho_{n\geq 2,h} = 2/(\pi n) + c\left(n^2-1\right)$, where c is an arbitrary constant. The only value of c consistent with the sum rules (C.11) is $c = 0$. Summarizing, we have

$$\eta_n = \left(n^2 - 1\right) + O(z)\,, \qquad n \geq 2\,, \tag{C.17a}$$

$$\rho_{n,h} = \frac{2z^2}{\pi n} + O(z^3)\,, \qquad n \geq 2\,, \tag{C.17b}$$

Therefore,

$$\rho_n = \frac{2z^2}{\pi n\,(n^2-1)} + O(z^3)\,, \qquad n \geq 2\,, \tag{C.18}$$

while ρ_1 can be computed using the $n=1$ BGT Eq. (2.66)

$$\begin{aligned}\rho_1(\lambda) &= s(\lambda) + (s * \rho_{2,h})(\lambda) - \rho_{1,h}(\lambda) \\ &= \frac{1}{2\pi}\Big\{1 + 4z\cos(2\lambda) + z^2[8\cos(4\lambda) - 3]\Big\} + \mathcal{O}(z^3)\,.\end{aligned} \tag{C.19}$$

Similarly, we can compute subleading orders of the expansion. The next-leading order vanishes for $n \geq 2$, while the next-next-leading order terms are reported in Eqs (5.82). As for the leading term, computing the GGE expansion involves the solutions of a set of recursion relations (one for η_n, another for $\rho_{n,h}$). Hence, the large-Δ expansion is technically more involved than the one for the quench action saddle-point state.

C.3 Large-Δ expansion for local correlators

In this appendix, we would like to summarize the basic formulas for computing the local correlators $\langle\sigma_1^z\sigma_2^z\rangle$ and $\langle\sigma_1^z\sigma_3^z\rangle$ as well as some intermediate results of their large-Δ expansion.

C.3.1 The nearest-neighbors correlator $\langle\sigma_1^z\sigma_2^z\rangle$

Recall that the correlator $\langle\sigma_1^z\sigma_2^z\rangle$ can be computed thanks to the Hellmann-Feynman theorem [4, 196]. We have (see Section 5.6)

$$\begin{aligned}\langle\boldsymbol{\rho}|\sigma_1^z\sigma_2^z|\boldsymbol{\rho}\rangle = 1 + 4\Bigg\{&\frac{\cosh\eta}{\sinh^2\eta}\langle\boldsymbol{\rho}|\hat{Q}_2/N|\boldsymbol{\rho}\rangle \\ &+ \sum_{k\in\mathbb{Z}}|k|\left[\frac{e^{-|k|\eta}}{2\cosh k\eta} + \tanh(|k|\eta)\left(\frac{e^{-|k|\eta} - \hat{\rho}_{1,h}(k)}{2\cosh k\eta}\right)\right] \\ &- \pi\int_{-\pi/2}^{\pi/2} d\lambda\,\rho_{1,h}(\lambda)\,h_1(\lambda)\frac{\partial}{\partial\lambda}s(\lambda)\Bigg\}\,,\end{aligned} \tag{C.20}$$

where $\langle\boldsymbol{\rho}|\hat{Q}_2/N|\boldsymbol{\rho}\rangle$ is proportional to the energy density of the state, $\hat{\rho}_{1,h}$ is the Fourier transform of $\rho_{1,h}$, while s is defined in Eq. (2.66b). The auxiliary function σ_1 satisfies the following set of equations

$$(\rho_n + \rho_{n,h})\,\sigma_n = [d_n - s*(d_{n-1} + d_{n+1})] + s*(\sigma_{n-1}\,\rho_{n-1,h} + \sigma_{n+1}\,\rho_{n+1,h})\,, \tag{C.21a}$$

with $\sigma_0 = d_0 = 0$. Here, d_n is defined as

$$d_n(\lambda) = \tilde{a}_n(\lambda) - \sum_{m=1}^{\infty}\tilde{a}_{nm} * \rho_m\,, \tag{C.21b}$$

