The macroeconomic consequences of carry trade gone wrong and borrower protection
Jakucionyte, E.

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Emerging Europe enjoyed massive capital inflows prior to the Great Financial Crisis, but these came to a sudden stop in 2008. This rapid reversal of fortunes created a major macroeconomic slowdown across the region. Moreover, accumulated debt and especially foreign currency debt became the undesirable legacy of these credit inflows. How countries in Emerging Europe coped with foreign currency debt during the ensuing recession and whether they could have done better is the main topic of this thesis. In contrast to the traditional approach, the thesis pays attention to household borrowing in addition to corporate borrowing, because households often borrowed in foreign currency even more actively than businesses. Finally, the thesis focuses on the macroeconomic implications of borrower protection for households, regardless of currency denomination. It shows that increasing borrower protection, namely decreasing lender recourse, brings ex-ante macroeconomic and welfare benefits.

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The Macroeconomic Consequences of Carry Trade Gone Wrong and Borrower Protection

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ter verkrijging van de graad van doctor aan de Universiteit van Amsterdam
op gezag van de Rector Magnificus prof. dr. ir. K.I.J. Maex
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They say ev’rything can be replaced,
Yet ev’ry distance is not near.
So I remember ev’ry face
Of ev’ry man who put me here.

Bob Dylan, “I shall be released”, 1967

Doing a PhD can be both rewarding and excruciating. Now, the journey approaching the end, I look back and see so many people who made the five years of PhD not only inspiring but also funkier and happier. I would like to thank to them, my teachers and colleagues.

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Co-author List

• Chapter 2, entitled “Debt Overhang and the Macroeconomics of Carry Trade” is a joint work with Prof. dr. Sweder van Wijnbergen, University of Amsterdam. Egle has developed the idea and the structure of the paper together with Sweder. Egle has developed the model and wrote the code for numerical simulations. She wrote up the results and was also involved in rewriting and editing the paper.

• Chapter 3, entitled “The Macroeconomics of Carry Trade Gone Wrong: Corporate and Consumer Losses in Emerging Europe” is a joint work with Prof. dr. Sweder van Wijnbergen, University of Amsterdam. Egle has developed the idea and the structure of the paper together with Sweder. Egle has developed the model and wrote the code for numerical simulations. She wrote up the results and was also involved in rewriting and editing the paper.
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Chapter 1

Introduction

Emerging Europe\(^1\) enjoyed massive capital inflows prior to the Great Financial Crisis, but these came to a sudden stop in 2008. This rapid reversal of fortunes created a major macroeconomic slowdown across the region. Moreover, accumulated debt and especially foreign currency debt became the undesirable legacy of these credit inflows. How countries in Emerging Europe coped with foreign currency debt during the ensuing recession and whether they could have done better is the main topic of this thesis. In contrast to the traditional approach, the thesis pays attention to household borrowing in addition to corporate borrowing, because households often borrowed in foreign currency even more actively than businesses. Finally, the thesis focuses on household credit, regardless of currency denomination. I provide additional evidence that increasing borrower protection, namely decreasing lender recourse, brings ex-ante macroeconomic and welfare benefits.

Borrowing in foreign currency in Emerging Europe became widespread due to a high presence of foreign banks since 1990s. Local subsidiaries of foreign banks heavily supplied credit to consumers and businesses in Emerging Europe, which made private credit the largest component of capital inflows in 2002-2007 (Bakker and Gulde (2010)). These subsidiaries offered foreign currency mortgages and loans at substantially lower interest rates compared to rates on credit in domestic currency. Households and businesses often underestimated exchange rate risk by relying

\(^1\)The economies of Central and Eastern Europe that transitioned from centrally planned economies to market economies in 1990s.
on the history of relatively stable local exchange rates and expectations of joining the Eurozone in the near future. As a result, consumers and businesses borrowed in euros and Swiss francs to take advantage of lower interest rates (Luca and Petrova (2008); Basso et al. (2011)) although that implied bearing exchange rate risk. In other words, they engaged in carry trade. The magnitude of this carry trade in Emerging Europe is well-illustrated by the fact that in the majority of countries foreign currency debt doubled over 2002-2007 and amounted to at least one half of total credit in 2008 (IMF (2013), p. 13).

When advanced economies went into recession, spillovers of the crisis reached Emerging Europe, making carry trade go wrong. At the end of 2008, foreign investors adjusted their expectations regarding the value of local currencies downward. This led to a round of depreciations of local currencies against the euro and the Swiss franc. Several countries with high stock of foreign currency debt experienced especially large depreciations of local currencies vis-à-vis their dominant foreign currency of borrowing. For instance, the Hungarian forint lost almost 30 percent of its value against the euro\textsuperscript{2}. The value of the Polish zloty and the Romanian leu dropped by a similar amount.

After the depreciation, high foreign currency debt in the household sector and the corporate sector became a macroeconomic problem which raised a question whether borrowers should bear the losses. Relieving borrowers of losses became an especially important topic in Hungary. The Hungarian government came up with a mortgage repayment scheme that shifted depreciation losses from indebted households to banks. The macroeconomic consequences of saving borrowers at the expense of banks, however, are not straightforward. Shifting losses to banks could have not only threatened financial stability but also increased the risk of parent banks reallocating some funds from their Hungarian subsidiaries. On November 4, 2011 ECB published an opinion warning about these potential risks (ECB (2011)). Despite the warnings, the Hungarian government went forward with the scheme. In 2014, after a second round of zloty depreciation against the Swiss franc, the president of Poland was considering a similar arrangement but later shelved the proposal (Waldoch

\textsuperscript{2} By March 2009, compared to September 2008.
A similar proposal was enacted in Croatia. In Romania an early mortgage repayment scheme was rejected as unconstitutional and no longer applies (Waldoch (2017b)).

The reversal of capital flows in Emerging Europe stood out from previous episodes of sudden stops. One important difference is that households did not engage in carry trade before the crisis in Latin America in the early 1980s or in Asia in the mid-1990s. However, in 2009 households in Emerging Europe accumulated large stocks of foreign currency debt. Household borrowing in Swiss francs was especially common in Poland and Hungary. I argue that excessive household debt deepens a recession through different channels than corporate debt and this difference needs to be acknowledged in the analysis. Household debt first affects aggregate demand for both consumption and housing, while corporate debt distorts firms’ investment decisions and in turn the supply of consumption goods directly. Also, aggregate demand losses triggered by high household debt are not exceptionally absorbed by domestic producers in an open economy but their effect on domestic production depends on the import structure of different goods. Last, financial frictions that make household default inefficient are potentially different from prevalent financial frictions in firms. For instance, consumption in Hungary rebounded in a few years, but investment did not, suggesting that Hungarian firms suffered from debt overhang but Hungarian households did not. Can it then still be that protecting consumers from foreign currency losses is more beneficial than protecting corporate borrowers? The answer to this question is particularly interesting given that none of the governments in the region considered saving firms but only households.

The discussion of who should bear unexpected exchange rate losses has not reached a consensus yet. This thesis joins the discussion by evaluating the macroeconomic consequences of shifting foreign currency losses from borrowers to banks. The inherent heterogeneity in the region is an argument in favor of a country-case analysis rather than building a generalized framework for Emerging Europe. I begin by focusing on corporate carry trades gone wrong for which I draw motivation from the Hungary case. Chapter 3 elaborates on consumer losses from foreign currency mortgages in Hungary.
In Chapter 4 I focus on the macroeconomic implications of reducing lender recourse. Recourse laws define lender’s losses from residential mortgage default, regardless of currency denomination. Borrower-friendly recourse laws reduce the risk of household debt overhang and provide consumption insurance for mortgage borrowers. I also show that reducing lender recourse can have positive macroeconomic effects. In the analysis I do not focus on Emerging Europe explicitly, but my motivation partially comes from the experience of the region. Emerging Europe had achieved a sufficiently high level of economic development and it has experienced rapid household credit growth ever since. Household credit growth is not likely to subside in the future either which signals the need of higher development of relevant institutions.

The rest of this introductory chapter is structured as a thesis outline. In the thesis outline I introduce, motivate and explain main insights of each of the three essays.

1.1 Thesis Outline

Debt Overhang and the Macroeconomics of Carry Trade

The depreciation of the Hungarian forint in 2009 left Hungarian borrowers with a skyrocketing value of foreign currency debt. The resulting losses worsened debt overhang in debt-ridden firms and eroded bank capital. This situation highlights the tight relationship between currency mismatch for borrowers and currency mismatch for banks. Currency mismatch that results from foreign currency liabilities exceeding foreign currency assets makes borrowers vulnerable to unexpected currency depreciation. In the same way, currency mismatch in banks makes banks vulnerable to unexpected fluctuations in the currency value. Banks can decrease currency mismatch on their balance sheets by lending in foreign currency, but this increases exchange rate risk for their borrowers and in turn implies higher credit risk to those banks. So, although Hungarian banks had partially isolated their balance sheets from exchange rate risk by extending foreign currency denominated loans, the ensuing debt overhang in borrowing firms exposed the banks to elevated credit risk. Chapter 2, based on Jakucionyte and van Wijnbergen (2017a), analyzes currency mismatch losses in different
sectors in the economy, and the macroeconomic consequences of reallocating losses from the corporate sector to the banking sector ex post.

We develop a small open economy New Keynesian DSGE model that accounts for the implications of domestic currency depreciation for corporate debt overhang and incorporates an active banking sector. The model is calibrated to the Hungarian economy. It shows that, in periods of unanticipated depreciation, allocating currency mismatch losses to the banking sector generates milder recession than if currency mismatch is placed at credit constrained firms. The government can intervene to reduce aggregate losses even further by recapitalizing banks and thus mitigating the effects of currency mismatch losses on credit supply.

The Macroeconomics of Carry Trade Gone Wrong: Corporate and Consumer Losses in Emerging Europe

Chapter 3, based on Jakucionyte and van Wijnbergen (2017b), takes the analysis of carry trades gone wrong one step further by introducing household currency mismatch losses. Therefore, transmission channels through household net worth, consumption and labor supply become relevant in explaining macroeconomic outcomes. To that end, we construct a New Keynesian DSGE model with debt overhang for corporate borrowers, monitoring costs for household mortgage debt and leverage constraints for banks.

The choice of financial frictions is motivated by the Hungarian recession. Private consumption rebounded after a few years, but investment did not which would suggest debt overhang for firms but not for households. However, the model is rich in details, so we can test the choice of some frictions by estimating it on data. We estimate the model on Hungarian data and find strong evidence in favor of corporate debt overhang rather than monitoring costs for corporate loans in the spirit of Bernanke et al. (1999). Introducing household debt improves model fit significantly.

Using model simulation, we show that making corporate borrowers bear currency risk results in worse macroeconomic outcomes than shifting currency mismatch losses to banks. Foreign currency mortgages to households, however, generate less recessionary outcomes than currency
mismatch in the banking sector. The effects of household currency mismatch losses are weaker compared to corporate currency mismatch losses for a couple of reasons. First, suddenly increased household debt does not affect aggregate supply of consumption goods directly but depresses aggregate demand for both housing and consumption goods. Therefore housing prices absorb a share of the negative effect that comes from household currency mismatch losses and production of consumption goods absorbs the rest of the negative effect. Second, consumption losses that arise from currency mismatch do not affect domestic production directly but also result in reduced demand for imported goods. Firms, on the other hand, use a smaller share of imported inputs compared to the share of imported goods in the household consumption basket and corporate losses are primarily reflected in the level of domestic production rather than foreign output.

The results suggest that even if shifting currency mismatch losses to Hungarian banks was a good idea, it was definitely shifting corporate losses rather than household losses that would have mitigated the recession.

Output and Welfare Gains from Non-recourse Mortgages

Establishing a personal bankruptcy institution in Emerging Europe is still both a theoretical and practical challenge, however, its importance has strengthened given increasing levels of household debt. In Chapter 4, I focus on the macroeconomic and welfare benefits of recourse laws. Chapter 4 is based on Jakucionyte (2017a).³

The conventional trade-off that arises for borrower protection is the one between providing better consumption insurance for borrowers and avoiding massive losses for banks so that credit supply would not contract too much. However, there is also a general equilibrium dimension to it. If borrower protection crowds out household credit, it may crowd in credit for businesses. Extending more corporate credit increases output and, as I show in the paper, creates welfare gains for the most valid calibration. I contribute to the literature by analyzing the effects of recourse laws in a general equilibrium setting and showing the role of housing prices in changing the credit allocation.

³Its older version is known as Jakucionyte (2017b).
I develop a general equilibrium model with mortgage default, housing and physical capital. Recourse laws are modeled by setting a particular level of earnings that defaulting households can exempt from creditors’ claims. Increasing the level of earnings exemptions creates positive output effects ex-ante for most cases. The size of the output effect, however, depends on the starting point of protection. If the initial level of protection is sufficiently high, output gains from increasing the exemption level (reducing lender recourse) are small. Equilibrium mortgage credit is already at a relatively high level, so the response of mortgage demand to higher protection is weak. Lenders respond by slightly reducing mortgage credit and shifting to business loans. For low levels of bankruptcy protection, when demand for mortgages is high, mortgage credit sharply increases in the level of earnings exemptions. Higher demand for housing boosts housing prices. Strong general equilibrium effects on housing prices increase savers’ income and savings and can mitigate capital crowding-out. So corporate credit and in turn output increase.

Better consumption insurance benefits borrowers, whereas positive output outcomes contribute to total welfare gains. The highest welfare is achieved when the level of protection is higher than one half of borrower’s earnings.
Chapter 2

Debt Overhang and the Macroeconomics of Carry Trade

2.1 Introduction

In the period leading up to the crisis Hungarian households and businesses exploited a favorable interest rate differential and ran up massive foreign currency debt. This carry trade was in expectation of low exchange rate volatility in the run-up to the anticipated adoption of the Euro in the near future. Both motives turned out to be wrong when in the first months in 2009 the Hungarian forint lost 26% of its value against the euro and even more against the Swiss franc\(^1\). The sharp depreciation of the forint considerably magnified the debt-to-GDP ratio; as a consequence the ratio of non-performing private loans increased sharply. Even those banks that shifted currency mismatch losses to borrowers by denominating loans in foreign currency did not escape: while avoiding foreign currency losses, they got increased credit risk in return.

We focus on Hungary as the most pronounced case of currency carry trade via corporate loans in Emerging Europe, but unhededged foreign currency borrowing in the private non-financial sector and substantial bank foreign debt were ubiquitous in the region (Bakker and Klingen (2012)). This

\(^{1}\) By March 2009, compared to September 2008.
motivates our focus on the macroeconomic implications of currency mismatch losses. In particular, what are the macroeconomic consequences of shifting exchange rate risk from borrowers to banks? Thus, besides the allocation of currency mismatch losses that reasonably resembles the Hungary’s case before 2009, we also study a counterfactual case with bank lending denominated in domestic currency only. In contrast to foreign currency loans, domestic currency denomination relieves domestic firms of currency mismatch and thus reduces potential debt overhang in the corporate sector, but at the expense of leaving banks with substantial funding from abroad with increased currency mismatch on their balance sheets. Resulting bank losses may impair the credit transmission channel as much as losses from non-performing loans in the former scenario. This trade-off is the topic of this paper.

We explore the macroeconomic consequences of this trade-off by developing a quantitative model with corporate debt overhang and an active banking sector facing financial frictions. We confirm that avoiding direct exposure to exchange rate fluctuations does not save banks from losses in times of domestic currency depreciation but we do show that, after unanticipated depreciation, the economy bears smaller aggregate losses if firms’ net worth is preserved by shifting currency mismatch losses to banks. Banks are in a better position to absorb currency mismatch losses because, in contrast to firms, they do not face default risk due to the various forms of insurance and bail-out provisions they are subject to. Even though banks are more leveraged than firms, unexpected bank losses affect borrowing conditions for firms and thus aggregate economic activity to a smaller extent than the investment distortion that can stem from a rising default probability in the firms’ sector. This conclusion relies on the fact that banks may expect to be rescued by either the government or parent banks, while a large number of financially constrained firms cannot expect to be nationalized or receive other types of financial support to prevent them from going bankrupt.

2 Corporate debt overhang in Hungary was as important as household debt overhang: in 2009 the share of corporate loans denominated in Swiss francs or euros was as high as the counterpart share in mortgages and amounted to more than fifty percent (Bank of Hungary (2012a)). In this paper we choose to look at borrowing firms rather than indebted households to distinguish between the very different impact effects and transmission channels of non-performing corporate loans problems and the macroeconomic problems triggered by non-performing mortgages. We address household bankruptcies triggered by the deteriorating value of domestic currency in a companion paper (Jakucionyte and van Wijnbergen (2017b)).
The second reason why allocating currency mismatch losses to firms generates larger real losses is that excessive corporate debt affects firms’ decisions as they occur and thus inflicts output losses directly, while bank losses affect aggregate economic activity with a lag and only after a share of the effect is absorbed by bank equity.

**Currency mismatch losses in Hungary**

The currency mismatch situation in Hungary was unavoidably shaped by financial vulnerabilities developed prior to the forint depreciation. Our focus on debt overhang as triggered (or intensified) by the forint depreciation is supported by the data. In the run up to the crisis more than one half of private loans were taken in Swiss francs or euros (Gulde and Giorgianni (2012)). Brown and Lane (2011) and Herzberg (2010) state that foreign currency borrowing in Emerging Europe was not large-scale and concentrated among exporting firms, but studies with access to firm-level data in Hungary cast doubt on the firms’ ability to hedge against the currency risk: Endrész et al. (2012) find that more than 82% of firms with foreign currency debt had no foreign currency revenue from exports, the survey of 698 Hungarian firms (Bodnár (2012)) discovers that also around 80% of foreign currency borrowers did not have a natural hedge. The weaker Hungarian forint resulted in significantly more bankruptcies among firms that borrowed in Swiss francs rather than Hungarian forints (Figure 2.2). Vonnák (2015) confirms that currency mismatch, and not the lending practices of Hungarian banks, contributed the most to the riskiness of foreign currency borrowers.

After 2008, foreign currency borrowers in Hungary were more likely to default and reduce investment (Endrész et al. (2012)). Foreign currency borrowers were not only riskier, but, as data analysis in Endrész et al. (2012) shows, also had sizable shares in aggregate variables such as investment and debt in Hungary. We notice that at the macro level the gap between private investment and profit shares in Hungary kept increasing: after 2008 investment declined by more and took longer to recover than the measure for corporate profitability (Figure 2.1). Apparently, Hungarian firms were unwilling to invest retained earnings for several years which is a strong indication of worsening debt overhang. In contrast to monitoring costs based models (like Bernanke et al. (1999)), debt overhang based approaches can explain prolonged under-investment in the recovery
environment. If firms perceive their chances to default on accumulated debt as sufficiently high, their private benefits from investing diminish (Myers (1977)). Recessions with investment falling below the socially optimal level of investment tend to be deeper and longer.

Currency mismatch both in the corporate sector and in the banking sector is at the heart of the problem. Both businesses and banks in Hungary borrowed in foreign currency (Hungarian Banking Association (2012)). The banks’ currency mismatch was reinforced by tight funding links between foreign parent banks and their subsidiaries in Hungary before the crisis. Moreover, isolation of currency mismatch losses in one sector is impossible due to the credit channel as banks are the main source of credit in the economy. This is common in all of Emerging Europe, where they intermediate up to 80% of total credit (World Bank (2015a); World Bank (2015b)). Passing on FX mismatch to bank borrowers would not really isolate the banks given their predominant position as providers of debt to non-financial firms: even if only borrowers would have faced currency mismatch, domestic currency depreciation would deteriorate the quality of such loans and banks would shrink credit supply anyhow faced with rising Non Performing Loans (NPL) ratio’s. Damage to the credit provision channel constituted the core of the ECB critique of the early repayment scheme of foreign currency mortgages with an artificially strong exchange rate instituted by the Hungarian Government for consumer mortgages, effectively shifting losses back to the lending banks: In 2011, against the advice of the ECB (ECB (2011)), the Hungarian government adopted such a scheme to aid debt-ridden households and forced banks to take massive losses\(^3\). In the authorities’ view, losses that extensive might have posed a real threat of interrupting credit provision in Hungary and casted doubt on saving borrowers at the expense of lenders (even when lenders are foreign-owned). Even though this policy targeted households, we take it as evidence for the importance of credit channel.

For bank losses to impair credit provision, bank funding costs and loan supply have to depend on bank performance. Indeed, banks are frequently leverage-constrained themselves during crises as their own access to funding depends on the riskiness of their balance sheets (e.g. Diamond

\(^3\) The estimated total bank losses from the early repayment scheme were around 1.1 billion euros or around 10% of total bank capital in Hungary (Reuters (2012); authors’ calculations).
and Rajan (2009)). The banking system in Hungary was well-capitalized in 2008 (IMF (2008)), however, liquidity shocks at the outbreak of the crisis changed the situation dramatically (Gulde and Giorgianni (2012)). The sudden dry-up of foreign funding caused a tightening of leverage constraints. To capture this channel, we introduce the second financial friction in the banking sector, namely a leverage constraint. We model it as an agency problem between banks and depositors following Gertler and Karadi (2011). The agency problem prevents banks from unlimited expansion of their balance sheets in good times. In bad times, non-performing loans in the corporate sector deplete bank equity so that the leverage constraint becomes tighter and leads to higher borrowing costs for banks. Eventually, the endogenous leverage constraint amplifies the drop in lending and economic activity. The feedback in bank lending is what makes the model structure complete and suitable to answer the research question formulated.

But what triggered the debt overhang situation to begin with? We look at the major shocks at the onset of the crisis in Hungary that could have led to domestic currency depreciation and so
magnified the domestic currency value of foreign currency loans. The chronology of the pre-crisis events in Emerging Europe points to external triggers instead of shocks of a local origin: despite severe domestic imbalances in emerging Europe, depreciation of local currencies followed spillovers from the looming economic crash in advanced economies rather than happening at the same time. Based on anecdotal evidence and data (Bakker and Klinge (2012)) we choose to look at three alternative (but not mutually exclusive) potential culprits: capital outflows, a drop in world demand for domestic exports and an increase in volatility in the markets.

We feed shocks into a small open economy New Keynesian model calibrated to Hungarian data. The international trade structure embedded in the model economy is similar to the set up used in Gali and Monacelli (2005), García-Cicco et al. (2014) and Adolfson et al. (2007). But the main new feature is the introduction of explicit debt overhang on the corporate level in the manner of famous paper Merton (1974) on pricing credit risk, where he shows that limited liability essentially implies a put option written by creditors to equity holders. Myers (1977) uses this approach to explore the concept of debt overhang and its impact on investment, also a key element of our paper4.

4Occhino and Pescatori (2015) follow a similar approach.
So we extend the endogenous leverage constraint model of Gertler and Karadi (2011) to include Merton (1974) like debt overhang on the corporate level with its associated moral hazard problems highlighted by Myers (1977).

The Merton put option approach to financial frictions between lender and borrower leads to another novelty in the paper. Despite using first-order approximation techniques to solve the resulting DSGE, volatility does have a first order impact on model outcomes because volatility shows up in the derivatives of that Merton put with respect to corporate investment and employment, in the same way volatility has an impact on general option derivatives ("the Greeks"), so we can use our model to study the impact of volatility shocks. The volatility related put option term in the financially constrained firms’ optimization problem drives a wedge between social and private benefits from investing. Besides modeling a shock to volatility of firms’ future profits, we endogenize volatility by incorporating uncertainty about prices: we simulate the model going back and forth between assumed and generated volatility until the two converge, thus endogenizing the overall volatility of corporate profits. The obtained volatility value contains more information about the propagation of a particular shock in our model and thus is superior to an arbitrarily calibrated value.

The debt overhang friction stems from a particular limited contractibility feature of the debt contracts in the model. Borrowing firms are subject to limited liability which skews incentives towards taking too much risk and rules out a risk-free debt contract from the menu of optimal contracts. Second, banks cannot write a contract on how the loans they extend will be used: the quantities of capital and labor are determined unilaterally by the firm after it has received the loan. In the event of adverse shocks, these frictions may create debt overhang and distort the firms’ choice of capital and labor demand.

The idea that risky debt makes firm reject valuable investment opportunities of course goes back to Myers (1977). Limited liability implies that debt is risky which may incentivize a sub-optimal investment strategy. Myers (1977) does not explore how the reduced value of the firm would affect firm’s borrowing costs, the idea that default risk feeds into the credit spread is formalized in Merton

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5 We thank Christian Stoltenberg for suggesting this numerical approach to endogenizing volatility.
(1974) who derives the credit spread as reflecting the unavoidable put option on the future assets of a debtor written out by the creditor to the equity holder. Our setup incorporates both seminal ideas: if debt is high enough, firms’ incentives to invest diminish and a default spread goes up reinforcing the mechanism.

Out of several explanations how debt can reinforce business cycle fluctuations, only debt overhang is suitable for our research problem. The costly state verification framework famously introduced in macroeconomics by Bernanke et al. (1999) just introduces an interest rate wedge, but because it allows lenders and borrowers to contract on investment and employment, avoids moral hazard and the associated debt overhang problems. A default wedge as in Gourio (2013) introduces corporate default effects on input providers instead of lenders and thus abstracts from the credit channel which is crucial in the Hungarian story. This paper is the first attempt to use the non-contractible investment approach to explain the role of excessive debt and foreign currency debt in particular in business cycle analysis.

The structure of the paper is as follows. We discuss related literature in section 2.2 and the model in detail in section 2.3, and show simulations in section 2.4. We discuss the results in section 2.5, while section 2.6 concludes.

2.2 Related literature

There is a lengthy corporate finance literature on debt overhang that starts with the seminal paper of Myers (1977). We contribute to the literature on macroeconomic consequences of debt overhang that were firstly examined in Lamont (1995). He argues that debt overhang can create strategic complementarities among investments of individual agents, thus potentially leading to multiple equilibria. Philippon (2010) studies the interaction between different indebted sectors in the model economy. The paper argues that debt overhang can create strategic complementarities between different economic sectors, namely, households and banks. In a closed economy, bailing out banks is efficient, while bailing out insolvent households means transferring funds to households that
made inefficient saving decisions. In an open economy, countries have an incentive to free ride on foreign recapitalization programs, therefore, international coordination is required. Besides the shared focus on the credit transmission channel in an indebted economy, we go beyond the analysis in Philippon (2010) and study the business cycle properties of the model economy and apply the concept of debt overhang to excessive foreign currency debt.

Our set up comes closer to Gomes et al. (2016) and Occhino and Pescatori (2014), who analyze the conduct of monetary policy in an environment with nominal debt. However, they focus on the effect of unanticipated inflation, while we focus on the debt overhang situation that arises after domestic currency depreciation.

There is a vast literature that explores foreign currency debt effects in the costly state verification framework as implemented in Bernanke and Gertler (1989) and Bernanke et al. (1999). Traditionally domestic currency depreciation invokes an expenditure switching effect that should stabilize demand for domestic goods. However, high foreign currency debt together with monitoring costs and sticky prices can potentially outweigh the expenditure switching effect and in turn make depreciations contractionary. Céspedes et al. (2004), Devereux et al. (2006), and Gertler et al. (2007a) study the depreciation effects on firms in a small open economy setting. They incorporate a model of investment in which net worth affects the cost of capital and allow firms to borrow in foreign currency. They argue that even with high foreign currency debt depreciations remain expansionary. A similar model is considered in Cook (2004) where it leads to the opposite conclusion. Cook (2004) attributes this discrepancy to the type of price stickiness. If, as in Céspedes et al. (2004), input prices are sticky but output prices are not, domestic currency depreciation lowers real wages and increases revenues. Thus, the increase in firms’ revenues might compensate for the soaring foreign currency debt and the depreciation remains expansionary. If, as in Cook (2004), output prices are sticky and input prices are not, revenues do not increase as fast as input costs and the depreciation can become contractionary. Despite the fact that these studies abstract from debt overhang, they emphasize the negative role of foreign currency debt and support our question too.
Empirical studies have established the relevance of financial frictions in explaining the macroeconomic outcomes. Without taking a stand on the prevalent financial friction, Towbin and Weber (2013) look at the data for 101 countries from 1974-2007 and show that high foreign currency debt increases the decline in investment in response to adverse external shocks. Kalemli-Özcan et al. (2015) advance further by studying firm-bank-sovereign linkages in Europe to weigh the role of several financial frictions. They find that debt overhang is more important in explaining weak investment relative to explanations focusing on weak bank and other weak firm balance sheet channels. Therefore, debt overhang also has on average better chances in explaining poor investment performance in Hungary compared to other financial frictions.

Another branch of the literature that we relate to is centered upon volatility shocks. A recent contribution by Christiano et al. (2014) attributes a significant share of business cycle fluctuations to idiosyncratic risk shocks fed through the time-varying idiosyncratic variance component. The variance component appears in the credit spread of entrepreneurs as in the costly state verification framework implemented in Bernanke et al. (1999). Thus the impact of the risk shock affects the credit spread rather than the default wedge in the firm’s investment decision.

2.3 Model

Our focus is on the interaction between FX losses induced debt overhang, undercapitalized banks and corporate investment and employment decisions. To that end we introduce a Merton (1974)/Myers (1977) like debt overhang friction\(^6\) in a model with leverage constrained banks in a small open economy context with foreign currency denominated private debt. The open sector with nominal rigidities generates realistic lending and output dynamics in the presence of foreign currency loans. We start the outline of the model by describing the more novel sections. We describe the more standard model blocks only briefly in the main text, all model details and associated derivations are in Appendix A.

\(^6\)See Occhino and Pescatori (2015) for a similar way of introducing corporate debt overhang, but in a closed economy model.
2.3.1 Financially constrained firms

Financially constrained firms live for two periods. Every period there is a new-born generation of firms and the total number of firms always constitute a continuum of mass one. The sequence of decisions for the generation $t$ of firms is presented in Table 2.3. In the first period firms buy two types of inputs, capital $k$ and labor $h$, and have to pay for a fraction $\rho$ in advance, which generates their demand for working capital and borrowing. Firms take loans in the first period of their lives. Production takes place in the next period. Firms may default in the next period too.

To pay in advance, a financially constrained firm $i$ uses two types of financing. First, it receives equity from households, $N_{\text{firms}}^{i,t}$. Second, it borrows from the bank an amount $L_{i,t}$ that consists of both domestic currency funds $L_{i,t}^D$ and foreign currency denominated funds $L_{i,t}^F$ such that $L_{i,t} = L_{i,t}^D + S_i L_{i,t}^F$ where $S_i$ is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by $\alpha^F$, so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

To borrow, the firm has to pledge a share $\kappa$ of future revenue as collateral where $0 < \kappa \leq 1$. We assume that the firm decides how much to borrow before shocks arrive and the prices of production
inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital minus the expected equity transfer from the household. It follows that in the beginning of period $t$ the following condition holds:

$$E_{t-1}\{l_{i,t}\} + E_{t-1}\{n_{i,t}^{firms}\} = E_{t-1}\{\rho (q_t k_{i,t} + w_t h_{i,t})\}$$ (2.1)

where $q_t$, $w_t$ and $rer_t$ denote the real price of capital, the real wage and the real exchange rate respectively. All three prices are expressed in units of composite goods. It follows that we define the real exchange rate as $S_t P_t^*/P_t$ where $S_t$ is the nominal exchange rate, $P_t$ is the price of composite goods and $P_t^*$ defines the price level of foreign composite goods. $n_{i,t}^{firms}$ stands for the real equity transfer from the domestic household, where $n_{i,t}^{firms} \equiv N_{i,t}^{firms} / P_t$. $l_{i,t}$ stands for the size of the total loan expressed in units of composite goods and is defined as $l_{i,t} \equiv L_{i,t} / P_t$. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm’s private incentives to invest in production inputs.

The amount of corporate equity available is a factor in determining the firms’ demand for funds and sets its “distance to default”. In bad times, a higher fraction of firms default, which decreases the total value of corporate net worth. The household pools retained earnings and distributes them to new-born firms equally. So in bad times new generations of firms receive less equity from the household, therefore to produce the same amount of goods they have to leverage up more and thus will face a higher default risk. Note that firms die after two periods and thus do not take into account profits further out in the future, which mutes the macroeconomic net worth effect to some extent. The first generation of firms that enters the scene after the shock makes its borrowing decision based on expectations about the value of its net worth, so the net worth effect materializes for future generations of firms only.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases the owner of the firm (the domestic household) steps in and transfers lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t} / P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on
them as the main source of finance. These funds enter the domestic household’s budget constraint as a lump-sum transfer and have no effect on either the household’s or the firm’s incentives.

Let the matured loan in units of composite goods be \( R_{i,t} \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}} \right) \), where \( R_{i,t} \) is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as \( l_{i,t}^D \equiv L_{i,t}^D / P_t \) and \( l_{i,t}^F \equiv L_{i,t}^F / P_t \). The contracted collateral is a fraction \( \kappa \) of firms’ revenue from selling goods and depreciated capital in the next period, \( p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \). \( p_{t+1}^R \) stands for the price of homogeneous goods, expressed in units of composite goods \( (p_{t+1}^R \equiv P_{t+1}^R / P_{t+1}) \). Then the decision of the financially constrained firm \( i \) born in period \( t \) whether to default or not is determined by the lower value:

\[
\min \left\{ R_{i,t} \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}} \right) , \quad \kappa \left( p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}
\]

(2.2)

where \( p_{t+1}^R y_{i,t+1}^R = p_{t+1}^R A_{i,t+1} \theta_{i,t+1} k_{i,t}^0 h_{i,t}^{1 - \sigma} \).

The firm \( i \) born in period \( t \) and endowed with corporate equity \( n_{i,t}^{\text{firms}} \) maximizes profits taking the loan as given. The firm maximizes the expected sum of future revenue from selling goods and depreciated capital subtracted by the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period \( t \) also enter the maximization problem and can be summarized as the difference between the loan plus equity (both \( n_{i,t}^{\text{firms}} \) and \( z_{i,t} \)) and working capital expenditure:

\[
\max_{\{k_{i,t}, h_{i,t}\}} \quad E_t \beta \Lambda_{t,t+1} \left\{ p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} - (1 - \rho) \frac{q_t k_{i,t} + w_t h_{i,t}}{\pi_{t+1}} \right\}
\]

\[- E_t \beta \Lambda_{t,t+1} \min \left\{ R_{i,t} \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}} \right) , \quad \kappa \left( p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}
\]

\[+ l_{i,t} + n_{i,t}^{\text{firms}} + z_{i,t} - \rho \left( q_t k_{i,t} + w_t h_{i,t} \right) \]
\[ E_{t-1} \{ l_{i,t} \} + E_{t-1} \{ n_{i,t}^{firms} \} = E_{t-1} \{ \rho (q_t k_{i,t} + w_t h_{i,t}) \} \]

The resulting first-order conditions are:

\[
k_{i,t} : \quad E_t \beta \Lambda_{t,t+1} \left\{ p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial k_{i,t}} + q_{t+1}(1 - \delta) - (1 - \rho) \frac{q_t}{\pi_{t+1}} \right\} \\
- E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{i,t})) \kappa \left( p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \right) \right\} \\
\frac{\partial cov}{\partial k_{i,t}}(\beta \Lambda_{t,t+1}, \min \left\{ R_{t,t}^R \frac{d_{i,t}}{\pi_{t+1}} + rer_{t+1} \frac{i_{t,t}^{F}}{\pi_{t+1}} , \kappa \left( p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}) \\
+ \rho q_t
\]

\[
h_{i,t} : \quad E_t \beta \Lambda_{t,t+1} \left\{ p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial h_{i,t}} - (1 - \rho) \frac{w_t}{\pi_{t+1}} \right\} \\
- E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{i,t})) \kappa \left( p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial h_{i,t}} \right) \right\} \\
\frac{\partial cov}{\partial h_{i,t}}(\beta \Lambda_{t,t+1}, \min \left\{ R_{t,t}^R \frac{d_{i,t}}{\pi_{t+1}} + rer_{t+1} \frac{i_{t,t}^{F}}{\pi_{t+1}} , \kappa \left( p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}) \\
+ \rho w_t
\]

where

\[
d_{2,t} = \frac{E_t \ln \left( \kappa \left( p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) - R_t^R rer_{t+1} \frac{i_{t,t}^{F}}{\pi_{t+1}} \right) - E_t \ln \left( R_t^R \frac{d_{i,t}}{\pi_{t+1}} \right) }{\sigma_y}, \quad d_{1,t} = d_{2,t} + \sigma_y
\]

\[7\] The derivation of the first-order conditions and the term \(d_{2,t}\) in particular are provided in Appendix A.2.1-A.2.2.
Note the similarity to the credit risk approach pioneered by Merton (1974): because of limited liability firms effectively receive a put option from creditors, ex ante this is priced in (that is where the credit risk comes from) but because investment and employment are not contractible in the debt contract, a moral hazard problem persists. The debt overhang friction introduces an additional term in otherwise standard demand functions for capital and labor: conditions incorporate a proxy for the default probability, \((1 - \Phi(d_{1,t}))\), that reduces a marginal product of capital and a marginal product of labor. Thus in this problem the default probability is what drives the wedge between social benefits from investing and private benefits from investing. When the default probability increases, private benefits would diminish and demand for labor and capital would shrink resulting in a lower level of working capital than a socially optimal one. Under-investment in working capital has negative and prolonged implications on aggregate variables: we can distinguish between static debt overhang effects and dynamic debt overhang effects. Static debt overhang results from a decline in demand for working capital which depresses aggregate demand on impact. Dynamic debt overhang occurs, if the indebted sector uses capital as input. Then sub-optimally lower demand for capital shrinks demand for investment. Lower investment today decreases capital stock available for production tomorrow which prolongs the economic recovery.

