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INSECURE DEBT[†]

Abstract

Does demand for safety create instability ? Secured (repo) funding can be made so safe that it never runs, but shifts risk to unsecured creditors. We show that this triggers more frequent runs by unsecured creditors, even in the absence of fundamental risk. This effect is separate from the liquidation externality caused by fire sales of seized collateral upon default. As more secured debt causes larger fire sales, it leads to higher haircuts which further increase the frequency of runs. While secured funding combined with high yield unsecured debt may reduce instability, the private choice of repo funding always increases it. Regulators need to contain its reinforcing effect on liquidity risk, trading off its role in expanding funding by creating a safe asset.

JEL Classification: G21 and G28
Keywords: bank runs, haircuts, repo and secured credit

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1 Introduction

Credit expansion during the recent financial boom was to a large extent funded by strong demand for safe dollar assets (Caballero and Krishnamurthy, 2009). Intermediaries sought to satisfy investors by repackaging loans in senior debt pools believed to be safe. Once their exposure to macro risk became manifest, banks and shadow banks suffered large unsecured debt runs. In contrast, secured financial credit (a repurchase agreement known as *repo*) kept being rolled over until the eve of default for Bear Sterns and Lehman. At the peak of the crisis, repo debt ran in the specific form of higher haircuts, playing a critical role in propagating financial stress (Gorton and Metrick, 2012). [1]

Since then, the role of secured financial credit has come under sharper focus. Can a contractual innovation aimed at greater safety produce instability? Secured funding is cheaper, safer and more stable than traditional unsecured funding. We show here how it contributes to runs by making unsecured debt feel insecure.

Our paper assesses the direct interaction of secured and traditional unsecured bank funding, in a setting where all debt is demandable and susceptible to strategic uncertainty. We follow the literature by explaining repo as a response to strong demand for absolute safety.[2] Repo funding is targeted to the most risk averse savers, historically willing to pay a safety premium (Krishnamurthy Vissing-Jorgensen, 2012). In our setting, secured lending may be made safe in any default, so that it is always rolled over in equilibrium. As it is less expensive, it is a desirable source of funding. In the absence of coordination issues, a risk redistribution to risk neutral unsecured lenders would be efficient. However, demandable bank funding introduces the possibility of multiple equilibria and runs (Diamond and Dybvig, 1984), driven by strategic complementarity. In line with this literature, we consider a set up with an unique run equilibrium in a global game setting (Morris and Shin, 2003; Goldstein and Pauzner, 2005).

[1] Krishnamurthy, Nagel and Orlov (2012) question the scale of the repo relative to the unsecured runs, based on flow of funds and tri-party repo data. Unfortunately, there is no precise data on the scale of bilateral repo lending.

[2] This view is also at the heart of recent work on shadow banking and safety traps (Gennaioli et al., 2013; Gorton and Ordoñez, 2014; Caballero and Fahri, 2013; Ahnert and Perotti, 2014.

In order to focus on instability, we assume no fundamental asset risk at maturity, so all runs are inefficient as they cause unnecessary liquidation.

In principle, the effect of secured credit on the risk of runs is ambiguous. It is cheaper and stable, and more of it reduces the volume of unsecured debt that may run. Yet as it receives the safer part of asset return, more secured debt leaves each unit of unsecured debt more exposed to risk. Our main result is that the private choice of secured funding causes more frequent unsecured runs. These occur because of a stronger strategic complementarity in running even in solvent states, because the bank assets are illiquid. There is both a direct and an indirect risk creation effect. The direct effect is due to increased risk concentration on each unit of unsecured debt, which shifts the threshold for a self protecting run (Goldstein and Pauzner, 2005). In this sense, an increase in safe debt makes other creditors more "insecure". Next we identify an indirect risk reinforcing effect arising from repo's reliance on financial collateral. Its potential illiquidity induces safety-conscious repo lenders to adjust haircuts, further concentrating risk and again shifting the run threshold.[3] Intermediaries without deposit insurance recognize the run risk caused by repo borrowing, but as it is inexpensive they may wish to attract some. We show that the private funding choice maximizes the volume of secured debt, and cause more runs and default risk than the social optimum. The effect of secured debt on the likelihood of run depends on asset liquidity and on the yield offered to demandable debt. A high rollover premium for unsecured debt reduces the frequency of runs, though the private choice prefers a higher average return at the cost of more runs. As it is intuitive, higher asset liquidity reduces instability. However, if the intermediaries pledges this liquidity to repo lenders, the chance of runs may actually increase.

Our result shows how secured debt may contribute to risk creation beyond the broadly accepted view of a risk externality associated to repo's bankruptcy privileges (Duffie and Skeel, 2012). To avoid a hasty termination of real projects, bankruptcy law forces an automatic stay on all lenders to ensure orderly resolution. As secured financial debt is exempt, it immediately

[3] This result echoes the liquidity risk effect present in all repo run models (Martin et al 2012, Oehmke 2014).

repossess and sell the pledged collateral upon default. In our setting, more repo funding increases the frequency of (unsecured) runs, thus causing more collateral sales. [4] Note that by itself, sales of repossessed collateral do not force faster liquidation of bank assets, as these are sold under orderly resolution. However, they increases its illiquidity, which in turn raises ex ante haircuts. Higher overcollateralization increases the chance of runs for all banks, a risk externality not internalized by individual intermediaries. [5]

Finally, we consider how the introduction ex nove of secured credit may undermine traditional deposit insurance, in a context where the bank is funded by insured deposits, uninsured wholesale debt, and secured debt. We show how such an innovation, when not properly anticipated by regulators, leads to increased instability and losses for the deposit insurance fund. This occurs not only because secured assets are subtracted from resources available in default, but because of losses caused by increased run frequency by unsecured creditors.

In conclusion, unregulated secured funding not just redistributes risk, but increases it by causing more runs and more early liquidation, a loss that should be traded off against its lower cost.

The optimal regulatory policy may suggest limiting the scale of secured funding, in order to reduce the frequency of runs and the associated liquidity externality. [6] It may also seek to increase the resilience of unsecured debt by increasing its rollover return above what a private intermediary would choose to do.

A more complete welfare assessment of secured debt should take into account its role in satisfying the demand for safety by some investors, and thus expanding funding supply to credit intermediaries. In the model we sidestep this welfare issue by assuming that agents have a safe storage option, but the issue becomes salient in a situation of excess demand for safety. Attracting very risk averse agents at a low cost may increase the scale of investment, both by

[4] On the main features of safe harbor and the associated incentives to resell quickly seized collateral, see Perotti (2011) and Duffie and Skeel (2012)}.

[5] In our framework the private choice of secured funding is already at the maximum, thus the liquidity externality affects the frequency of runs but does not contribute to a further increase in repo funding.

[6] This may be achieved by capping it, or by imposing a Pigouvian charge to balance the externality effect. The optimal balance of ratio vs charges depends on the degree of bank capitalization (Perotti and Suarez, 2011)

increasing the scale of funding and reducing the marginal required rate of return. In a more complete model with decreasing returns, attracting repo funding enables marginal projects to be funded, as in Ahnert and Perotti (2014).

Our approach offers a distinct contribution by showing the possibility of purely inessential runs. This result is derived by a realistic description of the bank default process that simplifies the solution of the global game. In our model, a bank may be declared in default once it runs out of liquid reserves, provided the value of its risky assets under immediate liquidation would fail to meet all remaining withdrawals.Once the bank exhausts its liquid assets it declares default and closes its doors.[7] The mandatory stay imposed by bankruptcy law enables liquidators to pursue an orderly resolution process, achieving a higher asset sale value.

Related literature

A literature has emerged since the crisis to analyse the behavior and consequences of repo lending, recognizing its origin in a strong demand for safety. Martin, Skeie and von Thadden (2013) study the dynamics of repo runs and their price impact, showing the critical role of collateral liquidity. Such liquidity risk may trigger inefficient runs, analogous to runs driven by poor coordination by demandable debtors (Diamond and Dybvig (1983), Goldstein and Pauzner (2005)). He and Xiong (2011) provide a dynamic model of runs when debt is staggered, where creditors' roll-over decision depends on beliefs about other creditors' subsequent roll-over choice. Kuong (2013) considers the case when unsecured debt responds to higher repo margins by demanding higher required return, and shows that the resulting higher leverage directly affects risk taking by borrowers. Auh and Sundaresan (2014) looks at the effect of repo illiquidity risk on long term debt. These papers do not compare repo with other claims of the same maturity, so there is no direct interaction effect. In our set up, repo emerges as the preferred choice by investors seeking absolute safety.[8]

[7] In contrast, in the traditional bank run approach withdrawing individuals may consume all assets. As a result, the payoff to a run is non monotonic in the signal.

[8] Ahnert et al. (2014) offer a related view, showing how asset encumbrances may affect confidence and raise unsecured funding costs.

We assume all debt is demandable, or in any case has a common rollover date, to measure the interaction independently from maturity effect. In principle there should be a rationale for demandable debt, such as contingent liquidity demand by traditional depositors. Consistent with this view, intermediaries in our model hold some liquid reserves to meet withdrawals. Our results do not depend on secured debt being demandable, as it is designed to be absolutely safe even in a run. Martin, Skeie and von Thadden (2013) propose that secured credit arise when asset values are non verifiable. Auh and Sundaresan (2014) argue that repo funding demands collateral to avoids violations of absolute priority.[9] They show that a firm may issue repo loans to save on the cost of long term debt, but will not issue too much when collateral liquidity is low, to avoid liquidation losses.

Gorton and Ordoñez (2014) elaborate on the view that information-insensitive claims arise to overcome adverse selection (Pennacchi and Gorton, 1999). Collateral runs may be triggered when it become information sensitive. Intriguingly, larger runs occur after a long positive period reduces the stock of public information.

A key driver in our approach is that safe claims are cheaper, as investors seeking absolute safety are willing to pay a safety premium. A financial pledge can be designed to avoid risk in all states. Such a strong investor preferences for safety has now been documented extensively (Gorton Lewellen Metrick (2012), Krishnamurthy Vissing-Jorgensen (2012)), and is leading to a new view of risk attitudes. Recent models assume some agents act as (locally) infinitely risk averse, either because of Knightian or salient beliefs, or in order to achieve a subsistence level of wealth in all states (Caballero Fahri (2013), Gennaioli et al (2013), and Ahnert and Perotti (2014)). Earlier work interpreted demand for safe assets as means for transaction services, with money in the utility function. Stein (2013) shows how this may lead to excess creation of bank demandable claims, and may lead to fire sales. Perotti and Suarez (2011) discuss the relative effectiveness of price- versus quantity liquidity rules when banks differ in their quality and risk incentives.