where

$$\tilde{a}_n(\lambda) = -\frac{n}{\pi} \sum_{k=1}^{\infty} \sin(2k\lambda) z^{nk} , \tag{C.21c}$$

$$\tilde{a}_{nm}(\lambda) = (1 - \delta_{nm}) \tilde{a}_{|n-m|}(\lambda) + 2\tilde{a}_{|n-m|+2}(\lambda) + \dots \\ \dots + 2\tilde{a}_{n+m-2}(\lambda) + \tilde{a}_{n+m}(\lambda) . \tag{C.21d}$$

The large-Δ expansion of the auxiliary functions σ_n does not present any difficulty. The first difference between the saddle-point state and the GGE manifests itself at the z^3 order in σ_1, as it can be seen by the expansions

$$\sigma_1^{\text{sp}}(\lambda) = -2\sin(2\lambda)z + 2\sin(4\lambda)z^2 - 2\sin(6\lambda)z^3 + \frac{3}{2}\sin(2\lambda)z^3 + O(z^4), \tag{C.22a}$$

$$\sigma_1^{\text{GGE}}(\lambda) = -2\sin(2\lambda)z + 2\sin(4\lambda)z^2 - 2\sin(6\lambda)z^3 + O(z^4) . \tag{C.22b}$$

This leads to a difference in the correlators only at the z^6 order, as stated in Eq. (5.88).

C.3.2 The next-to-nearest-neighbors correlator $\langle \sigma_1^z \sigma_3^z \rangle$

The correlator $\langle \sigma_1^z \sigma_3^z \rangle$ can be computed thanks to a conjecture proposed in Ref. [196]. However, it is necessary to compute two sets of auxiliary functions, and not only one as for $\langle \sigma_1^z \sigma_2^z \rangle$. Given $\eta_n = \rho_{n,h}/\rho_n$, let us define the functions $\rho_{n,h}^{(a)}$ and $\rho_n^{(a)} = \rho_{n,h}^{(a)}/\eta_n$ ($a = 0, 1, 2, \dots$), determined by the set of equations

$$\rho_{n,h}^{(a)}(\lambda) \left[1 + \eta_n^{-1}(\lambda)\right] = \delta_{n,1} \frac{\mathrm{d}^a}{\mathrm{d}\lambda^a} s(\lambda) + \left[s * \left(\rho_{n-1,h}^{(a)} + \rho_{n+1,h}^{(a)}\right)\right](\lambda) , \tag{C.23}$$

where $\rho_{0,h}^{(a)}(\lambda) = 0$. Notice that $\rho_{n,h}^{(0)} = \rho_{n,h}$ and $\rho_n^{(0)} = \rho_n$. Now, we are ready to introduce the functions $\sigma_n^{(a)}$ satisfying

$$(\rho_n + \rho_{n,h})\, \sigma_n^{(a)} = \left[d_n^{(a)} - s * \left(d_{n-1}^{(a)} + d_{n+1}^{(a)}\right)\right] + s * \left[\sigma_{n-1}^{(a)} \rho_{n-1,h} + \sigma_{n+1}^{(a)} \rho_{n+1,h}\right], \tag{C.24}$$

where $\sigma_0^{(a)}(\lambda) = d_0^{(a)}(\lambda) = 0$ and $d_n^{(a)}(\lambda) = \partial_\lambda^a \tilde{a}_n(\lambda) - \sum_{m=1}^{\infty} (\tilde{a}_{nm} * \rho_m^{(a)})(\lambda)$. For $a = 0$, $\sigma_n^{(a)}$ reduces to the function σ_n defined in Eq. (C.21a). Given these sets of auxiliary functions, $\langle \sigma_1^z \sigma_3^z \rangle$ can be expressed as

$$\langle \sigma_1^z \sigma_3^z \rangle = \langle \sigma_1^z \sigma_2^z \rangle - \tanh(\eta) \frac{4\Omega_{0,0} - \Omega_{0,2} + 2\Omega_{1,1}}{4} + \frac{\sinh^2(\eta)}{4} \Gamma_{1,2} . \tag{C.25}$$

