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DOI
10.1103/PhysRevLett.65.2642
Publication date
1990

Published in
Physical Review Letters

Classical Realization of a Strongly Driven Two-Level System

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(Received 5 September 1990)

A classical two-level system has been realized by coupling two propagation or two polarization modes of an optical ring resonator. This system can be driven by periodic modulation of either the coupling or the bare mode frequencies, at sufficient strength to violate the rotating-wave approximation (RWA) on resonance. Landau-Zener transitions, Rabi oscillation with non-RWA signature, and Autler-Townes doublets have been observed.

PACS numbers: 42.50.Hz, 32.80.Bx

A driven two-level system is usually associated with the realm of quantum physics. Examples are a spin-$\frac{1}{2}$ system or a two-level atom driven by an electromagnetic field. Interesting dynamical features which appear in this context are Rabi oscillations, Autler-Townes splittings, the validity of the rotating-wave approximation (RWA), Bloch-Siegert shifts, multiphoton transitions, Landau-Zener transitions, and the possibility of chaos.[1-7] It is well known[7] that features like these are not quantum mechanical in nature but can also appear in a purely classical context (except, of course, for those features associated with spontaneous emission). It is the purpose of the Letter to demonstrate a convenient *classical* implementation of a driven two-level system.

As usual, we describe the evolution of the two-level system by the action of a Hamiltonian matrix H_S upon a two-component state vector with complex amplitudes a and b,

$$i\frac{d}{dt}\begin{pmatrix}a\\b\end{pmatrix}=H_S\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}S & W\\W & -S\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}, \qquad (1)$$

where the frequency difference of the states is $2S$ and the states are coupled at a rate W. If we diagonalize H_S and plot the eigenfrequencies $\omega_\pm$ as a function of S, we find an avoided crossing: $\omega_\pm=\pm(S^2+W^2)^{1/2}$; see Fig. 1(a). We define the eigenvectors (stationary states) of H_S for $W=0$ as the S basis and those for $S=0$ as the W basis. Thus Eq. (1) was given in the S basis and a transformation to the W basis would convert H_S into a matrix H_W, in which the roles of the tuning S and coupling W are interchanged.

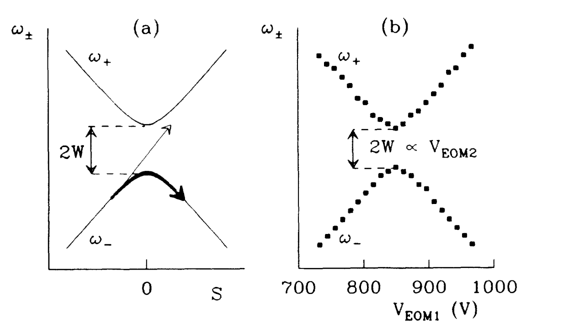

FIG. 1. (a) Avoided crossing governed by tuning parameter S and coupling parameter W. Arrows indicate adiabatic and nonadiabatic dynamic behavior. (b) Experimental realization of avoided crossing using the setup of Fig. 2(b).

In our optical implementation we consider a single longitudinal mode of an optical ring cavity. This mode has a twofold propagation degeneracy [clockwise (cw), counterclockwise (ccw)] as well as a twofold polarization degeneracy (x,y or σ^+,σ^-). The former degeneracy can be lifted by backscattering, and the latter by birefringence. In either case two modes with separate frequencies are obtained.[8] We restrict ourselves to the regime where the frequency splitting of the longitudinal mode is much smaller than the free spectral range of the ring cavity, so that a two-mode description is valid. These two modes, with amplitudes a and b, represent our two-level system. Two possible implementations are shown schematically in Fig. 2.

The ring cavity in Fig. 2(a) supports two coupled *propagation* modes. In contains a Faraday modulator FAR, sandwiched between two compensating quarter-wave plates QW1 and QW2. The "black box" QW1-FAR-QW2 acts like a nonreciprocal linear birefringent element;[9,10] i.e., its fast axis (say, y) for cw traversal is the slow axis for ccw traversal. Using y-polarized light, the black box defines the S modes as $|y,\mathrm{cw}\rangle$ and $|y,\mathrm{ccw}\rangle$, with the tuning parameter S proportional to the magnetic field inside FAR. These modes are coupled by the weakly reflecting etalon R, aligned perpendicular to the resonator axis. The coupling W is proportional to the