[9] In many cases it is ex post efficient to violate absolute priority rules, e.g. to ensure proper continuation incentives.

A key amplification effect may result from the special bankruptcy treatment for collateralized financial credit, the "safe harbor" privileges. This unique status creates a proprietary right directly enforceable on assets. This avoids risks such as excessive issuance or imperfect enforcement that dilute the value of any unsecured debt. Legal changes to the US and EU bankruptcy codes since 1978 steadily extended the privileges for secured credit and the scope for eligible collateral. [10] The key privilege is the ability of secured financial creditors to gain immediate access to the collateral (a unique privilege not enjoyed by any other claim, not even secured real credit). This super priority status supported during the boom the spectacular growth for the repo market as well as for all derivative contracts, which also rely on collateralized margins (Bolton and Oehmke, 2013). The bankruptcy privileges have drawn considerable attention since the crisis. Legal scholars question whether it is justified to grant superior bankruptcy privileges to repo and derivatives (e.g. Roe 2011). Bolton and Oehmke (2011) shows it leads to excess risk incentives in the use of derivatives, as they enable to shift risk to other creditors. Duffie and Skeel (2012) argue that in order to reduce the risk of fire sales, only cash-like collateral may be excluded from automatic stay. [11] Mandatory stay is the core instrument of orderly resolution in bankruptcy law, created to avoid the externality caused by uncoordinated asset stripping by creditors. The consequences of immediate repossession of collateral upon default became visible when Lehman Brothers collapsed. Within hours, hundred of billion in securities were repossessed, and immediately resold by risk averse repo lenders. The collapse in collateral prices and liquidity propagated the shock to the entire system. If repossession had occurred under a traditional orderly resolution plan, it would not have led to such rushed sales. Acharya, Anshuman and Vishwanthan (2012) argue for automatic stay provisions to avoid such fire sales. Perotti (2011) argues that safe harbor is what enables shadow banks to credibly promise liquidity on demand. By pledging the liquid component of assets, it replicates the banking model outside the regulated perimeter. Hanson, Stein, Shleifer and Vishny

[10] Privileges include exemptions from preference, netting, cross-default and eve-of-default norms (Perotti 2013).

[11] This is equivalent to a narrow shadow banking model, also invoked in Gorton and Metrick (2012).

(2014) show that banks are best at funding less risky but less liquid projects. As a result, "although traditional banks have more stable funding ... they create fewer money-like claims than shadow banks". In practice the distinction is not sharp, as the volume of repo funding issued by commercial banks is quite significant, and in fact rising rapidly in recent years. The degree of balance sheet encumbrance is thus rapidly becoming a key stability question for banking supervisors.

2 The Model

The economy lasts for three periods $t = 0, 1, 2$. It is populated by a bank and a continuum of lenders indexed by i. The intermediary has access to a project that needs one unit of funding in $t = 0$. It raises funds from lenders, each of whom is endowed with one unit. Lenders belong to one of two subsets. Some investors are risk neutral and demand a minimum expected return of $\gamma > 1$, reflecting their alternative option. A set of investors is infinitely risk averse, willing to lend if and only if they can be assured to be paid in full in all states. In exchange for this absolute safety, they accept a lower return equal to 1, the rate that they could earn on safe storage. The measure of each subset is sufficiently large such that the bank could in principle finance the project with only one type of lender.

- *Project*

For each unit invested, the project generates a return of $y_t(\omega)$ in $t = 1, 2$, where $\omega \in \{H, L\}$ is the aggregate state. With probability λ the state is high ($\omega = H$), and the project matures in $t = 1$: $y_1(H) = r > \gamma$ and $y_2(H) = 0$. With probability $1 - \lambda$, the state is low ($\omega = L$), and the project matures only in $t = 2$. In this case, early liquidation at $t = 1$ could be costly as the project has not fully developed its potential. The liquidation value has a safe component k plus an uncertain value θ, drawn from a uniform distribution on $\left[0, \overline{\theta}\right]$. Agents receive private signals on θ at the begin of time 1. Liquidating the unsafe component of the bank's assets requires a fixed cost c. A claim to the safe portion of this return may be securitized in order to be pledged to lenders seeking higher safety. If sold at time 1, financial collateral returns an ex ante known price $p \leq 1$, where the discount reflects limited interim liquidity. We initially treat p as a parameter, later we endogenize it as the outcome of aggregate sales.

If the project is allowed to mature in the low state, it generates a return of $y_2(L) = r$ if $\theta \geq c > 0$, and a return of $y_2(L) = \rho < 1$ if $\theta < c$. One interpretation for the fundamental risk is that, when the liquidation value of unsafe assets θ is smaller than the liquidation cost c, the bank becomes insolvent. We think of c as being small, such that the project is essentially riskless if allowed to mature. To ensure that depositors may be fully repaid in the low state when assets have a sufficently high liquidation value, we assume that $\overline{\theta} + pk > 1 + c$. This assumption and the dependence of $y_2(L)$ on θ create upper and lower dominance regions in our global game setup, respectively, which are needed to ensure equilibrium uniqueness. We also assume that r is sufficiently large that the project has positive NPV if discounted at the rate of return demanded by unsecured debt, even if the bank goes bankrupt in the low state

- *Lenders and Financing*

The bank raises funds by issuing a measure s of secure debt, and funding the residual $1 - s$ through unsecured debt with face value d. We will refer to them by the subscripts $\{U, S\}$. The project has a positive NPV for any funding choice. Because of rollover risk, however, the bank needs to provide superior safety to attract risk intolerant investors. Specifically, the bank securitizes the safe part of its return and pledges some of it as financial collateral to secured lenders, while retaining some.

We assume (realistically) that a fraction of financial collateral $k - \underline{k}$ must be retained by the bank, so the maximum amount that can be pledged to back secured debt is $\underline{k}$. An interpretation is a minimum reserve requirement, to ensure some liquidity to meet routine withdrawals.

Because financial collateral may be sold at a discount at time 1 (at a price $p \leq 1$), secured lenders demand a sufficient haircut h — excess collateral per unit of funding — at $t = 0$ to be sure of full repayment at $t = 1$. Provided the haircut satisfies $hp \geq 1$, secured lenders are always paid in full even in default, and never face losses.

As a result, unsecured debt in default has access to $k - hs$ units of financial collateral. Since a higher s reduces the amount of safe return available to repay unsecured debt withdrawals, it increases the probability that the bank survives in the event of a large run. For a given probability of runs, unsecured creditors may therefore require a higher face value to compensate for larger losses in a run.

- *Lenders' Information Structure*

All agents observe the state ω at the begin of $t = 1$. In the high state the project has matured so all claims are safe. In the event of a low state $\omega = L$, agents receive individual noisy signals on the early liquidation value of assets $\theta + k$. [12] This signal is given by

$$x_i = \theta + \sigma\eta_i, \tag{1}$$

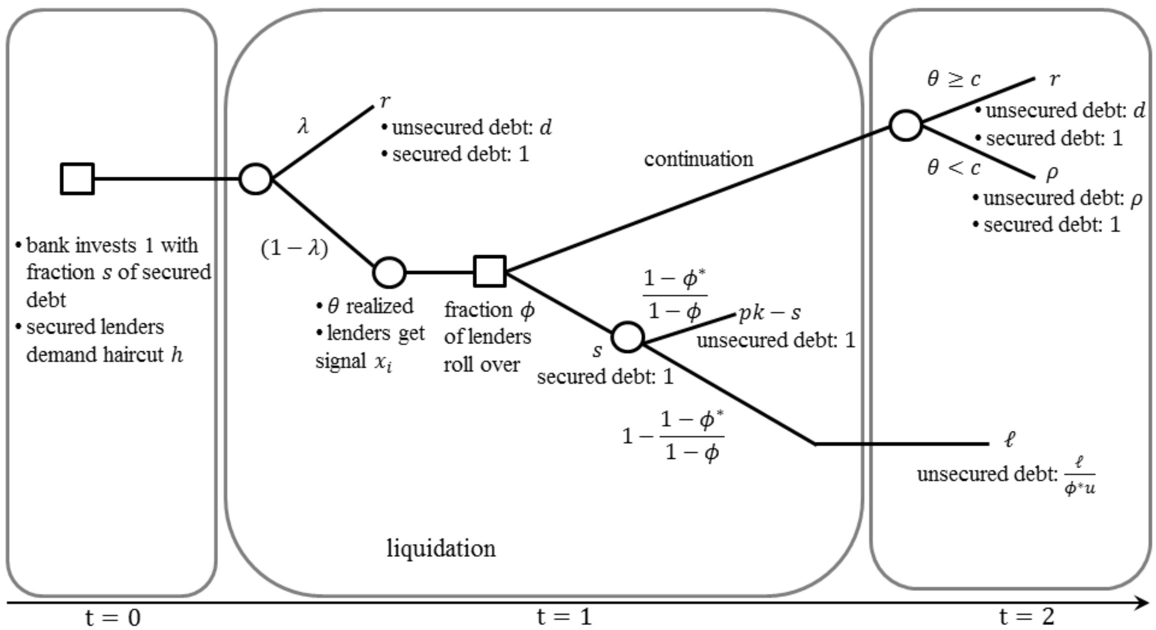
where $\sigma > 0$ and η_i are i.i.d. across players and uniformly distributed over $[-\epsilon, \epsilon]$.

- *Debt Rollover, Bank Runs, and Orderly Liquidation*

Since all claims are safe in the high state, we henceforth focus on the low state $\omega = L$. By design, secured agents are fully protected even in default, and have no incentive to run. In contrast, unsecured lenders may choose to withdraw the principal amount 1, upon receiving their private signals. In this case, the bank sells assets in order to meet withdrawals, starting from its reserves. If withdrawals exceed the value of reserves, the bank needs to liquidate a part of the project to satisfy the remaining withdrawing depositors. The liquidation process has a large fixed cost of c. If the amount not rolled over is greater than the maximum liquidation proceeds, the bank is immediately declared bankrupt. Here we deviate from the standard assumption that only the money paid out is recovered, using a realistic description of bank bankruptcy. Once a bank is declared in default, bankruptcy law forces a stay for all unsecured creditors, enabling orderly resolution at $t = 2$. At that point, any unpaid depositors are

[12] To be precise, θ is the liquidation value of the asset in excess of the safe component.