The second implication of the first-order conditions relates to the option structure as reflected by the definition of the function argument \(d_{2,t}\). The default probability directly depends on a volatility term \(\sigma^2_y\) which captures the variance of future profits. \(\sigma^2_y\) is given by

\[
\text{var} \left( \pi_{t+1} \left( \kappa \left( p_{t+1}^R Y_{t+1}^R + q_{t+1}(1 - \delta)k_{t,t} \right) - R_t^R \frac{r^{F_{t+1}}_{t,t}}{\pi^*_t} \right) \right)
\]

and depends on exogenous productivity shocks, working capital and endogenous volatility of prices and exchange rate value in the domestic economy. The first-order conditions imply that increased uncertainty about of future collateral value reduces firms’ chances to repay. Looming uncertainty
during the latest crisis\textsuperscript{8} highlights the importance of the volatility term in explaining borrowing conditions for firms and firms’ willingness to borrow and suggests that we cannot assume constant volatility without a loss of generality. Thus we model an exogenous shock to a volatility term to simulate increased uncertainty about financially constrained firms’ performance in the future as one of possible triggers of debt overhang.

Noteworthy, the default probability varies not only with stochastic components such as technology but with expected prices and exchange rates as well. This motivates our simulation exercise in which we simulate the model until the endogenously implied volatility of firms’ expected collateral value converges. This exercise allows us to incorporate the second-order characteristics of the economy and obtain a better estimate for the volatility term than an arbitrary calibrated value.

In the beginning of every period, after shocks take place and a fraction of firms default, the domestic household pools the remaining net worth from non-defaulted firms into aggregate net worth by following the aggregation rule:

\[
\begin{align*}
    n_{t}^{\text{firms}} &= \omega_{t}^{\text{firms}} \left( \rho_{t} R_{t}^{R} + q_{t} (1 - \delta) k_{t-1} - (1 - \rho) q_{t-1} k_{t-1} + w_{t-1} h_{t-1} \right) \\
    &\quad - \omega_{t}^{\text{firms}} \left( (1 - \Phi(d_{1,t-1})) \kappa \left( \rho_{t} R_{t}^{R} + q_{t} (1 - \delta) k_{t-1} \right) + \Phi(d_{2,t-1}) R_{t-1}^{R} \frac{I_{t-1}^{D}}{\pi_{t}} + \Phi(d_{1,t-1}) r e r_{t} \frac{I_{t-1}^{F}}{\pi_{t}} \right) \\
    &\quad + \iota_{t}^{\text{firms}} \cdot n_{t}^{\text{firms}}
\end{align*}
\]

Recall that \((1 - \Phi(d_{1,t-1}))\) proxies for the default rate (by the law of large numbers this is equal to the share of defaulted firms in the economy). Then the first term on the right hand side is aggregate firms’ revenue from production and selling depreciated capital minus the rest of the expenditure for working capital. The second term is the firms’ aggregate expenditure for repaying loans. The difference between the two gives financially constrained firms’ profits. The third term is the injection of new equity. We assume that the domestic households acts as distributor and

\textsuperscript{8} The implied volatility indexes for both European markets and Poland rocketed in the end of 2008, see the plot in Appendix A (Figure A.1). We do not have a measure for Hungary, however, the implied volatility index for Polish markets should serve as a satisfactory proxy for the markets’ risk perception for the Hungarian economy.
cannot divert pooled equity funds anywhere else. Also the existing equity can be increased only by the amount \( \ell^{\text{firms}} \cdot n^{\text{firms}} \) that is fixed and proportional to aggregate net worth in the steady state. Thus, this equity transfer does not depend on the household’s decision. \( \omega^{\text{firms}} \) is a fraction that is close but lower than unity. We assume that this parameter proxies for the equity management costs incurred by the household and use this parameter to calibrate the steady state corporate leverage to the one observed in the data.

### 2.3.2 Banks

Domestic households own all banks that operate in the domestic economy and lend to financially constrained domestic firms. We assume that there is a continuum of these banks and every period there is a probability \( \omega \) that a bank continues operating. Otherwise, the net worth is transferred to the owners of the bank, domestic households.

We assume that banks give loans to firms out of accumulated equity \( n_t \), domestic deposits \( d_t \) and foreign debt \( d_t^* \). A fraction of banks’ liabilities (foreign debt) is denominated in foreign currency which exposes banks to currency mismatch. Lending in foreign currency hedges the open currency position for banks\(^9\). However, shifting exchange rate risk to the credit constrained corporate sector increases the credit risk for banks. We consider two lending scenarios which have different implications for bank currency mismatch. First, banks lend in domestic currency only which creates currency mismatch on their balance sheets. The second scenario is described by bank lending in both foreign currency and domestic currency so that banks are relieved from currency mismatch. We will consider these two cases in the following discussion on shifting currency mismatch. The model with loans denominated in both currencies is described here, while the model with lending in domestic currency only is described in Appendix A.3.2.

The balance sheet constraint of a bank \( j \), expressed in units of composite goods, is given by

\(^9\) We calibrate the share of loans denominated in foreign currency such that banks do not have a zero open currency position in that case. This allows us to distinguish between the credit risk effects and the exchange rate risk effects.
\[ n_{j,t} + d_{j,t} + rer_t d_{j,t}^* = l_{j,t} \]

Banks pay a nominal domestic interest rate \( R_t \) on deposits and a nominal foreign interest rate \( R_t^* \xi_t \) on foreign debt. \( R_t^* \) follows a stationary AR(1) process. \( \xi_t \) denotes a premium on bank foreign debt. To ensure stationarity in the model, we assume that the premium depends on the level of foreign bank debt (as in Schmitt-Grohé and Uribe (2003)):

\[
\xi_t = \exp \left( \kappa \frac{rer_t d_{j,t}^* - rer \cdot d^*}{rer \cdot d^*} + \frac{\xi_t - \zeta}{\xi} \right) 
\]

(2.3)

where \( \zeta_t \) is an exogenous shock that follows a stable AR(1) process.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction \( \lambda L \) of assets, but if that happens the bank goes bankrupt (i.e. cannot continue). Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is equal to or higher than the value of what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet of the bank for given amount of equity.

Loan performance directly affects bank profits, loans to domestic financially constrained firms are the only asset on the banks’ balance sheet. When the default probability \( 1 - \Phi(d_{2,t}) \) for financially constrained firms increases, banks expect lower returns. High corporate leverage has similar consequences as it increases the size of loans for the same level of production and reduces firms’ chances to repay ceteris paribus. We define the expected return for the bank \( j \) as \( R_{j,t}^L \). The definition makes use of the derivation of the expected loan payment (see Appendix A.2.2) and in its final expression directly incorporates the default probability on corporate loans:

\[
E_t \left\{ \frac{R_{j,t}^L}{\pi_{t+1}} l_{j,t} \right\} = E_t \min \left\{ R_{j,t}^R \left( \frac{j_D}{\pi_{t+1}} + rer_{i+1} \frac{j_{i,t}^*}{\pi_{i+1}} \right), \kappa \left( p_{i+1}^R \right)_{j,t+1} + q_{i+1}(1 - \delta)k_{j,t} \right\}
\]
Or

\[
E_t \left\{ \frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\}
\]

\[
\equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^R y^R_{j,t+1} + (1 - \delta) q_{t+1} k_{j,t} \right) + \Phi(d_{2,t}) R^R_{j,t} l^D_{j,t} \pi_{t+1} + \Phi(d_{1,t}) R^R_{j,t} rer_{t+1} l^E_{j,t} \pi_{t+1} \right\}
\]

(2.4)

To facilitate further discussion, we define two components of the overall bank spread (actual rate charged to borrowers minus the cost of funds to the bank). The first is the default spread, measured as the difference in the actual interest rate charged on the loan and the expected return on the loan: \( E_t \left( R^R_{j,t} - R^L_{j,t} \right) / \pi_{t+1}. \) The higher is the spread, the more the bank charges to compensate for the default risk. Second, there is the component of the overall bank spread that depends on the banking friction: it captures the premium that arises due to the endogenous leverage constraint. This spread is given by the difference in the expected return on the loan to financially constrained firms and the expected funding costs to the bank: \( E_t \left( \frac{R^L_{j,t}}{\pi_{t+1}} - \frac{R^R_{j,t} \xi_t / \pi_{t+1}^{rer_{t+1}}}{rer_{t}^{rer_{t}}} \right). \) Note the role of real exchange rate changes in determining the expected costs of funding. So the overall credit spread is the sum of the default spread and the bank spread and is given by \( E_t \left( \frac{R^L_{j,t}}{\pi_{t+1}} - \frac{R^R_{j,t} \xi_t / \pi_{t+1}^{rer_{t+1}}}{rer_{t}^{rer_{t}}} \right). \) A higher credit spread reflects tighter borrowing conditions due to either one or both of the financial frictions.

Then the optimization problem of the bank \( j \) can be written as:

\[
V_{j,t} = \max \left\{ d_{j,t}, d^*_j, l_{j,t} \right\} E_t \left[ \beta \Lambda_{t,t+1} \left\{ (1 - \omega)n_{j,t} + \omega V_{j,t+1} \right\} \right]
\]

s.t.

\[
V_{j,t} \geq \lambda^L l_{j,t}, \quad \text{(Incentive constraint)}
\]

\[
n_{j,t} + d_{j,t} + rer_d^* = l_{j,t}, \quad \text{(Balance sheet constraint)}
\]

\[
n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^R_{t-1} \xi_{t-1}}{\pi_t} rer_d^* l_{j,t-1}
\]

(LoM of net worth)
The first-order conditions follow:

\[ d_{j,t} : (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1 - \omega) + \omega \nu_{2,t+1} \right\} \left( \frac{R_t}{\pi_{t+1}} \right) = \nu_{2,t} \]  

(2.5)

\[ d_{j,t}^* : (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1 - \omega) + \omega \nu_{2,t+1} \right\} \left( \frac{R_t^* \xi_t \text{rer}_{t+1}}{\pi_{t+1}^* \text{rer}_t} \right) = \nu_{2,t} \]  

(2.6)

\[ l_{j,t} : (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1 - \omega) + \omega \nu_{2,t+1} \right\} \left( \frac{R_{j,t}^L}{\pi_{t+1}} \right) = \lambda^L \nu_{1,t} + \nu_{2,t} \]  

(2.7)

\( \nu_{1,t} \) and \( \nu_{2,t} \) are the Lagrangian multiplier to the incentive constraint and the Lagrangian multiplier to the balance sheet constraint combined with the law of motion for equity, respectively.

Equations (3.21) and (3.22) govern the bank debt portfolio choice. Equation (3.21) presents the marginal cost to the bank from issuing one additional unit of deposits (the left hand side) in relation to the marginal benefit from increasing equity by one unit, \( \nu_{2,t} \) (the right hand side). The marginal cost from issuing one additional unit of foreign bank debt is compared to the marginal benefit from increasing equity on the right hand side of equation (3.22) and is adjusted for changes in the exchange rate value. The structure of these choice rules suggests that in equilibrium the bank has to be indifferent between taking deposits or issuing bank debt to foreign agents.

Equation (3.23) presents the relation between the marginal benefit to the bank from issuing one additional unit of loans (the left hand side) and the marginal cost (the right hand side). We see that in equilibrium one additional unit of loans earns the discounted risk adjusted return on loans. Firstly, this return has to increase in the marginal cost from issuing bank debt to finance the expansion of the balance sheet, \( \nu_{2,t} \). Secondly, due to the endogenous bank leverage constraint, the risk adjusted bank return on loans also increases in the share of divertable assets \( \lambda^L \) and the marginal loss to the bank creditor in the case of asset diversion, \( \nu_{1,t} \). Both terms proxy for the marginal cost associated
with the tighter incentive constraint. Moreover, the tighter leverage constraint increases the bank spread as well which translates into more credit tightening.

The first-order conditions hold together with complementary slackness conditions:

\[ v_{1,t} : \quad v_{1,t} \left( V_{j,t} - \lambda^L L_{j,t} \right) = 0 \]

\[ v_{2,t} : \quad v_{2,t} \left( \frac{R_{j,t-1}^L}{\pi_t} l_{j,t-1} - \frac{R_{t-1}^L}{\pi_t} d_{j,t-1} - \frac{R_{t-1}^*}{\pi_t} \xi_{t-1}^* rer_t d_{j,t-1}^* - rer_t l_{j,t}^* + d_{j,t} + rer_t d_{j,t}^* \right) = 0 \]

The set of equilibrium conditions also includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by domestic households and is assumed to be of the size \( \iota n \). Then

\[ n_t = \omega \left( \frac{R_{j,t-1}^L}{\pi_t} l_{t-1} - \frac{R_{t-1}^L}{\pi_t} d_{t-1} - \frac{R_{t-1}^*}{\pi_t} \xi_{t-1}^* rer_t d_{t-1}^* \right) + \iota n \]  

(2.8)

2.3.3 Financial sector support

Financial sector support is modelled as in Kirchner and van Wijnbergen (2016), we assume that the government can intervene during the crisis by injecting capital \( \tau_t^{FS} \) in the banks. We assign the following rule to the recapitalization of the financial intermediary \( j \):

\[ \tau_t^{FI} = \kappa_{FS} (\text{shock}_{t-l} - \text{shock}) n_{j,t-1}, \quad \kappa_{FS} > 0, \quad l \geq 0 \]

where \( n_{j,t-1} \) is the net worth of the intermediary from the previous period. The recapitalization can be immediate (\( l = 0 \)) or delayed (\( l > 0 \)). The variable \( \text{shock}_t \) equals the shock driving the crisis, e.g. the risk premium shock (\( \text{shock}_t \equiv \xi_t \)). We assume that the recapitalization is a gift from the
government and does not have to be repaid (van der Kwaak and van Wijnbergen (2014) explore the consequences of different payback rules).

Now the bank equity increases in the equity injection from the government besides being a function of loan returns and borrowing costs:

\[
n_{jt} = \frac{R_{jt-1}^l}{\pi_t} l_{jt-1} - \frac{R_{jt-1}^l}{\pi_t} d_{jt-1} - \frac{R_{jt-1}^*}{\pi_t^*} rer_t d_{jt-1}^* + \kappa_F S (\text{shock}_{t-1} - \text{shock}) n_{jt-1}
\]

Bank’s optimization problem would yield different results now. We present modified first-order conditions in Appendix A.3.3.

### 2.3.4 Households

We assume a representative household. The household has two alternatives to invest in: make deposits \(d_t\) in a bank or buy domestic bonds issued by the government, \(b_t\). The household supplies labor to a competitive labor market. The household has Greenwood-Hercowitz-Huffman (henceforth, GHH) preferences as in Greenwood et al. (1988), so labor supply does not depend on wealth. The household chooses a level of real consumption \(c_t\) and working hours \(h_t\) such that the following lifetime utility function is maximized:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1 - \gamma} \left( c_t - \frac{\chi (h_t)^{1+\varphi}}{1 + \varphi} \right)^{1-\gamma} \right) \quad \gamma, \chi, \varphi > 0
\]  

subject to the household’s budget constraint, expressed in units of composite goods:

\[
c_t + b_t + d_t = w_t h_t + \frac{R_{t-1}^l}{\pi_t} b_{t-1} + \frac{R_{t-1}^l}{\pi_t} d_{t-1} + \Pi_t - \Pi_t
\]

\(\pi_t\) denotes the composite goods price inflation. We assume that the household is indifferent between buying domestic bonds and making deposits, thus, \(R_t\) is nominal gross interest rate of both domestic bonds and deposits. The household owns all banks in the model economy and thus
receives lump-sum dividends, $\Pi_t$. Taxes $t_t$ enter the household’s budget constraint in a lump-
sum way as well. Lump-sum dividends from financially constrained firms are included in total
dividends $\Pi_t$. Lump-sum dividends from financially constrained firms consist of firms’ profits that
the household receives in the beginning in the period minus the equity that the household transfers
in the beginning of the period.

2.3.5 Production and Pricing

There are several types of firms in the domestic economy. It takes three types of firms to produce
domestic aggregate inputs for composite goods. First, there are the financially constrained firms
that combine purchased capital with labor and produce homogeneous goods. They were analyzed
in Section 3.1. Their homogeneous outputs are bought by retail firms who costlessly differentiate
the products bought and sell them as (local) monopolists, in Dixit and Stiglitz (1977) fashion. A
similar group of firms called importers differentiate foreign (imported) goods. A composite goods
producer buys the differentiated home goods and aggregates them into an aggregate domestic good
$y^H_t$ with associated price $p^H_t$. The same composite goods producer also buys imported differentiated
goods and aggregates them into a foreign aggregate good $y^F_t$. The corresponding aggregate price
level of foreign goods is $p^F_t$. All details of the derivations of the various first order conditions
optimization problems can be found in Appendix A.5. We discuss each step in more detail below.

Retail firms

Homogeneous goods produced by financially constrained firms are sold to domestic retail firms. A
domestic retail firm $j$ differentiates purchased inputs at no cost and sells at a monopolistic price
$p^H_t(j)$. We assume that only a fraction $(1 - \omega^H)$ of domestic retail firms can adjust prices every
period as in Calvo (1983). The fraction $\omega^H$ of remaining firms adjust past prices by the rate $\pi^H_{adj}$.
The aggregate price level that prevails in the retail sector is denoted by $p^H_t$. Differentiated goods
from the domestic retail sector, $y^H_t(j), \quad j \in (0, 1)$, are purchased by the composite goods producer.
Importers

Imported foreign goods undergo a differentiation process that is similar to what happens with domestic goods. The retailers differentiating foreign composite goods are called importers. Importers also exercise (local) market power and set prices in a staggered way, again as in Calvo (1983), which allows for incomplete exchange rate pass-through. Thus, \( (1 - \omega^F) \) of importers change their past prices to the optimal price at period \( t \). The fraction \( \omega^F \) of remaining firms adjust past prices by the rate \( \pi_{t}^{adj} \).

Composite goods producer

We assume that the composite goods producer has access to an aggregation technology and can assemble differentiated goods at no cost. First, the composite goods producer assembles differentiated domestic goods \( y^H_t(j) \forall j \) into domestic aggregate goods \( y^H_t \) and differentiated imported goods \( y^F_t(j) \forall j \) into foreign aggregate goods \( y^F_t \). She uses the following assembling technologies:

\[
y^H_t = \left( \int_0^1 y^H_t(j)^{1 - \frac{1}{\epsilon_H}} dj \right)^{\frac{\epsilon_H}{\epsilon_H - 1}},
\]

\[
y^F_t = \left( \int_0^1 y^F_t(j)^{1 - \frac{1}{\epsilon_F}} dj \right)^{\frac{\epsilon_F}{\epsilon_F - 1}}.
\]

Then she combines domestic aggregate goods and foreign aggregate goods into composite goods \( y^C_t \) with the aggregation technology that takes the taste parameter for foreign aggregate goods \( \eta \) as given:

\[
y^C_t = \left( (1 - \eta) \xi (y^H_t - e_x_t)^{\frac{1}{\epsilon_H}} + \eta \xi (y^F_t)^{\frac{1}{\epsilon_F}} \right)^{\frac{\epsilon}{\epsilon - 1}}
\]

Only a share of domestic aggregate goods is used to produce composite goods and the rest is exported, because we assume exports not to have imported content. Thus, exporters would export domestic aggregate goods rather than composite goods. \( \epsilon \) stands for elasticity of substitution.
between domestic aggregate goods and foreign aggregate goods. The composite good \( y^C_t \) is sold to the domestic household, the government and capital goods producers. Its associated price is \( P_t \).

**Capital producers**

Capital producers sell capital to financially constrained firms at the real competitive price \( q_t \) and buying the depreciated capital stock back next period. To restore the depreciated capital, capital producers add composite goods (investment) \( i_t \) as additional inputs to the depreciated capital stock by using a technology subject to investment adjustment costs \( \Gamma \left( \frac{i_t}{i_{t-1}} \right) \):

\[
k_t = (1 - \delta)k_{t-1} + \left(1 - \Gamma \left( \frac{i_t}{i_{t-1}} \right) \right) i_t
\]

where adjustment costs \( \Gamma \) equal:

\[
\Gamma \left( \frac{i_t}{i_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2
\]

**Exporters**

We assume that perfectly competitive exporters demand \( e x_t \) units of the domestic aggregate good \( y^H_t \), so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters. Exported goods consist of the domestic aggregate, so they do not use imported inputs.

Exports are sold at a price \( p^H_t / re_{rt} \) which is the price of domestic aggregate goods expressed in units of foreign composite goods. The foreign demand for domestic aggregate goods is price-sensitive:

\[
e x_t = \eta^* \left( \frac{p^H_t}{re_{rt}} \right)^{-\epsilon^*} y^*_t
\]

Consistent with the small open economy assumption, \( P^*_t \) and \( y^*_t \) are assumed to evolve exogenously.

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2.3.6 Government

We abstract from normative analysis of government policies and take government spending as exogenous. We assume that to finance a stochastic stream of real government expenditure $g_t$ and the bank recapitalization program $\tau_{FS}^t$, the government collects lump-sum taxes $t_t$ from the household and issues domestic bonds $b_t$. It has to satisfy the budget constraint (expressed in units of composite goods):

$$g_t + \tau_{FS}^t + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

We assume that taxes follow this rule:

$$t_t = t + \kappa^B (b_{t-1} - b) + \kappa^{FS} \tau_{FS}^t + e_t, \quad 0 < \kappa^B \leq 1, \quad 0 \leq \kappa^{FS} \leq 1$$

So a fraction $\kappa^{FS}$ of the recapitalization expenditure is covered by increasing the lump-sum tax and the remaining fraction $(1 - \kappa^{FS})$ is financed by issuing new government debt.

2.3.7 Monetary policy

The central bank conducts monetary policy by following the Taylor rule:

$$\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\gamma_R} \left( \frac{y^H_t}{\bar{y}^H} \right)^{(1-\gamma_R)\gamma_Y} \left( \frac{\pi^H_t}{\bar{\pi}^H} \right)^{(1-\gamma_R)\gamma_\pi} \exp(mp_t)$$

where $mp_t$ is a monetary policy shock and the domestic aggregate goods price inflation $\pi^H_t$ can be expressed as $\pi^H_t = p^H_t / p^H_{t-1} \pi_t$.

2.3.8 Market clearing

The domestic household, the government and capital producers buy composite goods. Therefore, the supply of composite goods $y^C_t$ has to satisfy the aggregate demand of domestic agents:
\[ y_t^C = c_t + i_t + g_t \]  

(2.15)

### 2.3.9 Current account and its components

Trade balance expressed in units of composite goods is given by:

\[ tb_t = p_t^H ex_t - m_t \]

where \( m_t \) denotes the value of imports and can be expressed as \( m_t \equiv rer_t D_t^F y_t^F \) (see Appendix A.9 for details).

So the current account is given by the sum of real trade balance and real net income from abroad. In units of composite goods the current account is given by:

\[ ca_t = tb_t + ni_t \]  

(2.16)

The domestic household owns banks that issue foreign debt \( d_t^* \). Banks are the only agents to borrow from abroad. Also, we assume that nobody in the domestic economy lends to foreign agents. As a result, real net income from abroad is negative and equal to minus payments of bank foreign debt. It follows that

\[ ca_t = tb_t - (R_{t-1}^* \xi_{t-1} - 1) rer_t \frac{d_{t-1}^*}{\pi_t^*} \]

In equilibrium the current account has to equal the capital account balance which is given by the change in bank foreign debt. The equilibrium condition is follows, expressing the change in foreign debt in units of composite goods as well to get:

\[ tb_t - (R_{t-1}^* \xi_{t-1} - 1) rer_t \frac{d_{t-1}^*}{\pi_t^*} = - \left( rer_t d_t^* - rer_t \frac{d_{t-1}^*}{\pi_t^*} \right) \]
2.4 Preliminaries to analyzing the model

2.4.1 Calibration

To employ the theoretical model for empirical simulation, all parameters are calibrated to Hungarian data. We list calibrated parameter values and targeted steady state values in Table A.1 in Appendix A. Parameters that are endogenously determined in steady state are $\beta, \chi, \eta^*, \kappa, \omega$ and $\pi^*$. $\chi$ is chosen such that average working hours in the steady is 0.3 as it is common in the literature. $\eta^*$ is chosen such that the ratio between the steady state foreign output and the domestic output is equal to the share of the Hungarian GDP in the EU GDP, namely 0.007. $\pi^*$ follows from satisfying the UIP condition in the steady state given the foreign nominal interest rate of 4.5 p.p. in annual terms. The most important ones of the rest of endogenously determined and calibrated parameters are discussed below.

The financial frictions we introduce bring a few additional parameters to calibrate. The debt overhang friction depends on the corporate default rate value in the steady state, $1 - \Phi(d_2)$. Due to 
*de facto* non-existent corporate bond market in Hungary, we choose to calibrate the steady state default probability to an average default frequency of corporate loans in Hungary over the period 2002-2007 as reported by the Bank of Hungary. This makes $1 - \Phi(d_2) \approx 0.03$. We choose the bankruptcy loss parameter $\kappa$ such that the steady state default probability in the model matches the data counterpart. The banking friction relies on the fraction of capital that can be diverted, $\lambda^L$, the proportional transfer to the entering bankers, $\iota$, and bank leverage in the steady state. We calibrate $\iota$ to 0.002 following the original paper of Gertler and Karadi (2011). Bank leverage matches the average bank leverage in the OECD data for year 2007. We make an adjustment to the average bank leverage of 8.6 in Hungary as reported by Bank of Hungary: we adjust for the average fraction of loans in total assets and get $8.6 \cdot 0.65 \approx 5.6$. The remaining parameter, $\lambda^L$, is chosen such that the lending spread in the steady state match the observed difference between nominal corporate loan interest rate and nominal corporate deposit rate in Hungary in 2001:Q1-2008:Q3 (data from the
Bank of Hungary). Our computations yield an annual lending spread of 2.7 p.p. It follows that \( L^L = 0.45 \).

We calibrate the share of foreign currency loans in total corporate loans to 0.6 to match the aggregate share of FX corporate loans in Hungary in 2007-2008 (Krekó et al. (2010)). For the model with loans of hybrid denomination we calibrate the steady state trade balance such that bank liabilities denominated in foreign currency would match foreign currency loans exactly.

We have also calibrated several steady state values using data from the Eurostat online database. The steady state annual inflation in Hungary over the period 2001:Q1-2008:Q3 was 5.9 p.p., we choose the discount factor \( \beta \) such that the steady state inflation in the model matches the data counterpart. The ratio of government spending to GDP, \( s^g \), is set to 0.22. The ratio of imported goods in domestic consumption is computed in the following way. We take the share of imports to GDP in Hungary (72.7 percent) over the period 2002:Q1-2008:Q4 and adjust it given the average import share in the Hungarian exports (56 percent; OECD (2017)). Since in our model exports are assumed to be of domestic origin entirely, we lower the observed import share in GDP by the amount of imports used in export production and get that the import share in domestic demand should constitute around 37 percent in our model. Thus we calibrate \( \eta \) to 0.37 to achieve the desired steady state share. For simplicity we set the steady state level of the nominal exchange rate to unity.

2.4.2 Endogenizing volatility

As we pointed in the financially constrained firms’ optimization problem, our model is capable of studying volatility effects. Besides modeling a shock to volatility of firms’ future profits, we can endogenize the volatility term by incorporating uncertainty about prices. We obtain the endogenized volatility value for future profits of financially constrained firms by simulating the theoretical model as long as the value converges. In this section we explain why the obtained volatility value is a better choice than an arbitrary calibrated value. We shortly describe the simulation procedure as well.
First order conditions that govern financially constrained firms’ behavior contain a proxy for
the default probability. The default probability depends not only on expected values of future
revenue and liabilities but on variances of future revenue and liabilities as well and, as a result
of endogenous prices, it varies not only with stochastic components such as technology but with
production prices and exchange rates as well. Therefore, we cannot postulate the variance of future
output or future liabilities to be an exogenous process dependent on technology and current state
variables only. The variance of endogenous variables is unknown, but we can obtain an estimate
from simulated series. In Appendix A.2.2 we derive what variance exactly we are interested in to
be able to compute the default probability and simulate the model:

\[ \sigma_{y,t+1}^2 = \text{var} \left( \ln \left( \pi_{t+1} \left( \kappa \left(p_{t+1}^R \pi_{t+1}^R + q_{t+1}(1 - \delta)k_t \right) - R_{t+1} \sigma_{e,t+1} \frac{l_{t+1}^F}{\pi_{t+1}^R} \right) \right) \right) \]

Hence to simulate the model we need a numerical value for \( \sigma_{y,t+1}^2 \) or, more precisely, \( \sigma_{y,t+1} \), where
\( \sigma_{y,t+1} = \sqrt{\sigma_{y,t+1}^2} \). We assume \( \sigma_{y,t+1} \) to be constant (\( \sigma_{y,t+1} = \sigma_y \)).

To find a value for \( \hat{\sigma}_y \) as close to the true value as possible we follow several steps:

1. Set a threshold level for convergence of the calibrated \( \hat{\sigma}_y \) to the value of \( \tilde{\sigma}_y \) that follows from
   the simulated time series generated by the model.

2. Choose an initial value for \( \hat{\sigma}_y \).

3. Simulate the model with the chosen value for \( \hat{\sigma}_y \).

4. Compute volatility of \( \tilde{y}_{t+1} \) from simulated time series and denote it by \( \tilde{\sigma}_y^2 \).

5. Compute the difference between the chosen value \( \hat{\sigma}_y \) and the simulated value \( \tilde{\sigma}_y \). If the
   difference is larger than the threshold value, set \( \hat{\sigma}_y = \tilde{\sigma}_y \) and repeat steps 3-5.

Converged values are presented in Table 2.1. We obtain estimates of the volatility value
generated by capital outflows shocks and a drop in world demand only. The exogenous volatility
shock sometimes prevents the simulation from converging because every new value shapes the
results of the next simulation (the shock effect directly depends on the simulated value in the last period). So instead of simulating to obtain the volatility estimate generated by the exogenous volatility shock we use the average volatility retrieved after a set of shocks hit the economy: the productivity shock, the risk premium shock and the world demand shock.

### 2.5 Results

In the following section we dissect the interaction of financial distress in the firms’ sector and losses in the banking sector. We begin by discussing the debt overhang friction in the firms’ sector and its consequences in the periods of unanticipated depreciation. Next we add the banking friction to the setup to see how leverage-constrained banks can amplify the shocks even further. The relative importance of the frictions is analyzed by comparing two scenarios of allocating currency mismatch losses. Given immense foreign bank funding flows in Emerging Europe, we assume that domestic banks issue debt denominated in foreign currency which creates currency mismatch unless banks match foreign currency liabilities with loans issued in foreign currency. In the latter case currency mismatch is shifted to domestic borrowers. We compare the model economy with bank lending

<table>
<thead>
<tr>
<th>Debt denomination</th>
<th>Banking friction</th>
<th>Shock</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FX &amp; domestic currency</td>
<td>No</td>
<td>Risk premium</td>
<td>0.1428</td>
</tr>
<tr>
<td>FX &amp; domestic currency</td>
<td>No</td>
<td>World demand</td>
<td>0.0568</td>
</tr>
<tr>
<td>FX &amp; domestic currency</td>
<td>No</td>
<td>All shocks</td>
<td>0.1459</td>
</tr>
<tr>
<td>Domestic currency</td>
<td>No</td>
<td>Risk premium</td>
<td>0.0678</td>
</tr>
<tr>
<td>Domestic currency</td>
<td>No</td>
<td>World demand</td>
<td>0.0591</td>
</tr>
<tr>
<td>Domestic currency</td>
<td>No</td>
<td>All shocks</td>
<td>0.0848</td>
</tr>
<tr>
<td>FX &amp; domestic currency</td>
<td>Endogenous leverage constraint</td>
<td>Risk premium</td>
<td>0.2148</td>
</tr>
<tr>
<td>FX &amp; domestic currency</td>
<td>Endogenous leverage constraint</td>
<td>World demand</td>
<td>0.0785</td>
</tr>
<tr>
<td>FX &amp; domestic currency</td>
<td>Endogenous leverage constraint</td>
<td>All shocks</td>
<td>0.2117</td>
</tr>
<tr>
<td>Domestic currency</td>
<td>Endogenous leverage constraint</td>
<td>Risk premium</td>
<td>0.1216</td>
</tr>
<tr>
<td>Domestic currency</td>
<td>Endogenous leverage constraint</td>
<td>World demand</td>
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<tr>
<td>Domestic currency</td>
<td>Endogenous leverage constraint</td>
<td>All shocks</td>
<td>0.1294</td>
</tr>
</tbody>
</table>
in domestic currency and bank lending in both foreign currency and domestic currency to explore which currency mismatch situation generates larger macroeconomic losses.

More plots for every shock discussed in the following section can be found in Appendix A. Here we present graphs with the most important variables only.

2.5.1 Debt overhang in the financially constrained firms’ sector

Borrowing in foreign currency makes domestic financially constrained firms prone to debt overhang whenever the domestic currency depreciates. If the expected value of debt indeed exceeds the expected collateral value, the indebted firm faces a higher chance of losing its collateral (future revenue) to creditors. The firm’s marginal benefits from investing diminish. In the setting with non-contractible investment, the rising possibility of default is enough to create a slump in output by decreasing investment. We consider exogenous events that may trigger domestic currency depreciation in a small open economy setup and thus increase the default probability: a country risk premium shock, a negative world demand shock and a shock to volatility of profits generated in the financially constrained firms’ sector.

Regardless of the denomination of corporate debt, the listed shocks are expected to bring an economic downturn by either dampening aggregate demand or supply. Accumulated foreign currency debt makes the corporate default probability depend not only on the aggregate level of economic activity but the degree of currency mismatch as well. Thus, whenever the domestic currency depreciates, foreign currency debt opens an additional contractionary channel that operates through even higher default probabilities and thus more intense debt overhang in the financially constrained firms’ sector.

Simulation results confirm our hypothesis that debt overhang amplifies adverse effects on aggregate variables more, if firms have their debt denominated in foreign currency rather than in domestic currency. In Figure 3.2 capital outflows, which we model by increasing a country risk premium on bank foreign debt, decrease demand for domestic currency and make it depreciate. To mute rising domestic inflation, the central bank responds by raising the domestic nominal interest
Figure 2.4: Country risk premium shock of 5 p.p. in the model without leverage-constrained banks.

rate and in turn creates a recession. The main driver behind it is the decline in consumption driven by the substitution effect. Currency mismatch for firms makes the recession deeper as investment in working capital decreases not only due to lower aggregate demand but also due to debt burden weighing on firms’ marginal benefits from investing. We observe that, if financially constrained firms borrow in foreign currency, a repayment probability is substantially lower, an interest rate on their loans rises higher and they post lower demand for labor and capital goods. Noteworthy, domestic currency depreciation not only distorts decisions ex-ante, but deprives firms of available funds ex-post: lower firms’ profits result in lower corporate worth and thus higher dependence on external funds which come at a now high default spread.

A decline in world demand for domestic exports, as exhibited in Figure 2.5, results in deflation. Domestic prices have to decrease so that the drop in external demand would be compensated by increased competitiveness. Domestic currency depreciates. Interesting enough, financially constrained firms face a lower default probability. They do not experience losses in corporate net worth which also contributes to higher demand for labor and capital. Consequently the effect on output is positive. The paradoxical result partially owes to the predetermined demand for
working capital as posted by financially constrained firms. When external demand for domestic goods declines, the domestic demand has to increase to absorb the idle output produced out of predetermined production inputs. Therefore, domestic prices drop sufficiently to make domestic consumers capable of consuming more. The increased domestic demand effect does not die immediately and the next period output grows to catch up with higher demand. To see that this is indeed the reason we modify the model so that labor demand can adjust immediately and output responds to changes in aggregate demand on impact (see Appendix A.2.4 for modeling details). Figure 2.6 shows how the drop in world demand becomes contractionary once labor demand can shrink in response to fewer orders for domestic goods from abroad.

Currency mismatch brings in more negative effects, however, the difference is relatively small, see Figure 2.5. It turns out that the resulting domestic currency depreciation is too small to increase the wedge between the value of debt and the collateral value for financially constrained firms. A higher depreciation is needed due to a relatively restrictive version of the debt overhang model. First, we model short-term debt which, in contrast to long-term debt, makes debt overhang fade away after the first period. Second, the timing of the firm’s optimization problem is such that firms learn about their net worth value after the borrowing amount is decided. Therefore, even though domestic currency depreciation triggers more defaults and thus reduces corporate net worth (Figure 2.5), the shock feedback through the corporate net worth comes with a delay. Third, firms die after two periods and do not take into account future profits which mutes the net worth effect to some extent as well. Shocks have to propagate through prices mostly and thus the exchange rate effect on firms’ performance in the future is limited.

In contrast to other shocks, the volatility shock primarily affects not the demand side but the supply side of the economy by making the firms’ future profits more uncertain. This has a direct effect on the default probability as the uncertainty magnifies the expected distance between the collateral value and the debt value. Then, for any debt burden and any productivity level, firms face lower chances to repay their debt and lenders respond by raising interest rates on corporate loans. Figure 2.7 depicts how in this case debt overhang weighs on the firms’ incentives to invest
Figure 2.5: World demand shock of 6.4% in the model without leverage-constrained banks.

and in turn the economy falls into a recession. The increased uncertainty of firms’ future profits has an indirect effect on household consumption by lowering income: firms post lower demand for labor and wages decrease. The substitution effect stimulates consumption as the central bank
Figure 2.7: Volatility shock of 100% in the model without leverage-constrained banks.

copes with the slump and the corresponding deflation by cutting the policy rate, however, this effect appears to be negligible. Overall, the volatility shock generates responses of relatively large magnitude, changes in investment are particularly large. Initially, foreign currency debt generates more contraction than accumulated domestic currency debt, however, after two periods the real exchange rate depreciation in the former cases subsides and depreciation-driven debt overhang loses its influence completely. The difference between the case with borrower currency mismatch and without it is negligible. Besides the reasons mentioned before, the volatility shock directly hits firms’ chances to repay and the depreciation effect becomes of the second order. In other words, the magnitude of the change in the default probability overshadows the risk related to the increased value of foreign currency debt.

Therefore, capital outflows can trigger domestic currency depreciation that increases currency mismatch in the corporate sector. Compared to firms borrowing in domestic currency only, the depreciation lands firms indebted in foreign currency in a more severe debt overhang situation. Under-investment and a deeper fall in output follow. The effects of the negative world demand shock and the increased exogenous uncertainty are less clear as they trigger an apparently insufficient
loss in the domestic currency value. Also, the volatility shock increases firms’ chances to default to an extent that depreciation effects get overshadowed and debt denomination loses its role in ranking the outcomes. The type of shocks appears to have important implications for the role of foreign currency debt and debt overhang.