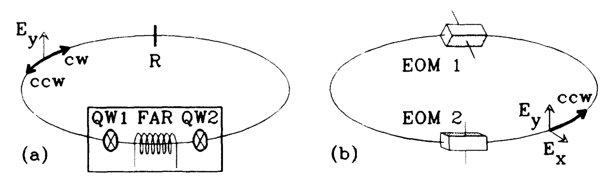

FIG. 2. Two optical implementations of a two-level system. (a) The propagation modes are tuned by a Faraday modulator FAR, sandwiched between two mutually compensating quarter-wave plates (QW) and coupled by eltalon R. (b) The polarization modes are tuned by electro-optic modulator EOM1 and coupled by modulator EOM2.

amplitude reflectivity of the etalon. The W modes are then $|y,\mathrm{cw}+\mathrm{ccw}\rangle$ and $|y,\mathrm{cw}-\mathrm{ccw}\rangle$, i.e., two standing-wave modes which are spatially dephased by $\pi/2$. The corresponding avoided crossing has been reported previously.[11]

An alternative is the ring cavity of Fig. 2(b) with two coupled *polarization* modes, using two electro-optic modulators (EOM). The axes of EOM1 are x and y, so that, using ccw-propagating light, the S modes are $|\mathrm{ccw},x\rangle$ and $|\mathrm{ccw},y\rangle$, with the tuning parameter S proportional to the electric field inside EOM1. The S modes are coupled by EOM2, with axes along $x+y$ and $x-y$; the W modes are now $|\mathrm{ccw},x+y\rangle$ and $|\mathrm{ccw},x-y\rangle$, with the coupling W proportional to the electric field inside EOM2. The experimentally observed mode structure, again revealing an avoided crossing, is shown in Fig. 1(b).

Dynamical features of our optical two-level system may be investigated by modulating a control parameter (S or W). For example, we can prepare the cavity for $t<0$, with $S=0$, in one of the W modes [eigenstate ω_- in Fig. 1(a)] and switch on, at $t=0$, an oscillating "driving field" $S=S(t)=S_0\sin(\omega t)$. We shall discuss two regimes: the Landau-Zener regime where $\omega \ll 2W$ and $S_0 \gtrsim W$, and the Rabi regime where $\omega \approx 2W$ and $S_0 \lesssim W$. In the Landau-Zener regime the driving field S slowly sweeps back and forth around zero with large amplitude [see Fig. 1(a)]. If the sweep is sufficiently slow, adiabatic following transfers the system periodically from one S mode to the other, i.e., from a positive slope to a negative slope in Fig. 1(a). The two S-mode intensities thus oscillate in antiphase, synchronously with the driving field $S(t)$. Such adiabatic behavior has been reported previously.[9] If the sweep of the driving field is faster, deviations from adiabatic behavior occur and the initially "empty" eigenstate (ω_+) is populated through Landau-Zener transitions.[12] As in a quantum beat this will show up as a beat at the instantaneous difference of the eigenfrequencies, $\omega_+(t)-\omega_-(t)$.

This latter behavior was investigated experimentally using the setup of Fig. 2(a). This ring cavity was described in detail in Ref. 9. A He-Ne amplifier with Brewster windows was used to compensate the optical losses for $|y\rangle$. Its gain was kept just below lasing threshold to avoid nonlinear dynamical effects. The amplitude reflectivity of R was 0.08, corresponding to $2W/2\pi=2.3$ MHz.[9-11] At $t<0$ ($S=0$) the cavity was prepared in one of the W modes by injecting the (y-polarized) beam of an external He-Ne laser resonant with this mode. At $t=0$ the injection beam was interrupted and an exponentially decaying ac magnetic field ($\omega/2\pi=0.44$ MHz, $S_0/2\pi=2.5$ MHz at $t=0$) was switched on by discharging a capacitor through the Faraday coil. Two photomultipliers were used to measure the cw and ccw intensities leaking out of the cavity, i.e., the S-mode intensities. Figure 3 shows a typical result: The two S-mode intensities ($I_{\mathrm{cw}}, I_{\mathrm{ccw}}$) oscillate in antiphase, in synchronism