Figure 1: Economy Timeline



treated equally. The bankruptcy process produces a final excess value equal to ℓ under orderly liquidation. Formally, let ϕ be the fraction of unsecured lenders that roll over in $t = 1$. The first depositors in the running queue are immediately paid out of liquid reserves (the retained collateral $k - sh$). If there are depositors left in the queue, the bank is declared bankrupt if and only if

$$u\left(1-\phi\right) > \theta - c + p\left(k - sh\right).$$

Here the left hand term indicates the face value demanded by running depositors, and the right hand side the amount available at $t = 1$, namely the net liquidation value of the unencumbered asset value plus the value of the retained collateral. The condition may be rewritten as stating that the bank is declared bankrupt if the net value of selling its non reserve assets exceeds the claims of the still unpaid withdrawing depositors. We assume that the excess value produced under orderly liquidation is never enough to fully repay all unsecured lenders: $\ell < 1 - pk$.

- *Lenders' Payoffs*

When the state is high, unsecured lenders always receive their face value d. In the low state, their payoffs depend on whether they choose to roll over or withdraw, and whether the bank survives in case of a run. Early withdrawals receive the principal 1 if the bank has enough liquidity. If it defaults, asset liquidation proceeds net of secured debt are distributed equally among all unpaid unsecured debt holders.

In a run, the random order of arrival implies that running depositors receive their full principal out of the liquid reserves with probability $\frac{1-\phi^*}{1-\phi}$, where ϕ^* is such that $(1-\phi^*)\,u = p\,(k-sh)$. That is, ϕ^* is the minimum fraction of unsecured lenders that needs to roll over in order for all withdrawers to receive full repayment out of reserves. With probability $1-\frac{1-\phi^*}{1-\phi}$, withdrawers receive $\frac{\ell}{\phi^* u}$, the orderly liquidation value of unencumbered assets scaled by the mass of remaining unsecured lenders.

In conclusion, the payoff of unsecured lenders who do not roll over in $t=1$ is

$$\pi_U^N(\phi,\theta) = \begin{cases} 1, & u\,(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k-sh) \\ \frac{1-\phi^*}{1-\phi} + \left(1 - \frac{1-\phi^*}{1-\phi}\right)\frac{\ell}{\phi^* u}, & u\,(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k-sh) \end{cases},$$

where $\mathbf{1}_{\{.\}}$ is the indicator function.

Unsecured lenders that roll over receive the face value of their loans if the bank does not go bankrupt. In bankruptcy, they are entitled to receive $\frac{\ell}{\phi^* u}$ out of the orderly liquidation value. That is,

$$\pi_U^R(\phi,\theta) = \begin{cases} \mathbf{1}_{\{\theta \geq c\}}d + \left(1 - \mathbf{1}_{\{\theta \geq c\}}\right)\rho, & u\,(1-\phi) \leq \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k-sh) \\ \frac{\ell}{\phi^* u}, & u\,(1-\phi) > \mathbf{1}_{\{\theta \geq c\}}\,(\theta - c) + p\,(k-sh) \end{cases}.$$

Notice that this payoff is decreasing in both s and h. Finally, we assume $r-1 \leq 1-\ell-pk$, which ensures the existence of a unique θ^* such that $\int_0^1 \left(\pi_U^R(\phi,\theta^*) - \pi_U^N(\phi,\theta^*)\right) d\phi = 0$. This is an additional global game assumption needed to reach equilibrium uniqueness. It has a natural interpretation in our context as it means that higher project return (higher r) must be associated with higher risk (lower $\ell + pk$).

3 Equilibrium Runs

In order to derive the optimal rollover decision, we first calculate the haircut h demanded by secured lenders. The haircut are set at $t = 0$ to make sure they are paid in full in the event of a run at $t = 1$. Recall that the sale price of financial collateral p is known as of time 0. Later we endogenies its value.

The payoff of each secured lenders in $t = 1$ in case of a run is $\pi_S = ph$. Therefore, the minimum haircut h^* demanded by unsecured lenders in $t = 0$ solves

$$ph^* = 1. \tag{2}$$

If the bank pledges sh^* units of financial collateral, secured lenders are completely safe as long as $sh^* = sp^{-1} \leq \underline{k}$. In turn, this is true only if $s \leq p\underline{k}$.

We now turn to unsecured lenders' rollover decision. From the previous section, we know that the bank is assessed to be bankrupt if and only if withdrawals $1 - \phi$ are sufficiently large:

$$(1 - \phi)\, u > \mathbf{1}_{\{\theta \geq c\}} (\theta - c) + p\,(k - sh^*) = \mathbf{1}_{\{\theta \geq c\}} (\theta - c) + pk - s, \tag{3}$$

Let $\Pi_U^R(\phi, \theta)$ be the net payoff of unsecured lenders who roll over relative to that of running. We have

$$\Pi_U^R(\phi, \theta) = \begin{cases} \mathbf{1}_{\{\theta \geq c\}} d + \left(1 - \mathbf{1}_{\{\theta \geq c\}}\right) \rho - 1, & u\,(1 - \phi) \leq \mathbf{1}_{\{\theta \geq c\}} (\theta - c) + p\,(k - sh) \\ -\frac{pk - s}{(1-\phi)(1-s)} \left(1 - \frac{\ell}{1 - pk}\right), & u\,(1 - \phi) > \mathbf{1}_{\{\theta \geq c\}} (\theta - c) + p\,(k - sh) \end{cases}. \tag{4}$$

Since unsecured lenders require a minimum expected return of $\gamma > 1$, the face value of unsecured debt d must be larger than 1. As a result, unsecured lenders face a complex coordination problem in their decision to roll over, which depends on their beliefs about both θ (fundamental uncertainty) and the fraction ϕ of lenders that rolls over (strategic uncertainty).

Suppose lenders follow a monotone strategy with a cutoff κ, rolling over if their signal is above κ and withdraw otherwise. Lender i's expectation about the fraction of lenders that roll over conditional on θ is simply the probability that any lender observes a signal above κ, that is, $1-\frac{\kappa-\theta}{\sigma}$. This proportion is less than z if $\theta \leq \kappa - \sigma(1-z)$. Each lender i calculates this probability using the estimated distribution of θ conditional on his signal x_i.

We rely now on the well known result in the literature of global games that as $\sigma \to 0$, this probability equals z for $x_i = \kappa$.[13] That is, the threshold type believes that the proportion of lenders that roll over follows the uniform distribution on the unit interval. Focusing on the situation when signals become nearly precise enables to highlight strategic uncertainty rather than uncertainty about θ. The equilibrium cutoff can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ.

Let θ^* be such cutoff. Since $\Pi_U^R(\phi,\theta)$ is negative for $\theta < c$, θ^* must be Then θ^* is the unique solution to

$$\int_0^{1-\frac{\theta^*-c+pk-s}{1-s}} \left[-\frac{pk-s}{(1-\phi)(1-s)}\left(1-\frac{\ell}{1-pk}\right)\right] d\phi + \int_{1-\frac{\theta^*-c+pk-s}{1-s}}^{1} (d-1)\, d\phi = 0. \qquad (5)$$

This leads us to Proposition 1.

Proposition 1 *In the limit $\sigma \to 0$, the unique equilibrium in $t=1$ has unsecured lenders following monotone strategies with threshold θ^* given by*

$$\theta^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{(1-s)}\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - (pk-s)\,, \qquad (6)$$

where all unsecured lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[14]

Proposition 1 allows us to derive the relation between the probability of bankruptcy and

[13] See Morris and Shin (2003) for a comprehensive discussion of the global games literature.

[14] $W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

the bank's financing policy.

Corollary 1 *The threshold θ^* has the following properties:*

(i) $\frac{\partial \theta^*}{\partial d} < 0$, $\frac{\partial^2 \theta^*}{\partial d^2} > 0$, *and* $\frac{\partial^2 \theta^*}{\partial s^2} < 0$.

(ii) There exists a cutoff $\underline{d}(s) \geq 1$ *such that* $\frac{\partial^2 \theta^*}{\partial s \partial d} \leq 0$ *for* $d \leq \underline{d}(s)$ *and* $\frac{\partial^2 \theta^*}{\partial s \partial d} > 0$ *for* $d > \underline{d}(s)$, *where* $\underline{d}(s) = 1$ *if and only if* $pk \geq \frac{1+s}{2}$.

(iii) For $s \in [0, pk)$, θ^* *is strictly decreasing in* s *for* $d \leq \underline{d}(0)$, *and is first strictly increasing, then strictly decreasing in* s *for* $d > \underline{d}(0)$, *where* $\underline{d}(0) = 1$ *if and only if* $pk \geq \frac{1}{2}$.

(iv) There exists a cutoff $\underline{s}(d) \in \left[\frac{\ell-(1-pk)^2}{\ell}, pk\right)$ *such that* θ^* *is strictly decreasing in* pk *for* $s \leq \underline{s}(d)$ *and is strictly increasing in* pk *for* $s > \underline{s}(d)$.

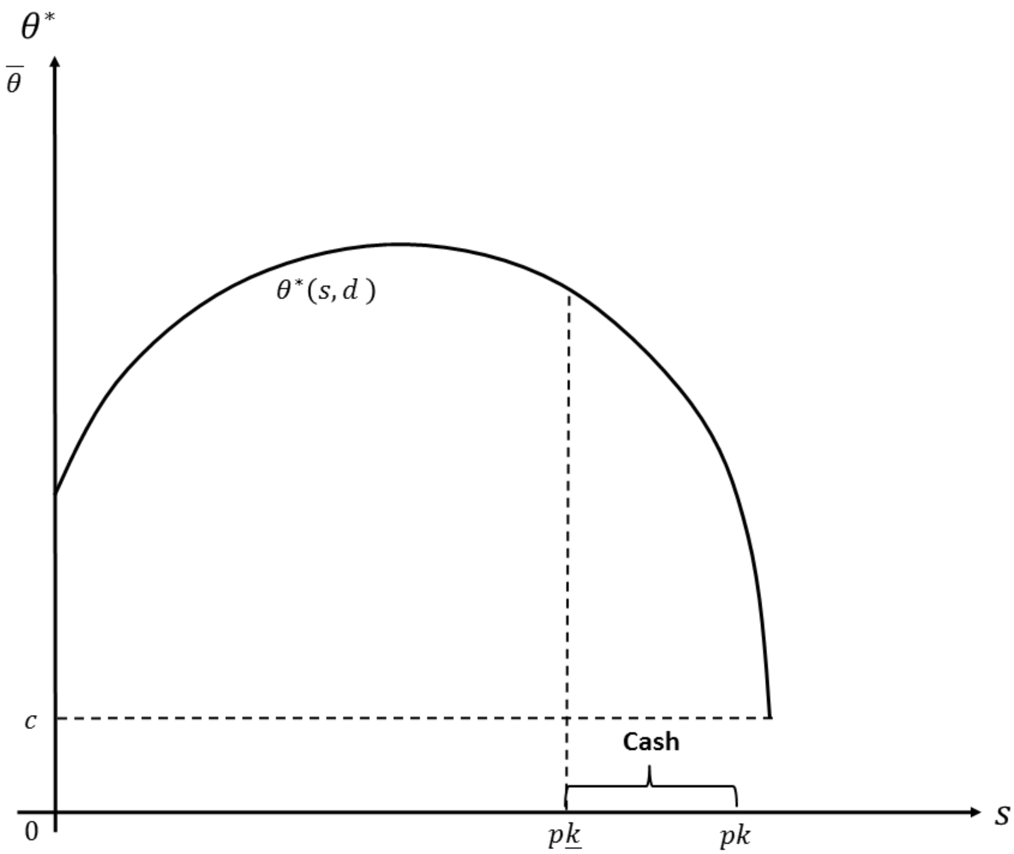
Note that W denotes the Lambert W function, defined in the Appendix.