2.5.2 Introducing leverage-constrained banks

The agency problem between banks and depositors generates an endogenous credit spread which tightens or improves borrowing conditions for banks depending on bank leverage. Highly leveraged banks face larger credit spreads on their debt. It follows that the credit spread moves countercyclically: in bad times non-performing loans deplete bank capital and bank leverage goes up.

Financial distress in the banking sector translates into worse borrowing conditions for the borrowing firms: the tighter endogenous leverage constraint and thus higher borrowing costs for banks make banks charge higher interest rates on loans issued to financially constrained firms. In bad times the binding bank leverage constraint amplifies initial losses in the economy.

In our experiment bank losses are triggered by currency mismatch losses placed in either the firms’ sector or the banking sector. Bank debt denominated in foreign currency exposes the banking sector to currency mismatch, so that domestic currency depreciation has an immediate negative effect on bank equity and leverage. If the bank lends in foreign currency as much as it borrows in foreign currency, the depreciation increases the value of both sides of the bank balance sheet and bank earnings do not deteriorate ceteris paribus. However, domestic currency depreciation triggers large losses for domestic firms that borrowed in foreign currency. Lower firms’ profits result in a higher ratio of non-performing loans and bank profits decline. Therefore, even if lending in foreign currency insulates the bank balance sheet from the exchange rate risk, a potentially higher increase in non-performing loans can still impair the credit transmission channel and worsen the recession.

This paper shows that, even if the model is enriched with the endogenous bank leverage constraint, aggregate losses are still smaller when corporate loans are denominated in domestic currency than when a share of debt is foreign currency loans.
Banks are in a better position to absorb currency mismatch losses because, in contrast to firms, they do not internalize default risk. Consequently, even though banks are more leveraged than firms, unexpected bank losses affect borrowing conditions for firms and thus aggregate economic activity to a smaller extent than the investment distortion that stems from the rising default probability in the firms’ sector. This assumption relies on the fact that banks may expect to be rescued by either the government or parent banks, while a large number of firms cannot expect to be nationalized or receive other types of financial support to prevent them from going bankrupt. The second reason why allocating currency mismatch losses to firms generates larger real losses is that firms burdened with debt decrease aggregate output and demand directly, while banks affect aggregate economic activity with a lag and only after a share of the effect is absorbed by bank equity.

We arrive at the previously described conclusion after simulating the same set of shocks as before for the extended model. After the risk premium shock or the world demand shock, foreign currency debt worsens firms’ chances to repay which generates larger output losses, see Figure 2.8. More non-performing loans deplete bank equity on impact and make banks ration credit for future borrowers. Over time, as the default frequency for firms goes down, banks replenish bank equity and the recession is contained. On the contrary, if banks face currency mismatch on their balance sheets, bank losses are smaller on impact but, since banks cannot switch to foreign currency lending later, the depreciation has a persistent negative effect on bank equity. Bank losses translate into persistent real losses for two additional reasons: bank cut lending to all firms rather than just troubled firms which constrains economic activity severely. Second, since banks accumulate equity out of retained earnings, even temporary bank losses can have a persistent effect on borrowing conditions in the economy. Nevertheless, we see that in the case of capital outflows magnified foreign currency debt and the related failures to repay offset bank gains from insulating their balance sheets from the exchange rate risk. Consequently foreign currency loans make domestic depreciation deepen the recession.

The drop in world demand for domestic goods also suggests that currency mismatch shifted to banks produce smaller aggregate losses in the short-run, however, it may generate a situation when
currency mismatch in banking inflicts more recessionary outcomes in the future. However, this is not obvious. The volatility shock makes the default probability skyrocket and the role of foreign currency debt is limited in ranking the outcomes.

Simulations show that, in the first period after the shock, banks charge a substantially higher default spread. In subsequent periods changes in the default spread are approximately the same regardless of the allocation of currency mismatch losses. It follows that corporate default risk determines borrowing costs for firms in initial periods and the bank leverage constraint dominates the dynamics of costs further in the future.

In closing this section, it is important to note that the assumption of the financially constrained firms’ exit after two-periods makes the effect of the debt overhang friction rather suspended in time. In contrast, banks incorporate their net worth dynamics in their optimization problem which makes bank losses have a prolonged effect on the economy. This can be considered as a bias towards the banking friction. The result that debt overhang nevertheless governs the dynamics of aggregate variables in the extended model lends more support to the importance of currency mismatch losses in the corporate sector in amplifying negative shocks than our model could offer. Even though the government should not underestimate the effects of bank losses derived from currency mismatch on
the bank balance sheets, our simulations show that increasing currency mismatch for banks at the expense of lowering currency mismatch for borrowers is likely to result in lower macroeconomic losses.

### 2.5.3 Bank recapitalization

Shifting currency mismatch losses to banks reduces debt overhang and, as we showed before, leads to most likely less recessionary macroeconomic outcomes. However, this implies saving financially constrained firms at the expense of the banking sector. Further we study the efficiency of a government intervention that aims at compensating for bank losses. We study the scenario where bank losses stem from bearing the exchange rate risk while financially constrained firms avoid currency mismatch altogether.

Financial sector support is modeled as a gift from the government to banks given in the form of an equity injection. Consider the case of capital outflows which generated the largest economic downturn in the series of our experiments. Figure 2.9 shows how full recapitalization of the banking sector after the increase in the country risk premium immediately relaxes the endogenous bank
leverage constraint and improves bank borrowing conditions. Banks cut credit supply by less and the economy undergoes a smaller recession than otherwise. Corporate loans increase by less in response to this policy because corporate net worth is replenished faster than the investment demand increases. The reason is the following. Financially constrained firms take the size of their net worth as given, therefore, higher net worth makes them demand fewer loans. However, investment demand is late to catch up with the increase in corporate net worth, because firms make borrowing decisions given their expectations of net worth value rather than the actual value. This assumption creates a lag in the net worth feedback to firms’ working capital expenditure. Nevertheless, banks cut lending spreads as loans become less risky. Financial support of 20% bank equity would yield similarly positive but smaller changes in aggregate outcomes.

Therefore, currency mismatch in the banking sector can be efficiently alleviated ex-post. Note-worthy here we abstract from the potential negative implications of government interventions such as increasing public debt during times of fiscal distress (van der Kwaak and van Wijnbergen (2014)).

2.6 Conclusions

Hungary’s experience after the fall in the domestic currency value in 2009 raised questions about the macroeconomic implications of allocating currency mismatch losses. We attempt to evaluate the consequences of shifting exchange rate risk from borrowers to banks: we weigh losses triggered by increased currency mismatch in the financially constrained firms’ sector against losses for banks, if banks bear currency mismatch instead. As almost everywhere in Emerging Europe banks heavily rely on foreign currency debt. This borrowing pattern exposes banks to currency mismatch, unless they lend in foreign currency and thus shift exchange rate risk to borrowers. Empirical evidence suggests that the forint depreciation amplified debt overhang in the private sector in Hungary and banks operating in Hungary were leverage-constrained. Therefore, to answer the research question, we develop a small open economy New Keynesian DSGE model with debt overhang in the corporate sector and the banking sector that operates under the endogenous leverage constraint.
The model, calibrated to the Hungarian economy, suggests that debt overhang in the corporate sector and losses at leverage-constrained banks are closely related and reinforce each other through the channel of credit provision. Nevertheless, we determine that capital outflows can trigger domestic currency depreciation that is large enough to strengthen debt overhang in the corporate sector and generate a large recession. Debt overhang and the related real losses dominate alternative losses from placing currency mismatch on the bank balance sheets. The result stems from the high power of the debt overhang distortion which, if strengthened, affects private investment to a larger extent than tighter borrowing conditions for firms that would alternatively result from currency mismatch losses attributed to highly leveraged banks. Besides this, firms burdened with debt decrease aggregate output and demand directly, while banks affect aggregate economic activity with a lag and only after a share of the effect is absorbed by bank equity. The results suggest that shifting exchange rate risk from borrowers to banks is most likely to have a positive effect on the depth and length of a recession.

To contain currency mismatch losses in the banking sector, the government can resort to bank recapitalization. We show that currency mismatch in the banking sector can be efficiently alleviated ex-post by injecting bank equity.

Our model abstracts from long-term debt and fully-fledged effects of corporate net worth which would potentially make the effects of adverse shocks more persistent and strengthen debt overhang in the corporate sector. Nevertheless, we still find macroeconomic outcomes to be in favor of placing currency mismatch in the banking sector rather than shifting to credit constrained firms. This context offers more support for our conclusions.

Our result should serve as an additional argument for why bank should bear currency risk besides such advantages as easier coordination of a few troubled banks than thousands of insolvent borrowers and the fact that, in contrast to firms in Emerging Europe, banks can access foreign exchange markets for hedging purposes.
Chapter 3

The Macroeconomics of Carry Trade Gone Wrong: Corporate and Consumer Losses in Emerging Europe

3.1 Introduction

Currency mismatches that result from foreign currency liabilities exceeding foreign currency assets make borrowers vulnerable to unexpected currency depreciation. Moreover, currency positions in one sector are not isolated from currency mismatch in other sectors. Currency mismatch for borrowers is related to currency mismatch in the banking sector which creates a trade-off. Banks with foreign currency liabilities can decrease currency mismatch on their balance sheets by lending in foreign currency, but this increases exchange rate risk for their borrowers and in turn implies higher credit risk for the lending banks.

The trade-off between currency mismatch for borrowers and currency mismatch for banks came to the fore in Emerging Europe when some local currencies depreciated by more than 30 percent in the beginning of 2009. In this way, a loss of investors’ confidence in local currencies transmitted the financial crisis to this region. A prominent example is Hungary. Like many other Emerging
European countries Hungarian households and firms had extensively engaged in currency carry trade since 2002. The Hungarian forint depreciated sharply in 2009 and the Hungarian government responded with a foreign currency mortgage repayment scheme that shifted currency mismatch losses from vulnerable borrowers to banks.

The economic rationale behind the mortgage repayment scheme relied on the story of economic imbalances. Although banks in Emerging Europe were not highly leveraged and did not engage in risky financial activities, the Great Recession in Emerging Europe was particularly deep. Accumulated domestic imbalances have played a role in prolonging the recession and foreign currency debt was one of them. However, the potential threat of such a scheme to financial stability attracted attention of international institutions ECB (2011). Similar schemes implemented in Croatia and suggested in Romania and Poland (see Waldoch (2017a) and Waldoch (2017b)). None of these countries considered a shift of currency mismatch losses from corporate borrowers to banks although corporates were highly indebted in foreign currency too. This paper focuses on foreign currency carry trades gone wrong for households and firms. The paper distinguishes between corporate currency mismatch losses and household currency mismatch losses and asks whether shifting currency mismatch losses from households or firms to banks can lead to better macroeconomic outcomes.

In an earlier paper, we indicated that insulating indebted firms from exchange rate risk delivers better macroeconomic outcomes after currency depreciation (Jakucionyte and van Wijnbergen (2017a)). Lower credit risk for banks lending in domestic currency offsets bank losses from an open currency position. Household currency mismatch losses were at least equally relevant in several countries in Emerging Europe. Although countries’ experience with potential currency mismatch losses varied not only by size of accumulated foreign currency debt but also by borrower type, most of countries had high shares of foreign currency mortgages, see for example IMF (2009), p. 21. In December 2008 Hungarian households and businesses had more than 60 percent of total credit denominated in foreign currency. In Poland, however, foreign currency debt was mostly concentrated in the household sector and amounted to 40 percent of total household credit.
Low incidence of corporate foreign currency debt would explain why shifting corporate currency mismatch losses was not considered in Poland. However, it is not clear whether shifting household currency mismatch losses is beneficial from the macroeconomic perspective. Moreover, it is not clear whether shifting corporate currency mismatch losses was less efficient from a macroeconomic perspective than shifting household currency mismatch losses in countries where both households and firms engaged in carry trade.

We argue that not only the magnitude of currency mismatch but also the identity of indebted sector can lead to different macroeconomic outcomes and suggest possibly different policies in terms of allocating foreign currency losses ex-post. Households in Emerging Europe were not likely to experience debt overhang as indicated in Brown and Lane (2011), but a prolonged decline in investment in some countries pointed to likely debt overhang for corporate borrowers. Therefore, we assume debt overhang for production firms only which implies that household debt would not distort household incentives to work. Also, differently from corporate debt, household debt does not affect aggregate supply directly. Moreover, elevated household debt inflicts consumption losses but, in an open economy, lower consumption means that households consume not only fewer domestic goods but also fewer imported goods. Therefore, household losses may have a lower effect on domestic output than corporate losses.

This paper models a small open economy where firms, households and leveraged banks borrow in foreign currency, creating several currency mismatches. To generate more realistic responses to shocks, we also introduce nominal price and wage rigidities, so we build a medium-scale New Keynesian DSGE model. The paper uses the Hungarian recession as a motivation for different financial inefficiencies in the model. Further, we discuss each of the financial frictions in detail.

Sluggish investment recovery in Emerging Europe raised the question of debt overhang in the corporate sector. When, at the end of 2008, the Hungarian forint lost more than 30 percent of its value against the euro and the Swiss franc, this created a situation of suddenly magnified corporate debt burden with implications for firms’ performance at both the micro and macro
levels\(^1\). Moreover, firms in Hungary rarely had access to natural hedges as confirmed by Endrész et al. (2012) and Bodnár (2012), which left them exposed to exchange rate risk. Default rates rose and firms cut investment. In 2015 investment level in Hungary was still below its level in 2005. The information friction that describes the debt burden effects and the resulting long and deep recessions is non-contractability. We assume non-contractibility for corporate debt and introduce the debt overhang distortion in this model to explain investment dynamics. When banks cannot contract on the level of investment, highly indebted firms choose to underinvest, because much of their investment value would be seized by creditors. The idea that risky debt makes firm reject valuable investment opportunities goes back to Myers (1977). Its importance in explaining the economic recovery in Europe was tested empirically in Kalemli-Özcan et al. (2015).

The lack of evidence in favor of debt overhang in the household sector as discussed in Brown and Lane (2011) motivates the choice of a different type of financial friction for indebted households. The amplification of adverse shocks through household net worth without distortions of labor supply or house investment decisions seems more appropriate given empirical evidence. The external finance premium for household mortgages in this model occurs because of the costly state verification problem as in Bernanke et al. (1999). The assumption is that banks do not observe housing quality shocks, but have to pay deadweight monitoring costs on defaulted mortgages, so they raise mortgage rates for all household borrowers ex-ante, resulting in a default premium on mortgage rates. The default premium is positively related to household leverage.

Bank losses can impair credit provision, if bank funding costs depend on bank performance. The banking system in Hungary was well-capitalized in 2008 (IMF (2008)), however, liquidity shocks at the outbreak of the crisis affected bank funding costs Bakker and Klingen (2012). The sudden dry-up of foreign funding caused a tightening of leverage constraints. To capture this channel, we introduce market-value bank leverage constraints. We model it as an agency problem between banks and depositors following Gertler and Karadi (2011). Lower bank equity increases the moral hazard problem and bank funding costs which translates into lower bank credit supply.

\(^1\) For a longer overview of the evidence on financial distress of Hungarian firms that borrowed in euros or Swiss francs and whether that could have had macroeconomic implications, see Jakucionyte and van Wijnbergen (2017a).
Empirical evidence in favor of currency mismatch driving changes in credit supply in Emerging Europe is reported in Fidrmuc and Kapounek (2016). Foreign banks that relied on foreign funds more responded to currency depreciation by cutting lending more than domestic banks.

The model resembles the currency mismatch situation in Hungary in the beginning of 2009, so we estimate the model on Hungarian data 2000:Q1-2016:Q3 using Bayesian estimation techniques to assess the chosen financial frictions. We first test the relevance of the debt overhang friction. We incorporate debt overhang applying a variant of Merton’s risk pricing model in Merton (1974) with predetermined debt as in Myers (1977). We compare such a model to the model that assumes costly state verification friction for corporate debt in a spirit of Bernanke et al. (1999). The model fit to Hungarian data is better with debt overhang suggesting that indeed a sluggish recovery is better explained by corporate debt overhang than rather debt cost factors in Bernanke et al. (1999). Introducing household debt improves model fit too. The model fit is better if we model indebted households rather than only saving households, suggesting a large role of household debt in explaining aggregate fluctuations in Hungary.

To gain insights into the currency mismatch losses allocation problem, we take the model with corporate debt overhang, household debt and leveraged banks and construct different scenarios dependent on which sector is assumed to face currency mismatch and compare aggregate outcomes after unexpected currency depreciation. Model simulation shows that shifting currency mismatch losses to banks has different implications dependent on whether households or production firms borrow in foreign currency. Our findings confirm potential gains from shifting currency mismatch losses from firms to banks. Domestic currency loans reduce corporate default risk so that incentives to invest become less distorted and boost economic activity. The boost is sufficient to counteract a fall in credit supply which comes from bank currency mismatch losses. Household currency mismatch losses, however, appear to have relatively small effects on output. If only household borrowers faced currency mismatch but production firms did not, shifting currency mismatch losses from banks to household borrowers leads to better macroeconomic outcomes. Better macroeconomic outcomes occur despite the elevated credit risk for foreign currency mortgages and larger
borrowers’ consumption losses after unexpected depreciation. Larger consumption losses appear insufficient to change the ranking of output outcomes for several reasons. For one reason, housing is a non-productive asset in our model and thus does not affect aggregate supply directly. Second, lower consumption has a smaller effect on domestic output than a drop in demand for production inputs because production inputs are less import-intensive than consumption. Although to produce one unit of investment goods takes the same proportion of imported goods as for consumption goods, corporate losses affect labor demand too not only investment. So when corporate losses weaken demand for labor, this is not offset by movements in trade balance, in contrast to a drop in consumption. Third, a high degree of consumption risk sharing in the model makes household debt generate relatively lower amplifications of aggregate fluctuations than one may expect given the empirical findings in for instance Mian et al. (2013). Given these characteristics of household debt, valuation gains from foreign currency mortgages for banks and thus higher credit supply offset output losses from elevated credit risk for foreign currency mortgages.

To sum up, simulation results suggest that shifting corporate losses back to banks would have mitigated the recession more effectively than shifting household losses. This is the opposite of what the Hungarian government actually did and also raises doubt about the efficiency of similar schemes implemented in Croatia and suggested in Romania and Poland (see Waldoch (2017a) and Waldoch (2017b)).

The structure of the paper is as follows. Section 3.2 presents related literature. We discuss the model in detail in section 3.3, and describe model parametrization, data and Bayesian estimation results in section 3.4. The discussion of the losses allocation problem is in section 3.5. Section 3.6 concludes.

**Related literature**

This paper is at the intersection of several strands of literature. We first borrow from the debt overhang framework in Myers (1977) that is a seminal paper in corporate finance. Second, this
paper relates to household default literature and especially papers that analyzed household default from the business cycles perspective. Third, there is a wide literature on the effects of liability dollarization. Below we briefly discuss related papers by first classifying them by common fields.

The idea that overindebted firms reject investment opportunities with a positive net present values goes back to Myers (1977). There are a few studies that provide different reasons for why renegotiation fails, for instance, Jensen and Meckling (1976), Hart and Moore (1995) and Bhattacharya and Faure-Grimaud (2001), to name a few. In our paper we assume that renegotiation is infeasible as well which is a necessary assumption for debt overhang, but we do not specify the reason behind the renegotiation failure.

There a few macroeconomic studies that laid ground for analyzing the interactions of debt overhang in different sectors of the economy. Lamont (1995) shows how self-fulfilling pessimistic expectations of individual investors can create multiple equilibria if levels of debt are high. Philippon (2010) also finds multiple equilibria but as a consequence of debt overhang in households and banks. Moreover, he finds that bailing out households is inefficient, in contrast to bailout out banks. This finding relies on the model assumption that households own the banks. So bank bailout decreases bank debt overhang and increases aggregate investment but additional taxes do not take away resources from households.

Several studies analyze debt overhang in the context of nominal debt. Gomes et al. (2016) finds that unanticipated deflation worsens debt overhang for corporate borrowers. So monetary policy with a purpose of alleviating debt overhang should aim at increasing inflation. Long term debt is a necessary condition for debt overhang in their paper. Occhino and Pescatori (2014) show that, in the presence of debt overhang, the balance sheet channel of monetary policy is important in determining the optimal monetary policy. However, none of these studies look at currency depreciation and magnified foreign currency debt value.

Foreign currency debt effects over a business cycle are partially offset by the expenditure switching effect. So studies that analyze liability dollarization in episodes of sudden stops account for nominal price rigidities. Examples include Cook (2004), Céspedes et al. (2004), Devereux et al.
(2006) and Gertler et al. (2007b). All of them, except for Cook (2004), argue that depreciations increase output even in the presence of sizable foreign currency debt. Cook (2004) arrives at the opposite conclusion because, differently from other papers, he assumes sticky output prices but flexible input prices. Then revenues of corporate borrowers with foreign currency liabilities do not increase as fast as input costs and the suddenly magnified value of debt can reduce their profits and depress macroeconomic outcomes. All of these studies assume a costly state verification problem for lenders as proposed in Townsend (1979) and use the modeling framework first implemented in Bernanke et al. (1999). Differently from corporate debt overhang, in a costly state verification framework lenders bear deadweight monitoring costs which increases borrowing costs for corporates, however, lenders can contract the level of investment. This implies that borrowers’ incentives do not get distorted and borrowers cut investment in response to higher costs only. As shown in Occhino and Pescatori (2015), these different assumptions also have implications for the quantitative effects of business cycle models. Debt overhang amplifies aggregate fluctuations more than the monitoring costs friction in Bernanke et al. (1999).

As for the household default, we come the closest to a real business cycles model with household default in Clerc et al. (2011). Clerc et al. (2011) feature household default, while the rest of the literature that analyzes the role of housing over a business cycle follows the tradition of Iacoviello (2005) and assumes household borrowing constraints. Similarly to the pioneering study Iacoviello (2005) we assume two types of households to accommodate household borrowing and saving in the equilibrium, however, then we follow Clerc et al. (2011) and assume household default. Household default in inefficient because of monitoring costs as in Bernanke et al. (1999). Both this paper and Clerc et al. (2011) assume a high degree of consumption risk sharing to facilitate aggregation of household net worth and tracking its dynamics across time. In contrast to this study, we assume different financial frictions for other participants of the credit market, firms and banks.

Incomplete markets literature, which analyzes household mortgage default, relaxes the assumption of consumption risk sharing. However, such studies mostly focus on analyzing government policies for housing markets (e.g. Jeske et al. (2013)) or the causes of the foreclosure crisis, e.g.
Chatterjee and Eyigungor (2015), Corbae and Quintin (2015), Garriga and Schlagenhauf (2009), etc. Aggregate uncertainty was introduced a far smaller number of studies and even then they focused on consumer default rather than mortgage default, thus, housing price dynamics were absent. For instance, Nakajima and Rios-Rull (2005) examine the effect of bankruptcy on amplifying aggregate shocks. Gordon (2015) evaluates the consequences of eliminating or restricting default in the presence and finds that aggregate risk substantially reduces welfare gains from eliminating default. Although these studies confirm the significant role of household debt in the presence of aggregate risk, however, the numerical apparatus for solving incomplete market models still lacks to address mortgage debt in a small open economy with a wide range of agents and nominal rigidities.

3.2 Model

The model is at the core a standard New Keynesian DSGE model, but introduces three types of financial frictions. Differently from Clerc et al. (2011), we do not apply a monitoring costs friction to all sectors, but rather select frictions based on empirical evidence. Household debt is priced at a higher rate due the default premium. To model the default premium, we choose the monitoring costs friction in a spirit of Bernanke et al. (1999) instead of debt overhang as in Myers (1977) due to the lack of empirical evidence in favor of household debt overhang in Emerging Europe. The default premium on household mortgage debt follows the modeling framework suggested in Bernanke et al. (1999). Due to large and persistent investment losses in Emerging Europe, corporate borrowers face an implicit debt overhang tax on investment as in Myers (1977) and in the later application to business cycles models in Occhino and Pescatori (2015). To make bank losses matter for aggregate outcomes, we introduce bank leverage constraints as in Gertler and Karadi (2011). Sectors that are modeled in a non-standard way are discussed before other segments of the model.
3.2.1 Households

Simultaneous occurrence of household borrowing and household deposits requires household heterogeneity, so we introduce two types of households. Households with a higher discount factor $\beta_p$ belong to the dynasty of patient households as opposed to the dynasty of impatient households ($\beta_I < \beta_P$). Patient households save. Impatient households borrow. Each dynasty has a continuum of measure-one members who are identical ex-ante and differ in idiosyncratic shocks $\omega_t$ ex-post. If the ex-post debt value exceeds the non-exempt collateral value, impatient households default on their loans.

3.2.2 Patient households

Every member of the dynasty of patient households supplies labor of unique type $i$ which gives households monopolistic power in wage setting and allows us to introduce nominal rigidity for wages. First, we solve the dynasty’s problem with respect to other choices the dynasty makes and later we elaborate on the monopolistic labor market and the modified first order condition for labor supply and wages.

In the beginning of each period the net worth of patient households is pooled resulting in complete intergenerational risk sharing. Given the pooled net worth, the patient dynasty chooses consumption $c^p_t$, housing stock $h^p_t$, and how much to save by putting deposits $d_t$ and buying government bonds $b_t$ such that her utility is maximized:

$$\max_{\{c^p_t, h^p_t, b_t, d_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \beta^p_t \right)^t v_t \left( \log \left( c^p_t \right) + A_h \log \left( h^p_t \right) - A_n \left( n^p_t \right)^{1+\sigma_n} \right)$$

where $v_t$ is an exogenous preference shock. The dynasty maximizes utility subject to the budget constraint:

$$c^p_t + q^h_t \left( h^p_t - h^p_{t-1} \right) + b_t + d_t \leq w^p_t n^p_t + \frac{R_{t-1}}{\pi_t} \left( b_{t-1} + d_{t-1} \right)$$

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$w_t^P$ is the real aggregate wage for patient households, $q_t^h$ is the real housing price and $n_t^P$ is labor demand. This is an aggregate budget constraint, therefore, labor types do not show up separately, although it can be shown that an individual budget constraint leads to the aggregate budget constraint. In deriving optimal wages, we will focus on the individual utility. Also after deflating variables we get composite consumption goods inflation $\pi_t$. $R_t$ denotes the domestic gross interest rate. First-order conditions follow:

\[ c_t^P : \quad \lambda_t^P = v_t \left( c_t^P \right)^{-\sigma_c} \]
\[ h_t^P : \quad v_t A_h \left( h_t^P \right)^{-\sigma_h} = \lambda_t^P q_t^h - \beta_t^P E_t \lambda_{t+1}^P q_{t+1}^h \]
\[ b_t : \quad E_t \beta_t^P \frac{\lambda_{t+1}^P}{\lambda_t^P} \frac{R_t}{\pi_{t+1}} = 1 \]
\[ d_t : \quad E_t \beta_t^P \frac{\lambda_{t+1}^P}{\lambda_t^P} \frac{R_t}{\pi_{t+1}} = 1 \]

$\lambda_t^P$ is the Lagrange multiplier to the budget constraint.

Only a share $(1 - \omega^W)$ of households will be allowed to adjust their wages in period $t$. This assumption leads to a utility maximization problem with respect to labor in which households equate their marginal disutility from work to marginal benefits (nominal wage $W_t^P$) for every household taking into account the probability of not being able to adjust wages in the future:

\[
\max_{\{W_t^P, i\}} \mathbb{E}_t \sum_{k=0}^{\infty} \left( \beta_t^P \omega_t^W \right)^k U_t^P \left( c_{t+k,i}^P, h_{t+k,i}^P, n_{t+k,i}^P \right) 
\]

subject to labor demand of type $i^2$:

\[
n_t^P = \left( \frac{W_{t,i}^P}{W_{t+k}^P} \right)^{-\epsilon_W} n_{t+k}^P
\]

---

\(^2\) Labor demand can be derived from a perfectly competitive labor packer’s problem $\max w_t^P l_t - \int_0^1 w_t^P n_t^P d_i$. 

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where \( n_t^P = \left( \int_0^1 \left( n_{t,i}^P \right) \frac{\epsilon_{W} - 1}{\epsilon_{W}} \, di \right)^{\frac{\epsilon_{W}}{\epsilon_{W} - 1}} \) is aggregate labor supply of patient households with labor supply elasticity among different types of labor \( \epsilon_{W} \) and \( W_t^P \) is aggregated nominal wage \( W_t^P = \left( \int_0^1 \left( W_{t,i}^P \right) \frac{1}{\frac{1}{\epsilon_{W}}} \, di \right)^{\frac{1}{\epsilon_{W}}}. \)

This yields a first order condition for the optimal wage:

\[
\mathbb{E}_t \sum_{k=0}^{\infty} \left( \beta_t^P \omega^W \right)^k \left( U_c^P \left( 1 + \frac{\partial n_{t+k,i}^P}{\partial W_{t,i}^P} W_{t,i}^P \right) \frac{W_{t,i}^P}{P_{t+k}} + U_n^P \frac{\partial n_{t+k,i}^P}{\partial W_{t,i}^P} W_{t,i}^P \right) = 0
\]

where

\[
\frac{\partial n_{t+k,i}^P}{\partial W_{t,i}^P} = -\epsilon_{W} \left( \frac{W_{t,i}^P}{W_{t+k}^P} \right)^{-\epsilon_{W} - 1} \frac{n_{t+k}^P}{W_{t+k}^P}
\]

The optimal nominal wage \( W_{t}^{P^*} \) is given by

\[
\left( \frac{W_{t}^{P^*}}{P_{t+k}} \right)^{1+\epsilon_{W}\sigma_{n}} = \frac{\epsilon_{W}}{\epsilon_{W} - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \left( \beta_t^P \omega^W \right)^k \left( A_n \left( W_{t+k}^P \right)^{\frac{\epsilon_{W}(1+\sigma_{n})}{\epsilon_{W}}} \left( n_{t+k}^P \right)^{(1+\sigma_{n})} \right)}{\mathbb{E}_t \sum_{k=0}^{\infty} \left( \beta_t^P \omega^W \right)^k \left( \left( n_{t+k}^P \right)^{\frac{\epsilon_{W}}{1+\sigma_{n}}} \left( c_{t,i}^P \right)^{-\sigma_{c}} \right)}
\]

### 3.2.3 Impatient households

In the beginning of every period the net worth of defaulted households is pooled with the net worth of non-defaulted households which implies complete intergenerational consumption risk sharing. However, the dynasty is not liable for the unpaid debt. Given the pooled net worth, the dynasty of impatient household maximizes utility of her members with respect to consumption \( c_t^I \), housing stock \( h_t^I \), how much to borrow from the bank \( (m_t) \) and what is the optimal default threshold \( \bar{\omega}_t \).

The debt variables define the optimal debt contract as in Bernanke et al. (1999). We provided an extensive solution to the household’s utility optimization problem and an optimal debt contract in the appendix B.1.
The household $i$ finds it optimal to default, if the collateral value is lower than the debt value. Given that a fixed fraction $\alpha^{FM}$ of the mortgage is in foreign currency, the household defaults if:

$$\omega_{i,t} \left( \frac{\xi^h q^h_t (1 - \delta^h) h^I_{i,t-1}}{\xi^h q^h_t (1 - \delta^h) h^I_{i,t-1}} \right) \leq R^M_{t-1} \left( \frac{\alpha^{FM} \text{rer}_t}{\pi_t^*} + \frac{1 - \alpha^{FM}}{\pi_t} \right) m_{i,t-1}$$

(3.1)

The left hand side of the inequality gives the value of the collateral (housing), which is affected by the housing price $q^h_t$, an idiosyncratic shock $\omega_t$ and the depreciation rate $\delta^h$. Parameter $\xi^h$ is an exogenous loan-to-value ratio. The right hand side describes the value of debt. $R^M_t$ is the nominal gross mortgage interest rate, rer$_t$ denote the real exchange rate and $\pi_t$ and $\pi_t^*$ denote consumer goods inflation in the domestic economy and the foreign inflation respectively. Thus, exchange rate depreciation boosts the debt value and has a positive effect on the default threshold increasing the default rate among indebted households.

The default threshold is given by the household-specific shock value $\omega_t$ such that:

$$\bar{\omega}_{i,t} \equiv \omega_{i,t} = \frac{R^M_{t-1} \left( \frac{\alpha^{FM} \text{rer}_t}{\pi_t^*} + \frac{1 - \alpha^{FM}}{\pi_t} \right) m_{i,t-1}}{\xi^h q^h_t (1 - \delta^h) h^I_{i,t-1}}$$

(3.2)

We can define the fraction of defaulted impatient households as $G_t$ and the fraction of household net worth attributed to the bank as $\Gamma_t$ such that the bank on average gets $\Gamma_t \xi^h q^h_{t+1} h^I_t$. Then the impatient dynasty maximizes utility by choosing consumption, housing, level of mortgage debt and leverage subject to the budget constraint of the dynasty:

$$c^I_t + q^h_t h^I_t \leq w^I_t n^I_t + (1 - \xi^h \Gamma_t) q^h_{t+1} h^I_{t-1} + m_t$$

(3.3)

and the participation constraint of the bank:

$$E_t \left[ (\Gamma_{t+1} - \mu_H G_{t+1}) \xi^h q^h_{t+1} h^I_t \right] = E_t \frac{R^M_t}{\pi_{t+1}} - m_t$$

(3.4)
where $E_t \bar{R}_t^M$ is the nominal expected return to the bank. Parameter $\mu_H$ defines monitoring costs incurred by banks. The creditor has to bear monitoring costs because a share of collateral gets lost in the spirit of the costly state verification literature starting with Townsend (1979) and Bernanke et al. (1999). The participation constraint ensures that the fraction of household net worth attributed to the bank, net of monitoring costs, is in expectation equal to bank’s expected returns on mortgages.

It can be shown, that first-order conditions are given by:

\begin{equation}
 c^I_t : \quad \lambda^I_t = v_t \left( c^I_t \right)^{-\sigma_e} 
\end{equation}

\begin{equation}
 h^I_t : \quad v_t A_h \left( h^I_t \right)^{-\sigma_h} = \lambda^I_t q^h_t - \beta^I E_t \lambda^I_{t+1} (1 - \zeta^h \Gamma_{t+1}) q^h_{t+1} - \Omega_t E_t \left( \Gamma_{t+1} - \mu_H G_{t+1} \right) \zeta^h q^h_{t+1} 
\end{equation}

\begin{equation}
 \bar{\omega}_{t+1} : \quad \beta^I E_t \lambda^I_{t+1} (\Gamma_{t+1})' \left( \zeta^h q^h_{t+1} h^I_t \right) = \Omega_t E_t \left( (\Gamma_{t+1})' - \mu_H (G_{t+1})' \right) \left( \zeta^h q^h_{t+1} h^I_t \right) 
\end{equation}

\begin{equation}
 m_t : \quad \Omega_t E_t \frac{\bar{R}_t^M}{\pi_{t+1}} = \lambda^I_t 
\end{equation}

The first-order conditions hold together with two slackness constraints:

\begin{equation}
 \lambda^I_t \left( w_t n_t^I + (1 - \zeta^h \Gamma_t) q^h_t h^I_{t-1} + m_t + t_t - c^I_t - q^h_t h^I_t \right) \geq 0, \quad \lambda^I_t \geq 0 
\end{equation}

\begin{equation}
 \Omega_t \left( (\Gamma_{t+1} - \mu_H G_{t+1}) \zeta^h q^h_{t+1} h^I_t - E_t \frac{\bar{R}_t^M}{\pi_{t+1}} m_t \right) \geq 0, \quad \Omega_t \geq 0 
\end{equation}

$\lambda^I_t$ and $\Omega_t$ are the Lagrange multiplier to the budget constraint and the bank’s participation constraint respectively.

Impatient households similarly to patient households have monopoly power in a labor market and can set wages. Similarly to patient households, we can show that optimal nominal wage $W^{I*}_t$ for impatient households equals:

\begin{equation}
 \left( \frac{W^{I*}_t}{P_{t+k}} \right)^{1+\omega \sigma_n} = \frac{\epsilon_W}{\epsilon_W - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta^I \omega^W)^k \left( A_n \left( W^{I*}_{t+k} \right)^{\epsilon_W (1+\sigma_n)} \left( n^I_{t+k} \right)^{(1+\sigma_n)} \right)}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta^I \omega^W)^k \left( \left( W^{I*}_{t+k} \right)^{\epsilon_W} n^I_{t+k} \left( c^I_{t,k} \right)^{-\sigma_e} \right)} 
\end{equation}
3.2.4 Production and Pricing

There are several types of firms in the domestic economy. First, there are the financially constrained firms that combine capital with labor to produce homogeneous goods. Second, their homogeneous outputs are bought by retail firms that costlessly differentiate purchased goods and sell them as (local) monopolists. A similar group of firms called importers differentiate foreign (imported) goods. Third, a composite goods producer buys the differentiated home goods and aggregates them into an aggregate domestic good \( y^H_t \) with associated price \( p^H_t \). The same composite goods producer also buys imported differentiated goods and aggregates them into a foreign aggregate good \( y^F_t \). The corresponding aggregate price level of foreign goods is \( p^F_t \). Finally, there are two different aggregation technologies for consumption composite goods and for investment composite goods. Each of them aggregates domestic aggregate goods with foreign aggregate goods to produce composite goods.

Since most of the layers of the production sector are standard, we discuss financially constrained firms below and leave the rest for Appendix B.3.

3.2.5 Financially constrained firms

Debt overhang can explain a prolonged slump in investment in Hungary because accumulated debt would diminish firms’ incentive to produce for a prolonged time. To introduce debt overhang in the model, we assume non-contractibility. Banks cannot contract upon the level of production inputs and firms can underinvest whenever their private benefits from production decrease below the social ones due to classic debt overhang reasons a lát Myers (1977). Financially constrained firms is the set of firms in our model that face debt overhang. Appendix B.2 provides the exhaustive transformation of nominal variables to real variables and the solution to firm’s optimization problem. Below, we provide a concise description.