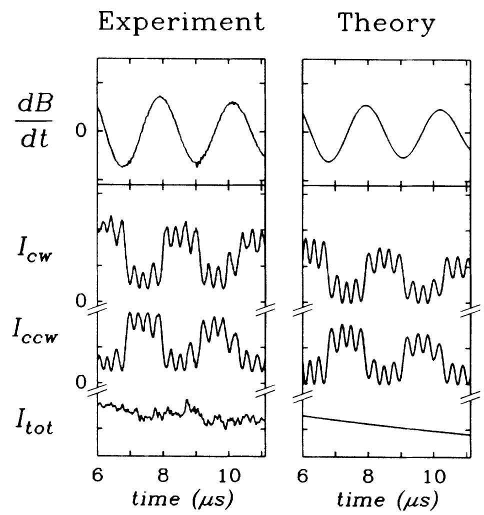

FIG. 3. Typical results in the Landau-Zener regime obtained with the setup of Fig. 2(a). The cw and ccw intensities I_{cw} and I_{ccw} oscillate in antiphase, in synchronism with the driving magnetic field B; the latter has been switched on at $t=0$. The fast modulation on top of this oscillation is the result of Landau-Zener transitions. The total intensity $I_{\mathrm{tot}}=I_{\mathrm{cw}}+I_{\mathrm{ccw}}$ decays slowly due to the finite photon lifetime in the cavity. The theoretical results were obtained by numerical integration of Eq. (1).

with the magnetic field. The high-frequency modulation on the signals is interpreted as due to nonadiabaticity, i.e., Landau-Zener transitions. We have observed that this modulation frequency decreases with time following the decay of the magnetic-field amplitude, in quantitative agreement with $\omega_+-\omega_-=2[W^2+S^2(t)]^{1/2}$. Also shown in Fig. 3 are the theoretical results obtained by numerically integrating Eq. (1) with an exponentially decaying driving field and accounting for optical losses by means of an additional damping term. They are in good agreement with the experiments. Note that Eq. (1) fully contains interference between successive Landau-Zener transitions; this interference is extremely important when a multiphoton resonance condition is met. Similar experimental results were obtained (not shown) when the gain was set *above* lasing threshold, leaving out the injection laser. This indicates that the nonlinear mode interaction in the He-Ne ring laser is negligible on the time scale of the experiment.[13]

For the Rabi regime ($\omega \approx 2W$, $S_0 \lesssim W$) it is instructive to use the W basis, where

$$H_W(t)=\begin{pmatrix} W & S_0\sin(\omega t) \\ S_0\sin(\omega t) & -W \end{pmatrix}. \tag{2}$$

This Hamiltonian is that of a two-level atom driven by a classical electromagnetic field.[14] Resonant transitions between the W modes are expected at the Rabi frequency $\Omega_R=(S_0^2+\Delta^2)^{1/2}$, where $\Delta \equiv \omega-2W$. For the experiment both setups shown in Fig. 2 were used. In the setup of Fig. 2(a) the gain was set above threshold, resulting in ring-laser action in one single W mode (standing-wave mode). An exponentially decaying ac magnetic field was switched on at $t=0$ and the detectors were set to record the W-mode intensities. To this end the cw and ccw beams (S modes) leaking out of the cavity were combined at a 50-50 beam splitter and their relative phase was adjusted in order to obtain the proper linear combinations at the output ports of the beam splitter (corresponding to intracavity standing waves). In Fig. 4 the measured intensity in the initially empty W mode is given for three values of the detuning $\Delta=\omega-2W$, with $\omega/2\pi=1.05$ MHz. The value of $2W$ was varied around that of ω by thermally tuning the reflectivity of etalon R. The intensity in the other W mode (not shown) was found to be complementary. Note that the Rabi oscillation frequency is chirped, because the magnetic-field amplitude ($\propto S$) decays exponentially. Also, the increase of the Rabi frequency with detuning is evident from Fig. 4. The inset in Fig. 4(b) shows a ripple on top of the Rabi oscillation which we interpret as the effect of the counter-rotating term in the driving field $S(t)$, showing that we violate the RWA. This is due to the fact that we do not satisfy the condition $\Omega_R \ll 2W$ [$\Omega_R/2\pi \approx 150$ kHz near $t\approx 0$ in the uppermost trace of Fig. 4(b)]. The theoretical results in Fig. 4, again obtained by numerical integration of Eq. (1), show good agreement, including the non-RWA ripple at a frequency $\omega+2W$.