The results of Corollary 1 can be more easily interpreted after rewriting (6):

$$\underbrace{\frac{\theta^* - c + (pk - s)}{(1-s)}}_{\text{unsecured debt recovery ratio}} = e^{-W\overbrace{\left(\frac{d-1}{\frac{pk-s}{(1-s)}\left(1-\frac{\ell}{1-pk}\right)}\right)}^{\text{rolling over net-benefit-to-cost ratio}}}. \tag{7}$$

Recall that a lower threshold is desirable, as it implies less frequent runs. From (7), the signal θ^* makes the threshold unsecured lender type just indifferent, balancing the recovery ratio in a run against the rollover premium $d-1$. The first term (the unsecured debt recovery ratio) captures a "probability" effect. That is, it measures the likelihood that the bank has enough resources to repay unsecured lenders in a run, which depends on the realized θ and the stock of unpledged collateral. The second term contains the rollover net-benefit-to cost ratio, which measures a "relative payoff" effect, the net benefits of rolling over when there is no bankruptcy relative to the losses incurred in a run.

Figure 2: Bankruptcy and Financing Policy



The corollary offers some insight on the comparative statics of the equilibrium run threshold θ^* and thus the frequency of runs.

The effect of the face value of unsecured debt d on θ^* is intuitive. Increasing it improves the payoff of rolling over for a given chance of default, and unambiguously reduces the probability of runs. [15] However, promising a large reward to unsecured lenders comes at the cost of the return to the bank in all solvent states. This observation is important to understand the bank's pricing choice.

A similar effect arises for a higher value of orderly liquidation ℓ, which affects only the relative payoff term. A higher recovery rate in default reduces the relative payoff to run.

- *The effect of repo credit on stability*

[15] It also leads to lower concavity of the threshold in s for d large enough, and thus a flattening of the curve.

The effect of secured debt s on θ^* is concave, but the sign of its derivative is ambiguous. More secured debt reduces the amount of collateral retained and available to withdrawers, decreasing the opportunity cost of rolling over. However, it also reduces the recovery ratio of unsecured debt, making the bank more likely to go bankrupt for any given fraction of funds withdrawn. The slope of the threshold curve in s at $s = 0$ depends on the rollover premium d. Since d needs to be above 1, the Corollary indicates that whenever asset liquidation risk is low (k is large, so that $pk \geq \frac{1+s}{2}$), a small increase in secured debt above $s = 0$ always leads to a higher risk of runs. In this case the threshold θ^* will be at first rising and then falling in the amount of secured debt. Figure 2 shows such a case when the probability of runs is first increasing in s, reflecting the dominance of the probability effect for lower levels of secured debt, then decreasing when the payoff effect becomes more prominent.[16] As d is set high (closer to the participation constraint of the bank), $\theta^*(s, d)$ shifts lower. Its intercept value is lower (and thus lower is the run risk) at $s = 0$ than at the maximum amount possible $s = p\underline{k}$. The threshold $\theta^*(s, d)$ may also be downward sloping from $s = 0$ when the rollover reward offered to unsecured debt d is very low and the asset liquidation value is risky (that is, k is sufficiently low).

4 Funding

This section examines the bank's initial funding choice (s, d). Because the project has positive NPV for any funding choice, we can focus on the stability tradeoff, excluding other effects of its financing structure.

The ex ante expected payoff of unsecured lenders as a function of its face value d is

$$V_U(s, d) = \lambda d + (1 - \lambda) \left[\left(\frac{\overline{\theta} - \theta^*(s, d)}{\overline{\theta}} \right) d + \frac{\theta^*(s, d)}{\overline{\theta}} \frac{pk - s + \ell}{1 - s} \right] \tag{8}$$

[16] Both $\frac{\theta^* - c + (pk - s)}{(1-s)}$ (the probability effect) and $e^{-W(s)}$ (the payoff effect) are decreasing and concave in s. However, as s goes to pk, the derivative of $\frac{\theta^* - c + (pk - s)}{(1-s)}$ converges to a finite number while that of $e^{-W(s)}$ goes to minus infinity. Therefore, for s large, the payoff effect dominates the probability effect.

The bank's expected payoff can be written as the return of the project of a solvent bank r net of financing costs and the expected deadweight loss $DW(s,d)$:

$$\begin{aligned} V_B(s,d) &= \lambda\left[r - d(1-s) - s\right] + (1-\lambda)\left(\frac{\overline{\theta} - \theta^*(s,d)}{\overline{\theta}}\right)\left[r - d(1-s) - s\right] \\ &= r - s - (1-s)V_U(s,d) - DW(s,d), \end{aligned} \tag{9}$$

where $DW(s,d)$ is the total payoff lost in the event of bankruptcy, that is

$$DW(s,d) = (1-\lambda)\frac{\theta^*(s,d)}{\overline{\theta}}(r - pk - \ell). \tag{10}$$

4.1 Socially Optimal Funding

As a benchmark, we characterize the optimal financing contract chosen by a social planner. The social planner chooses a pair (s,d) that maximizes the aggregate payoff subject to the participation constraint of the bank and unsecured lenders:

$$\max_{s,\,d}\; r - DW(s,d) \tag{11}$$

subject to

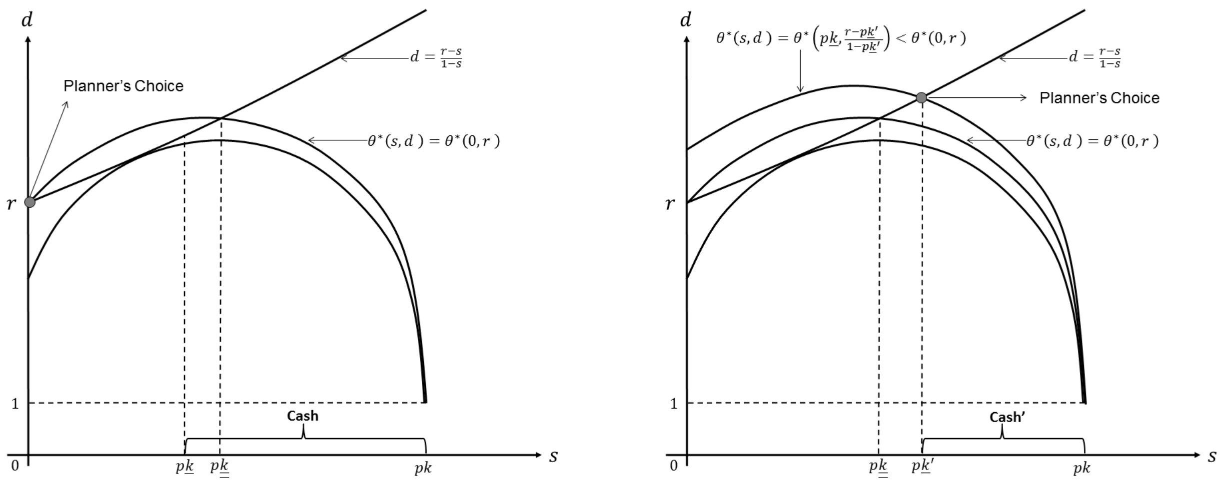
$$V_B(s,d) \geq 0,\; V_U(s,d) \geq \gamma,\; s \in [0, p\underline{k}]\,.$$

In other words, the optimal financing policy minimizes the deadweight loss subject to agents' participation constraints. Since $-DW(s,d)$ is increasing in d, the social planner would like to increase d as much as possible for any fraction of secured debt s.

Increasing d relaxes the lenders' participation constraint. However, the bank's participation constraint is binding at $d = \frac{r-s}{1-s}$. In addition, the bank's payoff is concave and decreasing at $d = \frac{r-s}{1-s}$, which implies that this is the maximum face value that can be chosen by the social planner.

To pin down the solution to the planner's problem, we make use of our assumption that r

Figure 3: Socially Optimal Funding



(a) No Secured Debt

(b) Maximum Secured Debt

is sufficiently large. Since the bank's participation constraint binds, it follows that the lenders' participation constraint does not bind. Moreover, since $d = \frac{r-s}{1-s} > r$ and θ^* is increasing in s for d sufficiently large, the deadweight loss is increasing in the fraction of secured debt. This is illustrated in Figure 3. Therefore, the social planner's optimal choice is to set $s = 0$, which results in a face value of $d = r$. Proposition 2 characterizes the socially optimal financing policy.

Proposition 2 *The socially optimal financing contract* (s^o, d^o) *requires the bank to issue either only unsecured debt* $(s^o, d^o) = (0, r)$, *or the maximum possible amount of secured debt* $(s^o, d^o) = \left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$. *There exists a cutoff* $\underline{\underline{k}} \in (0, k)$ *such that* $(s^o, d^o) = (0, r)$ *is socially optimal if and only if* $\underline{k} \leq \underline{\underline{k}}$.

It is worth noting that the result of Proposition 3 does not imply that secured debt could not add value if $\underline{k} \leq \underline{\underline{k}}$. If the project had positive NPV if and only if some secured debt is used $(r - \gamma < 0)$, then it could be financed only if some secured debt is used. Specifically, if

$$\lambda r + (1-\lambda)(pk + \ell) - (1 - p\underline{k})\gamma - p\underline{k} > 0,$$

then the project can be financed provided that the bank issues enough secured debt.

