The quench action approach to out-of-equilibrium quantum integrable models

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Gapless Néel-to-XXZ quench

In this section we present the GTBA equations of the GGE and QA approach for the Néel-to-XXZ quench in the gapless or quantum critical regime defined by $-1 < \Delta < 1$. One feature of coordinate Bethe Ansatz for the gapless regime is that not all string type are allowed, as certain strings render the wave function unnormalizable. Intuitively, this is because rapidities can only lie in the infinite strip $-\pi/2 < \mathrm{Re}(\lambda) \leq \pi/2$ and therefore longer strings can sometimes be reduced to shorter strings. Which strings are allowed depends on the value of the anisotropy Δ. For details, see e.g. Refs [13, 89].

We only consider the simplest case where $\Delta = \cos(\pi/\ell)$, with $\ell > 1$ an integer. In that case only ℓ string types are allowed. They are specified by a length n_j and a parity v_j and the allowed strings are

$$n_j = j\,, \qquad v_j = 1\,, \qquad \text{for} \quad j = 1, 2 \ldots, \ell - 1\,, \tag{F.1}$$

$$n_\ell = 1\,, \qquad v_\ell = -1\,. \tag{F.2}$$

The Bethe-Gaudin-Takahashi equations are given by

$$v_j \left[\rho_j(\lambda) + \rho_{h,j}(\lambda)\right] = a_{n_j}^{v_j}(\lambda) - \sum_{k=1}^{m_\ell} (T_{j,k} * \rho_k)(\lambda)\,, \tag{F.3a}$$

for $1 \leq j \leq \ell$, where

$$a_{n_j}^{v_j}(\lambda) = \frac{v_j}{\pi} \frac{\sin(\gamma n_j)}{\cosh(2\lambda) - v_j \cos(\gamma n_j)}\,, \tag{F.3b}$$

$$T_{j,k}(\lambda) = (1 - \delta_{n_j n_k}) a_{|n_j - n_k|}^{v_j v_k}(\lambda) + 2\, a_{|n_j - n_k|+2}^{v_j v_k}(\lambda) + \ldots \\ \ldots + 2\, a_{n_j + n_k - 2}^{v_j v_k}(\lambda) + a_{n_j + n_k}^{v_j v_k}(\lambda)\,. \tag{F.3c}$$

Note the factor v_j in front of the density functions. For strings with $v_j = -1$ one finds that if the quantum numbers $I_\alpha^j < I_\beta^j$, then $\lambda_\alpha^j > \lambda_\beta^j$.

To bring the BGT equations in partially decoupled form it is useful to work in Fourier space. The kernels become

$$\hat{a}^{+}_{n_j}(k) = \frac{\sinh\left[\frac{k}{2}(\pi - \gamma n_j)\right]}{\sinh\left(\frac{\pi k}{2}\right)} , \qquad \hat{a}^{-}_{n_j}(k) = -\frac{\sinh\left(\frac{k}{2}\gamma n_j\right)}{\sinh\left(\frac{\pi k}{2}\right)} , \tag{F.4}$$

where we used the Fourier conventions of the XXX spin chain. It should be noted that the identity for $\hat{a}^{+}_{n_j}$ is only valid if $0 < \gamma n_j < 2\pi$. For larger or smaller values one must mod out by 2π. Similarly, the identity for $\hat{a}^{-}_{n_j}$ is only valid if $-\pi < \gamma n_j < \pi$. This leads to the following set of identities for the convolution kernel $T_{j,k}$:

$$T_{j,k} = s * (T_{j-1,k} + T_{j+1,k}) + s\,(\delta_{j-1,k} + \delta_{j+1,k}) , \quad 1 \le j \le \ell - 2, \tag{F.5a}$$
$$T_{\ell-1,k} = s * T_{\ell-2,k} + s\,\delta_{\ell-2,k} , \qquad 1 \le k \le \ell - 1, \tag{F.5b}$$
$$T_{j,\ell} = s * (T_{j-1,\ell} + T_{j+1,\ell}) - s\,\delta_{j,\ell-2} , \qquad 1 \le j \le \ell - 2, \tag{F.5c}$$
$$T_{\ell,k} = -s * T_{\ell-2,k} - s\,\delta_{\ell-2,k} , \qquad 1 \le k \le \ell, \tag{F.5d}$$
$$T_{\ell,\ell} = s * T_{\ell-2,\ell-1} . \tag{F.5e}$$

We introduced the function

$$s(\lambda) = \frac{\ell}{\pi}\frac{1}{2\cosh(\ell\lambda)} , \qquad \hat{s}(\omega) = \frac{1}{2\cosh\left(\frac{\pi\omega}{2\ell}\right)} . \tag{F.6}$$

Using the above identities, one easily finds that the partially decoupled BGT equations are

$$\rho_{j,t} = s * (\rho_{j-1,h} + \rho_{j+1,h}) + s * \rho_{\ell}\,\delta_{j,\ell-2} , \qquad j = 1, \ldots, \ell-2 , \tag{F.7a}$$
$$\rho_{\ell-1,t} = s * \rho_{\ell-2,h} , \tag{F.7b}$$
$$\rho_{\ell,t} = s * \rho_{\ell-2,h} , \tag{F.7c}$$

where we defined $\rho_{0,h}(\lambda) = \delta(\lambda)$.