The quantities Ω_{ab} and Γ_{ab} are defined as

$$\Omega_{ab} = 4\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathrm{d}\mu\, s^{(b)}(-\mu) \left[(-1)^a a_1(\mu) + (-1)^{b+1} \rho_{1,h}^{(a)}\right] , \tag{C.26a}$$

$$\Gamma_{ab} = (-)^b 4\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathrm{d}\mu \Big[s^{(a+b)}(-\mu)\, \tilde{a}_1(\mu) + g^{(a+b)}(-\mu)\tilde{a}_1(\mu) + \tilde{g}^{(b)}(-\mu)\rho_{1,h}^{(a)}(\mu) - s^{(b)}(-\mu)\rho_{1,h}(\mu)\sigma_1^{(1)}(\mu)\Big] , \tag{C.26b}$$

where the superscript $^{(a)}$ stands for the a-th derivative with respect to λ, and

$$g(\lambda) = \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\tanh(k\eta)}{2\cosh(k\eta)} \cos(2k\lambda) , \tag{C.27a}$$

$$\tilde{g}(\lambda) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\tanh(k\eta)}{2\cosh(k\eta)} \sin(2k\lambda) . \tag{C.27b}$$

In order to compute $\langle \sigma_1^z \sigma_3^z \rangle$ we need $\rho_n^{(1)}$ (to compute $d_n^{(a)}$), $\rho_{1,h}^{(1)}$ and $\sigma_1^{(1)}$. For $\rho_{n,h}^{(1)}$ leading behavior is

$$\rho_{n,h}^{(1)\,\mathrm{sp}}(\lambda) \sim -32\, 6^{\frac{n-1}{2}}\, z^{2n+1} \sin^3(2\lambda) + O(z^{2n+2}) , \qquad n \text{ odd} , \tag{C.28a}$$

$$\rho_{n,h}^{(1)\,\mathrm{sp}}(\lambda) \sim -48\, 6^{\frac{n}{2}-1}\, z^{2n} \cos^3(\lambda) \sin(\lambda) + O(z^{2n+1}) , \qquad n \text{ even} , \tag{C.28b}$$

$$\rho_{1,h}^{(1)\,\mathrm{GGE}}(\lambda) \sim -\frac{32}{\pi}\, z^3 \sin^3(2\lambda) + O(z^4) , \tag{C.28c}$$

$$\rho_{n,h}^{(1)\,\mathrm{GGE}}(\lambda) \sim -\frac{12}{\pi}\, \frac{n+1}{n} z^{n+2} \sin(2\lambda) + O(z^{n+3}) , \qquad n \geq 2 . \tag{C.28d}$$

and the resulting expansion for $\sigma_1^{(1)}$ is thus

$$\sigma_1^{(1),\mathrm{sp}}(\lambda) = -4z\cos(2\lambda) + 8z^2 - 4z^3[\tfrac{5}{2}\cos(2\lambda) + \cos(6\lambda)] - z^4[2\cos(4\lambda) - 7] + O(z^5) , \tag{C.29a}$$

$$\sigma_1^{(1),\mathrm{GGE}}(\lambda) = -4z\cos(2\lambda) + 8z^2 - 4z^3[2\cos(2\lambda) + \cos(6\lambda)] - z^4[8\cos(4\lambda) + 2] + O(z^5) . \tag{C.29b}$$

Knowing the small-z expansions of the functions $\rho_{1,h}^{(a)}$, $a = 0, 1$, and $\sigma_1^{(1)}$, plugging them into Eqs (C.26), and afterwards the results into Eq. (C.25), gives finally the large-Δ expansions (5.87c) and (5.87d) of the next-to-nearest neighbor correlator.