Finally, we show that a higher collateral price p (similarly for k) reduces the probability of runs in the socially optimal funding arrangement.

Corollary 2 *Under the socially optimal funding structure* $(s^o, d^o) = (0, r)$*, the probability of bankruptcy is decreasing in* pk*.*

4.2 Private Funding Choice

The bank's problem is to choose a funding structure (s, d) that maximize its payoff subject to the participation constraint:

$$\max_{s,\, d} V_B(s, d) \tag{12}$$

$$\text{subject to}$$

$$V_U(s, d) \geq \gamma,\ s \in [0, p\underline{k}].$$

In choosing its optimal funding structure, the bank faces a tradeoff between the cost of financing and the expected deadweight loss. The cost of financing is decreasing in the face value of unsecured debt d. As the unsecured lenders' required payoff is greater than for secured lenders, increasing the proportion of secured debt reduces the average cost of financing. However, lower d makes runs more likely, which increases the expected deadweight loss.

Proposition 3 *The probability of bankruptcy under the socially optimal funding structure is always lower than under the bank's financing policy:* $\theta^*(s^o, d^o) < \theta^*(s^*, d^*)$*.*

The result in Proposition 3 follows from the following. Suppose that $\theta^o(s^o, d^o) \geq \theta^*(s^*, d^*)$. Since we assume that r is sufficiently large, the bank's payoff under (12) is greater than zero (the bank can garantee a positive payoff by choosing $s = 0$). But then a contrat (s^*, d) with d marginally greater than d^* satifies both participation constraints in (11) and results in $\theta^o(s^o, d^o) \geq \theta^*(s^*, d^*) > \theta^*(s^*, d)$. But this contradicts (s^o, d^o) being a solution to (11).

In graphic terms, because the private choice of d is lower than for the social planner, it produces an upward shift of the $\theta^*(s^*, d)$ curve with a higher intercept at $s = 0$. The curve also exhibit increasing concavity. In conclusions, the private choice of s^* is either equal or higher than the social optimum value. Even when it is equal, it is combined with a lower value for d, as shareholders prefer to earn more in solvent states than reducing further the chance of runs. This leads to a higher threshold $\theta^*(s^*, d)$, and thus more frequent runs than the social optimum.

Proposition 4 characterizes the optimal private funding choice.

Proposition 4 *The bank's financing policy is characterized as follows:*

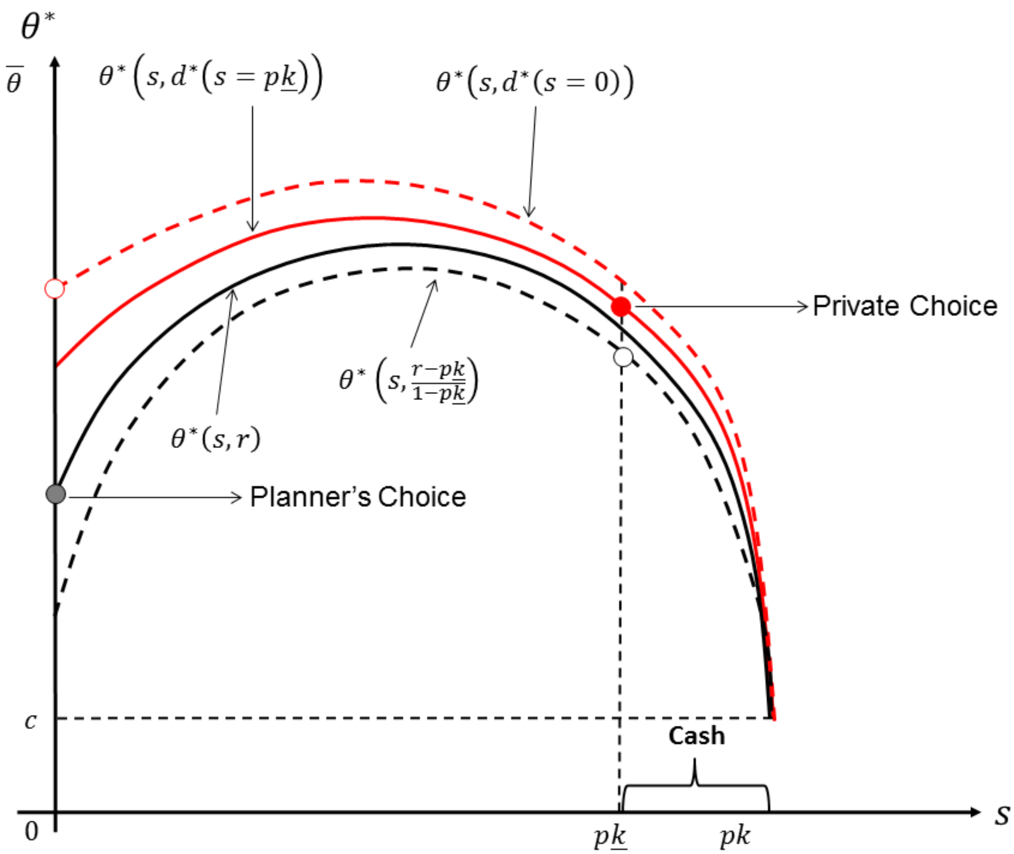
(i) There exists a cutoff $\lambda_1 \in [0, 1)$ such that, if $\lambda \geq \lambda_1$, the bank's financing policy (s^*, d^*) has the bank borrowing either by issuing only unsecured debt ($s^* = 0$) or by issuing the maximum possible amount of secured debt ($s^* = p\underline{k}$). The optimal face value of unsecured debt d^* is characterized by either $\mu^*[V_U(s^*, d^*) - \gamma] = 0$ or $-\frac{\partial DW(s^*,d^*)}{\partial d} = \frac{\partial V_U(s^*,d^*)}{\partial d}[1 - s^* - \mu^*]$, where μ^* is the Lagrange multiplier associated with unsecured lenders' participation constraint.

(ii) There exists a cutoff $\lambda_2 \in (0, 1)$ such that, if $\lambda > \lambda_2$, unsecured lenders' participation constraint binds, i.e., $V_U(s^*, d^*) - \gamma = 0$.

(iii) There exists a cutoff $\lambda_3 \in [0, 1)$ such that, if $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$, the bank borrows by issuing the maximum possible amount of secured debt ($s^* = p\underline{k}$). The optimal face value of unsecured debt d^* is characterized by unsecured lenders' breakeven condition $V_U(s^*, d^*) - \gamma = 0$.

The optimal funding structure is a corner solution because the bank's payoff is quasiconvex in s when λ is sufficiently high. The face value of unsecured debt balances the lower cost of funding against the higher expected deadweight loss from reducing d subject to the participation constraint. The condition for d^* is sufficient as we show that $V_B(s, d)$ is concave in d.

Figure 4: Private Funding Choice



The next result completes our comparative statics, showing that at the chosen funding structure the probability of runs is lower whenever the price of pledgeable assets p is higher.

Corollary 3 *If $\lambda > \max\{\lambda_1, \lambda_2, \lambda_3\}$, then the probability of runs under the bank's financing policy is decreasing in the price of the collateral p for $\ell \geq \frac{(1-pk)^2}{1-p\underline{k}}$.*

Thus when the liquidity of repo collateral in a fire sale is lower, the chance of runs in equilibrium increases.

5 Deposit Insurance

In this section, we extend our model to include the possibility that a third party, such as a regulator, provides deposit insurance (DI) to unsecuder lenders. Consistent with real practice, we model DI a minimum payment of $\pi \in [0, 1]$ for unsecured lenders in all states.

In the presence of DI, the payoff of unsecured lenders who do not roll over in $t = 1$ is

$$\pi_U^N(\phi,\theta) = \begin{cases} 1, & u(1-\phi) \le \mathbf{1}_{\{\theta \ge c\}}(\theta - c) + p(k - sh) \\ \frac{1-\phi^*}{1-\phi} + \left(1 - \frac{1-\phi^*}{1-\phi}\right)\max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & u(1-\phi) > \mathbf{1}_{\{\theta \ge c\}}(\theta - c) + p(k - sh) \end{cases},$$

while that of those who roll over is

$$\pi_U^R(\phi,\theta) = \begin{cases} \mathbf{1}_{\{\theta \ge c\}} d + \left(1 - \mathbf{1}_{\{\theta \ge c\}}\right)\max\{\rho,\pi\}, & u(1-\phi) \le \mathbf{1}_{\{\theta \ge c\}}(\theta - c) + p(k - sh) \\ \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}, & u(1-\phi) > \mathbf{1}_{\{\theta \ge c\}}(\theta - c) + p(k - sh) \end{cases}. \tag{13}$$

Therefore, unsecured lender's net payoff of rolling over relative to that of running is

$$\Pi_U^R(\phi,\theta) = \begin{cases} \mathbf{1}_{\{\theta \ge c\}} d + \left(1 - \mathbf{1}_{\{\theta \ge c\}}\right)\max\{\rho,\pi\} - 1, & u(1-\phi) \le \mathbf{1}_{\{\theta \ge c\}}(\theta - c) + p(k - sh) \\ -\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \max\left\{\frac{\ell}{\phi^* u}, \pi\right\}\right), & u(1-\phi) > \mathbf{1}_{\{\theta \ge c\}}(\theta - c) + p(k - sh) \end{cases}. \tag{14}$$

Similar to Diamond and Dybvig (1984), if the regulator provides full insurance, $\pi = 1$, then it is a dominant strategy to roll over regardless of the uncertain liquidation value of the assets θ and the fraction of unsecured lenders that roll over ϕ. That is, full insurance fully deters runs and achieves efficiency. If the amount of DI is such that $\pi \le \min\left\{\frac{\ell}{1-pk}, \rho\right\}$, the payoffs are the same as those without the presence of DI and all the previous results go through.