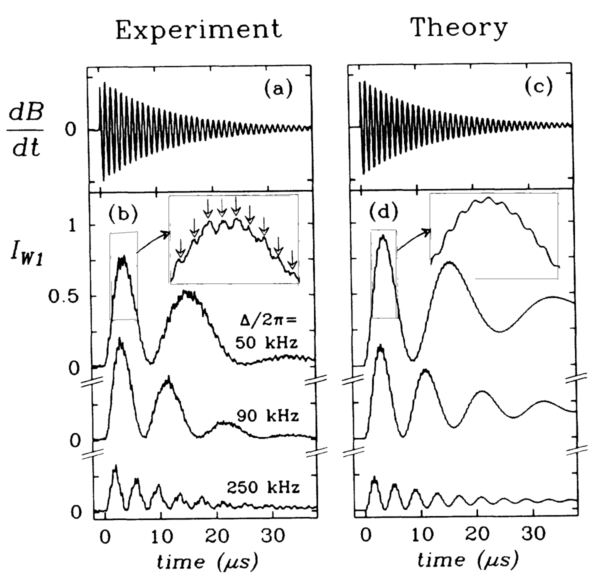

FIG. 4. Typical Rabi oscillations observed in the setup of Fig. 2(a). The (normalized) intensity I_{W1} of an optical mode $W1$ is shown for three values of the detuning $\Delta=\omega-2W$; the vertical scale is the same for the three traces. The theoretical curves were obtained by numerical integration of Eq. (1). The intensity of the other mode ($W2$) was $1-I_{W1}$. The insets correspond to a 5× enlarged time scale and display a ripple due to violation of the RWA.

The long-term discrepancy between theory and experiment is attributed to a residual nonlinear interaction and a differential loss rate of the two W modes. These effects are evidently slow on the time scale of the experiment.

The Rabi regime was also examined in the polarization setup [Fig. 2(b)]. No intracavity amplifier was used and EOM1 was driven by a continuous ac electric field ($\omega\approx 2W\approx 2\pi\times 9$ MHz). This driven system was probed by scanning the cavity length while injecting fixed-frequency He-Ne laser light. The polarization of the injected light was x in order to have equal projections on both W modes. We measured the light leaking out of the cavity without using an analyzer and observed that each W mode was split, by an amount Ω_R, in an Autler-Townes doublet (Fig. 5). The doublets are symmetric for zero detuning and asymmetric for nonzero detuning. The frequency spacing of the doublet centers equals $2W$. The diagram in Fig. 5 illustrates that, for $\Delta=0$, the Rabi frequency increases linearly with the ac voltage across EOM1, i.e., with the amplitude of the driving field S_0. These results can be easily understood in a dressed-level picture. For the experiments shown in Fig. 5, S_0 assumes values up to $\sim 2W$, so that the RWA is no longer valid and a Bloch-Siegert shift[1,7] should show up in Fig. 5. Because of the limited cavity finesse, however, the experimental accuracy was insufficient for its detection. Work is in progress to improve this situation and to observe also other non-RWA features. Some indications of multiphoton resonances[1,5,7] have already been observed.

In conclusion, we have demonstrated that the driven optical ring resonator represents a *classical* realization of a strongly driven two-level system ("optical atom").[14] Landau-Zener transitions, Rabi oscillations with non-RWA signature, and Autler-Townes doublets have been observed. The beauty of having a *macroscopic* implementation of a driven two-level system is that all relevant parameters can be closely controlled, in particular, dissipation (including its sign). This offers interest-

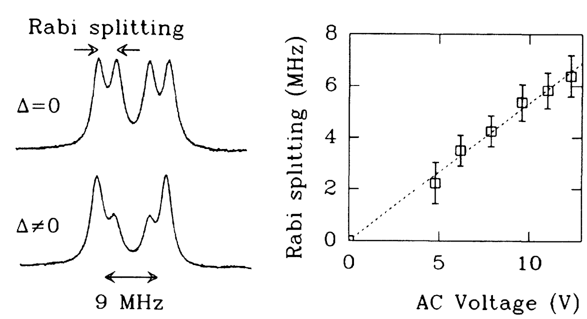

FIG. 5. Autler-Townes structure of driven optical two-level system for zero and nonzero detuning Δ. For $\Delta=0$ the diagram illustrates the linear dependence of the Rabi frequency on the driving field amplitude, i.e., the ac voltage across EOM1 in Fig. 2(b).

ing possibilities to resolve current issues in (dissipative) quantum dynamics.[2,3] It might also be possible to study chaos in our driven two-level system, induced by nonharmonic driving or by violation of the RWA.[4-6] This would then allow study of the transition from Hamiltonian to dissipative chaos.[6] In this context it is interesting to note that the Jaynes-Cummings model[1,5] may also be implemented by adding an electronic feedback loop in order to mimic the reaction of the optical field ("atomic inversion") on the rf driving field. We close by noting that a study of a quantized version of the present configuration offers a worthwhile challenge, both theoretically and experimentally. Three field oscillators may be quantized, namely, the two optical modes and the rf driving field. In particular, quantization of the optical modes may lead to interesting new phenomena.[4] In that case, of course, we no longer deal with a two-level system; the latter corresponds to the *classical* limit of the two optical fields.

This work is part of the research program of the Foundation for Fundamental Research on Matter (FOM) and was made possible by financial support from the Netherlands Organization for Scientific Research (NWO).

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