Financially constrained firms live for two periods. Every period there is a new-born generation of firms and the total number of firms always constitute a continuum of mass one. The sequence of
firms’ decisions is presented in Table 3.1. In the first period firms buy two types of inputs, capital $k$ and labor $n$, and have to pay for it in advance, while production takes place in the next period.

To pay in advance, a financially constrained firm $i$ borrows from the bank an amount $l_{i,t}$ that consists of both domestic currency funds $l_{i,t}^D$ and foreign currency denominated funds $l_{i,t}^F$ such that $l_{i,t} = l_{i,t}^D + rer_t l_{i,t}^F$, where $rer_t$ is the real exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by $\alpha^{FF}$, so that the firm can choose the size of the total loan but not denomination.

We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital: $E_{t-1} \{l_{i,t}\} = E_{t-1} \{qt k_{i,t} + w_t n_{i,t}\}$. $q_t$ and $w_t$ denote the real price of capital and the real wage respectively. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm’s private incentives to invest in production inputs. The actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases households step in and transfer lump-sum funds $z_{i,t}$ to cover the difference.

Let the matured loan in units of composite goods be $R_{i,t}^L \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}} \right)$, where $R_{i,t}^L$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks.
take place, therefore, the loan rate adjusts to clear the loan market. To borrow, the firm has to pledge a share $\kappa$ of future revenue as collateral where $0 < \kappa \leq 1$. Then the contracted collateral is a fraction $\kappa$ of firms’ revenue from selling goods and depreciated capital in the next period, $p_{t+1}L_i y_{t+1}^L + q_{t+1}(1 - \delta)k_{i,t}$. $p_{t+1}^L$ stands for the price of homogeneous goods, expressed in units of composite goods. The decision of the financially constrained firm $i$ born in period $t$ whether to default or not is determined by the lower value:

$$\min \left\{ R_{i,t}^L \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}^*} \right), \kappa \left( p_{t+1}^L y_{t+1}^L + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}$$ (3.12)

The firm produces its output by using the technology $y_{i,t+1}^L = z_{t+1} + \theta_{i,t+1} k_{i,t}^{\alpha} \left( A_{t+1} n_{i,t} \right)^{1-\alpha}$ where $z_{t+1}$ is a stationary aggregate technology shock, $A_{t+1}$ is a non-stationary aggregate technology shock and $\theta_{i,t+1}$ is a stationary idiosyncratic technology shock.

The firm $i$ born in period $t$ maximizes profits taking the loan as given. The firm maximizes the expected sum of future revenue from selling goods and depreciated capital subtracted by the debt payment. Financial flows received in period $t$ also enter the maximization problem and can be summarized as the difference between the loan plus equity ($z_{i,t}$) and working capital expenditure:

$$\max_{\{l_{i,t}, n_{i,t} \}} \{ p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta)k_{i,t} \}$$

$$- E_t \beta^P \left\{ N_{i,t+1}^P \left( \frac{l_{t,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{t,t}^F}{\pi_{t+1}^*} \right), \kappa \left( p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}$$

$$+ l_{i,t} + z_{i,t} - (q_{t_1} k_{i,t} + w_{i,t} n_{i,t})$$

s.t.

$$E_{t-1} \{ l_{i,t} \} = E_{t-1} \{ q_{t} k_{i,t} + w_{i,t} n_{i,t} \}$$ (3.13)

The resulting first-order conditions are$^3$:

$^3$The derivation of the first-order conditions and the term $d_{2,t}$ in particular are provided in Appendix B.2.1-B.2.2.
\[ k_{i,t} : \quad E_t \beta \Lambda_{t,t+1}^P \left\{ p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \right\} \]

\[ - E_t \beta \Lambda_{t,t+1}^P \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \right) \right\} \]

\[ \frac{\partial \text{cov} \left( \beta^P \Lambda_{t,t+1}^P, \min \left\{ R_{t,t}^L \left( \frac{i_{D,t+1}}{\pi_{t+1}} + rer_{t+1} \frac{i_{F,t}}{\pi_{t+1}} \right), \kappa \left( p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta) k_{i,t} \right) \right\} \right)}{\partial k_{i,t}} \]

\[ = q_t \quad (3.14) \]

\[ n_{i,t} : \quad E_t \beta \Lambda_{t,t+1}^P \left\{ p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial n_{i,t}} \right\} \]

\[ - E_t \beta \Lambda_{t,t+1}^P \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial n_{i,t}} \right) \right\} \]

\[ \frac{\partial \text{cov} \left( \beta^P \Lambda_{t,t+1}^P, \min \left\{ R_{t,t}^L \left( \frac{i_{D,t+1}}{\pi_{t+1}} + rer_{t+1} \frac{i_{F,t}}{\pi_{t+1}} \right), \kappa \left( p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta) k_{i,t} \right) \right\} \right)}{\partial n_{i,t}} \]

\[ = w_t \quad (3.15) \]

where

\[ d_{2,t} \equiv \frac{E_t \ln \left( \kappa \left( p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta) k_{i,t} \right) - R_t^L rer_{t+1} \frac{i_{D,t+1}}{\pi_{t+1}} \right) - E_t \ln \left( R_{t,t}^L \frac{i_{D,t+1}}{\pi_{t+1}} \right)}{\sigma_{F,t}} \quad (3.16) \]

\[ \text{and} \quad d_{1,t} = d_{2,t} + \sigma_{F,t} \quad (3.17) \]

The debt overhang friction introduces an additional term in otherwise standard demand functions for capital and labor: conditions incorporate a proxy for the default probability, \((1 - \Phi(d_{1,t}))\), that reduces a marginal product of capital and a marginal product of labor. Thus in this problem the default probability is what drives the wedge between social benefits from investing and private
benefits from investing. When the default probability increases, private benefits would diminish and demand for labor and capital would shrink resulting in a lower level of working capital than a socially optimal one. Under-investment in working capital has negative and prolonged implications on aggregate variables. Static debt overhang results from an immediate decline in demand for working capital which depresses investment on impact. However, there is a dynamic effect of debt overhang as well, because lower investment today decreases capital stock available for production tomorrow.

The second implication of the first-order conditions relates to the option structure as reflected by the definition of the function argument $d_{2,t}$. The default probability directly depends on a volatility term $\sigma^2_{F,t}$ which captures the variance of future profits. The first-order conditions imply that increased uncertainty about of future collateral value reduces firms’ chances to repay.

### 3.2.6 Banks

Bank losses can have macroeconomic implications if bank funding costs depend on the bank’s equity position. To achieve that we introduce endogenous bank leverage constraints. We assume that banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction $\lambda_t^B$ of assets, but if that happens the bank goes bankrupt (i.e. cannot continue).\(^4\) Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is equal to or higher than the value of what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet of the bank for given amount of equity.

Domestic households own all banks that operate in the domestic economy and lend to financially constrained domestic firms. We assume that there is a continuum of these banks and every period there is a probability $\omega^B$ that a bank continues operating. Otherwise, the net worth is transferred to the owners of the bank, domestic households. We assume that banks give loans to firms and households out of accumulated equity $n_t^B$, domestic deposits $d_t$ and foreign debt $d_t^*$. A fraction

\(^4\) $\lambda_t^B$ is an exogenous variable that can be affected by a stationary autoregressive process.
of banks’ liabilities (foreign debt) is denominated in foreign currency which exposes banks to currency mismatch. Lending in foreign currency hedges the open currency position for banks. However, shifting exchange rate risk to borrowers increases the credit risk for banks. We consider two lending scenarios which have different implications for bank currency mismatch. First, banks lend in domestic currency only which creates currency mismatch on their balance sheets. The second scenario is described by bank lending in both foreign currency and domestic currency so that banks are relieved from currency mismatch. We will consider these two cases in the following discussion on shifting currency mismatch. The model with loans denominated in both currencies is described here, while the model with lending in domestic currency only is described in Appendix B.6.2.

The balance sheet constraint of a bank $j$, expressed in units of composite goods, is given by

$$n_{j,t}^B + d_{j,t} + rer_d^*j_t = l_{j,t} + m_{j,t}$$  (3.18)

Banks pay a nominal domestic interest rate $R_t$ on deposits and a nominal foreign interest rate $R_t^*\xi_t$ on foreign debt. $R_t^*$ follows a stationary AR(1) process. $\xi_t$ denotes a premium on bank foreign debt such that:

$$\xi_t = \exp\left(\kappa_\xi \frac{(rer_d^*A_{t-1} - rer \cdot d^*)}{rer \cdot d^*} + \frac{\zeta_t - \zeta}{\zeta}\right)$$  (3.19)

$\zeta_t$ is an exogenous shock that follows a stable AR(1) process.

Corporate loan performance directly affects bank profits. When the default probability $(1 - \Phi(d_{2,t}))$ for financially constrained firms increases, banks expect lower returns. We define the expected return for the bank $j$ as $\tilde{R}_{j,t}^L$. The definition makes use of the derivation of the expected loan payment (see Appendix B.2.2) and directly incorporates the default probability on corporate loans:
\[ E_t \left\{ \frac{\tilde{R}_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \min \left\{ R_{j,t}^L \left( \frac{l^D_{j,t}}{\pi_{t+1}} + rer_{t+1} \frac{l^F_{j,t}}{\pi_{t+1}} \right), \kappa \left( \rho_{t+1}^L y_{j,t+1} + q_{t+1} (1 - \delta) k_{j,t} \right) \right\} \]

Or

\[ E_t \left\{ \frac{\tilde{R}_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left( \rho_{t+1}^L y_{j,t+1} + (1 - \delta) q_{t+1} k_{j,t} \right) + \Phi(d_{2,t}) R_{j,t}^L \frac{l^D_{j,t}}{\pi_{t+1}} + \Phi(d_{1,t}) R_{j,t}^L rer_{t+1} \frac{l^F_{j,t}}{\pi_{t+1}} \right\} \]

(3.20)

We know the expression for the expected return on mortgages \( \tilde{R}_{M}^L \) from the impatient households section. It is given by equation (B.6).

In the optimization problem of the bank \( j \) we use the stochastic discount rate of patient households because they own banks. Then the optimization problem can be written as:

\[ V_{j,t} = \max_{\{d_{j,t}, d_{j,t}^*, l_{j,t}, m_{j,t}\}} E_t \left[ (1 - \omega^B) n_{j,t+1}^B + \omega^B V_{j,t+1} \right] \]

s.t.

\[ V_{j,t} \geq \lambda_t^B (l_{j,t} + m_{j,t}), \quad \text{(Incentive constraint)} \]

\[ n_{j,t}^B + d_{j,t} + rer_{t} d_{j,t}^* = l_{j,t} + m_{j,t}, \quad \text{(Balance sheet constraint)} \]

\[ n_{j,t}^B = \frac{\tilde{R}_{t-1}^L m_{j,t-1}}{\pi_t} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R_{t-1}^L \xi_{t-1}}{\pi_t} rer_{t-1} d_{j,t-1}^* \quad \text{(LoM of net worth)} \]

The first-order conditions follow:

\[ d_{j,t} : (1 + \nu_{1,t}) \beta^P E_t \Lambda_{t+1}^P \left\{ (1 - \omega^B) + \omega^B v_{2,t+1} \right\} \left( \frac{R_t}{\Lambda_{t+1}} \right) = v_{2,t} \quad (3.21) \]
\begin{equation}
\begin{aligned}
d_{j,t}^* : & \quad (1 + \nu_{1,t}) \beta^P E_t \Lambda_{t,t+1}^P \left\{ (1 - \omega^B) + \omega^B \nu_{2,t+1} \right\} \left( \frac{R_{\bar{R}}^* \xi_t}{r}_{t+1} \right) = \nu_{2,t} \\
l_{j,t} : & \quad (1 + \nu_{1,t}) \beta^P E_t \Lambda_{t,t+1}^P \left\{ (1 - \omega^B) + \omega^B \nu_{2,t+1} \right\} \frac{\bar{R}_t^L}{\pi_{t+1}} = \lambda_t^B \nu_{1,t} + \nu_{2,t} \\
m_{j,t} : & \quad (1 + \nu_{1,t}) \beta^P E_t \Lambda_{t,t+1}^P \left\{ (1 - \omega^B) + \omega^B \nu_{2,t+1} \right\} \frac{\bar{R}_t^M}{\pi_{t+1}} = \lambda_t^B \nu_{1,t} + \nu_{2,t}
\end{aligned}
\end{equation}

\(\nu_{1,t}\) and \(\nu_{2,t}\) are the Lagrangian multiplier to the incentive constraint and the Lagrangian multiplier to the balance sheet constraint combined with the law of motion for equity, respectively.

Equations (3.21) and (3.22) govern the bank debt portfolio choice. Equation (3.21) presents the marginal cost to the bank from issuing one additional unit of deposits (the left hand side) in relation to the marginal benefit from increasing equity by one unit, \(\nu_{2,t}\) (the right hand side). The marginal cost from issuing one additional unit of foreign bank debt is compared to the marginal benefit from increasing equity on the right hand side of equation (3.22) and is adjusted for changes in the exchange rate value. The structure of these choice rules suggests that in equilibrium the bank has to be indifferent between taking deposits or issuing bank debt to foreign agents.

Equations (3.23) and (3.24) present the relation between the marginal benefit to the bank from issuing one additional unit of loans (the left hand side) and the marginal cost (the right hand side). We see that in equilibrium one additional unit of loans earns the discounted risk adjusted return on loans. Firstly, this return has to increase in the marginal cost from issuing bank debt to finance the expansion of the balance sheet, \(\nu_{2,t}\). Secondly, due to the endogenous bank leverage constraint, the risk adjusted bank return on loans also increases in the share of divertable assets \(\lambda_t^B\) and the marginal loss to the bank creditor in the case of asset diversion, \(\nu_{1,t}\). Both terms proxy for the marginal cost associated with the tighter incentive constraint. Moreover, the tighter leverage constraint increases the bank spread as well which translates into more credit tightening.
The first-order conditions hold together with complementary slackness conditions associated with the incentive constraint and the balance sheet constraint:

\[ v_{1,t} : \nu_{1,t} \left( V_{j,t} - \lambda_t^B \left( l_{j,t} + m_{j,t} \right) \right) = 0 \]

\[ \nu_{2,t} \left( \frac{R_t^L}{\pi_t} l_{j,t-1} + \frac{R_t^M}{\pi_t} m_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R_{t-1}^* \xi_{t-1} - \text{rer}_t^* d_{j,t-1} - \text{rer}_t^* l_{j,t} + d_{j,t} + \text{rer}_t^* d_{j,t} \right) = 0 \]

The set of equilibrium conditions includes the law of motion for aggregate net worth of banks. We assume that a fraction \((1 - \omega^B)\) of banks exit every period and are replaced by the same number of new banks. New banks get an equity transfer households equal to \(t^B n^B A_{t-1} \). Then aggregate bank net worth evolves as:

\[ n_t^B = \omega^B \left( \frac{R_{t-1}^L}{\pi_t} l_{t-1} + \frac{R_{t-1}^M}{\pi_t} m_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R_{t-1}^* \xi_{t-1} - \text{rer}_t^* d_{t-1} - \text{rer}_t^* l_{t} + d_{t} + \text{rer}_t^* d_{t} \right) + t^B n^B A_{t-1} \quad (3.25) \]

The incentive constraint belongs to equilibrium conditions too. It can be simplified to \(\nu_{2,t} n_t^B = \lambda_t^B \left( l_t + m_t \right)\) as we show in Appendix B.6.1.

### 3.2.7 Monetary policy

The central bank conducts monetary policy by following the Taylor rule:

\[ \frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\gamma_R} \left( \frac{\pi_t^H}{\pi^H} \right)^{(1-\gamma_R)\gamma_Y} \left( \frac{\pi_t^H}{\pi^H} \right)^{(1-\gamma_R)\gamma_s} \exp(mp_t) \quad (3.26) \]

where \(mp_t\) is a monetary policy shock and the domestic composite goods price inflation \(\pi_t^H\) can be expressed as \(\pi_t^H = p_t^H / p_{t-1}^H \pi_t\).
3.2.8 Current account

Trade balance expressed in units of domestic goods is given by:

\[ tb_t = p_t^H y_t^H - rer_t D_t^F y_t^F \]

We assume that nobody else in the domestic economy borrows from foreign agents or lends to them except for banks that issue foreign debt \( d_t^s \). As a result, in equilibrium the stock of foreign debt changes either due to trade balance or payments of foreign bank debt. The price of domestic currency (relative to foreign currency) adjusts to clear the foreign asset market. It follows that:

\[ tb_t - (R_{t-1}^* \xi_{t-1} - 1) rer_t \frac{d_{t-1}^s}{\pi_t^s} = - \left( rer_t d_t^s - rer_t \frac{d_{t-1}^s}{\pi_t^s} \right) \]

3.3 Bringing the model to data

Model simulation and estimation use a mixture of calibrated parameters and estimated parameters, since the sample size is too short to resort to a full fledged econometric estimation. Also, some parameters cannot be identified with macro data, for instance elasticity of substitution between varieties of goods. Below, we discuss model calibration and estimation explaining our choices and results.

3.3.1 Calibration

We calibrate parameters that correspond to steady state ratios and model elasticities. The sources and values of parameters are listed in Table B.1 in Appendix B. In most cases, we resort to values used in models estimated on Hungarian data or used to simulate the Hungarian economy. Calvo parameters for domestic prices and wages are taken from a DSGE model estimated on Hungary data (Jakab and Kónya (2016)). We set Calvo parameters for import prices equal to the ones for domestic prices in Jakab and Kónya (2016), because they do not model sticky import prices. Elasticities of
substitution are calibrated to the values chosen in a DSGE model estimated on Hungarian data in Jakab and Világi (2008). Labor supply elasticity among different types of labor is taken from Jakab and Világi (2008). Investment adjustment cost parameter is set to 13 following Jakab and Kónya (2016). Government consumption to GDP ratio is taken from Jakab and Világi (2008). The Taylor rule is calibrated to Jakab and Világi (2008), but since they do not augment the Taylor rule with the rate response to output, we set the response to output, denoted as $\gamma_y$ in our model, to 0.1.

In some cases we have to rely on papers that did not focus on the Hungarian economy because Jakab and Világi (2008) or Jakab and Kónya (2016) either differ too much from our model in terms of a production structure or do not model agents that we focus on. We calibrate household discount factors to values suggested in Iacoviello (2005), which is one of the first papers that introduced heterogeneous households and household borrowing in New-Keynesian DSGE models. The utility weight of housing in the utility function is also taken from Iacoviello (2005). Elasticity of substitution for exports is taken from Gali and Monacelli (2005). The bankers’ exit probability $\omega_B$ and the new equity parameter $\iota_B$ are calibrated as in Gertler and Karadi (2011). We calibrate the volatility of the idiosyncratic housing quality shock to approximately the value used in Clerc et al. (2011). They applied a monitoring costs friction to explain the household spread as we do in our model. The volatility of corporate revenue shock is set to the volatility of idiosyncratic technology shock used in Clerc et al. (2011) as well for the sake of uniformity across borrower types.

The housing stock is normalized to unity, so is the relative price of domestic aggregated goods $p^H$ and the technology shock $z$. Non-stationary technology growth in the steady state is set to GDP growth net of population growth in Hungary over 2000:Q1-2016:Q3.

Several parameters in the model correspond to steady state values in data. We need a share of domestic goods in total consumption in Hungary. Due to different structures of models, especially when it comes to goods aggregation, we have to resort to data rather than use parameters indicated in the existing studies on the Hungarian economy. We take the share of imports to GDP in Hungary (73 percent) over the period 2002:Q1-2008:Q4 and adjust it given the average import share in the Hungarian exports (56 percent; OECD (2017)). In our model exports are assumed to be of domestic
origin, so we lower the observed import share in GDP by the amount of imports used in export production (OECD (2017)) and get that the import share in domestic demand should constitute around 40 percent in our model. Thus we calibrate \( \eta_c \) and \( \eta_I \) to 0.6 to achieve the desired steady state share.

A few parameters are determined endogenously. \( A_n \) is chosen such that average working hours in the steady is 0.33. \( R \) follows from the Euler equation given \( \beta_P \). \( R^* \) follows from satisfying the UIP condition in the steady state given the quarterly foreign inflation of 0.4 p.p. \( \kappa \) is endogenously determined to match \( \Phi(d_2) \) expression to its calibrated value. The shares of foreign currency loans to households and corporates are calibrated based on Hungarian data as of 2008:Q4 (Bank of Hungary (2012a)). \( \Phi(d_2) \) matches the corporate loan spread of 0.7 p.p. \( \bar{\omega}^M \) matches the average quarterly household loan spread of 1 p.p. \( \lambda^B \) is calibrated to match the bank spread of 0.3 p.p. The spreads are constructed using data provided by Bank of Hungary. We elaborate on this procedure in the next section when we present financial data used in model estimation.

Finally, the utility function for patient households and impatient households is chosen such that the model preparation to match data would be easier. We elaborate on this in the next subsection.

### 3.3.2 Estimation

To carry on with the estimation exercise, we prepare the model to match observed variables. Usually the observed data cannot match the model variables, if a (stochastic) trend is present in the data. In our sample all variables are non-stationary too, except for interest rates. A large number of non-stationary variables can also be due to a small sample size though. To remedy the non-stationarity problem, we can match either growth rates of observed variables or HP-filtered levels of observed variables. We choose to avoid HP-filtering\(^5\) and match growth rates of observed non-stationary variables and levels of stationary variables as for instance Christiano et al. (2011). Then observation equations for non-stationary variables have to account for labor augmenting technology

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\(^5\) Using HP-filter is criticized for creating rather than finding cyclical properties and deteriorating prediction power of data, see for instance Hamilton (2017).
growth. Also, we transform equilibrium conditions by dividing non-stationary variables by labor augmenting technology level and get the set of equations listed in Appendix B.8. Exceptions to this transformation and relevant notation are discussed there too. This transformation has implications for the utility function. We choose a very simple utility function that assumes separately additive preferences. This facilitates model transformation to a stationary form considerably. For the same reason risk aversion parameters for consumption and housing are set to one and the Frisch elasticity for labor is set to one too.

We estimate parameters of the various stochastic processes using Hungarian macro and financial data from 2000:Q1-2016:Q3. The macro data set includes real GDP growth, consumption growth, investment growth, CPI inflation, nominal gross interest rate, real exchange rate, trade balance to GDP, foreign interest rate, foreign inflation, foreign real GDP growth. Financial variables include the corporate loan spread, the household spread and the bank spread. All quantities are in per capita terms. Also, after computing growth rates (if necessary), we demean all variables because most of these variables have substantially different trend growth rates in the data.\(^6\) The model, however, allows for one real trend growth rate. Definitions of data series and data sources are listed in Table B.3 in Appendix B.

The full model with all three financial frictions is estimated using 13 shocks: stationary productivity shock, non-stationary productivity shock, government spending, monetary policy, capital utilization, preference, country’s risk premium, foreign interest rate, foreign inflation, foreign GDP growth, housing quality shock volatility \(\sigma_{M,t}\), corporate profits volatility shock \(\sigma_{F,t}\) and the bank leverage tightness \(\lambda^B_t\). Model estimation also allows for measurement errors for all observed variables except interest rates. Measurement errors are calibrated to 10 percent of observed variance of particular time series. The size of measurement errors is on the conservative end of sizes used in Christiano et al. (2011) for Swedish data. We report measurement errors in Table B.2 in Appendix B.

\(^6\)We follow Christiano et al. (2011).
A brief comment on financial data is needed. To construct spreads, we use data provided by the Bank of Hungary (2012b). We construct the corporate loans spread by taking a difference between the average agreed interest rate of forint loans to non-financial corporations and the interest rate of forint deposits to households. The household spread is defined as a difference between the average agreed interest rate of forint loans for house purchase and the average interest rate of forint deposits to households. We take interest rates for forint loans and deposits because data on euro loans and deposits is available only for 2005:Q1-2016:Q3. Using data on euro loans would cut our sample size from 68 observations to 48. The rates are called average because they are weighted by the amount of new business. Both spreads use the household deposit rate as a base to make the spreads comparable to the bank spread which uses the household deposit rate as a base as well. Moreover, the deposit rate of non-financial corporations correlates with the deposit rate of households with a correlation coefficient of 0.99. All the deposit interest are for deposits with with up to 1 year maturity. The loan rate of non-financial corporations is a floating rate with up to 1 year initial rate fixation. To construct the bank spread, we take a difference between the average interest rates of unsecured forint interbank lending transactions (overnight rates) and the average household forint deposit rate with up to 1 year maturity.

We estimate autoregressive coefficients and standard deviations of shocks. The estimates are based on a double chain Metropolis-Hastings algorithm with 400,000 draws after a burn-in period of 200,000 draws and with acceptance rate set to 0.21. Table B.4 provides priors and posterior estimates. These estimates do not affect model simulation results, so we do not discuss them here and focus on model fit instead.

We do not use the estimated autoregressive coefficient for the country’s risk premium in model simulation because we focus on one episode of currency depreciation and the estimate is based on the whole sample. The currency depreciation episode in the beginning of 2009 was potentially triggered by the increase in the Hungary CDS by almost four percentage points. After the Hungarian CDS peaked on October 2008, it dropped by 1.5 percentage points in three quarters (see IMF (2009), p. 52). This development is substantially more in line with an autoregressive coefficient
of 0.8 rather than 0.99. So in model simulations we use $\rho_\zeta$ of 0.8. Also, the fact that the value of the estimate is close to 0.999 suggests that the estimate captures other shocks or movements in the model rather than the dynamics on the country’s risk premium alone.

3.3.3 Model fit

We use Bayesian estimation to evaluate model fit to Hungarian data. This way we can estimate the model with different sets of financial frictions and assess the role of different frictions based on model fit. Currency mismatch losses in a particular sectors amplify aggregate loses after unexpected depreciation through the introduced financial friction in that sector, so the relevance of currency mismatch losses in that sector can be evaluated in this way too. We also test alternative frictions. The corporate debt overhang friction is compared to the monitoring costs friction based on their fit to macro and financial data in Hungary.

The estimated fit of different models is presented in Table 3.1. The estimated log-likelihood values are based on a double chain Metropolis-Hastings algorithm with 400,000 draws after a burn-in period of 200,000 draws and with acceptance rate set to 0.21. We compare models with different assumptions about household debt based on how well they fit Hungarian macro data and the corporate default spread series of Hungarian firms. The results suggest that introducing household borrowers improves the fit significantly: the log-likelihood of the model with both savers and borrowers is significantly higher than with saving households only. The Bayes factor computed for this comparison exceeds 20 which is an indication of a significant difference between the models. Importantly, the fit improves although we do not use the model to fit the household spread. It follows that household debt improves fit on macro data and not only household spread data fit.

The debt overhang friction has an alternative, if financial distress for firms arises from unobserved firm-specific productivity shocks and thus uniformly higher borrowing costs but not from distorted incentives of indebted firms. In other words, if the monitoring costs explanation fits the data better than the moral hazard problem, the debt overhang friction should not be used. Although
Table 3.1: Marginal likelihood of different models.

<table>
<thead>
<tr>
<th>HH types</th>
<th>Fin. friction for firms</th>
<th>Fin. friction for banks</th>
<th>Fin. shocks</th>
<th>Fin. data</th>
<th>Log-likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>DO</td>
<td>-</td>
<td>$\sigma_{F,t}$</td>
<td>firm spread</td>
<td>-2326.9</td>
</tr>
<tr>
<td>S&amp;B</td>
<td>DO</td>
<td>-</td>
<td>$\sigma_{F,t}$</td>
<td>firm spread</td>
<td>-2304.6</td>
</tr>
<tr>
<td>S&amp;B</td>
<td>DO</td>
<td>-</td>
<td>$\sigma_{F,t}$, $\sigma_{M,t}$</td>
<td>firm and HH spreads</td>
<td>-2531.7</td>
</tr>
<tr>
<td>S&amp;B</td>
<td>BGG</td>
<td>-</td>
<td>$\sigma_{F,t}$, $\sigma_{M,t}$</td>
<td>firm and HH spreads</td>
<td>-2839.3</td>
</tr>
<tr>
<td>S&amp;B</td>
<td>DO</td>
<td>GK</td>
<td>$\sigma_{F,t}$, $\sigma_{M,t}$, $\lambda^B_{t}$</td>
<td>firm, HH and bank spreads</td>
<td>-2397.5</td>
</tr>
<tr>
<td>S&amp;B</td>
<td>BGG</td>
<td>GK</td>
<td>$\sigma_{F,t}$, $\sigma_{M,t}$, $\lambda^B_{t}$</td>
<td>firm, HH and bank spreads</td>
<td>-2900.1</td>
</tr>
</tbody>
</table>

Note: Marginal likelihood was computed using the Laplace approximation at the posterior mode. All estimations use macro data. Macro data includes: real GDP growth, consumption growth, investment growth, CPI inflation, nominal gross interest rate, real exchange rate, foreign interest rate, foreign inflation, foreign real GDP growth. In the model the firm spread is the default spread on corporate loans defined as $R_{L,t}^+ - R_t/\pi_{t+1}$. The household spread is the default spread and mortgages defined as $R_{M,t}^+ - R_t/\pi_{t+1}$. The bank spread is defined as $R_{L,t}^B - R_t/\pi_{t+1}$.


Anecdotal evidence points to the relevance of debt overhang in Hungary, we also test this prediction by comparing the model with the debt overhang friction to the one that features the monitoring costs friction for businesses instead. The latter follows modeling tradition started in Bernanke et al. (1999). The description of this alternative model is provided in Appendix B.7.

We first test the fit of the debt overhang friction for the model without leveraged banks but with household debt. We take these models to data and estimate on the macro data set, the corporate spread data and the household spread data. The debt overhang friction explains aggregate fluctuations of real variables and financial variables better. The log-likelihood of the model with the debt overhang friction is higher by more than 300 points. We also compare the fit of corporate frictions by allowing an interaction with bank leverage. Besides a financial friction for corporates, both models used for this comparison feature indebted households and leveraged banks. We again take these models to the data and estimate them on the macro data set, the corporate spread data, the household spread data and the bank spread. The estimation results are presented in the last two rows of Table 3.1. The results show that, if the interaction between corporate financial inefficiency and banks’ equity position is allowed, corporate debt overhang still fits data better than the monitoring
costs approach of Bernanke et al. (1999) for corporate loans. Moreover, the debt overhang friction outperforms the monitoring costs friction more than when we compared models without leveraged banks.

3.4 Results

This section analyzes the currency losses allocation problem. First, we prepare the ground for it. We explore how household debt denominated in foreign currency can lead to recessionary outcomes in times of currency depreciation and compare it to the downsides of corporate foreign currency loans. Currency mismatch in the household sector and currency mismatch in the corporate sector trigger a recession after a depreciation through different channels, so to understand them better we first discuss these currency mismatches separately.

In the next subsection we introduce bank leverage constraints to create wider ranging effects of bank currency mismatch losses and analyze different currency mismatch scenarios.

The following discussion uses selected output plots. A more complete set of figures of each of the experiment is presented in Appendix B.

3.4.1 Household FX debt vs. corporate FX debt

*Household FX mismatches*

To explore the outcomes of foreign currency mortgages to households, we model a scenario with 80 percent of mortgages in foreign currency\(^7\) and a scenario with domestic currency mortgages only. The former scenario exposes households to currency mismatch and the latter would keep them insulated from exchange rate risk. Unexpected currency depreciation is triggered by an unexpected increase in the country’s risk premium by three percentage points. Currency mismatch in the household sector creates an additional channel of how depreciation can create aggregate losses. Figure 3.2 presents the model simulation results for the two scenarios and shows the difference

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\(^7\) This fraction is chosen based on the share of foreign currency mortgages in Hungary in 2008:Q4.
Figure 3.2: Country’s premium shock and currency mismatch for households.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Corporate loans in both cases are issued in domestic currency only.

In outcomes which results from the household currency mismatch. The figure plots aggregate real variables and financial variables that describe the financial situation of households and firms, namely default spreads and credit volume.

When the value of domestic currency declines, the domestic value of household debt increases, deteriorating the financial position of household borrowers. By assumption households do not have export revenue or any other hedge against currency risk, so an unexpected increase in the debt value is not offset by gains Figure 3.2 shows that the presence of foreign currency debt increases the household default spread by 2 percentage points after a shock to the country’s risk premium. This suggests that foreign currency debt magnifies financial frictions on the households’ side: without a hedge against exchange rate risk, the magnified debt value increases household leverage making default more attractive.

The increased probability of household default affects aggregate variables through two main channels, consumption and labor supply. Consumption decreases, but labor supply of borrowing households increases, see Figure 3.2. The consumption response is directly related to the decline in
The unexpectedly higher value of debt affects household net worth negatively so borrowers have to cut consumption to meet debt payments and smooth housing purchases. Since the higher probability of default raises borrowing costs and reduces the availability of mortgage credit, impatient households have to use more of their net worth to acquire the same amount of housing and thus substitute some consumption goods to mitigate the decline cash available for housing purchases. Despite lower consumption, borrowers’ demand for housing declines and this affects housing prices negatively (see Figure (B.1) in the appendix B) which reinforces the decline in borrowers’ net worth.

The negative effect on household net worth also has a dynamic side to it. We had assumed that the dynasty of impatient households pools net worth at the end of each period and then shares it equally among its members in the beginning of the next period. Hence, a reduction in household net worth in the first period after the unexpected shock leads to a lower total household net worth. Due to asymmetric information, a decline in total household net worth tightens borrowing conditions for all borrowers in future periods. Then throughout time until the shock and the associated depreciation dies out, households struggle to get mortgage credit and have to cut consumption in order to smooth the forced drop in housing purchases.

Borrowers also adjust their labor supply in response to more expensive mortgage credit in order to smooth consumption and housing after the shock. Of course only impatient households increase their labor supply since only they are hit by the borrowing cost shock. Increasing labor supply and thus wage income marginally improves net worth. Importantly, higher labor supply is the second-order effect on equilibrium working hours because lower aggregate demand reduces labor demand and this effect is stronger than the increase in labor supply.

Lower net worth of household borrowers creates spillover effects to other sectors of the economy. We observe that the presence of foreign currency debt and thus stronger financial frictions on the households’ side slightly reduce investment and working hours. The decline in investment and labor demand is partially offset by the increased export demand. The improved competitiveness of the economy increases exports while imports decline due to consumers switching from imported
Figure 3.3: Country’s premium shock and currency mismatch for firms.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mortgages in both cases are extended in domestic currency only.

goods to domestic goods. In total, the trade balance to GDP ratio improves. Figure 3.2 shows that the trade balance increases relatively to GDP by more if household debt is denominated in foreign currency. This is the result of lower borrower’s consumption and thus lower demand for imported goods. The decline in output is also cushioned by a marginally smaller decline in consumption of patient households or the savers. Savers’ consumption declines in response to higher real domestic interest rates and higher prices, but in case of foreign currency debt savers’ consumption declines by less. Lower borrowers’ consumption depresses the economy and the interest rate which contains savers’ willingness to save.

**Corporate FX mismatches**

Different transmission channels would dominate, if currency mismatch was not in the household sector but in the corporate sector. In our model financially constrained firms produce goods out of capital and labor and finance working capital by taking loans denominated in foreign currency. Unexpected currency depreciation would slow down the economy through worsening the financial
situation of these firms but not through the direct effects on consumption. To illustrate the additional harm that foreign currency corporate debt creates after depreciation we again consider two scenarios. One scenario assumes that 60 percent of corporate debt is denominated in foreign currency\(^8\) and another scenario allows for only 5 percent of debt denominated in foreign currency.\(^9\) The difference between outcomes of the two scenarios can be attributed to corporate currency mismatch. Figure 3.3 presents the results of the scenario where the economy is hit with the unexpected increase in the country’s premium.

Investment is the most affected component of aggregate demand: Figure 3.3 shows that consumption barely reacts to the presence of corporate currency mismatch while investment does respond more. The reason is that the shock to the country’s risk premium creates currency mismatch losses in the corporate sector but does not directly affect consumers yet. Currency mismatch in the corporate sector amplifies aggregate losses because it worsens corporate debt overhang. Corporate debt overhang manifests in reduced capital purchases and labor hiring. Currency mismatch strengthens the debt overhang friction in the financially constrained firms’ sector because the distance to default decreases not only due to lower aggregate demand for firms’ production but also due to magnified corporate debt value. Corporate default probability increases by more, if firms face currency mismatch, see Figure 3.3, and firms cut their purchases of capital and labor hiring by more. A ten percentage points higher measure of default probability in the corporate sector results in about 0.5 percent larger decline in investment. Elasticity of investment to the corporate default probability is relatively low but it does not affect the ranking of outcomes in this experiment.

The volume of corporate loans also indicates higher financial distress in the corporate sector indebted in foreign currency. Initially the value of corporate loans increases because currency depreciation of over ten percent increases the debt value accordingly. However, in the second period after the shock, loans decline in response to higher corporate borrowing costs. The value of loans decreases in the second period as much as in the case with domestic currency loans

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\(^8\) This fraction is chosen based on the share of foreign currency corporate debt in Hungary in 2008:Q4.

\(^9\) We set 5 percent foreign currency share rather than zero because the zero share would make the distance to default expression in equation (3.16) explode.
despite a currency depreciation of around ten percent. Thus, the volume of loans must go down by approximately six percent more with foreign currency corporate loans.\textsuperscript{10}

Output loss due to corporate currency mismatch is small but positive. One of the reasons why corporate losses do not create larger spillovers to other sectors is that firms take one-period loans only and they cease existing after two periods which limits the effect that overindebted firms can have on aggregate demand.

Both types of households do not change their consumption response if corporate currency mismatch is introduced. As Jakucionyte and van Wijnbergen (2017a) shows, the spillover effect would be stronger, if we introduce corporate equity that is pooled and transferred from exiting firms to new firms. Then higher debt value not only reduces profits and makes more firms default this period but also reduces funds for future firms. New firms need to borrow more to produce the same amount of goods and thus have to leverage up more. This would amplify the investment response but would not change the current ranking of outcomes.