We are thus left with the following two cases: $\min\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi \le \max\left\{\frac{\ell}{1-pk}, \rho\right\}$ and $\max\left\{\frac{\ell}{1-pk}, \rho\right\} < \pi < 1$. As before, the equilibrium cutoff θ_{DI}^* can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ:

$$\int_0^{1-\frac{\theta_{DI}^*-c+pk-s}{1-s}} \left[-\frac{pk-s}{(1-\phi)(1-s)}\left(1 - \max\left\{\frac{\ell}{1-pk}, \pi\right\}\right)\right] d\phi + \int_{1-\frac{\theta_{DI}^*-c+pk-s}{1-s}}^1 (d-1)\, d\phi = 0. \tag{15}$$

This leads us to Proposition 5:

Proposition 5 *Suppose* $\min\left\{\frac{\ell}{1-pk},\rho\right\}<\pi<1$*. In the limit* $\sigma\to 0$*, the unique equilibrium in* $t=1$ *has unsecured lenders following monotone strategies with threshold* θ^* *given by*

$$\theta^*_{DI}=(1-s)\,e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\max\left\{\frac{\ell}{1-pk},\pi\right\}\right)}\right)}+c-(pk-s)\,, \tag{16}$$

where all unsecured lenders roll over if $\theta>\theta^*$ *and do not roll over if* $\theta<\theta^*$*.*

The results in Corollary 4 below follow from Proposition 5.

Corollary 4 *If* $\pi=1$*, then there is no run in the presence of DI. If* $\pi\leq\min\left\{\frac{\ell}{1-pk},\rho\right\}$*, the probability of bankruptcy with DI and without DI are the same:* $\theta^*_{DI}=\theta^*$*. If* $\min\left\{\frac{\ell}{1-pk},\rho\right\}<\pi<1$*, the probability of bankruptcy with DI is at least as low as that without DI:* $\theta^*_{DI}=\theta^*$ *for* $\pi\leq\frac{\ell}{1-pk}$ *and* $\theta^*_{DI}<\theta^*$ *for* $\pi>\frac{\ell}{1-pk}$*, in which case* θ^*_{DI} *is strictly decreasing in* π*.*

The results above show that for any given private funding choice, an increase in the level of DI from π to $\pi'>\pi$ reduces the probability of bankruptcy (provided that π is sufficiently large). The natural question that arises is whether the same result holds taking into the dependence of the bank's funding choice on the level of DI.

If the high state is sufficiently likely (λ large enough), then Proposition 4 tells us that the bank issues the maximum possible amount of secured debt, $s^*=p\underline{k}$, and the face value of unsecured debt is determined by unsecured lenders' participation constraint $V_U\left(p\underline{k},d^*;\pi\right)=\gamma$. An increase in π, *directly reduces* the probability of bankruptcy as $\theta^*\left(p\underline{k},d^*;\pi'\right)<\theta^*\left(p\underline{k},d^*;\pi\right)$, which increases unsecured lenders' expected payoff $V_U\left(p\underline{k},d^*;\pi'\right)>V_U\left(p\underline{k},d^*;\pi\right)=\gamma$. Thus, the bank's is able to reduce the face value of debt to $d^{*\prime}<d^*$ such that $V_U\left(p\underline{k},d^{*\prime};\pi'\right)=V_U\left(p\underline{k},d^*;\pi\right)=\gamma$, which *indirectly increases* the probability of bankruptcy: $\theta^*\left(p\underline{k},d^{*\prime};\pi\right)>\theta^*\left(p\underline{k},d^*;\pi\right)$. Corollary 5 below shows that the direct effect dominates when λ is sufficiently large.

Corollary 5 *Suppose* $\frac{\ell}{1-pk}<\pi<1$ *and* λ *is sufficiently large. Then under the private funding* *choice with DI, both the face value of unsecured debt* d^* *and the probability of bankruptcy* $\theta^*(p\underline{k}, d^*)$ *are strictly decreasing in* π.

The intuition behind the result in Corollary 4 is simple. If the high state is sufficiently likely, then unsecured lenders' ex ante payoff is highly sensitive to the face value of usecured debt. Therefore, small drops in the face value d rapidly offset the gains brought about by decreases in probability of bankruptcy. As a result, the bank is unable to significantly reduce the face value of unsecured debt following an increase in the level of DI.

6 Conclusion

We contrast the effect of demand for absolute safety that drives repo funding on credit and bank stability. Our focus is on the interaction of repo and unsecured lender behavior, in a context where repo is designed to have no risk in all states of nature.

We show that the lower cost of repo funding may enable marginal projects to get funded. However, secured financial credit has both a direct and an indirect risk shifting effect on other claimants, causing more frequent runs of other lenders even in states with no fundamental risk. The direct effect is due to increased risk concentration on each unit of unsecured debt, which exacerbates the incentive to run. The indirect effect arise from the risk of collateral illiquidity (Martin, Skeie and von Thadden (2013)), which forces more asset pledges to repo lenders and reinforces the first effect. Intermediaries will recognize the increased risk of repo borrowing, but will still seek to attract some as it is inexpensive. More frequent runs affect the liquidity of financial collateral. By forcing higher haircuts, this in turn reinforces the chance of a run.

In equilibrium, the reliance on secured funding is excessive relative to the social optimum from a stability perspective. Its cost may lower discount rates for marginal projects, and thus expand credit, just as lower interest rates do. This may have a procyclical effect on credit volume as well as on risk incentives.

In related work we seek to refine our result on inessential runs, adopting the definition of bank default developed in this paper. This seems to offer a simpler solution for bank run equilibria, as the payoff to a run is no longer a function of the signal. A natural question to study in that context is the effect of liquid reserves, an important regulatory question under the new Basel III framework for bank regulation. Finally, we plan to work on the effect of longer maturity assets, by applying the novel approach to dynamic global games.

A broad issue in future research is the effect of the volume of encumbered assets on stability. Public information is limited because of lack of disclosure. This reinforces market segmentation between traditional bank funding and its secured transactions, such as derivatives (Acharya and Bisin, 2013). Such imprecise information may create Knightian uncertainty and self fulfilling panics (Caballero Khrisnamurthy, 2008), even before private information becomes information sensitive (Gorton and Ordoñez, 2014).

Appendix

Proof of Proposition 1. Goldstein and Pauzner (2000) and Morris and Shin (2003) prove this result for a general class of global games that satisfies the following conditions: (i) for each $\theta \in \mathbb{R}$, there exists $\underline{\phi} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that $\Pi_U^R(\phi, \theta) > 0$ if $\phi > \underline{\phi}$ and $\Pi_U^R(\phi, \theta) < 0$ if $\phi < \underline{\phi}$; (ii) $\Pi_U^R(\phi, \theta)$ is nondecreasing in θ; (iii) there exists a unique θ^* that satisfies $\int_0^1 \Pi_U^R(\phi, \theta^*)\, d\phi = 0$; (iv) there exists $\overline{\theta}$, $\underline{\theta}$, and $\epsilon > 0$ such that $\Pi_U^R(\phi, \theta) \leq -\epsilon$ for all $\phi \in [0,1]$ and $\theta \leq \underline{\theta}$ and $\Pi_U^R(\phi, \theta) > \epsilon$ for all $\phi \in [0,1]$ and $\theta \geq \overline{\theta}$; (v) continuity of $\int_0^1 w(\phi)\, \Pi_U^R(\phi, \theta)\, d\phi$ with respect to signal x_i and density w; and (vi) the noise terms η_i are i.i.d. across players and uniformly distributed over some interval $[-\epsilon, \epsilon]$. Except for (iii), $\Pi_U^R(\phi, \theta)$ clearly satisfies (i), (ii), (iv) and (v). Condition (vi) is assumed in the model setup.

We now show that (iii) is also satisfied. Let $\Delta(\theta; s, d) \equiv \int_0^1 \Pi_U^R(\phi, \theta)\, d\phi$. Since $\Delta(\theta; s, d) < 0$ for all (s, d) and $\theta < c$, then if θ^* exists it must be that $\theta^* \geq c$. Moreover, since $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$, we must show that $\Delta(c; s, d) \leq 0$ for all (s, d) (otherwise for some (s, d) we have $\Delta(\theta; s, d) \geq \Delta(c; s, d) > 0$ for all $\theta \geq c$ and no θ^* would satisfy $\Delta(\theta^*; s, d) = 0$). It is straightforward to show that (a) $\Delta(c; s, d)$ is strictly increasing in d, (b) d is bounded by $\frac{r-s}{1-s}$ (in which case the bank's participation constraint binds), (c) $\Delta\left(c; s, \frac{r-s}{1-s}\right)$ is decreasing in s if $\frac{r-1}{1-\ell-pk} \leq \frac{1}{pk}$, and (d) that $\Delta(c; 0, r) \leq 0$ if $e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} \geq pk$. Therefore, for $\frac{r-1}{1-\ell-pk} \leq 1$ we have

$$e^{-\frac{r-1}{1-\frac{\ell}{1-pk}}} > 1 - \frac{r-1}{1-\frac{\ell}{1-pk}} = (1-pk)\left(1 - \frac{r-1}{1-\ell-pk}\right) + pk \geq pk,$$

which implies that for all (s, d) we have $\Delta(c; s, d) \leq \Delta\left(c; s, \frac{r-s}{1-s}\right) \leq \Delta(c; 0, r) \leq 0$. In addition, for all (s, d) we have $\Delta(\theta; s, d) > 0$ for θ sufficiently large such that there exists $\theta^* \geq c$ that satisfies $\Delta(\theta^*; s, d) = 0$. Finally, there is a unique such θ^* as $\Delta(\theta; s, d)$ is strictly increasing in θ for $\theta \geq c$.