F.1 The GTBA equations

The Yang-Yang entropy has the same form as for the gapped regime, for even- as well as odd-parity strings. The GTBA equations for the Néel-to-XXZ quench are given by

$$\log(\eta_j) = 2\,n_j\,[2\log(2) - h] + g_j + \sum_{k=1}^{\ell} v_k\,T_{j,k} * \log\left(1 + \eta_k^{-1}\right) , \tag{F.8}$$

for $1 \le j \le \ell$ and where h is the usual Lagrange multiplier. The driving terms are

$$g_j = g^{v_j}_{n_j} = \sum_{k=0}^{n_j-1} \log\left[\frac{sh_{n_j-1-2k}ch_{n_j-1-2k}sh_{-n_j+1+2k}ch_{-n_j+1+2k}}{th^{v_j}_{n_j-2k}th^{v_j}_{-n_j+2k}}\right] , \tag{F.9a}$$

where

$$th_n = \frac{sh_n}{ch_n}\,, \quad sh_n(\lambda) = \sinh\left(\lambda + \frac{i\gamma n}{2}\right)\,, \quad ch_n(\lambda) = \cosh\left(\lambda + \frac{i\gamma n}{2}\right)\,. \tag{F.9b}$$

Note the negative powers in the denominator for odd-parity strings. Using the identities (F.5) the partially decoupled GTBA equations can be derived,

$$\begin{aligned} \log(\eta_j) &= g_j - s*(g_{j-1}+g_{j+1}) + s*\log\left[(1+\eta_{j-1})(1+\eta_{j+1})\right] && \text{(F.10a)}\\ &\quad + s*\log\left(1+\eta_\ell^{-1}\right)\,\delta_{j,\ell-2}\,, \\ \log(\eta_{\ell-1}) &= \ell\left[2\log(2)-h\right] + g_{\ell-1} - s*g_{\ell-2} + s*\log\left(1+\eta_{\ell-2}\right)\,, && \text{(F.10b)}\\ \log(\eta_\ell) &= \ell\left[2\log(2)-h\right] + g_\ell + s*g_{\ell-2} - s*\log\left(1+\eta_{\ell-2}\right)\,, && \text{(F.10c)} \end{aligned}$$

for $j = 1, \ldots, \ell-2$, where $\eta_0(\lambda) = 0$ and $g_0(\lambda) = 0$. Since this is a finite set of integral equations one does not need to worry about truncation in numerical analysis.

F.2 The GGE and $\rho_{1,h}$

The GTBA equations for the GGE are given by

$$\begin{aligned} \log(\eta_2) &= s*\log\left[\left(1+\frac{\rho_1^h}{\rho_1}\right)(1+\eta_3)\right]\,, && \text{(F.11a)}\\ \log(\eta_j) &= s*\log\left[(1+\eta_{j-1})(1+\eta_{j+1})\right] \qquad j=3,\ldots,\ell-2\,, && \text{(F.11b)}\\ \log(\eta_{\ell-1}/h) &= s*\log\left(1+\eta_{\ell-2}\right) - \log\left(1+\eta_\ell^{-1}\right)\,, && \text{(F.11c)}\\ \log(\eta_\ell/h) &= -s*\log\left(1+\eta_{\ell-2}\right) - \log(1+\eta_{\ell-1}^{-1})\,, && \text{(F.11d)} \end{aligned}$$

which can be solved in combination with the BTG Eqs (F.7) and knowledge of $\rho_{1,h}$ (see Section 5.2). The latter is obtained from the generating function

$$\Omega_{\Psi_0}(\lambda) = \lim\nolimits_{\text{th}} \frac{-1}{N}\langle\Psi_0|\hat{\tau}^{-1}\left(\tfrac{i\eta}{2}+\lambda\right)\partial_\lambda\hat{\tau}\left(\tfrac{i\eta}{2}+\lambda\right)|\Psi_0\rangle\,, \tag{F.12}$$

through

$$\rho_{1,h}^{\Psi_0}(\lambda) = a_1^+(\lambda) - \frac{i}{2\pi}\left[\Omega_{\Psi_0}\left(\lambda+\tfrac{i\pi}{2\ell}\right) + \Omega_{\Psi_0}\left(\lambda-\tfrac{i\pi}{2\ell}\right)\right]\,. \tag{F.13}$$

For the Néel-to-XXZ quench we can use the generating function that was computed in the gapped regime [126] and immediately translate this quantity to the gapless regime ($\gamma = \pi/\ell$):

$$\Omega_{\text{Néel}}(\lambda) = \frac{i\sin(2\gamma)}{\cos(2\gamma)+1-2\cosh(2\lambda)}\,. \tag{F.14}$$

Substitution leads to the final result

$$\rho_{1,h}^{\text{Néel}}(\lambda) = \frac{1}{\pi} \frac{\sin(\gamma)^3 \sinh(2\lambda)^2 [\cosh(2\lambda) - \cos(\gamma)]^{-1}}{[\cosh(2\lambda)^2 - 2\cos(\gamma)^3 \cosh(2\lambda) + \cos(\gamma)^4 + \cos(\gamma)^2 - 1]}, \quad \text{(F.15)}$$

whose XXX limit is as expected,

$$\lim_{\gamma \to 0} \left(\gamma \, \rho_{1,h}^{\text{Néel}}(\gamma\lambda) \right) = \frac{1}{\pi} \frac{32\lambda^2}{1 + 28\lambda^2 + 112\lambda^4 + 64\lambda^6} \, . \quad \text{(F.16)}$$