In contrast to household currency mismatch, corporate currency mismatch has negligible effects on the trade balance to GDP ratio. The trade balance to GDP ratio marginally decreases when firms face currency mismatch losses because debt overhang shrinks production and exports go down. Imports are little effected because capital is mostly produced with domestic goods. Imported inputs for consumption do not respond either because the spillover effect of corporate debt overhang on consumption is very small.

\textit{FX mismatches in household and corporate sectors}

Combining currency mismatch in the household sector with currency mismatch in the production sector increases the total effect on output and can potentially strengthen the channels through reinforcing. We again construct a scenario with 80 percent of mortgages in foreign currency and 60 percent of corporate loans in foreign currency and comparing it to the economy where all non-financial borrowers have only domestic currency liabilities. Figure 3.4 plots aggregate variables and

\textsuperscript{10}We get six percent by taking ten percent currency depreciation and weighting by a 60 percent share of foreign currency loans in total corporate loans.
Figure 3.4: Country’s premium shock and currency mismatch for all borrowers.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mixed denomination mortgages and loans mean that 80% of mortgages and 60% of loans is dominated in foreign currency. In the domestic currency case, 5% corporate loans and zero of mortgages are denominated in foreign currency.

financial variables for these two cases. The shock in consideration is the country’s risk premium. As expected, larger foreign currency debt in absolute amount leads to larger output losses. Losses are caused by lower consumption and lower investment demand. Lower investment is mainly driven by stronger debt overhang in the corporate sector, while lower household net worth and the related higher external finance premium for households is behind the larger decline in consumption. These two channels were identified in the two previous experiments.

The effect of currency mismatch on working hours is negative which suggests that corporate debt overhang restricts labor demand by more, if firms face currency mismatch. The negative outcome on labor occurs despite higher labor supply posted by household borrowers. Impatient households supply more labor to afford more housing in the environment with household currency mismatch and thus more expensive mortgage credit. Output losses depress the real domestic interest rate only initially but this is enough for savers to cut consumption marginally less as their willingness to save becomes lower.
The open sector generates the traditional expansionary effect of currency depreciation by stimulating exports and switching demand away from imports and so dampens the decline in output to some extent. Introducing household currency mismatch creates a negative effect on consumption and this improves trade balance more. Figure 3.4 shows that the marginal effect of currency mismatch on the trade balance is positive but it dies after one quarter. The faster decay is partially due to corporate currency mismatch which strengthens corporate debt overhang. Stronger debt overhang depresses production and reduces supply of exports. To sum up, the interaction of currency mismatch in different sectors results in larger negative effect on output. However, we do not find evidence that combining currency mismatch in different sectors leads to interactions triggering multiplier effects.

3.4.2 Allocating currency mismatch losses to banks

So far, currency mismatch losses were allocated to borrowers. A direct wealth effect on consumers came through the fall in ex-post returns on bank assets and the subsequent decline in bank equity values. These wealth losses are small because they are second order effects. But capital losses for banks have much more wide ranging effects because are operating under leverage constraints. Therefore, we introduce endogenous market-value leverage constraints for banks. The leverage makes banks vulnerable to unexpected losses. When bank equity gets wiped out, bank creditors become unwilling to lend as much as before. This results in a tightening of the leverage constraint for banks and a reduction of credit supply will follow. Bank leverage constraints magnify the negative side of insulating borrowers from currency mismatch by shifting the losses to banks.

Unexpected currency depreciation increases the value of bank liabilities but not the value of domestic currency loans to households or businesses, so bank equity decreases. Therefore, in times of currency depreciation banks that lent in domestic currency face higher exchange rate risk, lower equity and tighter bank leverage constraints compared to the alternative economy where banks lent in foreign currency. Since bank losses affect macroeconomic variables through changes in credit supply, bank currency mismatch losses become important for the dynamics of output. We evaluate
whether the downside of shifting currency mismatch losses to banks is large enough to offset the benefits of domestic currency credit to households and businesses as discussed in the previous section. We show that under reasonable calibration bank currency mismatch losses can be large enough to result in worse macroeconomic outcomes.

The introduction of bank leverage constraints follows Gertler and Karadi (2011) so the decline in net worth is related to bank borrowing costs through the moral hazard problem. In their setup lower bank equity implies that bankers become more likely to divert assets. Bank creditors take this into account and require a compensation for potential diversion which increases bank borrowing costs and tightens bank leverage constraints.

We start the analysis by focusing on household foreign currency debt as a way to shift currency mismatch losses from banks to borrowers. By assumption banks always have a share of liabilities denominated in foreign currency because they borrow from both domestic households and foreign households. Our calibration implies that when banks issue foreign currency mortgages, the share of
their assets that is denominated in foreign currency becomes higher than the share of their foreign currency liabilities. Then unexpected currency depreciation creates gains for banks and their profits increase. We compare this scenario to the case when banks lend in domestic currency only and do not reap valuation gains from unexpected depreciation, on the contrary they face currency mismatch losses. Currency depreciation is triggered by an increase in the country’s risk premium by three percentage points. Figure 3.5 plots the responses of aggregate real variables and financial variables to the country’s risk premium shock for the two scenarios.

Results suggest that shifting depreciation losses to household borrowers has a positive effect on bank equity. Figure 3.5 shows that, if households borrow in foreign currency, bank equity increases by more. Relaxed bank leverage constraints result in a lower bank spread which is expressed as the difference between the expected real return on corporate loans and the real expected deposit rate. The bank spread is decreasing despite the increasing mortgage default spread which suggests that the bank financial situation improves despite the elevated credit risk which arises from currency mismatch losses in the household sector.

Household currency mismatch has a negative effect on borrowers’ consumption, demand for housing and total consumption. We observe that higher currency risk in the household sector contributes to a significantly higher mortgage default spread. More expensive mortgages make borrowers cut demand for housing and dampen housing prices (see Figure (B.4) in the appendix B), which reinforces the decline in borrowers’ net worth. Borrowers’ consumption is affected by currency mismatch as well. Currency depreciation erodes household net worth and leaves less cash available for consumption. Consumption is also affected by more expensive mortgage credit: lower household net worth makes mortgages more costly and households have to substitute consumption for housing. Despite lower consumption, better financial situation in the banking sector results in higher total credit supply. In Figure 3.5 corporate loans decrease by less if households face currency mismatch. The corporate default spread is lower too, if currency mismatch losses are shifted from banks to household borrowers. Investment decreases by less as now financially constrained firms

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In equilibrium the expected return on corporate loans is equal to the expected returns on mortgages.
Figure 3.6: Country’s premium shock and currency mismatch for firms with leverage-constrained banks.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mortgages in both cases are extended in domestic currency only.

can afford more working capital. Higher investment together with improved trade balance evidently offsets the negative effect of consumption on output. Hence, in this case shifting currency mismatch losses to household borrowers is beneficial from the macroeconomic perspective or at least delivers similar output outcomes as shifting currency mismatch losses to banks.

Bank losses play a smaller role in ranking macroeconomic outcomes in case of corporate foreign currency debt. Figure 3.6 shows that insulating businesses from unexpected currency depreciation leads to better outcomes than allowing business to bear currency risk. We observe that output is higher mainly due to higher working hours whereas consumption is not affected by the currency mismatch in the corporate sector which is in line with the results without bank leverage constraints. The role of investment in determining output outcomes, however, is not so clear as without bank leverage constraints. The way bank currency mismatch losses affect aggregate outcomes resembles the channels observed with foreign currency mortgages. Shifting currency mismatch losses to banks decreases their equity, increases the bank spread and contracts credit supply. Figure 3.6
confirms that bank currency mismatch losses are also associated with lower mortgage credit. Therefore, although the way bank currency mismatch losses affect aggregate outcomes resembles to the channels with foreign currency mortgages, the total effect has smaller bearings on affecting output outcomes.

There are several reasons why bank losses play a smaller role in ranking macroeconomic outcomes in case of corporate foreign currency debt than in case of foreign currency mortgages. First, currency mismatch in the corporate sector has stronger effects on the supply side of the economy, not only the demand side, while housing is not a productive asset and does not affect output directly. Second, the effects of corporate default are less weakened by the trade balance adjustment. Although to produce one unit of investment goods takes the same proportion of imported goods as for consumption goods, corporate losses affect labor demand too not only investment. So when corporate losses weaken demand for labor, this is not offset by movements in trade balance, in contrast to a drop in consumption. Third, valuation gains from denominating corporate loans in foreign currency are barely present. Corporate loans constitute a smaller share of total bank credit than mortgages, also a smaller share of corporate credit is allowed to be in foreign currency, so fixing 60 percent of corporate loans to be denominated in foreign currency is sufficient to hedge the bank open currency position but insufficient to make the bank reap valuation gains after unexpected currency depreciation.

Finally we analyze the case where both corporate debt and household debt is partially denominated in foreign currency. On one hand, valuation gains should be even stronger as now the share of total assets in foreign currency exceeds the share of bank foreign currency debt more. On the other hand, as we saw before, credit risk associated with corporate loans leads to large macroeconomic losses if businesses have to borrow in foreign currency. Figure 3.7 shows that, if we allow for both household and corporate debt to be partially denominated in foreign currency, output losses from unexpected depreciation are approximately the same regardless of who bears exchange rate risk. Thus, increasing the share of total foreign currency debt in the economy increases bank valuation gains to such an extent, that this case leads to approximately similar or even better output outcomes.
Figure 3.7: Country’s premium shock and currency mismatch for all borrowers with leverage-constrained banks.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mixed denomination mortgages and loans mean that 80% of mortgages and 60% of loans is dominated in foreign currency. In the domestic currency case, 10% corporate loans and zero of mortgages are denominated in foreign currency.

after depreciation. Output outcomes are not worse or even better despite stronger debt overhang in the corporate sector and consumption losses.

To sum up, the only case where shifting currency mismatch losses to borrowers delivered worse (output) outcomes was with currency mismatch losses shifted to corporate borrowers. Given a high degree of consumption risk sharing, household default risk does not create sufficiently strong spillover effects to output. Household debt creates consumption losses but they are partially offset by improvements in trade balance. Also, housing is a non-productive asset so it does not affect aggregate supply directly. Corporate debt denominated in foreign currency, on the other hand, results in depressed corporate investment and labor supply which leads to a fall in production and aggregate demand. These results suggest that from a macroeconomic point of view it is better for regulators to prevent shifting currency mismatch losses to corporate borrowers but can shift them to consumers.
3.5 Conclusions

This paper analyzes whether shifting currency mismatch losses from borrowers and corporate borrowers to banks is better from a macroeconomic point of view. We find that only shifting losses from corporate borrowers to banks leads to macroeconomic gains in the Hungarian economy. Therefore, shifting corporate losses rather than household losses back to banks would have mitigated the recession more effectively. This is paradoxically the opposite of what the Hungarian government actually did.

Our findings follow from a medium-sized New-Keynesian DSGE model with three types of financial frictions and risky foreign currency debt. Bayesian estimation shows the relevance of debt overhang in explaining Hungarian data. The debt overhang friction improves model fit considerably and performs better than a costly state verification friction (as in Bernanke et al. (1999)). Also, household debt is important in explaining aggregate fluctuations, because model fit is better when we introduce indebted households.

We construct different currency mismatch scenarios and analyze them by simulating the model. The findings suggest that shifting currency mismatch losses to banks has different implications dependent on whether households or production firms borrow in foreign currency. Household currency mismatch losses have relatively small macroeconomic effects. So shifting currency mismatch losses to household borrowers is beneficial from the macroeconomic perspective or at least delivers similar output outcomes as shifting currency mismatch losses to banks.

In model simulation insulating corporate borrowers from unexpected currency depreciation leads to significantly better outcomes than allowing corporate borrowers to bear currency risk. The main reason is that corporate default delivers larger losses to the economy than household default. Corporate default distorts investment incentives and affects not only demand but also the supply side of the economy directly. Household losses do not directly affect aggregate supply. Also, when households contract consumption, they consume fewer imported goods, which improves the trade balance. So consumption losses do not fully translate into losses for domestic producers.
Chapter 4

Output and Welfare Gains from Non-recourse Mortgages: The Role of General Equilibrium Effects

4.1 Introduction

Recourse laws provide consumption insurance to defaulting mortgage holders, but allowing for general equilibrium effects creates spillovers to aggregate variables too. This paper analyzes the macroeconomic effects of recourse laws through the reallocation between residential mortgage credit and corporate credit. I show that macroeconomic and welfare outcomes may depend on the level of risk in the economy and the starting level of protection. But increasing the level of borrower protection provided by recourse laws is welfare increasing for the most valid calibration.

Mortgage credit is a secured debt. However, housing price depreciation can cause the house value to fall below the borrowed amount. Recourse laws define what are the losses for the lender when the primary collateral (housing) is insufficient to repay the mortgage fully. Non-recourse mortgages or mortgages with mild recourse limit the borrower’s liability to the value of her collateral, i.e. the mortgaged house. If mortgages are recourse loans, mortgage debt of an individual in default
would not be forgiven but the individual can be sued to collect other assets. Insolvent debtor’s wage income or other assets can be potentially seized to compensate the creditor for the depreciation of the house value, so I use milder recourse and a higher level of borrower protection interchangeably. From a creditor’s perspective, a low level of borrower protection (recourse mortgages) increases his returns in bad states compared to a high level of borrower protection (mild recourse mortgages). Thus on average lenders may be incentivized to extend more mortgage credit at the expense of other loans, including corporate loans, if mortgages are recourse. If creditors’ response is strong enough, then one would expect crowding-out of other credit than mortgage credit. I call the spillover effect of borrower protection on other types of assets the portfolio reallocation effect. The portfolio reallocation effect relates the change in mortgage credit to the change in physical capital, therefore, the resulting credit allocation may have far-reaching implications for the macroeconomic outcomes and welfare.

The empirical literature has not explored whether borrower protection can generate spillover effects to other types of assets than residential mortgage credit, so I present suggestive empirical evidence in favor of the portfolio reallocation effect in Table 4.1. I use Federal Deposit Insurance Corporation (FDIC) bank-level data to compute average bank credit shares. The sample features 7795 banks over more than four years (2001:Q1-2005:Q3). Ghent and Kudlyak (2011) classify 11 US states as non-recourse states. I use their classification to group banks into two groups where I compute averages of bank credit shares for each category of credit in each group. The table reports t-statistics to test whether differences in credit shares across the two groups of states are statistically significant. All shares differ significantly across the groups except for the ratio of consumer loans to total loans. States that allow recourse tend to have on average higher residential real estate mortgage ratios, lower commercial and industrial loans ratios and lower commercial real estate ratios compared to non-recourse states. This suggests that in recourse states banks issue more residential mortgages at the expense of other credit. This evidence supports the hypothesis that

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1 I abstract from other types of bankruptcy protection that would matter for unsecured debt because this paper focuses on secured debt only.

<table>
<thead>
<tr>
<th>Credit as a ratio to total bank assets</th>
<th>Recourse states</th>
<th>Non-recourse states</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential real estate loans</td>
<td>22.0</td>
<td>16.7</td>
<td>1.85</td>
</tr>
<tr>
<td>Commercial and industrial loans</td>
<td>8.5</td>
<td>10.8</td>
<td>-2.29</td>
</tr>
<tr>
<td>Consumer loans</td>
<td>12.1</td>
<td>11.2</td>
<td>0.14</td>
</tr>
<tr>
<td>Commercial real estate loans</td>
<td>11.7</td>
<td>16.6</td>
<td>-2.30</td>
</tr>
</tbody>
</table>

Note: The table presents average bank credit shares in the US, computed for individual banks over the period 2001:Q1-2005:Q3 from FDIC data. The distinction between recourse states and non-recourse states is based on the classification of Ghent and Kudlyak (2011). They classify Alaska, Arizona, California, Iowa, Minnesota, Montana, North Carolina, North Dakota, Oregon, Washington, and Wisconsin as non-recourse states. The last column present t-statistic for the hypothesis of no difference between credit shares in recourse states and non-recourse states.

the bank response to potentially higher returns on mortgages in recourse states can lead to higher mortgage credit and lower corporate credit.

The aim of this paper is to provide a theoretical analysis of what could be the mechanism behind the portfolio reallocation effect of recourse laws and whether the effect could be large enough to matter for aggregate outcomes. Also, if the theory suggests different predictions than the correlations presented in Table 4.1, are any explanations for that? For simplicity, I focus on corporate loans (physical capital loans) as the only available other option for lenders and I develop a theoretical framework with a bank portfolio choice between mortgage credit and corporate credit and two types of risk, housing value shocks and earnings shocks. The model structure is such that the mortgage default risk is priced in and determines the supply side of the credit equilibrium. Banks are perfectly competitive.

In the setup with housing value shocks only, I show that borrower protection in the form of milder recourse has benefits even in case of resulting more frequent (strategic) household defaults. Strategic default occurs if the indebted individual chooses to default because the housing value is below the loan value but not because she would not have resources left for consumption after debt repayment. By definition milder recourse increases financial benefits from default and in equilibrium strategic default becomes more frequent. In such a setup milder recourse results in higher equilibrium mortgage credit. However, due to general equilibrium feedback effects on housing prices lenders do not have to increase mortgage credit at the expense of corporate loans but can lend more in total. Since higher supply of corporate loans means more finance available for
credit-constrained firms and in turn higher capital stock, wage income in the economy rises. Both borrowing and saving households benefit from higher income.

The crowding-out of capital does not take place because of an offsetting general equilibrium effect: housing prices increase, saver’s income and in turn savings go up and more funds become available for intermediation. I assume that the saving household is the owner of the housing stock, so when more generous borrower protection increases borrowers’ demand for housing, the saver experiences income gains from higher housing prices and saves more. Thus milder recourse can substantially increase welfare depending on the strength of feedback effects on housing prices. This channel (through housing prices) would not be present with unsecured credit only and motivates the development of a theoretical model for my research question in particular rather than extrapolating the existing findings for unsecured credit as in Li and Sarte (2006).

The portfolio reallocation effect is mitigated by the general equilibrium effect on housing prices even if the model has features a higher level of risk. If I allow for earnings shocks to household income, forced default becomes more frequent. Then the presence of low earnings shocks justifies borrower protection as a consumption insurance tool in forced default cases. The added dimension of earnings uncertainty especially strengthens borrower’s demand for mortgages. For a low starting level of borrower protection, a marginal increase in the level of protection would lead to a strong positive demand effect on mortgages and higher mortgage credit. This however would still not crowd out capital but rather increase it due to the general equilibrium feedback effect to housing prices. The increase in capital is stronger than with housing value shocks only because the general equilibrium effect is stronger as well.

The general equilibrium effect is the strongest for very low levels of protection and for sufficiently high values of earnings exemptions it is weak enough to be offset by the conventional portfolio reallocation effect. For high values of borrower protection (or almost non-recourse mortgages), higher borrowing costs offset higher demand for mortgages and mortgage credit even declines, crowding in capital. Thus equilibrium outcomes depend not only on the level of risk in the economy but the starting level of protection as well.
I find that the highest welfare is achieved when the recourse law exempts approximately between 58 and 72 percent of borrower’s earnings. Beyond this point, the increase in capital levels off while borrowing costs continue increasing, thus reducing expected utility for both the saver and the borrower. However, the model is not calibrated to match the US mortgage market data, so this finding should be taken with caution.

My main contribution to the literature is the analysis of recourse policies and the evaluation of welfare outcomes while acknowledging general equilibrium effects. Most of quantitative studies with housing markets and equilibrium mortgage default pursued the analysis of different government policies (e.g. Jeske et al. (2013)) or the causes of the foreclosure crisis (e.g. Chatterjee and Eyigungor (2015), Corbae and Quintin (2015), Garriga and Schlagenhauf (2009), etc.) in the US but did not discuss recourse policies that could mitigate defaults. The exceptions are Mitman (2016), Corbae and Quintin (2015) and Hatchondo et al. (2015). Mitman (2016) finds that recourse has a small effect on mortgage default rates and Corbae and Quintin (2015) and Hatchondo et al. (2015) find that recourse creates welfare gains. Differently from Mitman (2016) they also assume that seeking recourse is costless for the lender and that mortgages are long-term assets. None of the studies mentioned consider recourse policies with feedback to production and this is potentially one of the reasons why, in contrast to my findings, they do not find welfare gains from non-recourse.

Section 4.2 elaborates on the connections of this paper to the literature. Section 4.3 presents the model. Section 4.4 discusses the results. I split section 4.4 to overview the effects on credit allocation and welfare outcomes separately. Section 4.5 provides robustness checks. Section 4.6 lists future extensions and section 4.7 concludes.

4.2 Related literature

The theory of borrower protection explains the main trade-off as the competing demand and supply effects (Athreya (2002)). It is key in my model too. A higher level of protection encourages demand for mortgage credit: borrower’s consumption is better insured against bad states of nature. The
negative supply effect would manifest in tighter borrowing conditions. Empirical evidence on this matter, however, is not conclusive. Ghent and Kudlyak (2011) provide empirical evidence that borrowers with houses valued at more than $200,000 were 32 percent more likely to default in non-recourse states than in recourse states. However, they do not research the effect of allowing the lender recourse on credit supply. Gropp et al. (1997), Pence (2006) and Lin and White (2001) provide evidence in favor of the dominating supply effect. However, Severino and Brown (2016) find a decline in unsecured credit after BAPCPA\(^2\) made filing for bankruptcy more expensive and subject to tighter conditions in 2005. If the supply effect was dominating, then an increase in credit should have occurred. In my model, crowding-in of capital can occur only if the supply effect dominates the demand effect. I indeed show that it is the case for sufficiently high values of borrower protection and argue that the demand effect dominates if there is a high value of consumption insurance.

I contribute to the personal bankruptcy literature by highlighting the role of the bank portfolio choice between mortgages and corporate loans in the presence of mortgage return uncertainty and the ensuing general equilibrium effects. So far only models with unsecured consumer debt took into account the portfolio reallocation effects. Li and Sarte (2006) show that a decrease in the level of personal bankruptcy protection can be followed by an increase in capital stock because unsecured credit becomes lower. Chatterjee and Gordon (2012), however, argue that capital stock would be unaffected because of a reduction in savings. However, as it follows from my findings, the results from the unsecured credit literature cannot be extrapolated to residential mortgage credit because the interplay between housing prices and household investment decisions play a large role in aggregate outcomes.

Most of the studies that model equilibrium mortgage default abstract from recourse vs. no-recourse considerations. Several quantitative studies develop general equilibrium models that are capable of describing keys characteristics of the housing and mortgage market in the US. Jeske et al.\(^2\) The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) is a legislative act that reformed the bankruptcy code in the US. It was passed on April 14, 2005. It made more difficult to file bankruptcy under Chapter 7 so that some consumers may instead use Chapter 13.
(2013) use such a model to study the effects of government housing market policy on the housing market. Corbae and Quintin (2015) focus on the role of high leverage mortgage in contributing to the foreclosure crisis in the US and Chatterjee and Eyigungor (2015) analyze different shocks that led to the foreclosure crisis in the US and assign the highest contribution to the financial friction shock. Garriga and Schlagenhauf (2009) do not consider endogenous housing prices and distinguish from other papers by modeling mortgages as long-term assets. They demonstrate that a house price shock can account for most of the spike in foreclosure rates in the US.

Recourse laws in a quantitative framework are studied in Corbae and Quintin (2015) and Mitman (2016). Corbae and Quintin (2015) have a section on the implications of non-recourse mortgages on the foreclosure crisis. They find that making all mortgages recourse and assuming zero recourse transaction costs would have limited the mortgage default spike by approximately 20 percent. Therefore, similarly to my findings, the non-recourse aspect matters for housing demand and housing prices. In contrast to my findings, Corbae and Quintin (2015) finds that introducing non-recourse mortgages costs 0.5 percent of agents’ lifetime consumption. The likely reason behind this discrepancy is that I consider endogenous housing prices and allow for physical capital accumulation and Corbae and Quintin (2015) do not. These two assumptions, as I show, can make recourse welfare-decreasing in my setup. The strength of their study is the analysis of distributional implications. They find that recourse makes mortgages more affordable to low-income agents while my model assumes ex-ante identical households and potentially overstates the losses from recourse. Mitman (2016) broadens the household asset portfolio and shows how an endogenous household choice between unsecured debt and secured debt helps to explain variation of bankruptcy rates and foreclosure rates in the US. The paper features recourse vs. non-recourse differences and analyze the effects that the changes in bankruptcy and foreclosure policies in the US had on foreclosures and bankruptcy rates. But the study abstracts from physical capital accumulation and keeps housing prices fixed.

The above-mentioned studies focus on developments in the housing and mortgage markets and naturally they feature endowment economies with the exception of Jeske et al. (2013). The feedback
effect of personal bankruptcy regulation into production is not analyzed. My framework comes close to one of the model extensions in Jeske et al. (2013) though. A general equilibrium model with foreclosures in Jeske et al. (2013) is extended with physical capital accumulation to show that the main results still hold and housing subsidies are even more welfare-reducing than in an endowment economy because they limit the accumulation of physical capital. The paper abstracts from non-recourse mortgages and assumes that borrower’s liability is limited to the collateral though. The effects of earnings exemptions which I analyze in my paper can be understood as an expected subsidy to borrowers paid by banks in bad states of nature. Similarly to Jeske et al. (2013), I find that in cases when higher earnings exemptions (“subsidies to borrowers”) lead to lower capital stock, this creates substantial welfare losses. In contrast to Jeske et al. (2013), my study lacks in a quantitative aspect as I do not match model parameters to the mortgage market data. However, I study the non-linear effects of earnings exemptions on the credit allocation.

4.3 Model

The main goal of this model is to illustrate the mechanism through which recourse laws impact the credit allocation in the economy. The model assumes only a few actors in the economy: two households, a perfectly competitive production firm and a bank. By assumption the two households differ in their discount factors which allows for the positive supply and positive demand for savings. Importantly, intermediation of savings requires a special technology that is available to the banker only. The bank collects deposits and lends to one of the households and the production firm. The banker is a part of the saving household and thus transfers profits or losses to this household at the end of every period. The saving household also owns the housing stock so he can sell some of it to the borrowing household and generate additional income in this way.

The economy exists for two periods and agents introduced in period 1 die in period 2. Therefore, there is only one generation in the economy. It is a closed economy with production and fixed housing supply. The borrowing household takes a mortgage to finance housing purchases in period
1 and can default in period 2. There are two sources of uncertainty in the model: the housing value shock $\zeta$ and the earnings shock $z$ with commonly known probabilities. The shocks are independent and occur in period 2 only. Their distributions are discussed in more detail in the parameters section.

After default the household loses her housing (through foreclosure) and incurs fixed rental costs. The rental market is exogenous to the model. The housing price clears the housing market and the risk-free rate clears the savings market.

The model generates both strategic default and non-strategic default. Default is strategic if the household defaults although she could repay the loan out of her wage and still enjoy consumption that is strictly higher than $c_{\min}$. In the calibration $c_{\min} = h^r$ which implies that the household would have higher than zero consumption after paying the exogenous rent cost $h^r$.

The timing of events ensures that borrower protection can affect the credit allocation ex-ante and the borrower’s default decision ex-post.

- **Period 1**
  1. The saver and the borrower receive an income endowment.
  2. The saver receives a housing stock endowment which he can sell to the borrower.
  3. The firm is set up and plans its production in the next period.
  4. The bank receives deposits from the saver and faces demand for mortgage credit and corporate credit.
  5. Demand for credit is satisfied at the price that ensures a zero profit for the bank.
  6. Households consume, the borrower buys housing.

- **Period 2**
  1. Uncertainty with respect to earnings shocks and housing value shocks is resolved.
  2. Households supply labor at the predetermined wage inelastically.
3. The borrower chooses whether to default and the lender transfers realized returns to the saver.

4. The firm transfers profits/losses to the saver and exits.

5. Households consume out of their wages and proceeds from housing.

6. The defaulted household incurs rental costs.

I solve model with backwards induction using terminal values indicated in the model description. The solution algorithm and the baseline set of parameters are given in Appendix C.

### 4.3.1 Households

Households are of two types. The household with a higher discount factor $\beta_S$ is the saver as opposed to the borrowing household ($\beta_B < \beta_S$).

**Saver**

In the beginning of period 1 the saver receives an endowment $y^S$. Also he owns the fixed housing stock $h$. The household chooses how much of this utility-generating asset he wants to sell to the other household at the market price $q$ and how much does he want to keep for himself. Next to the housing stock, this household has access to saving through putting deposits $d_1$ to the bank. Labor supply is inelastic. The household chooses consumption $c^S_1$, housing $h^S_1$ and how much to save by putting deposits $d_1$ such that her value function is given by

$$V^S_1 = \max_{\{c^S_1, h^S_1, d_1\}} \left( \frac{\left(c^S_1\right)^{1-\sigma} \left(h^S_1\right)^{\sigma}}{1-\sigma} - 1 \right) + \beta_S V^S_2 \left(d_1, h^S_1; \zeta, z \right)$$

s.t. $c^S_1 + q \cdot h^S_1 + d_1 \leq y^S + q \cdot h$

Note that the value function depends on future shocks to the household’s income and housing value. Although housing shocks are aggregate, households can receive different earnings shocks.
In period 2 the saver receives labor income \( z_2w_1n^S \) that is affected by the realization of the earnings shock \( z_2 \), gross returns on deposits \( (1 + r_1) d_1 \) and a lump-sum profit transfer from firms and the bank \( \pi_2 \). The saver owns all firms and the bank, thus, all profits/losses get attributed to him at the beginning of period 2. The saver simply consumes everything in period 2:

\[
V^S_2 = \max \left\{ \frac{(c^S_2)^{1-\sigma_h} (h^S_2)^{\sigma_h} \frac{1-\sigma}{1-\sigma_h}}{1-\sigma} - 1 \right\}
\]

\[s.t.\quad c^S_2 \leq z_2w_1n^S + (1 + r_1) d_1 + \xi_2 h^S_1 + \pi_2 \]

and \( h^S_2 = h^S_1 \)

The budget constraint reflects that in period 2 housing becomes a value-generating asset: it takes its terminal value determined by the realization of the housing value shock \( \xi_2 \) and contributes to the saver’s income. (Only in period 1 the housing is purchased at the market-clearing price \( q_2 \).)

**Borrower**

Household default may arise because of two reasons. The housing value shock is the direct source of uncertainty that the collateral value and can trigger default. The setup allows for the household to default if her financial benefits from default are high enough: the collateral value falls below the debt value and the costs associated with default are not high enough to make up for the difference between the collateral value and the debt value. This would resemble the housing market bust.

The second driver of the default decision can be an earnings shock. A sufficiently negative shock to the borrower’s earnings may make the indebted household incapable of meeting the payment. Although earnings shocks reduce income that household can use to repay the mortgage, the earnings shock cannot trigger default without housing shocks because in the framework with two periods only the household could always repay by selling the housing asset. However, in the setup where I allow for both type of shocks, a realization of low earnings can strengthen incentives to default and also affect the borrowing decision significantly. This largely resembles mortgage default due
to earnings shocks (e.g. job loss, sickness) combined with low housing prices. Therefore, in the model results section I consider the case with only housing shocks present and the case with both earnings and housing value shocks.

I formalize the setup by letting the borrowing household choose consumption $c_1^B$, housing stock $h_1^B$ and how much to borrow from the bank ($m_1$) such that her value function $V_1^B$ is maximized:

$$V_1^B = \max_{\{c_1^B, h_1^B, m_1\}} \left( \frac{(c_1^B)^{1-\sigma} (h_1^B)^{\sigma h_1}}{1-\sigma} + 1 \right) + \beta V_2^B \left( h_1^B, m_1; \zeta, z \right)$$

subject to the budget constraint:

$$c_1^B + q \cdot h_1^B \leq y_1^B + m_1$$

For simplicity I assume that the household borrows the maximum allowed amount given by $\rho = \tilde{\xi} h_1^B / (1 + r_1^m)$. $\rho$ ($0 < \rho < 1$) is an exogenous LTV ratio, $\tilde{\xi}$ denotes an expected value of the housing shock $\zeta$ and $r_1^m$ stands for the mortgage net interest rate. The household pledges her housing $h_1^B$ which can be seized in case of default in period 2. Low values of the housing shock would reduce the value of the housing asset possibly triggering household default (though not necessarily). It is important to note that housing value in period 2 is exogenous for the sake of simplicity.

Higher earnings exemptions imply a higher level of borrower protection. Earnings exemptions matter in case of insolvency because, if household income, comprised of wage income and the value of housing assets, is insufficient to repay the mortgage, the bank can claim non-exempt earnings. Then higher earnings exemptions mean that the borrower can keep more of her earnings in case of default.

The maximum level of earnings that the household can keep in case of default is denoted by $\kappa$. Then $\max \{w_1 n^B - \kappa, 0\}$ from insolvent household’s earnings is seized by the creditor together with housing assets in exchange for forgiving mortgage debt. A positive parameter $\kappa$ implies higher borrower protection: if the borrower’s wage income is below the maximum exemption level $\kappa$, the household keeps her wage. Otherwise, the borrower has to give wage above the exemption level to the bank. The setup loosely corresponds to recourse laws in the US: if the value of the house
is insufficient to repay the mortgage, the bank can seize some but not all of other assets (including cash) to cover the losses. \( \kappa = 0 \) approximately corresponds to recourse laws in some states in the US, assuming that the minimum consumption level that has to be guaranteed is zero.

Further I formalize maximization problems for different decisions of the borrower. If the borrower does not default, the value function for period 2 conditional on repayment \( V_{2}^{BP} \) is the following:

\[
V_{2}^{BP} = \max_{\{c_2^B\}} \left( \left( c_2^B \right)^{1-\sigma_h} \left( h_2^B \right)^{\sigma_h} \right)^{1-\sigma} - 1
\]

s.t. the budget constraint which gives residual consumption equal to the household wage income which is left after repaying the mortgage and receiving the housing value:

\[
c_2^B \leq z_2 w_1 n^B + \zeta_2 h_1^B - \left( 1 + r_1^m \right) m_1
\]

and \( h_2^B = h_1^B \)

The value for the borrower after default is given by utility from consumption and rental services. Since the household loses his claim on housing value, she has to incur fixed renting costs \( h_r^f \). It follows that the borrower’s value in period 2 conditional on default is given by \( V_{2}^{BD} \):

\[
V_{2}^{BD} = \max_{\{c_2^B\}} \left( \left( c_2^B \right)^{1-\sigma_h} \left( h_r^f \right)^{\sigma_h} \right)^{1-\sigma} - 1
\]

s.t. \( c_2^B \leq \min \{ z_2 w_1 n^B, \kappa \} - h_r^f \)

The household’s choice whether to default or not is determined by the maximum value:

\[
V_{2}^B = \max \{ V_{2}^{BP}, V_{2}^{BD} \}
\]
4.3.2 Production

Firms live for two periods in total. They buy capital $k_1$ in period 1 but produce in period 2 only. For simplicity, labor supply $n_1$ is inelastic. It follows that both capital and wage is predetermined. I assume that firms need to finance capital expenditure by taking a loan from the bank. Earnings shocks in this economy are idiosyncratic, so firm’s labor supply and production revenue are not affected and firms always repay their loans. Firms use a perfectly competitive technology:

$$y_2 = k_1^{\alpha} n_1^{1-\alpha}$$

Every firm solves the following maximization problem in the first period of their lives:

$$\max_{k_1, n_1} E_t \left\{ y_2 + (1 - \delta)k_1 - w_1 n_1 - \left(1 + r_1^l\right) l_1 \right\}$$

s.t. \quad l_1 = k_1, \quad n_1 = n

$l_1$ denotes loan taken to finance capital. For simplicity we assume that capital depreciates fully ($\delta = 1$). It follows that competitive wages and returns on capital would satisfy

$$\alpha k_1^{\alpha-1} n_1^{1-\alpha} - 1 = r_1^l$$

and

$$(1 - \alpha)k_1^{\alpha} n_1^{\alpha} = w_1$$

4.3.3 Banking sector

A representative competitive bank decides on the volume of loans to households and firms and deposits such that it breaks even in expectation. The bank is owned by the saving household so realized profits/losses are transferred to the household in the beginning of the second period.
In period 1 the banks expects a net return \( \bar{r}_2^m \) on the mortgage and a return \( \bar{r}_2^l \) on the corporate loan. The indicator \( I (\text{repaid}) \) denotes the borrowing household’s decision to repay debt. The firm always repays. The expected return on the mortgage reflects that in case of default the bank will seize the non-exempt earnings together with housing:

\[
E_1 \left( 1 + \bar{r}_2^m \right) m_1 = E_1 \left\{ I (\text{repaid}) \left( 1 + r_1^m \right) m_1 \right\} + E_1 \left\{ (1 - I (\text{repaid})) \left( \max\{z_2 w_1 n^B - \kappa, 0\} + \zeta h_1^B \right) \right\}
\]

The expected return on the corporate loan is simply the interest rate times the borrowed amount:

\[
E_1 \left( 1 + \bar{r}_2^l \right) l_1 = \left( 1 + r_1^l \right) l_1
\]

It can be shown that in equilibrium required returns on both mortgage credit and corporate credit should account for both the bank funding costs \( r_1 \):

\[
E_1 \bar{r}_2^m = r_1, \quad r_1^l = r_1
\]

(4.2)

### 4.3.4 Market clearing

In equilibrium total savings in the economy have to equal total demand for loans:

\[
m_1 + l_1 = d_1
\]

The housing market clears at the price \( q \) such that when

\[
h_1^B + h_1^S = h
\]

The rest two conditions ensure labor market clearing:

\[
n_1 = n
\]
\[ n = n^B + n^S \]

### 4.4 Results

Results are obtained by calibrating the model and solving it with backward induction. Most of the parameters are standard and reported in Appendix C. There I also describe the chosen shock values and the associated probabilities. Robustness of the results with respect to key parameters is explored in the next section.