For the derivation of the cutoff θ^*, note that after integrating the left-hand side of (5) we obtain

$$\frac{pk-s}{1-s}\left(1 - \frac{\ell}{1-pk}\right) \ln \frac{\theta^* - c + pk - s}{1-s} + \frac{\theta^* - c + pk - s}{1-s}(d-1) = 0. \tag{A.1}$$

After some algebra, (A.1) can be rewritten as

$$\frac{d-1}{\frac{pk-s}{1-s}\left(1 - \frac{\ell}{1-pk}\right)} = -\ln \frac{\theta^* - c + pk - s}{1-s} e^{-\ln \frac{\theta^* - c + pk - s}{1-s}}. \tag{A.2}$$

Let $W(\cdot)$ be the inverse function of $y = xe^x$ for $x \geq -1$ (known as the Lambert W function), that is, $x = W(y)$. Combined with (A.2) this implies

$$\theta^* = (1-s)\, e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)} + c - (pk - s)\,.$$

■

Proof of Corollary 1. Implicitly differentiating $y = W(y)\, e^{W(y)}$ results in

$$W'(y) = \frac{W}{(W+1)\, y} = \frac{e^{-W(y)}}{1+W(y)} > 0,$$
$$W''(y) = W'^2 \left(\frac{-2-W}{1+W}\right) < 0.$$

This allows us to compute

$$\frac{\partial\theta^*}{\partial s} = 1 - \frac{1-\frac{\ell}{1-pk}}{d-1} W \left(1 - \frac{1}{W+1}\frac{1-pk}{1-s}\right),$$
$$\frac{\partial^2\theta^*}{\partial s^2} = W'' \frac{(1-pk)^2}{(pk-s)^3} \frac{d-1}{\left(1-\frac{\ell}{1-pk}\right)} < 0,$$
$$\frac{\partial\theta^*}{\partial d} = \frac{-(1-s)\, e^{-W} W'}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)} < 0,$$
$$\frac{\partial^2\theta^*}{\partial d^2} = \frac{(1-s)\, e^{-W}\left(W'^2 - W''\right)}{\left[\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)\right]^2} > 0,$$
$$\frac{\partial\theta^*}{\partial k} = -1 + \frac{p}{d-1}\frac{W^2}{W+1}\left[1 - \frac{\ell(1-s)}{(1-pk)^2}\right].$$

Since $\lim_{s\to pk} \frac{\partial\theta^*(s,d)}{\partial s} = -\infty$ and $\theta^*(s,d)$ is strictly concave in s, it follows that $\theta^*(s,d)$ is strictly decreasing in s if $\frac{\partial\theta^*(0,d)}{\partial s} \leq 0$, and first strictly increasing, then strictly decreasing in s if $\frac{\partial\theta^*(0,d)}{\partial s} > 0$. Note that

$$\begin{aligned}
\lim_{d\to 1} \frac{\partial\theta^*(s,d)}{\partial s} &= 1 - \left(1 - \frac{\ell}{1-pk}\right)\left(1 - \frac{1-pk}{1-s}\right) \lim_{d\to 1} \frac{W}{d-1} \\
&= 1 - \left(1 - \frac{\ell}{1-pk}\right)\left(1 - \frac{1-pk}{1-s}\right) \lim_{d\to 1} \frac{W' \frac{1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}}{1} \\
&= 1 - \left(\frac{pk-s}{pk-s}\right) = 0.
\end{aligned}$$

Because the above limit is true for any s, we have that $\frac{\partial \theta^*(0,d)}{\partial s}$ goes to zero as $d \to 1$. We also have that

$$\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} = \frac{1 - \frac{\ell}{1-pk}}{(d-1)^2} W^2 \left[\frac{1}{W+1} - \frac{1-pk}{1-s} \frac{W+2}{(W+1)^3} \right], \tag{A.3}$$

whose sign is determined by the the term inside the brackets.

If $pk = \frac{1+s}{2}$, then $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} = 0$ for $d = 1$ and $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} > 0$ for $d > 1$. If $pk > \frac{1+s}{2}$, $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d}$ is positive for all $d \geq 1$. Thus, if $pk \geq \frac{1+s}{2}$, it follows that $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} > 0$ for all $d > \underline{d} = 1$. This, in turn, implies that $\frac{\partial \theta^*(0,d)}{\partial s} > 0$ whenever $pk \geq \frac{1}{2}$.

If $pk < \frac{1+s}{2}$, then $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} = 0$ when $W(s,d) = \frac{-(1+pk-2s)+\sqrt[2]{(pk)^2 - pk(6-4s)+5-4s}}{2}$. The term inside brackets in (A.3) is strictly increasing in $W(s,d)$, which in turn is strictly increasing in d. Since $W(s,d)$ grows without bounds as d increases and $W(s,1) = 0$, it follows that there exists $\underline{d}(s) > 1$ such that $W(s, \underline{d}(s)) = \frac{-(1+pk-2s)+\sqrt[2]{(pk)^2 - pk(6-4s)+5-4s}}{2}$. Therefore, $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} > 0$ for $d > \underline{d}(s)$ and $\frac{\partial^2 \theta^*(s,d)}{\partial s \partial d} \leq 0$ for $d \leq \underline{d}(s)$. This implies that $\frac{\partial \theta^*(0,d)}{\partial s} > 0$ for $d > \underline{d}(0)$ and $\frac{\partial \theta^*(0,d)}{\partial s} \leq 0$ for $d \leq \underline{d}(0)$.

Finally, we have that

$$\begin{aligned} \frac{\partial \theta^*(s,r)}{\partial (pk)} &= -(1-s) W' e^{-W\left(\frac{d-1}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\right)} \frac{(d-1)\left[\ell(1-s) - (1-pk)^2\right]}{\left[(pk-s)(1-pk-\ell)\right]^2} - 1 \\ &= -\frac{1}{(d-1)(1-s)} \frac{W^2}{W+1} \left[\frac{\ell(1-s) - (1-pk)^2}{(1-pk)^2} \right] - 1, \end{aligned}$$

which is negatrive if $\frac{\ell-(1-pk)^2}{\ell} \geq s$. If $\frac{\ell-(1-pk)^2}{\ell} < s$, then $\frac{\partial \theta^*(s,r)}{\partial (pk)}$ is positive for s close enough to pk, and negative for s close enough to $\frac{\ell-(1-pk)^2}{\ell}$. Therefore, there exists $s^* \in \left(\frac{\ell-(1-pk)^2}{\ell}, pk\right)$ such that $\frac{\partial \theta^*(s,r)}{\partial (pk)} > 0$ for $s > s^*$ and $\frac{\partial \theta^*(s,r)}{\partial (pk)} < 0$ for $s < s^*$. ■

Proof of Proposition 2. We first show that the bank's participation constraint must bind at a solution (s^o, d^o). Suppose not, that is, $V_B(s^o, d^o) > 0$. The aggregate payoff $r - DW(s,d)$ is increasing in d, while the bank's payoff is either one of the following: (1) decreasing, or (2) increasing and then decreasing. The latter follows from the fact that

$$\overline{\theta} \frac{\partial V_B(s,d)}{\partial d} = -(1-s)\left[\overline{\theta} - (1-\lambda)\theta^*\right] - (1-\lambda)(r - d(1-s) - s)\frac{\partial \theta^*}{\partial d} - \left(\overline{\theta} - 1\right)(1-s)$$

is negative for $d = \frac{r-s}{1-s}$. If $\frac{\partial V_B(s,d)}{\partial d} \leq 0$ for all d, then $V_B(s,d)$ is monotone decreasing. If $\frac{\partial V_B(s,d)}{\partial d} > 0$ for some d', then there exists d'' such that $\frac{\partial V_B(s,d)}{\partial d} = 0$. Since $V_B(s,d)$ is strictly concave in d, $\frac{\partial V_B(s,d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(s,d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = \frac{r-s}{1-s}$. Therefore, the social planner can increase d^o

until $V_B(s^o, d^o)$ binds: this increases the aggregate payoff while still satifying the constraints, which contradicts (s^o, d^o) being a solution.

The result that the bank's participation constraint must be binding along with our assumption that r is sufficiently large assures that the unsecured lenders' participation constraint does not bind. The social planner's problem is therefore

$$\min_{s,\, d} \theta^*(s, d)$$

subject to

$$r - s - d(1 - s) = 0,\ s \geq 0,\ s \leq p\underline{k}.$$

The first order necessary conditions (FOC) are

$$\begin{aligned}
\frac{\partial \theta^*}{\partial s} - \lambda(d-1) - \mu_1 + \mu_2 &= 0, \\
\frac{\partial \theta^*}{\partial d} + \lambda(1-s) &= 0, \\
r - s - d(1-s) &= 0, \\
\mu_1 s &= 0, \\
\mu_2 \left[p\underline{k} - s\right] &= 0, \\
\mu_1, \mu_2 &\geq 0.
\end{aligned}$$

We now show that an interior optimum does not exist. Suppose not, i.e., there exists an interior optimum (s^o, d^o). In this case, $\mu_1 = \mu_2 = 0$ and it must be that $-\frac{\frac{\partial \theta^*(s^o,d^o)}{\partial s}}{\frac{\partial \theta^*(s^o,d^o)}{\partial d}} = \frac{d-1}{1-s}$, which yields

$$\frac{W}{W+1} \frac{1-s}{pk-s} = e^W - 1.$$

This implies that a feasible decrease in s^* of Δs such that $\frac{\Delta d}{\Delta s} = \frac{d-1}{1-s}$ decreases $\theta^*(s^o, d^o)$:

$$\begin{aligned}
&\frac{\partial\theta^*(s^o,d^o)}{\partial s}\Delta s + \frac{\partial\theta^*(s^o,d^o)}{\partial s}\Delta d \\
&= e^{-W}\left[e^W - 1 - \frac{1-pk}{pk-s}\frac{W}{W+1}\right]\Delta s - \frac{(1-s)e^{-W}W'}{\frac{pk-s}{1-s}\left(1-\frac{\ell}{1-pk}\right)}\frac{d-1}{1-s}\Delta s \\
&= e^{-W}\left[e^W - 1 - \frac{1-s}{pk-s}\frac{W}{W+1} + \frac{W}{W+1}\right]\Delta s - e^{-W}\frac{W}{W+1}\Delta s \\
&= e^{-W}\left[e^W - 1 - \frac{1-s}{pk-s}\frac{W}{W+1}\right]\Delta s - e^{-W}\frac{W}{W+1}\Delta s + \frac{W}{W+1}\Delta s \\
&= \frac{W}{W+1}\left(1-e^{-W}\right)\Delta s < 0,
\end{aligned}$$

which contradicts (s^o, d^o) being a solution.

The previous result shows that $\theta^*(0,r) < \theta^*\left(s', \frac{r-s'}{1-s'}\right)$ for any given interior candidate $s' \in (0, p\underline{k})$, which implies that that an optimum has either $s = 0$ or $s = p\underline{k}$. From the proof of Proposition 1, we know that $\theta^*(0,r) > c$. Since $\lim_{\underline{k}\to k}\theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right) = c$, there exists a $\underline{\underline{k}} \in \left(\frac{s'}{p}, k\right)$ such that $\theta^*(0,r) \leq \theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $\underline{k} \leq \underline{\underline{k}}$ and $\theta^*(0,r) > \theta^*\left(p\underline{k}, \frac{r-p\underline{k}}{1-p\underline{k}}\right)$ for $\underline{k} > \underline{\underline{k}}$. ■

Proof of Corollary 2. Follows from Corollary 1 (iv). ■

Proof of Proposition 3. See discussion in text. ■

Proof of Proposition 4. We first show (i). We use the Principle of Iterated Suprema to break the bank's problem into two stages, that is, we solve $\max_{d\in D}\left[\max_{s\in S} V_B(s,d)\right]$, where $S = [0, p\underline{k}]$ and $D = \{d : V_U(s^*(d), d) \geq \gamma\}$.