This section explores the effects of increasing the level of borrower protection on the credit allocation. The section is divided into two subsections to describe the credit allocation effects first and continue with the description of welfare effects. Initially the credit allocation is discussed while assuming that the economy experiences housing value shocks only so that the general equilibrium effects could be presented without delving into consumption insurance or the non-linearities in the mortgage credit response. After, I include earnings risk as well and exhibit the resulting differences in the credit allocation. Welfare outcomes are discussed in the second subsection.

An exogenous earnings exemption level \((\kappa)\) proxies for the level of borrower protection. The bank can seize earnings which are above the exemption level \(\kappa\), i.e. \(\max\{w_1 n^B - \kappa, 0\}\). Hence, higher \(\kappa\) implies a higher level of borrower protection.

#### 4.4.1 Effect on the credit allocation

The resulting credit allocation depends on the first-order effects and the second-order effects. An increase in the level of borrower protection (higher \(\kappa\)) directly translates into an additional expected loss to the bank because the bank can claim less of insolvent borrower’s earnings. This would create a negative first-order supply effect on mortgage credit. Another first-order effect, the demand effect of higher borrower protection, is positive because the individual is insured from bad states better thus would like to borrow more. The equilibrium outcome in the mortgage credit market will depend on which of the two effects will dominate.
Changes in mortgage credit will create the second-order effect on corporate loans and thus output. This is what I would call the portfolio effect: higher (lower) mortgage credit would crowd out (crowd in) corporate credit.

**Housing value shocks**

This section shows that in the model with housing value shocks the demand effect of borrower protection can be strong enough to result in higher mortgage credit. However, higher mortgages do not have to crowd out capital if rising housing prices increase savers’ income sufficiently. This is the general equilibrium effect that I explain in detail below.

Figure 4.1 plots the values of macroeconomic variables across different levels of earnings exemptions $\kappa$. The maximum $\kappa$ considered here is 0.6 because wages given baseline calibration are always below 0.6 and increasing the level further than that do not change the model dynamics. The figure plots default rates (total $\delta$ and non-strategic $\delta^n$), the mortgage interest rate spread $r^m_1 - r_1$, mortgage volume $m_1$, borrower’s housing stock $h^B_1$, housing price $q$ and capital $k_1$.

When the level of borrower protection becomes high enough, the household finds it beneficial to default in bad states. Figure 4.1 shows how the default rate $\delta$ in period 2 turns positive for $\kappa > 0.57$ which is almost 98 percent of borrower’s wages. Below this value, the household always repays because the earnings level is insufficiently generous and default is too costly. Aggregate variables are inactive as long as default risk is zero because earnings exemptions can affect aggregate variables only through the default risk.

All defaults are strategic defaults ($\delta^n = 0$). Housing value shocks do not affect household’s wage income and given this calibration the household could repay out of her wage income and still enjoy positive consumption even in cases with the least valuable housing stock. The highest default risk corresponds to the maximum level of $\kappa$ but it does not increase in $\kappa$ monotonically because the number of housing shock realizations is only two and the household defaults only for the lower value of the housing value shock. This why the maximum default rate is equal to the probability of the lower value of the housing shock ($p_L^\varepsilon = 0.05$). Banks respond to higher default
Figure 4.1: Mortgage total default rate, non-strategic default rate, the mortgage spread, mortgages, housing, housing price and capital stock.

Note: The first row plots total mortgage default rate $\delta$ and non-strategic mortgage default rate $\delta^\alpha$ (left panel) and the mortgage spread (right panel). Both are expressed in percentage points. The mortgage spread is defined as a difference between the mortgage rate and the risk-free rate. The second row plots mortgages $m_1$ and borrower’s housing $h_1^B$. The third row plots housing price $q$ (relative to the consumption price) and capital $k_1$. The horizontal axis measures different levels of earnings exemptions $\kappa$.

risk and thus higher expected losses by lending less. Tightening borrowing conditions in response to higher default risk are evident from the increasing interest rate spread for mortgages. The spread is defined as the difference between the mortgage rate and the risk-free rate, $(r^m_1 - r_1)$.

In equilibrium, the demand effect turns out to be the dominant force in the dynamics of mortgages. Higher interest rates do not prevent the equilibrium mortgage credit from increasing in the level of borrower protection and mortgages increase in volume for the right end of the chosen interval for $\kappa$, see Figure 4.1. The rise in mortgage credit starts at the level of earnings exemption at which default risk becomes positive ($\kappa > 0.57$), so mortgage credit starts increasing when the household actually makes use of the exemption in bad states. The earnings exemption level $\kappa > 0.57$ implies that more than 80 percent of earnings are shielded from the lender (in this calibration equilibrium $w_1n^B$ is always below 0.6) in case of default. Thus the household chooses to default on her mortgage only under very generous borrower protection.
With rising mortgage credit, the borrowing household affords to purchase more housing stock. Figure 4.1 shows how borrower’s housing purchases approximately follow the mortgage dynamics. The housing price increases to accommodate higher borrower’s housing demand and clear the housing market.

The increasing generosity of borrower protection affects not only the mortgage market equilibrium but the credit allocation as well. However, it turns out that although the increasing level of borrower protection increases mortgage credit, capital loans are not crowded out. I find the opposite to the conventional portfolio effect. Figure 4.1 shows how capital increases in the level of borrower protection for $\kappa > 0.57$.

The portfolio reallocation effect is resumed if the housing market does not clear. Figure 4.2 compares the values of mortgages, capital, housing and deposits to the values they would take if the housing price was fixed. The remaining of the subsection discusses why given a fixed housing price higher mortgage credit leaves fewer funds to corporate lending and thus capital and why this is not the case with the endogenous housing price.

If the housing market clears, higher borrower’s demand for housing results in a higher housing price because the saving household sells more housing to the borrower and keeps less housing stock to himself. Figure 4.2) shows how housing stock at the savers declines in $\kappa$. Higher housing price affects saver’s wealth positively and allows the saver to increase savings in order to smooth his consumption over time. Deposits rise, as depicted Figure 4.2. Consequently, the bank does not have to shift from corporate lending due to limited funds but can actually increase lending to all borrowers, both households and firms. Therefore, if the housing market clears, this gives rise to the general equilibrium effect which, if strong enough, dominates the portfolio reallocation effect.

If the housing price was fixed, saver would not sell more housing to the borrower and his wealth would remain unchanged. Deposits supply remains constant across exemption levels, see Figure 4.2. Then the traditional portfolio effect would remain: capital loans would have to decrease in response to increasing mortgage credit.
Figure 4.2: Comparison of mortgages, capital, saver’s investment and deposits based on whether the housing price is fixed.

Note: The first row plots mortgages $m_1$ and mortgages in the model with a fixed housing price $m^q_1$ (left panel). Right panel compares capital $k_1$ to capital in the model with a fixed housing price $k^q_1$. The second row compares saver’s housing $h^S_1$ to saver’s housing in the model with a fixed housing price $h^{Sq}_1$ (left panel). Right panel compares deposits $d_1$ to deposits in the model with a fixed housing price $d^q_1$. All is expressed in consumption units. The horizontal axis measures different levels of earnings exemptions $\kappa$.

The relevance of this result depends on how much the housing price can contribute to savers’ income. If valuation gains from owning housing are small, savers would increase deposits by less. Also, the model may overestimate the housing demand feedback effect to housing prices. If borrowers constitute a small share of total housing demand then their ability to influence housing prices would be limited. With housing production, the housing supply would adjust to satisfy higher housing demand dampening the effect on prices too. Both of these characteristics could be explored in a more detailed model.

**Both housing value and earnings shocks**

Adding uncertainty to household wage earnings makes mortgages more risky. Consequently, the borrowing household who receives a low earnings shock may choose to default at housing value states that did not trigger default in the setup without earnings risk. In this section I show that introducing more uncertainty does not extinguish the generaequilibrium effect and rising mortgages
Figure 4.3: Mortgage total default rate, non-strategic default rate, the mortgage spread, mortgages, housing, housing price and capital stock.

![Graphs showing mortgage default rates and other economic variables.]

**Note:** The first row plots total mortgage default rate $\delta$ and non-strategic mortgage default rate $\delta^n$ (left panel) and the mortgage spread (right panel). Both are expressed in percentage points. The mortgage spread is defined as a difference between the mortgage rate and the risk-free rate. The second row plots mortgages $m_1$ and borrower’s housing $h_1^B$. The third row plots housing price $q$ (relative to the consumption price) and capital $k_1$. The horizontal axis measures different levels of earnings exemptions $\kappa$.

...can prevail with higher capital as before. Differently from the setup with housing value shocks only, earnings shocks increase the non-strategic default rate. Earnings shocks combined with housing value shocks can reduce household income significantly and repaying the mortgage now can result in negative consumption. Higher rate of forced defaults underscores the role of borrower protection in providing consumption insurance. The increased importance of consumption insurance provided by earnings exemptions results in even higher demand effect and the rise in mortgage credit is higher than if wage income is risk-free.

Figure 4.3 plots the values of macroeconomic variables in the economy with earnings risk across different levels of earnings exemptions $\kappa$. The figure plots default rates (total $\delta$ and non-strategic $\delta^n$), the mortgage interest rate spread $r_1^m - r_1$, mortgage volume $m_1$, housing $h_1^B$, housing price $q$ and capital $k_1$. 

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Introducing earnings shocks restricts household’s ability to repay in bad states and makes household default more frequent in economies with low levels of borrower protection. The increased total default risk is presented in Figure 4.3. As previously, default risk increases in the earnings exemption level $\kappa$, and the mortgage rate spread follows it, but different exemption levels now corresponds to higher default risk than without earnings shocks. For a comparison, $\kappa = 0.2$ corresponded to zero default risk in the previous setup but given the presence of earnings uncertainty there is already 2 percent default risk at $\kappa = 0.2$. The right panel shows that the borrowing costs (the mortgage spread) start rising for lower exemption levels as compared to the previous setup: previously it became positive only for $\kappa > 0.57$ and now it exceeds zero when $\kappa > 0.17$.

Despite the clear tendency of the increasingly more expensive mortgage credit, for lower values of earnings exemptions demand for mortgages still dominates the supply effect. In Figure 4.3 equilibrium mortgage credit rises in the level of borrower protection for the first half of the $\kappa$ interval. The jump in the equilibrium mortgage credit corresponds to larger than zero default risk, i.e. when the indebted household actually uses consumption insurance in bad states. Higher risk significantly increases demand for mortgages, because this is the only tool to insure against low consumption in bad states. In the economy with risky earnings mortgages rise by almost 100 percent, up from the increase of 11 percent in the setup with housing shocks only (Figures 4.1 and 4.3).

For higher values of earnings exemptions ($\kappa > 0.23$) mortgage credit remains almost stable and even declines a bit suggesting that the supply effect almost counteracts higher demand for mortgages when $\kappa$ is sufficiently high. Under the chosen parametrization, mortgages decline by 0.2 percent when $\kappa$ increases from 0.23 to 0.46 and remain at that level. Ignoring the dynamics in the mid-interval and comparing the zero level of earnings exemptions with the maximum level of earnings exemptions would lead to the wrong conclusion that increasing the level of borrower protection always results in the dominating demand effect. The non-linear responses of mortgages to the earnings exemptions level thus prove to be important for the results.
Figure 4.4: Comparison of mortgages, capital, saver’s investment and deposits based on whether the housing price is fixed.

Note: The first row plots mortgages $m_1$ and mortgages in the model with a fixed housing price $m^q_1$ (left panel). Right panel compares capital $k_1$ to capital in the model with a fixed housing price $k^q_1$. The second row compares saver’s housing $h^S_1$ to saver’s housing in the model with a fixed housing price $h^q_{S1}$ (left panel). Right panel compares deposits $d_1$ to deposits in the model with a fixed housing price $d^q_1$. All is expressed in consumption units. The horizontal axis measures different levels of earnings exemptions $\kappa$.

The increase in borrower’s demand for housing would be possibly lower if the borrowing household had access to unsecured credit. Unsecured credit would help her insure consumption and secured credit would partially lose its importance as the only way to insure consumption against bad states.

The results on credit allocation from the previous section prevail in the setup with risk earnings too. I use Figure 4.4 to show that the credit allocation again crucially depends on the strength of general equilibrium effects. Figure 4.4 compares the values of mortgages, capital, housing and deposits to the values they would take if the housing price was fixed.

The conventional portfolio reallocation effect is again overshadowed by suddenly higher saver’s income and his decision to smooth consumption by saving more through deposits. Then, although mortgages sharply increase in the first half of the earnings exemption interval, capital is not crowded out. If the housing price clears the housing market, higher saver’s wealth increases deposits and allows the bank to increase lending to all borrowers, both households and firms. Fixed housing
prices crowd out capital because housing prices do not affect the saver’s wealth and his savings remain stable across the interval for earnings exemption values. Then the bank has to shift from corporate loans to mortgage credit in order to satisfy its resource constraint.

Notably, the general equilibrium effect is the strongest for very low levels of protection, therefore, if the starting level of borrower protection is already high (in my setup that is $\kappa > 0.23$), the effect on prices would be limited. For sufficiently high levels of earnings exemptions, the conventional portfolio effect is present suggesting that the effect on income is too small to offset direct effects on the bank portfolio composition. I observe that for high values of earnings exemptions ($\kappa > 0.23$), a marginal increase in earnings exemptions leads to a marginal decrease in mortgages combined with an increase in capital by 0.6 percent. This finding is qualitatively in line with the presented US data in Table 4.1.

### 4.4.2 Welfare effects

In this section I show that increasing the level of earnings exemptions can be welfare increasing, particularly due to the associated general equilibrium effect in housing prices. Further, the section discusses how borrower protection affects main contributors to higher welfare, namely higher consumption and higher consumption insurance.

The borrowing household benefits from higher borrower protection because of better consumption insurance and higher consumption opportunities. Consumption insurance is proxied here with a variance of consumption in period 2 where lower variance relative to income implies higher insurance. Borrower protection has a positive effect on consumption insurance for two reasons. Firstly, a positive exemption level protects borrower’s consumption in default. If the household repays, her consumption is lower due to higher interest rates associated with higher earnings exemptions. Higher consumption in bad states and lower consumption in good states contribute to lower variance of consumption and thus better insurance.

Borrower protection also leads to higher equilibrium consumption. The general equilibrium effect into housing prices boosts savers’ wealth and in equilibrium leads to higher savings and
higher equilibrium production. The saving household does not consume the whole increase in income but saves too because borrower’s default reduces saver’s profits in period 2. Although higher consumption contributes to higher welfare, some of the welfare gains may be offset because if consumption rises unevenly across states, for instance, if consumption in good states increases more. In the further discussion, the general equilibrium effect that drives higher consumption is disentangled to understand welfare outcomes better.

Figure 4.5 presents evidence of higher consumption and better consumption insurance for the borrower. To separate the general equilibrium effect, I again distinguish between the values of variables under the endogenous housing price and the values if the fixed housing price was fixed. The first row presents variables for the economy with housing value shocks only. Expected borrower’s consumption in period 2 $E c_{2}^{Bh}$ (left panel) and the variance of borrower’s consumption in period 2 $var(c_{2}^{Bh})$ (right panel) are compared with their counterparts if the housing price is fixed, $E c_{2}^{Bhq}$ and $var(c_{2}^{Bhq})$. The second row provides similar comparison but for the economy with both housing value shocks and earnings risk. It plots expected borrower’s consumption in period 2 $E c_{2}^{B}$ together with consumption when the housing price is fixed, $E c_{2}^{Bq}$ (left panel). The variance of borrower’s consumption in period 2 $var(c_{2}^{B})$ is plotted together with variance when the housing price is fixed, $var(c_{2}^{Bq})$, in the right panel.

The left panel in the first row demonstrates that under housing value shocks the expected borrower’s consumption $E c_{2}^{Bh}$ in period 2 rises. The rise can be attributed to a higher housing price and higher wages. The variance of consumption declines for higher values of $\kappa$, exactly when the household default becomes an equilibrium outcome. Therefore, as expected higher exemptions provide better consumption insurance. If the general equilibrium effect is silenced by assuming a fixed housing price, expected borrower’s consumption in period 2 does not increase suggesting that all equilibrium gains in consumption are entirely through resulting higher savings and higher capital. Switching off the general equilibrium effect, however, does not affect consumption insurance as shown in the right panel.
Figure 4.5: Borrower’s consumption in period 2 and variance of consumption with housing value shocks and with both housing value shocks and earnings shocks.

Note: The first row plots variables in the economy with housing shocks only. The left panel in the first row plots expected borrower’s consumption in period 2 $E_c^{Bh}$ and expected consumption in period 2 $E_c^{Bhq}$ when the housing price is fixed. The right panel provides the variance of borrower’s consumption in period 2 when the housing market clears, $\text{var}(c_2^{Bh})$, and the one with the fixed housing price, $\text{var}(c_2^{Bhq})$. The second row provides the same plots but for the economy with both housing value shocks and risky earnings, that’s why the superscript $h$ disappears. Again in each panel the variables are compared to the ones that result if the housing price was fixed. The horizontal axis measures different levels of earnings exemptions $\kappa$.

When both housing investment and earnings are risky, allowing for a higher the level of exemptions boosts the borrower’s expected consumption in period 2 $E_c^B$ (the second row, left panel). The rise again can be entirely attributed to the general equilibrium effect into housing prices because if the housing price is fixed, consumption even declines, see $E_c^{Bq}$. When the housing price increases to clear the housing market, the rise in expected consumption is even sharper and occurs already for lower levels of exemptions than with housing shocks only. The reason is that when earnings are risky, equilibrium default occurs for lower $\kappa$ and thus the general equilibrium effect is triggered for lower $\kappa$ than with housing value shocks only. This raises equilibrium income and in turn equilibrium consumption for lower $\kappa$ too.

The variance of consumption, however, either does not decline in the level of exemptions as demonstrated by the series $\text{var}(c_2^B)$. However, if the general equilibrium effect is silenced by
imposing the fixed housing price, the variance of consumption in the economy with risky earnings decreases. This is demonstrated by the dynamics of \( \text{var}(c_{2q}^B) \). The result is in line with the idea that borrower protection increases consumption in bad states and decreases consumption in good states. The general equilibrium effect mitigates this result by increasing consumption in goods states. Higher interest rate under higher level of exemptions would decrease consumption in good states (states when the borrower repays), however, the general equilibrium effect raises income to such a level that consumption rises even in good states despite higher borrowing costs. The general equilibrium effect does not necessarily increase consumption in bad states as well because in some cases the borrower’s consumption after default is given by the exemption level \( \kappa \) and thus the the general equilibrium effect into prices does not have an even effect on consumption across states. Higher consumption in good states rather than in all states inflates the variance of consumption. Therefore, although a higher earnings exemption level decreases the variance of consumption for a given level of consumption, increasing the level of consumption increases the resulting variance.

The case with housing value shocks is an exception in this sense, because, as we saw, the general equilibrium effect did not matter for the variance. There are two reasons for that. First, the general equilibrium is especially strong with risky earnings and in other cases it does not raise consumption enough to inflate the variance of consumption. Second, in the economy with risk-free earnings household default occurs for very high levels of exemptions only and in all cases the borrower’s earnings fall below the exemption level (i.e. \( w^B n < \kappa \)). This keeps the one-to-one mapping between household’s consumption and wage and thus the general equilibrium effect increases borrower’s consumption in all states without affecting the variance of consumption. When earnings are risky, the borrower defaults for very low \( \kappa \) too and in those cases her consumption is capped by the exemption level and income excess of \( \kappa \) is garnished by the creditor. Although there are just a few instances of such cases, this is enough to mitigate a decrease in variance.

To sum up this discussion, the borrowing household benefits from higher borrower protection in the form of higher consumption and lower variance of consumption given the level of consumption. However, if the general equilibrium effect is strong, the variance of consumption does not necessarily
Figure 4.6: Expected utility for the borrower, the saver and in total with housing value shocks and both housing value shocks and earnings shocks.

Note: The first row concerns the economy with housing value shocks. The borrower’s expected utility $E_u^{BH}$ is compared to the borrower’s expected utility if the housing price is fixed $E_u^{Bhq}$. The analogous comparison is done for the saver ($E_u^{S}$ vs. $E_u^{Sq}$) and for total utility ($E_u^{h}$ vs. $E_u^{hq}$). The second row concerns the economy with both housing value shocks and earnings shocks. The borrower’s expected utility $E_u^{B}$ is compared to the borrower’s expected utility if the housing price is fixed $E_u^{Bq}$. The analogous comparison is done for the saver ($E_u^{S}$ vs. $E_u^{Sq}$) and for total utility ($E_u$ vs. $E_u^{q}$). The horizontal axis measures different levels of earnings exemptions $\kappa$.

decrease. Then welfare gains for the borrower come mainly through higher consumption rather than better consumption insurance.

The rest of the section discusses welfare outcomes in the model relating them to the results on consumption and consumption insurance. It will show that, despite more opportunities to insure consumption, borrower protection is not necessarily welfare-increasing. In the economy with risk-free earnings, borrower protection is welfare-increasing only if it does not result in crowding-out of capital stock.

Figure 4.6 plots expected utility for the agents. The figure exhibits expected utility for the borrower, for the saver and total expected utility for two types of models: the model with an endogenous housing price and the model with a fixed housing price. The first row corresponds to the economy with housing value shocks only and the second row shows values computed for the economy with risky earnings and risky housing values.
With housing value shocks only, the welfare outcome depends highly on the general equilibrium effect. If the housing price is fixed and thus saver’s income is not positively affected by higher housing demand, capital is crowded out by higher mortgage credit. In such an environment the expected borrower’s utility does not increase in the level of borrower protection despite better consumption insurance, as presented in the first row of Figure 4.6. Higher borrowing costs offset the benefits of smaller fluctuations in income. Total expected utility approximately follows the expected borrower’s utility suggesting that the economy with housing value shocks does not benefit from borrower protection substantially unless it raises the capital stock. Notably, when the housing price is endogenous all agents experience utility gains: higher capital increases wages and all agents afford more consumption.

In the model with risky earnings on top of housing shocks, however, the consumption insurance provided by higher borrower protection becomes sufficiently valuable to create welfare gains even if capital is crowded out. When a fixed housing price prevents capital from rising and even crowds it out as was shown in Figure 4.4, increasing earnings exemptions can still create welfare. Figure 4.6 (second row, left panel) exhibits that the borrower’s expected utility increases in $\kappa$ when $\kappa > 0.2$ even under a fixed housing price. However, the borrower experiences welfare losses if the protection is increased from very low levels. For $\kappa \in (0, 0.2)$, the increase in earnings exemptions results in the sharp crowding-out of capital and the borrower suffers from shrinking income. Total expected utility follows the dynamics of the borrower’s expected utility except that it is shifted marginally up by the saver’s utility. The saver experiences welfare gains from higher equilibrium income as well and these gains seemingly offset lower profits from banks when defaults become more frequent. If the housing price is allowed to clear the housing market, a higher exemption level unambiguously leads to welfare gains, the reason again being a higher resulting capital stock and higher equilibrium income.

What level of borrower protection brings the highest welfare? An endogenous housing price brings welfare gains to all agents and expected total utility rises for the larger part of the earnings exemptions interval, as shown in Figure 4.6. However, when $\kappa > 0.43$ the increase in expected
utility levels off and even declines suggesting that the highest level of welfare is reached in the neighborhood of $\kappa \in (0.35, 0.43)$. That is approximately between 58 and 72 percent of borrower’s wage income. However, I do not match the US mortgage market data, so this finding should be interpreted with caution.

4.5 Robustness checks

This section explores the robustness of results with respect to several key parameters. I choose a risk aversion parameter, the loan-to-value ratio and banker’s preferences as key characteristics that can affect the strength of the mortgage response to the level of earnings exemptions.

4.5.1 Risk aversion

The role of borrower protection in providing consumption insurance governs the mortgage response to the level of earnings exemptions, as discussed in the previous section. The risk aversion parameter
Figure 4.8: Mortgages and capital for different LTV ratios ($\rho$).

Note: The figure plots mortgages $m_1$ and capital $k_1$ in the economy with housing shocks and earnings shocks for different LTV ratios $\rho$. Mortgages and capital are measured in consumption units. The horizontal axis measures different levels of earnings exemptions $\kappa$.

$\sigma_c$ is varied to capture the value of consumption insurance and its importance for the results. Higher $\sigma_c$ means higher relative risk aversion of both the saver and the borrower.

Figure 4.7 plots mortgages and capital under different assumptions about the value of $\sigma_c$. Lower relative risk aversion indeed makes the mortgage response to the level of earnings exemptions weaker. Since under lower risk aversion the value of consumption insurance is lower, mortgage demand would be lower as well. The capital increase is also lower as the strength of the general equilibrium effects that increases capital depend is directly related to mortgage demand and equilibrium mortgage credit.

4.5.2 Loan-to-value ratio

The loan-to-value ratio for mortgages is exogenous and thus the non-monotonic relationship between default rates and the level of protection cannot be generated. A higher level of exemptions in this model unambiguously leads to higher default. In Hatchondo et al. (2015) the household can adjust
Figure 4.9: Mortgages and capital for different banker’s risk aversion parameter $A$.

Note: The figure plots mortgages $m_1$ and capital $k_1$ in the economy with housing shocks and earnings shocks for different banker’s risk aversion parameter $A$. Mortgages and capital are measured in consumption units. The horizontal axis measures different levels of earnings exemptions $\kappa$.

her leverage to overcome the higher default possibility and a higher level of exemptions may even decrease the default risk because households would be borrowing less relative to their assets.

I vary the exogenous LTV ratio to exhibit that given the relationship between the exemption level and default rates does not reverse in my setup for realistic exogenous LTV values but becomes weaker if the LTV ratio is sufficiently low. Figure 4.8 plots mortgages and capital under different assumptions about the value of $\rho$. When the exogenously imposed LTV ratio becomes low enough, household default is not present anymore because the debt is low enough to be repaid. This results in non-response mortgage credit and thus stable capital across different values of $\kappa$. For higher values of LTV ratio the mortgage response to the level of exemptions is positive but weaker if $\rho$ is lower than in the baseline parametrization and thus the household leverage is lower.

### 4.5.3 Bank manager’s risk aversion

Finally, I investigate whether different assumptions about the banking problem affect the strength of the supply effect of borrower protection. Allowing for the banker to be risk averse modifies the
banking problem as described in the Appendix, section C.1. The key change is that the banker now takes into account not only expected returns but also the uncertainty associated with those returns. Increasing the level of borrower protection makes bank returns on mortgages not only lower in bad states but also increases the variance of returns on mortgages relative to the case when the bank could get compensated for (almost) all unpaid mortgage debt by garnishing wages. Under a higher variance of returns the bank lends less to the household at a given interest rate because the banker does not get compensated for the risk enough. The bank’s response would make the first-order supply effect of borrower protection stronger.

Figure 4.9 plots the resulting mortgages and capital. The banker’s risk aversion parameter $A = 0$ corresponds to the case when the banker is risk-neutral and $A = 50$ implies risk aversion. Indeed the bank shrinks the credit supply to the household stronger if the bank manager is risk averse. However, this happens for higher values of earnings exemptions only, i.e. when the demand effect is already weaker. Notably, for higher values of earnings exemptions uncertainty of returns on mortgages is also stronger because the bank loses more in bad states as compared to low levels of borrower protection when the bank can get compensated for (almost) all debt. Since decreasing mortgage credit crowds-in corporate loans for high levels of exemptions, the risk aversion assumption makes mortgages decline by more and capital increase by more. Thus, the supply effect is indeed stronger if the bank manager accounts for the uncertainty of mortgage returns.

4.6 Discussion and future extensions

My findings can be reconciled with the data presented in Table 4.1, if borrower’s housing demand has sufficiently small effects on housing prices. This could be the case due to several reasons, for instance, if housing supply is especially elastic or borrower’s constitute a small number of house buyers, to name a few. Also, in my model the general equilibrium effect is the strongest for very low levels of protection. For high levels of protection, a marginal increase in the level of borrower protection makes mortgage credit decline and capital loans increase, resulting in the conventional
The consumption insurance provided by borrower protection is a driver behind higher mortgage demand in the economy with earnings shocks compared to the economy with housing value shocks only. Exploring the role of the utility function for the results would be useful to strengthen the
consumption insurance argument in the paper, especially, if I considered a utility function that allows to separate a risk aversion parameter and an intertemporal substitution parameter, Epstein-Zin preferences.

Employing households to produce housing goods may have a quantitative effect on my findings. Once the increase in the level of borrower protection reduces mortgage demand, for some values of earnings exemptions housing demand would decrease too. This would have a negative effect on wage income from working at the housing sector and act as an opposite effect to the increase in wage income received from financially constrained firms. However, given that the share of housing production in total output is usually less than half, the decrease in wage income received from the housing sector would be unlikely to offset the increase in wage income from the rest of the economy.

The model assumes exogenous rental market. The interaction between rental market and the housing market could be very enriching and help targeting the data.

4.7 Conclusions

Using a general equilibrium model with mortgage default, I illustrate how an increase in the level of borrower protection (milder recourse) can create benefits even in the case of strategic household default.

The welfare results largely depend on the general equilibrium effect which manifests in the positive feedback effect into savers’ wealth and higher savings. I explore it under two scenarios. First, I consider an economy with housing value shocks only and then the economy with risky earnings and housing value shocks. I show that with housing value shocks the general equilibrium effect determines that an increase in the level of borrower protection does not necessarily crowd out corporate lending but can even increase it. Higher lending to financially constrained firms increases capital stock and leads to the positive feedback effect into production. This results in total output and welfare gains for both the borrower and the saver despite that household default is strategic.
Introducing earnings uncertainty does not offset the general equilibrium effect and higher mortgages can prevail with higher capital as before. Differently from the setup with housing value shocks only, earnings shocks increase the non-strategic default rate because of substantially higher total risk in the economy. Then raising the level of earnings exemptions results in an even higher demand effect and a higher rise in mortgage credit because households value housing more. Notably, the general equilibrium effect is the strongest for very low levels of protection and for sufficiently high values of earnings exemptions it is weak enough to result in the conventional portfolio reallocation effect.

Output gains from milder recourse contribute to total welfare significantly. The highest welfare is achieved when the level of earnings exemptions reaches approximately between 58 and 72 percent of borrower’s earnings. Beyond this point, the capital stock levels off, but borrowing costs continue increasing and reducing the value of consumption insurance.
Chapter 5

Summary

This thesis focuses on the interaction of the macroeconomy, excessive private debt and underlying bankruptcy institutions. First, the thesis explores how Emerging Europe coped with foreign currency debt during the Great Financial Crisis, when local currencies depreciated sharply against the euro and the Swiss franc. The reallocation of depreciation losses from borrowers to banks can lead to different macroeconomic implications dependent not only on the size of debt but also the borrower type. Thus, the analysis focuses on different potential ex-post policies after unexpected currency depreciation: (i) shifting corporate currency mismatch losses to banks and (ii) shifting consumer currency mismatch losses to banks. The thesis continues by considering borrower protection laws, namely recourse laws, and their effect on macroeconomic and welfare outcomes, regardless of debt denomination.

In Chapter 2, I analyze the policy of shifting corporate currency mismatch losses from firms to banks. The key novelty in the model is the explicit modeling of corporate debt overhang in line with the finance literature (Merton (1974)). I calibrate the model to the Hungarian economy which experienced a sudden depreciation episode in the beginning of 2009. I show that insulating corporate borrowers from exchange rate risk and landing losses to banks results in better macroeconomic outcomes. The reason behind is that lending in foreign currency reduces bank exchange rate risk but raises credit risk creating bank equity losses. Therefore, banks cannot avoid losses entirely.

1 It is based on the joint work with Prof. Sweder van Wijnbergen.
but the size of losses depends on the strength of corporate frictions. Corporate debt overhang amplifies aggregate shocks substantially. So reducing losses for overindebted firms leads to better macroeconomic outcomes despite bank losses from an open currency position. The results suggest that one of the reasons why six years after the crisis the Hungarian economy was still experiencing sluggish recovery may well have been the policy of shifting currency mismatch losses to corporate borrowers. This backfired not only by distorting firms’ incentives to invest downwards, but also in higher credit risk for banks, so the policy did not succeed in protecting banks either.

In the analysis I also show that, to contain currency mismatch losses in the banking sector, the government can resort to bank recapitalization. Recapitalizing banks as a policy response would have reduced aggregate losses and would have mitigated the effects of currency mismatch losses on credit supply.

In Chapter 3\(^2\), I again address the macroeconomic consequences of carry trades gone wrong but now I also account for household currency mismatch losses. Household currency mismatch losses were relevant in several countries in Emerging Europe, especially in Hungary and Poland, where about half of mortgages were denominated in Swiss francs.

Although I choose the financial frictions based on the Hungary’s experience during the recession, I also estimate the model on Hungarian data to evaluate the relevance of the chosen frictions. I find strong evidence in favor of corporate debt overhang rather than monitoring costs for corporate loans in the spirit of Bernanke et al. (1999). The Bernanke et al. (1999) based dynamics of borrower net worth and the associated borrowing costs cannot explain aggregate fluctuations in the Hungarian economy as well as the financial distortion for firms’ incentives to invest and hire. Moreover, introducing household debt together with corporate debt improves model fit significantly, suggesting that household debt played a significant role in explaining aggregate fluctuations too.

Using the calibrated/estimated model, I investigate the losses allocation problem. Making corporate borrowers bear currency risk results in worse macroeconomic outcomes than shifting currency mismatch losses back to banks. This result confirms the findings presented in Chapter

\(^2\) It is based on the joint work with Prof. Sweder van Wijnbergen.
2. However, foreign currency mortgages to households generate less recessionary outcomes than currency mismatch in the banking sector. This is because household debt does not affect aggregate supply directly. Also, consumption losses do not affect domestic producers directly but the effect on domestic producers depends on the import structure.

The results suggest that shifting corporate losses back to banks (and recapitalizing them if necessary) would have mitigated the recession better than shifting households losses. Paradoxically, the Hungarian government did the opposite.

In Chapter 4, I explore the general equilibrium arguments for increasing borrower protection. I focus on a particular type of protection, namely recourse laws, which targets residential mortgages. A higher level of protection in this case means milder recourse laws. Then in case of default and for negative home equity, a lender can claim a smaller share of borrower’s wage income. Due to general equilibrium effects, increasing borrower protection creates output gains for most of cases but the size of the effect on output depends on the starting level of protection and the level of risk in the economy. The novel result is that, for low initial levels of bankruptcy protection, both mortgage credit and corporate credit increase in response to higher borrower protection, creating large output and welfare gains. Mortgage credit increases sharply because mortgages become less risky from the borrower’s perspective. The increase in mortgage demand creates strong general equilibrium effects through housing prices: higher housing prices boost savers’ income and savings so that banks can intermediate more credit to both households and firms. Thus, the general equilibrium effect mitigates crowding-out of capital and higher capital increases equilibrium income for both savers and borrowers. The output effect also depends on the level of risk in the economy. The higher is the level of total risk, the higher are output gains from milder recourse for all levels of earnings exemptions.

The associated general equilibrium effects contribute to total welfare gains from higher borrower protection significantly. The highest welfare is achieved when the level of earnings exceptions for mortgage borrowers amounts to more than one half of borrower’s earnings.
The results offer support for strengthening borrower protection because milder recourse laws not only provide more consumption insurance but also generate positive general equilibrium effects. This finding is important for Emerging Europe too; although the region has experienced rapid financial development in the last two decades, the development of relevant institutions lagged behind.
Appendix A

A.1 Tables and figures

Figure A.1: Implied volatility indexes.

Sources: EURO STOXX 50 Volatility Indices database and the courtesy of the blog 'Volatility Futures & Options', available at http://onlyvix.blogspot.nl/2013/03/polands-volatility-index.html.
Table A.1: Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household’s discount factor</td>
<td>0.9970</td>
<td>matches $\pi = 1.059$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient in GHH preferences</td>
<td>1.6</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Labour supply elasticity</td>
<td>8</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.34</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>E.o.S. between domestic and imported goods</td>
<td>1.5</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\epsilon^H$</td>
<td>E.o.S. between varieties of domestic goods</td>
<td>6</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\epsilon^F$</td>
<td>E.o.S. between varieties of imported goods</td>
<td>6</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\epsilon^*$</td>
<td>E.o.S. for exports</td>
<td>1.5</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\theta^H$</td>
<td>Calvo parameter, domestic goods</td>
<td>0.75</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\theta^F$</td>
<td>Calvo parameter, imported goods</td>
<td>0.75</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Share of $x^F$ in $y^C$</td>
<td>0.37</td>
<td>matches avg. imports share of 37%</td>
</tr>
<tr>
<td>$\eta^*$</td>
<td>Share of $e x$ in $y^*$</td>
<td>0.0033</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Investment adjustment cost parameter</td>
<td>13</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>Tax feedback parameter for government debt</td>
<td>0.05</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Technology in SS</td>
<td>1</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation in SS</td>
<td>1.059</td>
<td>avg. in the data in annual terms</td>
</tr>
<tr>
<td>$p^H$</td>
<td>Relative price of $x^H$ in SS</td>
<td>1</td>
<td>calibrated</td>
</tr>
<tr>
<td>$n$</td>
<td>Working hours in SS</td>
<td>0.3</td>
<td>calibrated</td>
</tr>
<tr>
<td>$S$</td>
<td>Nominal exchange rate in SS</td>
<td>1</td>
<td>calibrated</td>
</tr>
<tr>
<td>$y^*$</td>
<td>Total foreign output in SS</td>
<td>104</td>
<td>calibrated</td>
</tr>
<tr>
<td>$R$</td>
<td>Risk-free rate in SS</td>
<td>1.073</td>
<td>avg. in the data in annual terms</td>
</tr>
<tr>
<td>$R^*$</td>
<td>Foreign interest rate in SS</td>
<td>1.045</td>
<td>calibrated</td>
</tr>
<tr>
<td>$s^e$</td>
<td>Gov. consumption/ GDP in SS</td>
<td>0.22</td>
<td>avg. in the data</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>Foreign inflation rate</td>
<td>1</td>
<td>from RER definition in SS</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Risk premium on international bonds in SS</td>
<td>1.01</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\kappa_G$</td>
<td>Elasticity of country risk to net asset position</td>
<td>0.001</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Exogenous shock to the bond premium in SS</td>
<td>1</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Interest rate smoothing</td>
<td>0.766</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\alpha_I$</td>
<td>Interest policy rule (inflation)</td>
<td>1.375</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\alpha_v$</td>
<td>Interest policy rule (output)</td>
<td>0.2</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\rho_{\sigma}$</td>
<td>Volatility shock autoregr. coeff.</td>
<td>0.9</td>
<td>Occhino and Pescatori (2015)</td>
</tr>
<tr>
<td>$\rho_{\rho}$</td>
<td>World demand shock autoregr. coeff.</td>
<td>0.43</td>
<td>Jakab and Kónya (2016)</td>
</tr>
<tr>
<td>$\rho_{\xi}$</td>
<td>Risk premium shock autoregr. coeff.</td>
<td>0.66</td>
<td>Jakab and Kónya (2016)</td>
</tr>
<tr>
<td>$1 - \Phi(d_2)$</td>
<td>Corporate default rate in SS</td>
<td>0.03</td>
<td>avg. in the data</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Fraction of working capital to be paid in advance</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha_f$</td>
<td>Share of FX loans</td>
<td>0.6</td>
<td>avg. in the data</td>
</tr>
<tr>
<td>$\theta_{firms}$</td>
<td>Proportional transfer to the entering firms</td>
<td>0.002</td>
<td>calibrated</td>
</tr>
<tr>
<td>$lev_{firms}$</td>
<td>Bank leverage in SS</td>
<td>3.3</td>
<td>avg. in the data</td>
</tr>
<tr>
<td>$\lambda^L$</td>
<td>Fraction of capital that can be diverted</td>
<td>0.45</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Proportional transfer to the entering bankers</td>
<td>0.002</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$lev$</td>
<td>Bank leverage in SS</td>
<td>5.6</td>
<td>avg. in the data</td>
</tr>
</tbody>
</table>
Figure A.2: Country risk premium shock in the model without leverage-constrained banks.
Figure A.3: World demand shock in the model without leverage-constrained banks.
Figure A.4: World demand shock in the model without leverage-constrained banks when labor demand is not predetermined.
Figure A.5: Volatility shock in the model without leverage-constrained banks.
Figure A.6: Country risk premium shock in the model with leverage-constrained banks.
Figure A.7: World demand shock in the model with leverage-constrained banks.
Figure A.8: Volatility shock in the model with leverage-constrained banks.
A.2 Financially constrained firms

A.2.1 Solving the financially constrained firms’ profit maximization problem with FX loans

To pay in advance, a financially constrained firm \( i \) uses two types of financing. First, it receives equity from households, \( N^{\text{firms}}_{i,t} \). Second, it borrows from the bank an amount \( L_{i,t} \) that consists of both domestic currency funds \( L^D_{i,t} \) and foreign currency denominated funds \( L^F_{i,t} \) such that \( L_{i,t} = L^D_{i,t} + S_t L^F_{i,t} \) where \( S_t \) is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by \( \alpha^F \), so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

To borrow, the firm has to pledge a share \( \kappa \) of future revenue as collateral where \( 0 < \kappa \leq 1 \). We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital minus the expected equity transfer from the household. It follows that in the beginning of period \( t \) the following condition holds:

\[
E_{t-1} \left\{ L_{i,t} \right\} + E_{t-1} \left\{ N^{\text{firms}}_{i,t} \right\} = E_{t-1} \left\{ \rho \left( Q_t k_{i,t} + W_t h_{i,t} \right) \right\} \quad (A.1)
\]

Or, in units of composite goods associated with price \( P_t \),

\[
E_{t-1} \left\{ l_{i,t} \right\} + E_{t-1} \left\{ n^{\text{firms}}_{i,t} \right\} = E_{t-1} \left\{ \rho \left( q_t k_{i,t} + w_t h_{i,t} \right) \right\} \quad (A.2)
\]

\( q_t, w_t \) and \( rer_t \) denote the real price of capital, the real wage and the real exchange rate respectively. We express all three prices are expressed in units of composite goods. It follows that we define \( q_t \) as \( Q_t/P_t \), \( w_t \) as \( W_t/P_t \) and the real exchange rate as \( S_t P^*_t/P_t \) where \( S_t \) is the nominal exchange rate, \( P_t \) is the price of composite goods and \( P^*_t \) defines the price level of foreign composite goods. \( n^{\text{firms}}_{i,t} \) stands for the equity transfer from the domestic household, where \( n^{\text{firms}}_{i,t} \equiv N^{\text{firms}}_{i,t} / P_t \). \( l_{i,t} \) stands for the size of the total loan expressed in units of composite goods and is defined as \( l_{i,t} \equiv L_{i,t} / P_t \). After the loan is taken, shocks
materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm’s private incentives to invest in production inputs.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases the owner of the firm (the domestic household) steps in and transfers lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t}/P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on them as the main source of finance. These funds enter the domestic household’s budget constraint as a lump-sum transfer and have no effect on either the household’s or the firm’s incentives.

Let the matured loan in units of composite goods be $R_{i,t}^R \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}} \right)$, where $R_{i,t}^R$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as $l_{i,t}^D \equiv L_{i,t}^D/P_t$ and $l_{i,t}^F \equiv L_{i,t}^F/P_t^*$. The contracted collateral is a fraction $\kappa$ of firms’ revenue from selling goods and depreciated capital in the next period, $p_{t+1}^R y_{t,t+1}^R + q_{t+1}(1-\delta)k_{i,t}$. $p_{t+1}^R$ stands for the price of homogeneous goods, expressed in units of composite goods ($p_{t+1}^R \equiv P_{t+1}/P_t^*$). Then the decision of the financially constrained firm $i$ born in period $t$ whether to default or not is determined by the lower value:

$$\min \left\{ R_{i,t}^R \left( L_{i,t}^D + S_{t+1} L_{i,t}^F \right), \ k \left( p_{t+1}^R y_{t,t+1}^R + q_{t+1}(1-\delta)k_{i,t} \right) \right\}$$

(A.3)

Deflating by $P_{t+1}$ gives the expression in units of composite goods:

$$\min \left\{ R_{i,t}^R \left( \frac{l_{i,t}^D}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F}{\pi_{t+1}} \right), \ k \left( p_{t+1}^R y_{t,t+1}^R + q_{t+1}(1-\delta)k_{i,t} \right) \right\}$$

(A.4)

where $p_{t+1}^R y_{t,t+1}^R = p_{t+1}^R A_{t+1} \theta_{i,t+1} k_{i,t+1}^{\alpha} h_{i,t+1}^{1-\alpha}$. The firm $i$ born in period $t$ and endowed with corporate equity $N_{i,t}^{firms}$ maximizes profits taking the loan as given. The firm maximizes expected profits given by future revenue from selling goods and depreciated capital minus the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period $t$ also enter the maximization problem and can be summarized as the difference between the loan plus equity and working capital expenditure:

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The resulting first-order conditions are:
\[ k_{i,t} : \quad E_t \beta \Lambda_{t,t+1} \left\{ p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial k_{i,t}} + q_{t+1} (1 - \delta) - (1 - \rho) \frac{q_t}{\pi_{t+1}} \right\} \]

\[ - E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial k_{i,t}} + q_{t+1} (1 - \delta) \right) \right\} \]

\[ = \partial \text{cov} \left( \beta \Lambda_{t,t+1}, \min \left\{ R_{t,t} \left( \frac{d_{1,t}}{\pi_{t+1}} + r_{r+1} \frac{i_{t,t}}{\pi_{t+1}} \right), \kappa \left( p_{t+1}^R \frac{y_{t+1}^R}{\pi_{t+1}} + q_{t+1} (1 - \delta) k_{i,t} \right) \right\} \right) \frac{\partial k_{i,t}}{\partial k_{i,t}} + q_t \]

\[ h_{i,t} : \quad E_t \beta \Lambda_{t,t+1} \left\{ p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial h_{i,t}} - (1 - \rho) \frac{w_t}{\pi_t} \right\} \]

\[ - E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial h_{i,t}} \right) \right\} \]

\[ = \partial \text{cov} \left( \beta \Lambda_{t,t+1}, \min \left\{ R_{t,t} \left( \frac{d_{1,t}}{\pi_{t+1}} + r_{r+1} \frac{i_{t,t}}{\pi_{t+1}} \right), \kappa \left( p_{t+1}^R \frac{y_{t+1}^R}{\pi_{t+1}} + q_{t+1} (1 - \delta) k_{i,t} \right) \right\} \right) \frac{\partial h_{i,t}}{\partial h_{i,t}} + w_t \]

where

\[ d_{2,t} = \frac{E_t \ln \left( \kappa \left( p_{t+1}^R \frac{y_{t+1}^R}{\pi_{t+1}} + q_{t+1} (1 - \delta) k_{i,t} \right) - R_{t,t} R_{r+1} \frac{i_{t,t}}{\pi_{t+1}} \right) - E_t \ln \left( R_{t,t} \frac{i_{t,t}}{\pi_{t+1}} \right)}{\sigma_y}, \quad d_{1,t} = d_{2,t} + \sigma_y \]

The derivation of \( d_{2,t} \) is given in the next section and results for the first-order conditions are given by equations (A2.1) and (A2.2).

The first-order conditions hold together with the ex-ante budget constraint:

\[ E_{t-1} \left\{ l_{i,t} \right\} + E_{t-1} \left\{ n_{i,t}^{firm} \right\} = E_{t-1} \left\{ \rho \left( q_{t} k_{i,t} + w_t h_{i,t} \right) \right\} \]
In the beginning of the next period, after shocks take place and a fraction of firms default, the domestic household pools the remaining net worth from non defaulted firms into aggregate net worth by the following aggregation rule:

\[ n_f t = \omega_f t \left( p_t R_t + q_t (1 - \delta) k_{t-1} - (1 - \rho) \frac{q_{t-1} k_{t-1} + w_{t-1} \pi_t}{\pi_t} \right) \]

\[-\omega_f t \left( (1 - \Phi(d_{1,t-1})) \kappa \left( p_t R_t + q_t (1 - \delta) k_{t-1} \right) + \Phi(d_{2,t-1}) R_{t-1} \frac{\lambda^p - 1}{\pi_t} + \Phi(d_{1,t-1}) r_{er_t} \frac{\lambda^F - 1}{\pi_t} \right) + \iota_f t \cdot n_f t \]

Recall that \((1 - \Phi(d_{1,t-1}))\) proxies for the default rate (by the law of large numbers this is equal to the share of defaulted firms in the economy). Then the first term on the right hand side is aggregate firms’ revenue from production and selling depreciated capital minus the rest of the expenditure for working capital. The second term is the firms’ aggregate expenditure for repaying loans. The difference between the two gives financially constrained firms’ profits. The third term is the injection of new equity. We assume that the domestic household acts as distributor and cannot divert pooled equity funds anywhere else. Also the existing equity can be increased only by the amount \(\iota_f t \cdot n_f t\) that is fixed and proportional to aggregate net worth in the steady state. Thus, this equity transfer does not depend on the household’s decision. \(\omega_f t\) is a fraction that is close but lower than unity. We assume that this parameter proxies for the equity management costs incurred by the household and use this parameter to calibrate the steady state corporate leverage to the one observed in the data.

**A.2.2 Derivation of the default probability**

We need to compute the expected value of the firm’s payment function (we abstract from indices \(i\) for the sake of brevity):

\[ E_t \min \left\{ R_t^R \left( \frac{1D}{\pi_{t+1}} + r_{er_{t+1}} \frac{1F}{\pi_{t+1}} \right), \kappa \left( p_{t+1} R_{t+1} + q_{t+1} (1 - \delta) k_t \right) \right\} \]
To simplify, we re-order the terms in the following way:

\[ E_t \min \left\{ R_t^R \frac{l_t^D}{\pi_{t+1}}, \kappa \left( p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_t \right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}} \right\} + E_t R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}} \]

Further we focus on the first term only, since it defines the default decision and contains all necessary prices too:

\[ E_t \min \left\{ R_t^R \frac{l_t^D}{\pi_{t+1}}, \kappa \left( p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_t \right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}} \right\} \]

Define \( \bar{y}_{t+1} \equiv \pi_{t+1} \left( \kappa \left( p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_t \right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}} \right) \), where

\[ \bar{y}_{t+1} \sim \text{log-normal} \left( \mu_{\bar{y}_{t+1}}, \sigma_{\bar{y}}^2 \right) \]

Then the modified minimum function can be re-written as

\[ E_t \min \left\{ R_t^R l_t^D, \bar{y}_{t+1} \right\} \]
Further, 

\[
E_t \min \{ R_t^{R_D}, \ \bar{y}_{t+1} \} = R_t^{R_D} \Pr \left( R_t^{R_D} < \bar{y}_{t+1} \right) + \left( 1 - \Pr \left( R_t^{R_D} < \bar{y}_{t+1} \right) \right) E_t \left( \bar{y}_{t+1} \mid \bar{y}_{t+1} < R_t^{R_D} \right) \\
= R_t^{R_D} \Pr \left( R_t^{R_D} < \bar{y}_{t+1} \right) + \int_{0}^{R_t^{R_D}} \frac{\bar{y}_{t+1} dF(\bar{y}_{t+1})}{1 - \Pr \left( R_t^{R_D} < \bar{y}_{t+1} \right)} \\
= R_t^{R_D} \Pr \left( R_t^{R_D} < \bar{y}_{t+1} \right) + \int_{0}^{R_t^{R_D}} \bar{y}_{t+1} dF(\bar{y}_{t+1}) \\
= R_t^{R_D} \int_{0}^{\infty} \frac{1}{\bar{y}_{t+1} \sigma_y \sqrt{2\pi}} e^{-\frac{(\ln(\bar{y}_{t+1}) - \mu_y)^2}{2\sigma^2}} d(\bar{y}_{t+1}) + \int_{0}^{R_t^{R_D}} \frac{1}{\sigma_y \sqrt{2\pi}} e^{-\frac{(\ln(\bar{y}_{t+1}) - \mu_y)^2}{2\sigma^2}} d(\bar{y}_{t+1}) \\
= R_t^{R_D} \Phi \left( \frac{\ln(\bar{y}_{t+1}) - \mu_y}{\sigma_y} \right) \int_{0}^{\infty} R_t^{R_D} + \int_{0}^{R_t^{R_D}} \frac{1}{\sigma_y \sqrt{2\pi}} e^{-\frac{(\ln(\bar{y}_{t+1}) - \mu_y)^2}{2\sigma^2}} d(\bar{y}_{t+1}) \\
= R_t^{R_D} \left( 1 - \Phi \left( \frac{\ln(\bar{y}_{t+1}) - \mu_y}{\sigma_y} \right) \right) - \frac{1}{2} e^{\frac{\sigma^2}{2}} \text{erf} \left( \frac{-\ln(\bar{y}_{t+1}) + \mu_y + \frac{\sigma^2}{2}}{\sqrt{2\sigma_y}} \right) R_t^{R_D} \\
= R_t^{R_D} \Phi \left( \frac{\mu_y - \ln(\bar{y}_{t+1})}{\sigma_y} \right) + \frac{1}{2} E_t(\bar{y}_{t+1}) \left( \text{erf} \left( \frac{\ln(\bar{R_t^{R_D}} - \mu_y - \sigma_y^2}{\sqrt{2\sigma_y}} \right) + 1 \right) \\
= R_t^{R_D} \Phi \left( \frac{\mu_y - \ln(\bar{y}_{t+1})}{\sigma_y} \right) + E_t(\bar{y}_{t+1}) \Phi \left( \frac{\ln(\bar{R_t^{R_D}} - \mu_y - \sigma_y^2}{\sigma_y} \right) \\
= R_t^{R_D} \Phi \left( \frac{\mu_y - \ln(\bar{R_t^{R_D}})}{\sigma_y} \right) + E_t(\bar{y}_{t+1}) \left( 1 - \Phi \left( \frac{\mu_y - \ln(\bar{R_t^{R_D}})}{\sigma_y} + \sigma_y \right) \right) \\
\\
The expression can be simplified as \\

\[
E_t \min \{ R_t^{R_D}, \ \bar{y}_{t+1} \} = (1 - \Phi(d_{1,t})) E_t(\bar{y}_{t+1}) + \Phi(d_{2,t}) R_t^{R_D} \\
\\
where \\

\[
d_{2,t} = \frac{\mu_y - \ln(\bar{R_t^{R_D}})}{\sigma_y}, \quad d_{1,t} = d_{2,t} + \sigma_y \\
\\
where \\

\[
\mu_y = E_t \ln(\bar{y}_{t+1}) \\
\\
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or
\[ d_{2,t} = \frac{E_t \ln (\bar{y}_{t+1}/\pi_{t+1}) - \ln \left( R_t^R / \pi_{t+1} l_t^D \right)}{\sigma_y}, \quad d_{1,t} \equiv d_{2,t} + \sigma_y \]

Recall that \( \bar{y}_{t+1} = \pi_{t+1} \left( k \left( p_{t+1}^R y_{t+1}^R + q_{t+1} (1 - \delta) k_t \right) - R_t^R \pi_{t+1} \pi_{t+1} l_t^D \right) \) so it can be substituted back to get complete expressions. Then \( \sigma_y^2 = \text{var} (\bar{y}_{t+1}) = \text{var} \left( \pi_{t+1} \left( k \left( p_{t+1}^R y_{t+1}^R + q_{t+1} (1 - \delta) k_t \right) - R_t^R \pi_{t+1} \pi_{t+1} l_t^D \right) \right) \).

To solve for the first-order conditions, we differentiate the expected loan payment w.r.t. \( k_t \):

\[
\frac{\partial E_t \min \left\{ R_t^R l_t^D, \bar{y}_{t+1} \right\}}{\partial k_t} = (1 - \Phi(d_{1,t})) \frac{\partial E_t \bar{y}_{t+1}}{\partial k_t}
- E_t \bar{y}_{t+1} \frac{\partial \Phi(d_{1,t})}{\partial d_{1,t}} \frac{\partial d_{1,t}}{\partial k_t} + R_t^R l_t^D \frac{\partial \Phi(d_{2,t})}{\partial d_{2,t}} \frac{\partial d_{2,t}}{\partial k_t}
= (1 - \Phi(d_{1,t})) \frac{\partial E_t \bar{y}_{t+1}}{\partial k_t}
\]

where the proof of the last expression comes from by using \( \frac{\partial d_{1,t}}{\partial k_t} = \frac{\partial d_{2,t}}{\partial k_t} \) and computing the following:

\[
-E_t (\bar{y}_{t+1}) \Phi'(d_{1,t}) + R_t^R l_t^D \Phi'(d_{2,t})
= -e^{\ln(E_t \bar{y}_{t+1})} \Phi'(d_{1,t}) + e^{\ln(R_t^R l_t^D)} \Phi'(d_{2,t})
= -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{1,t}^2} + e^{\ln(R_t^R l_t^D)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2}
= -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( d_{2,t}^2 + 2 d_{2,t} \sigma_y + \sigma_y^2 \right)} + e^{\ln(R_t^R l_t^D)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2}
= -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{-\left( d_{2,t} \sigma_y + \frac{1}{2} \sigma_y^2 \right)} + e^{\ln(R_t^R l_t^D)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2}
= -e^{\ln(E_t \bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{-\left( E_t \ln(\bar{y}_{t+1}) - \ln(R_t^R l_t^D) + \frac{1}{2} \sigma_y^2 \right)} + e^{\ln(R_t^R l_t^D)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2}
= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \left[ -e^{\ln(E_t \bar{y}_{t+1})} e^{-\left( E_t \ln(\bar{y}_{t+1}) - \ln(R_t^R l_t^D) + \frac{1}{2} \sigma_y^2 \right)} + e^{\ln(R_t^R l_t^D)} \right]
= -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{\ln(R_t^R l_t^D)} + e^{\ln(R_t^R l_t^D)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2}
= 0,
\]

where such expressions are used as

\[ E_t \ln (\bar{y}_{t+1}) = \ln (E_t \bar{y}_{t+1}) - \frac{1}{2} \sigma_y^2 \]

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and the definition of the variable \( d_{1,t} \). Substituting a definition for \( \bar{y}_{t+1} \) back gives

\[
\frac{\partial E_t}{\partial k_t} \min \left\{ \frac{R_i^R}{\pi_{t+1}^i}, \kappa \left( p_{t+1}^R z_{t+1} + q_{t+1}(1 - \delta) k_i \right) - R_i^R r e_{t+1}^i \frac{l_i^F}{\pi_{t+1}^i} \right\} = (1 - \Phi(d_{1,t})) \frac{\partial E_t}{\partial k_t} \kappa \left( p_{t+1}^R z_{t+1} + q_{t+1}(1 - \delta) k_i \right)
\]  

(A2.1)

Similarly it can be showed that

\[
\frac{\partial E_t}{\partial h_t} \min \left\{ \frac{R_i^R}{\pi_{t+1}^i}, \kappa \left( p_{t+1}^R z_{t+1} + q_{t+1}(1 - \delta) k_i \right) - R_i^R r e_{t+1}^i \frac{l_i^F}{\pi_{t+1}^i} \right\} = (1 - \Phi(d_{1,t})) \frac{\partial E_t}{\partial h_t} \kappa \left( p_{t+1}^R z_{t+1} \right)
\]  

(A2.2)

### A.2.3 Solving the financially constrained firms’ profit maximization problem with domestic currency loans

Now the matured loan in units of composite goods is \( R_{i,t}^l \equiv R_{i,t}^l \pi_{t+1}^i \). The loan is denominated in domestic currency and \( R_{i,t}^R \) is the nominal gross interest rate on the loan. The contracted collateral is a fraction \( \kappa \) of firms’ revenue from selling goods and depreciated capital in the next period. In units of composite goods the contracted collateral can be expressed as \( p_{t+1}^R z_{t+1} + q_{t+1}(1 - \delta) k_{i,t} \). Then the decision of the financially constrained firm \( i \) born in period \( t \) whether to default or not is determined by the lower value:

\[
\min \left\{ R_{i,t}^l \kappa^i, \kappa \left( p_{t+1}^R z_{t+1} + q_{t+1}(1 - \delta) k_{i,t} \right) \right\}
\]

As previously, \( p_{t+1}^R z_{t+1} = p_{t+1}^R \theta_{i,t+1} k_{i,t}^\alpha h_{i,t}^{1-\alpha}, p_{t+1}^R = p_{t+1}^R / P_{t+1} \) and \( q_{t+1} = Q_{t+1} / P_{t+1} \).

Financial flows received in period \( t \) also enter the maximization problem and can be summarized as the difference between the loan plus equity (both \( N_{i,t}^{\text{firms}} \) and \( Z_{i,t} \)) and working capital expenditure expressed in
units of composite goods:

\[
\max_{\{k_{i,t}, h_{i,t}\}} E_t \beta \Lambda_{t,t+1} \left\{ \frac{p_{t+1}^{R} y_{i,t+1}^{R} + Q_{t+1}(1 - \delta) k_{i,t} - (1 - \rho) (Q_t k_{i,t} + W_t h_{i,t})}{P_{t+1}} \right\}
\]

\[- E_t \beta \Lambda_{t,t+1} \min \left\{ \frac{R_{t+1}^{R} L_{i,t}}{P_{t+1}} \right\}
\]

\[
= - E_t \beta \Lambda_{t,t+1} \left\{ \kappa \left( \frac{p_{t+1}^{R} y_{i,t+1}^{R} + Q_{t+1}(1 - \delta) k_{i,t}}{P_{t+1}} \right) \right\}
\]

s.t.

\[
\frac{E_{t-1} \left\{ L_{i,t} + N_{i,t}^{firms} \right\}}{P_t} = \frac{E_{t-1} \left\{ \rho (Q_t k_{i,t} + W_t h_{i,t}) \right\}}{P_t}
\]

Using the previously introduced definitions yields

\[
\max_{\{k_{i,t}, h_{i,t}\}} E_t \beta \Lambda_{t,t+1} \left\{ \frac{p_{t+1}^{R} y_{i,t+1}^{R} + q_{t+1}(1 - \delta) k_{i,t} - (1 - \rho) \frac{q_t k_{i,t} + w_t h_{i,t}}{\pi_{t+1}}}{\pi_{t+1}} \right\}
\]

\[- E_t \beta \Lambda_{t,t+1} \min \left\{ \frac{R_{t+1}^{R} l_{i,t}}{\pi_{t+1}} \right\}
\]

\[
= - E_t \beta \Lambda_{t,t+1} \left\{ \kappa \left( \frac{p_{t+1}^{R} y_{i,t+1}^{R} + q_{t+1}(1 - \delta) k_{i,t}}{\pi_{t+1}} \right) \right\}
\]

s.t.

\[
E_{t-1} \left\{ l_{i,t} \right\} + E_{t-1} \left\{ n_{i,t}^{firms} \right\} = E_{t-1} \left\{ \rho (q_t k_{i,t} + w_t h_{i,t}) \right\}
\]

The resulting first-order conditions are:

\[
k_{i,t} : E_t \beta \Lambda_{t,t+1} \left\{ \frac{p_{t+1}^{R} \partial y_{i,t+1}^{R}}{\partial k_{i,t}} + q_{t+1}(1 - \delta) - (1 - \rho) \frac{q_t}{\pi_{t+1}} \right\}
\]

\[- E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{i,t})) \kappa \left( \frac{p_{t+1}^{R} \partial y_{i,t+1}^{R}}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \right) \right\}
\]

\[
= \frac{\partial \text{cov} \left( \beta \Lambda_{t,t+1}, \kappa \left( p_{t+1}^{R} y_{i,t+1}^{R} + q_{t+1}(1 - \delta) k_{i,t} \right) \right)}{\partial k_{i,t}}
\]

\[+ \rho q_t
\]

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\[
\begin{align*}
  h_{i,t} & : E_t \beta \Lambda_{t,t+1} \left\{ p_{t+1}^R \frac{\partial y_{i,t+1}^R}{\partial h_{i,t}} - (1 - \rho) \frac{w_{t}}{\pi_{t+1}} \right\} \\
  & \quad - E_t \beta \Lambda_{t,t+1} \left\{ (1 - \Phi(d_{1,t})) \kappa \left(p_{t+1}^R \frac{\partial y_{i,t+1}^R}{\partial h_{i,t}} \right) \right\} \\
  & \quad = \frac{\partial \text{cov} \left( \beta \Lambda_{t,t+1}, \min \left\{ R_{i,t}^R, \kappa \left(p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right\} \right)}{\partial h_{i,t}} \\
  & \quad + \rho w_{t}
\end{align*}
\]

where

\[
d_{2,t} \equiv E_t \ln \left( \kappa \left(p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right) - E_t \ln \left(p_{t+1}^R \frac{h_{i,t}}{\pi_{t+1}} \right), \quad d_{1,t} = d_{2,t} + \sigma_y
\]

and \( \sigma_y^2 = \text{var} \left( \pi_{t+1} \kappa \left(p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1 - \delta)k_{i,t} \right) \right) \).

The first-order conditions hold together with the ex-ante budget constraint:

\[
E_{t-1} \left\{ l_{i,t} \right\} + E_{t-1} \left\{ n_{i,t}^{firms} \right\} = E_{t-1} \left\{ \rho \left(q_{t} k_{i,t} + w_{t} h_{i,t} \right) \right\}
\]

### A.2.4 Model with flexible labor demand

In simulation exercises, when we relax the assumption of predetermined labor supply, we make the following modifications to the model. Firstly, we assume that the only input for financially constrained firms’ production is capital. Second, we introduce a new layer of production firms and call them intermediate firms. These firms combine financially constrained firms’ production with labor and sell homogeneous goods to domestic retail firms. The novel type of firms is not subject to financial frictions.

Then the financially constrained firm’s problem changes accordingly. The firm’s borrowing decision depends on the firm’s expected working capital needs such that in the beginning of period \( t \) the following condition holds:

\[
E_{t-1} \left\{ L_{i,t} \right\} + E_{t-1} \left\{ N_{i,t}^{firms} \right\} = E_{t-1} \left\{ \rho \left(Q_{t} k_{i,t} \right) \right\}
\]
Or, units of composite goods,

\[ E_{t-1} \{ l_{i,t} \} + E_{t-1} \left\{ n_{t,t}^{firms} \right\} = E_{t-1} \left\{ \rho \left( q_{t,k_{i,t}} \right) \right\} \]

Definition of \( p_{t+1}^R \) changes in the following way: \( p_{t+1}^R y_{t+1}^R = p_{t+1}^R A_{t+1} \theta_{i,t+1} k_{i,t} \).

After shocks take place, the generation of firms \( t \) will solve the profit maximization problem taking the loan as given. They will sell goods at the competitive price \( P_{t+1}^R \) which is defined \( p_{t+1}^R \), if expressed in units of composite goods. The profit optimization problem of a financially constrained firm \( i \) will be the following (the numeraire is the composite good):

\[
\max_{\{k_{i,t}, h_{i,t}\}} E_i \beta \Lambda_{t+1} \left\{ \frac{p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_{i,t} - \left(1 - \rho \right)q_{t,k_{i,t}}}{\pi_{t+1}} \right\} \\
- E_i \beta \Lambda_{t+1} \min \left\{ \frac{R_{t+1}^R l_{i,t}}{\pi_{t+1}}, \kappa \left( \frac{p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_{i,t}}{\pi_{t+1}} \right) \right\} \\
+ \lambda_{i,t} + n_{t,t}^{firms} + z_{i,t} - \rho \left( q_{t,k_{i,t}} \right)
\]

s.t.

\[ E_{t-1} \{ l_{i,t} \} + E_{t-1} \left\{ n_{t,t}^{firms} \right\} = E_{t-1} \left\{ \rho \left( q_{t,k_{i,t}} \right) \right\} \]

The corresponding first-order condition is:

\[ k_{i,t} : E_i \beta \Lambda_{t+1} \left\{ p_{t+1}^R \frac{\partial y_{t+1}^R}{\partial k_{i,t}} + (1-\delta)q_{t+1} - \left(1 - \rho \right)q_{t,k_{i,t}} \right\} \]

\[
\frac{\partial E_i \beta \Lambda_{t+1} E_i}{\partial k_{i,t}} \min \left\{ \frac{R_{t+1}^R l_{i,t}}{\pi_{t+1}}, \kappa \left( \frac{p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_{i,t}}{\pi_{t+1}} \right) \right\} \\
= \frac{\partial \text{cov} \left( \beta \Lambda_{t+1}, \min \left\{ \frac{R_{t+1}^R l_{i,t}}{\pi_{t+1}}, \kappa \left( \frac{p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_{i,t}}{\pi_{t+1}} \right) \right\} \right)}{\partial k_{i,t}} + \rho q_i
\]

If we substitute the expression for the expected value of loan repayment, we get:
\[ k_{i,t} : \quad E_t \beta \Lambda_{t,t+1} \left( p_{t+1} R \frac{\partial y_{i,t+1}^R}{\partial k_{i,t}} + q_{t+1} (1 - \delta) - (1 - \rho) \frac{q_t}{\pi_{t+1}} \right) \]

\[ - E_t \beta \Lambda_{t,t+1} \left( (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1} R \frac{\partial y_{i,t+1}^R}{\partial k_{i,t}} + q_{t+1} (1 - \delta) \right) \right) \]

\[ \partial \text{cov} \left( \beta \Lambda_{t,t+1}, \min \left( \frac{R_{t+1}^R}{\pi_{t+1}}, \kappa \left( p_{t+1} R + q_{t+1} (1 - \delta) k_{i,t} \right) \right) \right) = \frac{\partial}{\partial k_{i,t}} \]

\[ = \rho q_t \]

where

\[ d_{2,t} \equiv \frac{E_t \ln \left( \kappa \left( p_{t+1} R + q_{t+1} (1 - \delta) k_{i,t} \right) \right) - E_t \ln \left( \frac{R_{t+1}^R}{\pi_{t+1}} \right)}{\sigma_y} \]

\[ d_{1,t} = d_{2,t} + \sigma_y \]

\[ \sigma_y^2 \text{ is given by } \text{var} \left( \pi_{t+1} \kappa \left( p_{t+1} R + q_{t+1} (1 - \delta) k_{i,t} \right) \right). \]

Homogeneous goods produced by financially constrained firms are purchased as inputs by the new layer of competitive producers, intermediate producers. Intermediate producers hire labor and combine it with homogeneous goods produced by financially constrained firms by using the following technology:

\[ y_I = \left( y_R^I \right)^{\alpha} h_I^{1-\alpha} \]

Recall that financially constrained firms’ aggregate production function now is given by: \( y_R^I = A_t k_{t-1} \).

Produced goods are sold to retail firms at the nominal price \( P_I^I \) immediately after production takes place. This gives two equilibrium conditions that can be derived from profit maximization with respect to inputs:

\[ y_R^I : \quad p_R^I = p_I^I \alpha \left( y_R^I \right)^{\alpha-1} h_I^{1-\alpha} \]

\[ h_I : \quad w_I = p_I^I (1 - \alpha) \left( y_R^I \right)^{\alpha} h_I^{-\alpha} \]

In derivations we defined the following relative prices: \( p_I^I \equiv P_I^I / P_I, p_R^I \equiv P_R / P_I \) and \( w_I \equiv W_I / P_I \).
Marginal costs of the retail firms changes from being the price of financially constrained firms’ goods to the price of intermediate goods.

### A.3 Solving the banks’ optimization problem

#### A.3.1 Lending in foreign currency and domestic currency with a fixed denomination structure

The domestic household owns all banks that operate in the domestic economy and lend to financially constrained firms. We assume that there is a continuum of these banks and every period there is a probability \( \omega \) that a bank continues operating. Otherwise, the net worth is transferred to the owner of the bank, the domestic household. We assume that banks give loans out of accumulated equity \( N_t \), deposits \( D_t \) and foreign debt \( D^*_t \). The balance sheet constraint of a bank \( j \), expressed in units of composite goods, is given by

\[
\frac{N_{j,t} + D_{j,t} + S_t D^*_t}{P_t} = \frac{L_{j,t}}{P_t}
\]

\( L_{j,t} \) consists of both domestic currency funds \( L^D_{j,t} \) and foreign currency denominated funds \( L^F_{j,t} \) such that

\[
L_{j,t} = L^D_{j,t} + S_t L^F_{j,t}
\]

where \( S_t \) is the nominal exchange rate.

Banks pay a nominal domestic interest rate \( R_t \) on deposits and a nominal foreign interest rate \( R^*_t \xi_t \) on foreign debt. \( R^*_t \) follows a stationary AR(1) process. \( \xi_t \) denotes a premium on bank foreign debt.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction \( \lambda^L \) of assets, but if that happens the bank goes bankrupt (i.e., cannot continue).