The next step is to show that $V_B(s,d)$ is quasiconvex in s if λ sufficiently high, which implies that there is not interior solution to problem $\max_{s\in S} V_B(s,d)$. This is done by showing that $\frac{\partial V_B(s,d)}{\partial s} = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s} - \frac{\partial DW(s,d)}{\partial s}$ is a single crossing function, which is equivalent to $V_U - 1 - (1-s)\frac{\partial V_U}{\partial s}$ and $-\frac{\partial DW}{\partial s}$ satisfying signed-ratio monotonicity (Qua and Strulovici, 2012). Two functions $f(s)$ and $g(s)$ satisfy signed-ratio monotonicity if whenever $f(s) > 0$ and $g(s) < 0$, $-\frac{g(s)}{f(s)}$ is decreasing and whenever $f(s) < 0$ and $g(s) > 0$, $-\frac{f(s)}{g(s)}$ is decreasing. We take $g(s) = -\frac{\partial DW(s,d)}{\partial s}$ and $f(s) = V_U(s,d) - 1 - (1-s)\frac{\partial V_U(s,d)}{\partial s}$. Since $f(s)$ is always positive, we only need to consider the case in which $g(s) < 0$. In this case, it must be that $-\frac{g(s)'f(s)-g(s)f(s)'}{f(s)^2}<0$. We have

$$\overline{\theta}\left[-g\left(s\right)'f\left(s\right)+g\left(s\right)f\left(s\right)'\right]=$$

$$\left(1-\lambda\right)\theta^{*\prime\prime}\left(r-pk-\ell\right)\left\{\left(d-1\right)\left[\overline{\theta}-\left(1-\lambda\right)\theta^{*}\right]+\left(1-\lambda\right)\theta^{*\prime}\left[d\left(1-s\right)-\left(pk-s+\ell\right)\right]\right\}$$

$$-\left(1-\lambda\right)\theta^{*\prime}\left(r-pk-\ell\right)\left\{\left(1-\lambda\right)\theta^{*\prime\prime}\left[d\left(1-s\right)-\left(pk-s+\ell\right)\right]-2\left(1-\lambda\right)\theta^{*\prime}\left(d-1\right)\right\}$$

$$=\left(1-\lambda\right)\left(r-pk-\ell\right)\left(d-1\right)\left[\theta^{*\prime\prime}\left(\overline{\theta}-\left(1-\lambda\right)\theta^{*}\right)+2\left(1-\lambda\right)\theta^{*\prime}\theta^{*\prime}\right].$$

The sign of the above expression is determined by the term inside brackets, which is strictly decreasing in λ. For any given s, it is negative if λ is sufficiently close to 1, so that we are left with two possibilities: either it is nonpositive for all λ, or there exists $\lambda\left(s\right)\in\left(0,1\right)$ such that it is nonpositive if $\lambda\geq\lambda\left(s\right)$ and positive if otherwise. If the former is true for all s, then $V_B\left(s,d\right)$ is quasiconvex if $\lambda\geq\lambda_1=0$. Suppose there exists s such that the latter is true and denote X the set of all such s. Then $V_B\left(s,d\right)$ is quasiconvex if $\lambda\geq\lambda_1=\sup\left\{\lambda\left(s\right):s\in X\right\}$. Combining both cases we have that there exists a cutff $\lambda_1\in[0,1)$ such that $V_B\left(s,d\right)$ is quasiconvex if $\lambda\geq\lambda_1$, which in turn implies that we must have a corner solution: $s^*\in\{0,p\underline{k}\}$.

We now turn to the problem $\max_{d\in D}V_B\left(s^*,d\right)$. The first order necessary conditions (FOC) are

$$-\frac{\partial DW\left(s^*,d\right)}{\partial d}=\frac{\partial V_U\left(s^*,d\right)}{\partial d}\left[1-s^*-\mu\right],\tag{A.4}$$

$$\mu\left[V_U\left(s^*,d\right)-\gamma\right]=0,\tag{A.5}$$

$$V_U\left(s^*,d\right)\geq\gamma,\tag{A.6}$$

$$\mu\geq0.\tag{A.7}$$

To conclude the proof of (i) we need to show that any d satisfying the FOC is a global maximizer. This follows from

$$\overline{\theta}\frac{\partial V_B^2\left(s,d\right)}{\partial d^2}=2\left(1-s\right)\left(1-\lambda\right)\frac{\partial\theta^*}{\partial d}-\left(1-\lambda\right)\left(r-d\left(1-s\right)-s\right)\frac{\partial^2\theta^*}{\partial d^2}<0,$$

which implies that $V_B\left(s^*,d\right)$ is (strictly) concave in d.

We now show (ii). Note that

$$\frac{\partial DW\left(s^{*},d\right)}{\partial d} = \frac{\left(1-\lambda\right)}{\overline{\theta}}\frac{\partial\theta^{*}\left(s^{*},d\right)}{\partial d}\left(r-pk-\ell\right), \tag{A.8}$$

$$\frac{\partial V_{U}\left(s^{*},d\right)}{\partial d} = 1-\frac{\left(1-\lambda\right)}{\overline{\theta}}\left[\theta^{*}\left(s^{*},d\right)+\frac{\partial\theta^{*}\left(s^{*},d\right)}{\partial d}\left(d-\frac{pk-s^{*}+\ell}{1-s^{*}}\right)\right]. \tag{A.9}$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.4) approach 0 ((A.8) approximates 0) and $1 - s^*$ ((A.9) converges to 1), respectively. Since s is bounded above by $p\underline{k} < 1$, the right-hand side of (A.4) is bounded away from 0. Therefore, there are only two possibilities: either the left-hand side of (A.4) (strictly decreasing in λ) is smaller than the right-hand side (strictly increasing in λ) for all λ, or there exists $\lambda\left(s^{*},d\right) \in (0,1)$ such that the left-hand side of (A.4) is smaller than the right-hand side if $\lambda > \lambda\left(s^{*},d\right)$ and at least as great if otherwise. If the former is true for all d, then (A.4) can only be satified if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_2 = \sup\left\{\lambda\left(s^{*},d\right) : d \in Y\right\}$, then (A.4) can only be satified if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_2 \in (0,1)$ such $\mu > 0$ if $\lambda > \lambda_2$, which in turn implies that $V_U\left(s^{*},d\right) - \gamma = 0$ (from (A.5)).

We finally show (iii). Suppose $\lambda > \max\left\{\lambda_1, \lambda_2\right\}$. In this case we know from (i) that there are two possible candidates for the bank's choice of secured debt: either $s^* = p\underline{k}$ or $s^* = 0$. We also know that unsecured lenders' participation constraint binds. Therefore, the bank's implied payoffs are given by

$$V_{B}\left(p\underline{k},d^{*}\left(p\underline{k}\right)\right) = r-p\underline{k}-\left(1-p\underline{k}\right)\gamma-DW\left(p\underline{k},d^{*}\left(p\underline{k}\right)\right), \tag{A.10}$$

$$V_{B}\left(0,d^{*}\left(0\right)\right) = r-\gamma-DW\left(0,d^{*}\left(0\right)\right). \tag{A.11}$$

The difference is given by

$$V_{B}\left(p\underline{k},d^{*}\left(p\underline{k}\right)\right)-V_{B}\left(0,d^{*}\left(0\right)\right) = p\underline{k}\left(\gamma-1\right)-\left[DW\left(p\underline{k},d^{*}\left(p\underline{k}\right)\right)-DW\left(0,d^{*}\left(0\right)\right)\right], \tag{A.12}$$

which is positive for λ sufficiently close to 1. Thus, there are two cases to consider: either (A.12) (strictly increasing in λ) is nonnegative for all $\lambda \geq \lambda_3 = 0$, or there exists $\lambda_3 \in (0,1)$ such that (A.12) is nonnegative if $\lambda > \lambda_3$, and negative if $\lambda < \lambda_3$. Therefore, we conclude that if $\lambda > \max\left\{\lambda_1, \lambda_2, \lambda_3\right\}$, then the bank's financing policy has the bank borrowing by issuing only secured debt ($s^* = p\underline{k}$) and d^* is such that $V_U\left(s^{*},d\right) - \gamma = 0$. ■

Proof of Corollary 3. Differentiating the cutoff θ^* relative to p when $s = p\underline{k}$ and

$d = d^*$ gives

$$\frac{\partial \theta^*\left(p\underline{k}, d^*\right)}{\partial p} = -\underline{k}e^{-W} - \left(k - \underline{k}\right) -$$
$$W'\left(1 - p\underline{k}\right)e^{-W}\left(\frac{-\left(d^* - 1\right)}{\left[\frac{p(k-\underline{k})}{1-p\underline{k}}\left(1 - \frac{\ell}{1-pk}\right)\right]^2}\right)\left[\frac{-pk\ell\left(1 - p\underline{k}\right) + \left(1 - pk - \ell\right)\left(1 - pk\right)}{\left(k - \underline{k}\right)^{-1}\left(1 - kp\right)^2\left(1 - p\underline{k}\right)^2}\right],$$

which is negative if the term inside the brackets is negative, that is, if $\ell \geq \frac{(1-pk)^2}{1-p\underline{k}}$. ■

Proof of Proposition 4. Proof is analogous to that of Proposition 1. ■

Proof of Corollary 4. See discussion in text. ■

Proof of Corollary 5. For λ large enough, unsecured lenders' participation constraint binds: $V_U\left(p\underline{k}, d^*; \pi\right) = \gamma$. Thus, the overall change in θ^* caused by an increase in π can be found by differentiating both sides with respect to π, whihc yields:

$$\frac{\partial \theta^*\left(d, \pi\right)}{\partial d}d' + \frac{\partial \theta^*\left(d, \pi\right)}{\partial \pi} = \frac{\frac{(1-\lambda)\theta^*(d,\pi)}{\overline{\theta}}\left[d' - \frac{1-pk}{1-p\underline{k}}\right] - d'}{\frac{(1-\lambda)}{\overline{\theta}}\left[-\left(d - 1\right) - \frac{1-pk}{1-p\underline{k}}\left(1 - \pi\right)\right]}.$$

Since the denumerator on the right-hand side is negative and $d' < 0$, the overall expression is negative for λ large enough. ■

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