The only asset on the banks’ balance sheet is loans to financially constrained firms, thus, the expected nominal return of the bank \( j \) is defined as \( R^{L}_{j,t} \) and given by:

\[
E_t \left\{ R^{L}_{j,t}L_{j,t} \right\} = E_t \min \left\{ R^{R}_{j,t} \left( L^D_{j,t} + S_{t+1}L^F_{j,t} \right), \, \kappa \left( P_{t+1}y^{R}_{j,t+1} + Q_{t+1}(1-\delta)k_{j,t} \right) \right\}
\]

Or, units of composite goods,

\[
E_t \left\{ \frac{R^{L}_{j,t}}{\pi_{t+1}}l_{j,t} \right\} = E_t \min \left\{ R^{R}_{j,t} \left( \frac{l^D_{j,t}}{\pi_{t+1}} + rer_{t+1} \frac{l^F_{j,t}}{\pi_{t+1}} \right), \, \kappa \left( p_{t+1}y^{R}_{j,t+1} + q_{t+1}(1-\delta)k_{j,t} \right) \right\}
\]
Then the optimization problem of the bank $j$ can be written as:

$$V_{j,t} = \max_{\{d_{j,t}, d_{j,t}^*, l_{j,t}\}} \ E_t \left[ \beta \Lambda_{t+1} \left( (1 - \omega) \frac{N_{j,t+1}}{P_{t+1}} + \omega V_{j,t+1} \right) \right]$$

s.t.

$$V_{j,t} \geq \lambda^L l_{j,t},$$  \hspace{1cm} \text{(Incentive constraint)}

$$\frac{N_{j,t} + D_{j,t} + S_t D_{j,t}^*}{P_t} = L_{j,t},$$  \hspace{1cm} \text{(Balance sheet constraint)}

$$n_{j,t} + d_{j,t} + \text{rer}_t d_{j,t}^* = l_{j,t},$$  \hspace{1cm} \text{(Balance sheet constraint)}

$$n_{j,t} = \frac{R_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{j,t-1}}{\pi_t} d_{j,t-1} - \frac{R_t}{\pi_t} \xi_{t-1} + \frac{R_t^*}{\pi_t} d_{j,t-1}^* \text{rer}_t d_{j,t-1}^*,$$  \hspace{1cm} \text{(LoM of net worth)}

Lagrangian of the problem can be formulated as:

$$L = (1 + \nu l_{j,t}) E_t \beta \Lambda_{t+1} \left( (1 - \omega) \left( \frac{R_{j,t}}{\pi_{t+1}} l_{j,t} - \frac{R_{j,t}}{\pi_{t+1}} d_{j,t} - \frac{R_t^*}{\pi_{t+1}} \xi_{t-1} + \frac{R_t^*}{\pi_{t+1}} d_{j,t}^* \right) \right) + \omega V_{j,t+1}$$

$$- \nu 1, t \lambda^L l_{j,t}$$

$$+ \nu 2, t \left( \frac{R_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{j,t-1}}{\pi_t} d_{j,t-1} - \frac{R_t^*}{\pi_t} \xi_{t-1} + \frac{R_t^*}{\pi_t} d_{j,t-1}^* - l_{j,t} + d_{j,t} + rer_t d_{j,t}^* \right)$$
This gives the first-order conditions:

\[
\begin{align*}
l_{j,t} : & \quad (1 + v_{1,t}) \beta_E \Lambda_{t,t+1} \left\{ (1 - \omega) \left( \frac{R^L_{j,t}}{\pi_{t+1}} \right) + \omega \frac{\partial V(.)}{\partial l_{j,t}} \right\} = \lambda^L v_{1,t} + v_{2,t} \\
d_{j,t} : & \quad (1 + v_{1,t}) \beta_E \Lambda_{t,t+1} \left\{ (1 - \omega) \left( \frac{R_{t}}{\pi_{t+1}} \right) - \omega \frac{\partial V(.)}{\partial d_{j,t}} \right\} = v_{2,t} \\
d^*_{j,t} : & \quad (1 + v_{1,t}) \beta_E \Lambda_{t,t+1} \left\{ (1 - \omega) \left( \frac{R^*_{t} \xi_{t}}{\pi^*_{t+1}} \right) - \omega \frac{\partial V(.)}{\partial d^*_{j,t}} \right\} = v_{2,t} rer_t 
\end{align*}
\]

with complementary slackness conditions:

\[
\begin{align*}
v_{1,t} : & \quad v_{1,t} \left( V_{j,t} - \lambda^L l_{j,t} \right) = 0 \\
v_{2,t} : & \quad v_{2,t} \left( \frac{R^L_{j,t-1}}{\pi_{t-1}} l_{j,t-1} - \frac{R_{t-1}}{\pi_{t-1}} d_{j,t-1} - \frac{R^*_{t-1} \xi_{t-1}}{\pi^*_{t-1}} rer_t d^*_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t} \right) = 0 
\end{align*}
\]

Further, the first-order conditions can be expressed as

\[
\begin{align*}
l_{j,t} : & \quad (1 + v_{1,t}) \beta_E \Lambda_{t,t+1} \left\{ (1 - \omega) + \omega v_{2,t+1} \right\} \left( \frac{R^L_{j,t}}{\pi_{t+1}} \right) = \lambda^L v_{1,t} + v_{2,t} \\
d_{j,t} : & \quad (1 + v_{1,t}) \beta_E \Lambda_{t,t+1} \left\{ (1 - \omega) + \omega v_{2,t+1} \right\} \left( \frac{R_{t}}{\pi_{t+1}} \right) = v_{2,t} \\
d^*_{j,t} : & \quad (1 + v_{1,t}) \beta E \Lambda_{t,t+1} \left\{ (1 - \omega) + \omega v_{2,t+1} \right\} \left( \frac{R^*_{t} \xi_{t}}{\pi^*_{t+1}} \right) = v_{2,t} rer_t
\end{align*}
\]

Besides these first-order conditions, the set of equilibrium conditions includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankruupted banks and the new
worth of new banks. The new equity is injected by the domestic household and is assumed to be of the size \( \omega n \). Then

\[
n_t = \omega \left( \frac{R^L_{j,t-1}}{\pi_t} l_{t-1} - \frac{R^c_{t-1}}{\pi_t} d_{t-1} - \frac{R^p_{t-1}}{\pi^*_{t}} \rho_{t-1} d^*_t \right) + \omega n
\]

To include the incentive constraint in the equilibrium conditions, we have to redefine it by using the value of marginal utility from increasing assets by one unit and the value of marginal disutility from increasing debt by one unit. It follows from the previously derived results that the value of the bank \( j \) can also be defined as:

\[
V_{j,t} = \left( \lambda^L \nu_{1,t} + \nu_{2,t} \right) l_{j,t} - \frac{\nu_{2,t}}{1 + \nu_{1,t}} d_{j,t} - \frac{\nu_{2,t}}{1 + \nu_{1,t}} \rho_{t-1} d^*_t + \lambda^L \nu_{1,t} l_{j,t}
\]

\[
\Rightarrow V_{j,t} = \frac{\nu_{2,t}}{1 + \nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1 + \nu_{1,t}} l_{j,t}
\]

Then we can modify the incentive constraint as

\[
\frac{\nu_{2,t}}{1 + \nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1 + \nu_{1,t}} l_{j,t} \geq \lambda^L l_{j,t}
\]

\[
\Rightarrow \nu_{2,t} n_{j,t} \geq \lambda^L l_{j,t}
\]

### A.3.2 Lending in domestic currency only

Now the only asset on the banks’ balance sheet is domestic currency loans extended to financially constrained firms, thus, the expected nominal return of the bank \( j \) is defined as \( R^L_{j,t} \) and given by:

\[
E_t \left\{ R^L_{j,t} L_{j,t} \right\} = E_t \min \left\{ R^R_{j,t} L_{j,t}, \kappa \left( \rho_{t+1} R^R_{j,t+1} + Q_{t+1}(1 - \delta)k_{j,t+1} \right) \right\}
\]

Or, in units of composite goods,
\[
E_t \left\{ \frac{R_{L,t}^t}{\pi_{t+1}^t} l_{j,t} \right\} \equiv E_t \min \left\{ R_{R,t}^t \frac{l_{j,t}}{\pi_{t+1}^t}, \kappa \left( p_{t+1}^R j_{t+1}^R + q_{t+1} (1 - \delta) k_{j,t} \right) \right\} \\
\Rightarrow E_t \left\{ \frac{R_{L,t}^t}{\pi_{t+1}^t} l_{j,t} \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^R j_{t+1}^R + (1 - \delta) q_{t+1} k_{j,t} \right) + \Phi(d_{2,t}) R_{L,t}^t \right\} \quad (A.6)
\]

The rest of derivations for the bank’s optimization problem remain the same.

### A.3.3 Financial sector support

This segment of the model closely follows Kirchner and van Wijnbergen (2016). We assume that the government can intervene during the crisis by injecting capital \( \tau_t^{FS} \) to the financial sector. We assign the following rule to the recap of the financial intermediary \( j \):

\[
\tau_t^{FI} = \kappa_{FS} (\text{shock}_{t-1} - \text{shock}) n_{j,t-1}, \quad \kappa_{FS} > 0, \quad l \geq 0
\]

where \( n_{j,t-1} \) is the net worth of the intermediary from the previous period. The recap can be immediate \((l = 0)\) or delayed \((l > 0)\). We introduce a new variable \( \text{shock}_t \) that coincides with the variable driving the crisis, e.g. the risk premium shock \((\text{shock}_t \equiv \xi_t)\). We assume that the recap is a gift from the government and does not have to be repaid.

Then the optimization problem of the financial intermediary \( j \) as defined in the previous section can be modified to

\[
V_{j,t} = \max_{l_{j,t},d_{j,t},d_{j,t}^*} E_t \left[ \beta \Lambda_{t+1} \left\{ (1 - \omega)n_{j,t+1} + \omega V_{j,t+1} \right\} \right]
\]

s.t.

\[
V_{j,t} \geq \lambda^L l_{j,t}, \quad \text{(Incentive constraint)}
\]

\[
n_{j,t} + d_{j,t} + r e_t d_{j,t}^* = l_{j,t}, \quad \text{(Balance sheet constraint)}
\]

\[
n_{j,t} = \frac{R_{L,t}^t}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R_{t-1}^R \xi_{t-1}}{\pi_t^t} r e_t d_{j,t-1}^* + \kappa_{FS} (\text{shock}_{t-1} - \text{shock}) n_{j,t-1} \quad \text{(LoM of net worth)}
\]
Further, the first-order conditions can be expressed as

$$L = (1 + \nu_{1,t})\beta_t \Lambda_{x,t+1} \left\{ (1 - \omega) \left( \frac{R_{j,t}^L}{\pi_{t+1}} l_{j,t} - \frac{R_t}{\pi_{t+1}} d_{j,t} - \frac{R_t D_{j,t}^*}{\pi_t} \nu \Lambda_{x,t+1} - \nu \Lambda_{x,t+1} - \nu \Lambda_{x,t+1} + \nu \Lambda_{x,t+1} \right) + \omega V_{j,t+1} \right\}

- \nu_{1,t} \lambda^L l_{j,t}

+ \nu_{2,t} \left( \frac{R_{j,t-1}^L}{\pi_t} l_{j,t-1} - \frac{R_t}{\pi_t} d_{j,t-1} - \frac{R_t D_{j,t-1}^*}{\pi_t} \nu \Lambda_{x,t-1} - \nu \Lambda_{x,t-1} + \nu \Lambda_{x,t-1} + \nu \Lambda_{x,t-1} \right)\nu_{2,t} d_{j,t} + \nu_{2,t} D_{j,t}^*

This gives the first-order conditions:

$$l_{j,t} : (1 + \nu_{1,t})\beta_t \Lambda_{x,t+1} \left\{ (1 - \omega) \left( \frac{R_{j,t}^L}{\pi_{t+1}} + \nu \Lambda_{x,t+1} - \nu \Lambda_{x,t+1} + \nu \Lambda_{x,t+1} \right) + \omega \frac{\partial V()}{\partial l_{j,t}} \right\} = \lambda^L v_{1,t} + \nu_{2,t}

d_{j,t} : (1 + \nu_{1,t})\beta_t \Lambda_{x,t+1} \left\{ (1 - \omega) \left( \frac{R_{j,t}^L}{\pi_{t+1}} + \nu \Lambda_{x,t+1} - \nu \Lambda_{x,t+1} + \nu \Lambda_{x,t+1} \right) - \omega \frac{\partial V()}{\partial d_{j,t}} \right\} = \nu_{2,t}

D_{j,t}^*: (1 + \nu_{1,t})\beta_t \Lambda_{x,t+1} \left\{ (1 - \omega) \left( \frac{R_{j,t}^L}{\pi_{t+1}} + \nu \Lambda_{x,t+1} + \nu \Lambda_{x,t+1} \right) - \omega \frac{\partial V()}{\partial D_{j,t}^*} \right\} = \nu_{2,t}

with complementary slackness conditions:

$$\nu_{1,t} : \nu_{1,t} \left( V_{j,t} - \lambda^L l_{j,t} \right) = 0$$

$$\nu_{2,t} : \nu_{2,t} \left( \frac{R_{j,t-1}^L}{\pi_t} l_{j,t-1} - \frac{R_t}{\pi_t} d_{j,t-1} - \frac{R_t D_{j,t-1}^*}{\pi_t} \nu \Lambda_{x,t-1} - \nu \Lambda_{x,t-1} + \nu \Lambda_{x,t-1} + \nu \Lambda_{x,t-1} \right) = 0$$

Further, the first-order conditions can be expressed as

$$l_{j,t} : (1 + \nu_{1,t})\beta_t \Lambda_{x,t+1} \left\{ (1 - \omega) + \omega \nu_{2,t+1} \right\} \frac{R_{j,t}^L}{\pi_{t+1}} l_{j,t} + (1 + \nu_{1,t})\beta_t \Lambda_{x,t+1} (1 - \omega) \nu \Lambda_{x,t+1} - \nu \Lambda_{x,t+1} - \nu \Lambda_{x,t+1} + \nu \Lambda_{x,t+1} + \nu \Lambda_{x,t+1}$$

$$= \nu_{1,t} \lambda^L + \nu_{2,t}$$
\[ d_{j,t} : (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \{(1 - \omega) + \omega \nu_{2,t+1}\} \frac{R_t}{\pi_{t+1}} + (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1}(1 - \omega) \kappa_{FS} (\text{shock}_{t-1} - \text{shock}) \]
\[ = \nu_{2,t} \]

\[ d^*_j : (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1} \{(1 - \omega) + \omega \nu_{2,t+1}\} \frac{R^*_t \xi_t \Pi_{t+1}}{\pi^*_t \Pi_{t+1}} + (1 + \nu_{1,t}) \beta E_t \Lambda_{t,t+1}(1 - \omega) \kappa_{FS} (\text{shock}_{t-1} - \text{shock}) \]
\[ = \nu_{2,t} \]

Aggregate net worth evolves as

\[ n_t = \omega \left[ \frac{R^*_{j,t-1}}{\pi_t} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R^*_{j,t-1}}{\pi_t} \xi_t \Pi_{t-1} d^*_{t-1} + \kappa_{FS} (\text{shock}_{t-1} - \text{shock}) n_{t-1} \right] + \text{in} \]

### A.4 Household’s problem

We assume a representative household. The household has two alternatives to invest in: make deposits \( D_t \) in a bank or buy bonds issued by the government, \( B_t \). The household supplies labor to a competitive labor market. The household has Greenwood-Hercowitz-Huffman (henceforth, GHH) preferences as in Greenwood et al. (1988), so labor supply does not depend on wealth. The household chooses a level of real consumption \( c_t \) and working hours \( h_t \) such that the following lifetime utility function is maximized:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \gamma} \left( c_t - \frac{\chi (h_t)^{1+\psi}}{1 + \varphi} \right)^{1-\gamma} \quad \gamma, \chi, \varphi > 0 \]  
(A.7)

subject to the household’s budget constraint:

\[ C_t + B_t + D_t = W_t h_t + R_{t-1} B_{t-1} + R_{t-1} D_{t-1} + P_t \Pi_t - T_t \]

The budget constraint, expressed in units of composite goods, is given by

\[ c_t + b_t + d_t = w_t h_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{R_{t-1}}{\pi_t} d_{t-1} + \Pi_t - t_t \]  
(A.8)
\( \pi_t \) denotes the composite goods price inflation, \( c_t \equiv C_t / P_t, w_t \equiv W_t / P_t, b_t \equiv B_t / P_t, d_t \equiv D_t / P_t, t_t \equiv T_t / P_t. \)

We assume that the household is indifferent between buying bonds and making deposits, thus, \( R_t \) is nominal gross interest rate of both bonds and deposits. The household owns all banks in the model economy and thus receives lump-sum dividends, \( \Pi_t \). Taxes \( t_t \) enter the household’s budget constraint in a lump-sum way as well. Lump-sum dividends from financially constrained firms are included in total dividends \( \Pi_t \).

Lump-sum dividends from financially constrained firms consist of firms’ profits that the household receives in the beginning in the period minus the equity that the household transfers to the firms in the beginning of the period (in response to liquidity shortage, if there is any):

\[
\Pi_t^{firms} = \omega^{firms} \left( p_t R_t^R + q_t (1 - \delta) k_{t-1} - (1 - \rho) \frac{q_{t-1} k_{t-1} w_{t-1} h_{t-1}}{\pi_t} \right) - \omega^{firms} \left( \kappa (1 - \Phi(d_{1,t-1})) (p_t R_t^R + q_t (1 - \delta) k_{t-1}) + \Phi(d_{2,t-1}) R_t^R \frac{1}{\pi_t} + \Phi(d_{1,t-1}) rer_t \frac{1}{\pi_t} \right) - t_t^{firms} \cdot n^{firms} - z_t
\]

The final result follows from the definition of aggregate corporate net worth given in the section for financially constrained firms.

The household’s optimization problem gives first-order conditions:

\[
\lambda_t = \left( c_t - \frac{\chi (h_t)^{1+\varphi}}{1 + \varphi} \right)^{-\gamma}
\]

\[
w_t = \chi (h_t)^{\varphi}
\]

\[
E_t \beta \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} = 1
\]

We denote \( \Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \) where \( \lambda_t \) is the Lagrangian multiplier to the household’s budget constraint.
A.5 Production and Pricing

There are several types of firms in the domestic economy. It takes three types of firms to produce domestic aggregate inputs for composite goods. First, there are the financially constrained firms that combine purchased capital with labor and produce homogeneous goods. They were analyzed in Section 3.1. Their homogeneous outputs are bought by retail firms who costlessly differentiate the products bought and sell them as (local) monopolists, in Dixit and Stiglitz (1977) fashion. A similar group of firms called importers differentiate foreign (imported) goods. A composite goods producer buys the differentiated home goods and aggregates them into an aggregate domestic good \( y^H_t \) with associated price \( p^H_t \). The same composite goods producer also buys imported differentiated goods and aggregates them into a foreign aggregate good \( y^F_t \). The corresponding aggregate price level of foreign goods is \( p^F_t \). All details of the derivations of the various first order conditions optimization problems can be found in the supplementary appendix D. We discuss each step in more detail below.

The structure of the production sector is exhibited in Figure A.9.
A.5.1 Retail firms

Homogeneous goods produced by financially constrained firms are sold to domestic retail firms. We assume that there is a continuum of domestic retail firms. A domestic retail firm $j$ differentiates purchased inputs at $p^R_t$ and sells at a monopolistic price $p^H_t(j)$. Differentiated goods from the domestic retail sector, $y^H_t(j), j \in (0, 1)$, are purchased by the composite goods producer.

Retail firms are subject to sticky prices as in Calvo (1983), so every period $(1 - \omega^H)$ of them adjust prices to the optimal reset price $P^#_t(j)$. Then the profit of a retail firm $j$ that is allowed to adjust its price in period $t$ is thus given by $(P^#_t(j) - P^R_t) y^H_t(j)$. The rest of retail firms adjust their past prices by the rate $\pi^adj = \pi$.

Then the aggregate price level of retail goods $P^H_t$ is defined as

$$P^H_t = \left( (1 - \omega^H) \left( p^H_t \right)^{1-\epsilon_H} + \omega^H \left( \frac{p^H_t}{p^H_t} \right)^{1-\epsilon_H} \right)^{1/(1-\epsilon_H)}$$

Define

$$\tilde{p}^H_t \equiv \frac{p^H_t}{P^H_t} \quad (B.1)$$

It follows that

$$1 = (1 - \omega^H) \left( \tilde{p}^H_t \right)^{1-\epsilon_H} + \omega^H \left( \frac{p^H_t}{p^H_t} \right)^{1-\epsilon_H}$$

Re-writing in terms of relative prices with respect to the price level of composite goods $P_t$ such that $p^H_t = P^H_t / P_t$ gives

$$1 = (1 - \omega^H) \left( \tilde{p}^H_t \right)^{1-\epsilon_H} + \omega^H \left( \frac{p^H_t}{p^H_t} \right)^{1-\epsilon_H}$$

As a result, a retail firm $j$ solves the optimization problem how to set the optimal price $P^#_t(j)$ conditional on not changing it in the future that be be formalized as:

$$\max_{P^H_t(j)} E_t \sum_{s=0}^{\infty} \left( \omega^H \right)^s B^s \Lambda_{t+s} \left( \frac{p^H_t(j) \left( \prod_{j=1}^{s} \pi^adj_t + j \right) - P^R_t}{P^H_t} \right) y^H_{t+s}(j)$$

s.t. demand for retail goods (equation (A.10))

$$y^H_t(j) = \left( \frac{P^H_t(j) \left( \prod_{j=1}^{s} \pi^adj_t + j \right)}{p^H_t} \right)^{-\epsilon_H} y^H_t$$
Define \( p_t^R \equiv \frac{p_t^R}{T_t} \):

\[
\max_{p_t^\#(j)} E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t,t+s} \left( \frac{p_t^\#(j)}{p_{t+s}^H} \left( \prod_{j=1}^{j=s} \frac{p_t^R}{p_{t+s}^H} \right) - p_t^R \right) y_{t+s}^H(j)
\]

s.t. demand for retail goods

\[
y_t^H(j) = \left( \frac{p_t^\#(j)}{p_{t+s}^H} \right)^{-\epsilon_H} y_t^H
\]

\[
\Rightarrow \max_{p_t^\#(j)} E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t,t+s} \left( \frac{p_t^\#(j)}{p_{t+s}^H} \left( \prod_{j=1}^{j=s} \frac{p_t^R}{p_{t+s}^H} \right) - p_t^R \right) y_{t+s}^H
\]

We take a derivative w.r.t. \( p_t^\#(j) \) and rearrange terms:

\[
E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t,t+s} \left( 1 - \epsilon_H \right) p_{t+s}^H \left( p_t^\#(j) \right)^{-\epsilon_H} \left( \prod_{j=1}^{j=s} \frac{p_t^R}{p_{t+s}^H} \right)^{-\epsilon_H} - \epsilon_H p_{t+s}^R \left( p_t^\#(j) \right)^{-\epsilon_H - 1} \left( \prod_{j=1}^{j=s} \frac{p_t^R}{p_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H = 0
\]

\[
\Rightarrow p_t^\#(j) = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t,t+s} \left( p_{t+s}^H \right)^{\epsilon_H} \left( \prod_{j=1}^{j=s} \frac{p_t^R}{p_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H}{E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t,t+s} \left( p_{t+s}^H \right)^{\epsilon_H} \left( \prod_{j=1}^{j=s} \frac{p_t^R}{p_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H} = 0
\]
A.5.2 Importers

We assume that there is a continuum of monopolistically competitive importers. They buy a variety $j$ of foreign goods $y^F_t(j)$ at price $P_t$ and sell it to the composite goods producer at a nominal price $P^H_t(j)$, expressed in domestic currency.

Every period there is a fraction $(1 - \omega^F)$ of importers who can adjust their prices, in Calvo (1983) fashion. The set of importers who can adjust the price choose it such that their profits are maximized. The rest of importers adjust their past prices by the rate $\pi^\text{adj}_t = \pi$. As a result, an importer $j$ solves the optimization

$$
\Rightarrow \frac{p^{Hj}_t(j)}{p^H_t} = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t+s} \left(\frac{\bar{R}}{\bar{H}} \left(\frac{p^H_{t+s}}{p^H_t}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^\text{adj}_{t+j}\right)^{-\epsilon_H} \gamma^H_{t+s}\right)}{E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t+s} \left(\frac{p^H_{t+s}}{p^H_t}\right)^{\epsilon_H-1} \left(\prod_{j=1}^{j=s} \pi^\text{adj}_{t+j}\right)^{1-\epsilon_H} \gamma^H_{t+s}}
$$

Since $\tilde{p}_t^H \equiv p^{Hj}_t(j) / p^H_t$,

$$
\Rightarrow p^H_t = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t+s} \left(\frac{\bar{R}}{\bar{H}} \left(\frac{p^H_{t+s}}{p^H_t}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi^\text{adj}_{t+j}\right)^{-\epsilon_H} \gamma^H_{t+s}\right)}{E_t \sum_{s=0}^{\infty} (\omega^H)^s \beta^s \Lambda_{t+s} \left(\frac{p^H_{t+s}}{p^H_t}\right)^{\epsilon_H-1} \left(\prod_{j=1}^{j=s} \pi^\text{adj}_{t+j}\right)^{1-\epsilon_H} \gamma^H_{t+s}}
$$

where

$$
F^H_{1,t} = p^R_t y^H_t + E_t \omega^H \beta \Lambda_{t+1} \left(\frac{p^H_{t+1} \pi^\text{adj}_{t+1}}{p^H_t \left(\prod_{j=1}^{j=s} \pi^\text{adj}_{t+j}\right)}\right)^{\epsilon_H} F^H_{1,t+1}
$$

and

$$
F^H_{2,t} = p^H_t y^H_t + E_t \omega^H \beta \Lambda_{t+1} \left(\frac{p^H_{t+1} \pi^\text{adj}_{t+1}}{p^H_t \left(\prod_{j=1}^{j=s} \pi^\text{adj}_{t+j}\right)}\right)^{\epsilon_H-1} F^H_{2,t+1}
$$

We assume that there is a continuum of monopolistically competitive importers. They buy a variety $j$ of foreign goods $y^F_t(j)$ at price $P_t$ and sell it to the composite goods producer at a nominal price $P^H_t(j)$, expressed in domestic currency.

Every period there is a fraction $(1 - \omega^F)$ of importers who can adjust their prices, in Calvo (1983) fashion. The set of importers who can adjust the price choose it such that their profits are maximized. The rest of importers adjust their past prices by the rate $\pi^\text{adj}_t = \pi$. As a result, an importer $j$ solves the optimization
problem how to set the optimal price \( P_t^{\#F} (j) \) conditional on not changing it in the future:

\[
\max_{P_t^{\#F} (j)} E_t \sum_{s=0}^{\infty} (\omega^F)^s \beta^s \Lambda_{t,t+s} \left( \frac{p_t^{\#F} (j) \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)}{P_{t+s}} - S_{t+s} P_{t+s}^r \right) y_{t+s}^F (j) 
\]

s.t.

\[
y_t^F (j) = \eta \left( \frac{p_t^{\#F} (j) \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)}{P_t^F} \right)^{\epsilon} y_t^F
\]

Since \( rer_t = S_t P_t^r / P_t \),

\[
\max_{P_t^{\#F} (j)} E_t \sum_{s=0}^{\infty} (\omega^F)^s \beta^s \Lambda_{t,t+s} \left( \frac{p_t^{\#F} (j) \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)}{P_{t+s}} - rer_{t+s} \right) y_{t+s}^F (j) 
\]

s.t.

\[
y_t^F (j) = \eta \left( \frac{p_t^{\#F} (j) \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)}{P_t^F} \right)^{\epsilon} y_t^F
\]

In analogy to the problem of retail firms, we maximize expected profits and rearrange terms. Since all importers who can adjust their price set the same optimal price, \( P_t^{\#F} (j) = P_t^{\#F} \forall j \). After introducing a variable \( \tilde{p}_t^F \), which is defined as

\[
\tilde{p}_t^F = \frac{p_t^{\#F}}{P_t^F},
\]

we can show that the optimal price-setting equation follows as

\[
\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{E_t \sum_{s=0}^{\infty} (\omega^F)^s \beta^s \Lambda_{t,t+s} \left( \frac{p_t^{\#F} \prod_{j=1}^{s} \pi_{t+j}^{adj}}{P_t^F} \right)^{\epsilon_F} \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)^{-\epsilon_F} y_{t+s}^F}
\]

\[
\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{F_{2,t}^F}
\]

where

\[
F_{1,t}^F = rer_t y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left( \frac{p_{t+1}^{\#F} \prod_{j=1}^{s} \pi_{t+j}^{adj}}{P_t^F \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F} F_{1,t+1}^F
\]
and
\[ F_{2,t}^F = p_t^F y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left( \frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right)} \right)^{\varepsilon_F - 1} F_{2,t+1}^F \]

Deriving an aggregate price level of imported goods' produces the following expression: 
\[ 1 = (1 - \omega^F) \left( \hat{p}_t^F \right)^{1 - \varepsilon_F} + \omega^F \left( \frac{p_{t+1}^F \pi_{t+1}^{adj}}{p_t^F \pi_t} \right)^{1 - \varepsilon_F}. \]

### A.5.3 Price dispersion

We define the price dispersion for retail goods as
\[ D_t^H \equiv \int_0^1 \left( \frac{p_t^H(j)}{p_t^H} \right)^{-\varepsilon_H} \, dj \]

\((1 - \omega^H)\) of firms update prices to the same optimal price \(\hat{p}_t^H\) and \(\omega^H\) of firms adjust the last period’s price with the adjustment term \(\pi_t^{adj}\). This gives
\[
D_t^H = \int_0^{1 - \omega^H} \left( \frac{p_t^H(j)}{p_t^H} \right)^{-\varepsilon_H} \, dj + \int_{1 - \omega^H}^1 \left( \frac{p_t^H(j)}{p_t^H} \right)^{-\varepsilon_H} \left( \frac{p_{t-1}^H \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right) - \varepsilon_H}{p_t^H \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right) - \varepsilon_H} \right) \, dj
\]
\[
= (1 - \omega^H) \left( \frac{p_t^H}{p_t^H} \right)^{-\varepsilon_H} + \int_{1 - \omega^H}^1 \frac{p_{t-1}^H \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right) - \varepsilon_H}{p_t^H \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right) - \varepsilon_H} \, dj
\]
\[
= (1 - \omega^H) \left( \frac{p_t^H}{p_t^H} \right)^{-\varepsilon_H} + \omega^H \left( \frac{p_{t-1}^H \left( \prod_{j=1}^{s} \pi_{t+j}^{adj} \right) - \varepsilon_H}{\pi_t p_t^H} \right) D_{t-1}^H
\]

In analogy, the price dispersion of importers’ goods is given by
\[ D_t^F \equiv \int_0^1 \left( \frac{p_t^F(j)}{p_t^F} \right)^{-\varepsilon_F} \, dj \]
and it follows a rule

\[ D_t^F = (1 - \omega^F) \left( p_t^F \right)^{-\epsilon^F} + \omega^F \left( \frac{p_t^F}{\prod_{j=1}^s \pi_t^F} \left( \Pi_{j=1}^s \pi_{t+j}^F \right) \right)^{-\epsilon^F} D_{t-1}^F \]

### A.5.4 Composite goods producer

The composite goods producer combines domestic aggregate goods and foreign aggregate goods into composite goods and sells them to the household, the government and capital goods producers. We define the supply of composite goods as \( y_t^C \). Its associated price is \( P_t \). The demanded amount of production inputs, namely, domestic aggregate goods and foreign aggregate goods, is denoted as \( x_t^H \) and \( x_t^F \) respectively.

**Domestic aggregate goods.** Domestic aggregate goods \( y_t^H \) result from assembling retailers’ production \( y_t^H(j) \) for \( j \in [0, 1] \), each bought at price \( P_t^H(j) \), expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of retail goods be \( P_t^H = \left( \int_0^1 (P_t^H(j))^{1-\epsilon_H} dj \right)^{1/(1-\epsilon_H)} \), expressed in domestic currency. Then it follows that the demand for retail goods is given as a solution to the problem

\[
\max_{y_t^H(j)} \left\{ P_t^H y_t^H - \int_0^1 P_t^H(j) y_t^H(j) dj \right\}
\]

subject to the assembling technology

\[ y_t^H = \left( \int_0^1 y_t^H(j)^{1-\epsilon_H} dj \right)^{\epsilon_H/(1-\epsilon_H)} \]

and to the market clearing constraint that says that domestic aggregate goods are used as input by the composite goods producer and face foreign demand \( e x_t \):

\[ y_t^H = x_t^H + e x_t \]

As a result, optimal demand for retail goods of variety \( j \) is given by

\[ y_t^H(j) = \left( \frac{p_t^H(j)}{p_t^H} \right)^{-\epsilon_H} y_t^H \]  \hspace{1cm} (A.10)
Foreign aggregate goods. Foreign aggregate goods \( y^F_t \) result from assembling importers’ production \( y^F_t(j) \) for \( j \in [0, 1] \), each bought at price \( P^F_t(j) \), expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of importers’ goods be \( P^F_t \equiv \left( \int_0^1 (P^F_t(j))^{1-\epsilon_F} dj \right)^{1/(1-\epsilon_F)} \), expressed in domestic currency. Then it follows that the demand for importers’ goods is given as a solution to the problem

\[
\max_{y^F_t(j)} \left\{ P^F_t y^F_t - \int_0^1 P^F_t(j) y^F_t(j) dj \right\}
\]

subject to the assembling technology

\[
y^F_t = \left( \int_0^1 y^F_t(j) \frac{1}{y^F_t} dj \right)^{\frac{\epsilon_F}{\epsilon_F - 1}}
\]

and to the market clearing constraint that says that all foreign aggregate goods are used to satisfy the demand of the composite goods producer:

\[
y^F_t = x^F_t
\]

As a result, optimal demand for importers’ production of variety \( j \) is given by

\[
y^F_t(j) = \left( \frac{P^F_t(j)}{P^F_t} \right)^{-\epsilon_F} y^F_t
\]

and demand for foreign aggregate goods clears \( x^F_t = y^F_t \).

Composite goods. Given inputs \( x^H_t \) and \( x^F_t \), composite goods are assembled with the aggregation technology

\[
y^C_t \equiv \left( (1 - \eta) \frac{1}{\epsilon} (x^H_t)^{\frac{\epsilon-1}{\epsilon}} + \eta \frac{1}{\epsilon} (x^F_t)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}
\]

where \( \epsilon \) stands for elasticity of substitution between domestically produced goods and imported goods. A parameter \( \eta \) proxies for openness of the home economy.

The composite goods producer operates in a perfectly competitive market, so she maximizes profits \( P_t y^C_t - P^H_t x^H_t - P^F_t x^F_t \) subject to the technology (B.23). This boils down to two demand conditions:

\[
x^H_t = (1 - \eta) \left( \frac{P^H_t}{P_t} \right)^{-\epsilon} y^C_t
\]
\[ x_t^F = \eta \left( \frac{p_t^F}{p_t} \right)^{-\epsilon} y_t^C \]

Further, we introduce relative prices \( p_t^F \equiv P_t^F / P_t \) and \( p_t^H \equiv P_t^H / P_t \) and get

\[ x_t^H = (1 - \eta) \left( p_t^H \right)^{-\epsilon} y_t^C \quad (\text{A.13}) \]

and

\[ x_t^F = \eta \left( p_t^F \right)^{-\epsilon} y_t^C \quad (\text{A.14}) \]

### A.5.5 Capital producers

Capital producers participate in the domestic economy by selling capital to financially constrained firms at the real competitive price \( q_t \), and buying the depreciated capital stock back next period. To restore the depreciated capital, capital producers add composite goods (investment) \( i_t \) as additional inputs to the depreciated capital stock by using the technology subject to investment adjustment costs \( \Gamma \left( \frac{i_t}{i_{t-1}} \right) \):

\[ k_t = (1 - \delta)k_{t-1} + \left( 1 - \Gamma \left( \frac{i_t}{i_{t-1}} \right) \right) i_t \quad (\text{A.15}) \]

where adjustment costs \( \Gamma \) equal:

\[ \Gamma \left( \frac{i_t}{i_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \]

Capital producers maximize profits, expressed in units of composite goods, subject to the production technology by choosing an optimal level of investment:

\[
\max_{i_t} \beta E_t \Lambda_{t,t+1} \left\{ (1 - \rho) \frac{q_t}{\pi_{t+1}} k_t \right\} + pq_t k_t - q_t (1 - \delta) k_{t-1} - i_t
\]

s.t.

\[ k_t = (1 - \delta)k_{t-1} + \left( 1 - \Gamma \left( \frac{i_t}{i_{t-1}} \right) \right) i_t \quad (\text{A.17}) \]
The optimization problem takes into account the share of capital purchases paid immediately $\rho$ as opposed to the share of the payment $(1-\rho)$ delayed to the next period which makes it slightly different from a standard optimization problem solved by competitive capital producers.

Optimizing gives the demand function for investment:

$$
\frac{1}{q_t} = \rho \left(1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) - \rho \gamma \left(\frac{i_t}{i_{t-1}} - 1\right) + \rho \gamma \beta E_t \Lambda_{t,t+1} \frac{q_{t+1}}{q_t} \left(\frac{i_{t+1}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right)^2
$$

$$
+ (1-\rho) \beta E_t \Lambda_{t,t+1} \frac{q_{t+1}}{q_t} \left(1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) - \gamma \left(\frac{i_t}{i_{t-1}} - 1\right) + (1-\rho) \gamma \beta E_t \Lambda_{t,t+2} \frac{q_{t+1}}{q_t} \left(\frac{i_{t+1}}{i_t} - 1\right) \left(\frac{i_{t+1}}{i_t}\right)^2
$$

(A.18)

### A.5.6 Exporters

We assume that perfectly competitive exporters demand $e x_t$ units of the domestic aggregate good $y_t^H$, so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters. Exported goods consist of the domestic aggregate, so they do not use imported inputs.

We abstract from modelling trade barriers. Hence, the rest of the world demands $e x_t$ units of domestic aggregate goods at a price $P_t^{H*} = p_t^H / S_t$, which is the price of domestic aggregate goods expressed in units of foreign composite goods. We assume that all economies in the world are identical and their demand for domestic aggregate goods can be aggregated and expressed relative to world output $y_t^*$. The foreign demand for domestic aggregate goods is price-sensitive:

$$
e x_t = \eta^* \left(\frac{p_t^H}{r_{er_t}}\right)^{-\epsilon} y_t^*
$$

(A.19)

Consistent with the small open economy assumption, $P_t^{*}$ and $y_t^*$ are assumed to evolve exogenously. $\eta^*$ is the foreign households’ taste parameter for domestic aggregate goods. $\epsilon^*$ defines the elasticity of substitution between domestic aggregate goods and goods produced in other economies.
A.6 Government

The government collects lump-sum taxes $T_t$ from the household and issues domestic bonds $B_t$ to finance a stochastic stream of nominal government expenditure, $G_t$, and the bank recap $P_t \tau_{t}^{FS}$. Therefore, it satisfies the budget constraint:

$$G_t + P_t \tau_{t}^{FS} + R_{t-1} B_{t-1} = T_t + B_t$$

Given $g_t \equiv G_t / P_t$, $b_t \equiv B_t / P_t$ and $t_t \equiv T_t / P_t$, the budget constraint can be expressed in units of composite goods as

$$g_t + \tau_{t}^{FS} + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

Taxes in units of composite goods follow this tax rule:

$$t_t = t + \kappa^B (b_{t-1} - b) + \kappa^{FS} \tau_{t}^{FS} + \epsilon_t, \quad 0 < \kappa^B \leq 1, \quad 0 \leq \kappa^{FS} \leq 1$$

The rule tells that a share $\kappa^{FS}$ of the recap expenditure is covered by increasing the lump-sum tax and the rest (a share $(1 - \kappa^{FS})$) is financed with new government debt.

A.7 Central bank

The central bank conducts monetary policy by following the Taylor rule:

$$\frac{R_t}{R} = (\frac{R_{t-1}}{R})^{\gamma_R} \left( \frac{\pi_t^H}{\pi_t^{H \text{bar}}} \right)^{(1-\gamma_R)\gamma_y} \left( \frac{\pi_t^H}{\pi_t^{H \text{bar}}} \right)^{(1-\gamma_R)\gamma_x} \exp(mp_t)$$

(A.20)

$mp_t$ is a monetary policy shock and the domestic aggregate goods price inflation $\pi_t^H$ can be expressed as $\pi_t^H = p_t^H / \pi_{t-1}^H$. 

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A.8 Market clearing

The domestic household, the government and capital producers buy composite goods. Therefore, the supply of composite goods $y^C_t$ has to satisfy the aggregate demand of domestic agents:

$$y^C_t = c_t + i_t + g_t \quad \text{(A.21)}$$

A.9 Current account and its components

First, we derive an expression for aggregate nominal imports $M_t$ in units of domestic currency.

We aggregate importers’ demand for foreign composite $y^F_t(j) \forall j \in (0, 1)$ that is priced at $P_t^*$ and use the nominal exchange rate $S_t$ to convert to domestic currency:

$$M_t = \int_0^1 S_t P_t^* y^F_t(j) dj$$

Further we use the derived demand function (A.11) to get

$$M_t = \int_0^1 S_t P_t^* \left( \frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon_F} y^F_t$$

Define the price dispersion of importers’ goods as $D_t^F \equiv \int_0^1 \left( \frac{P_t^F(j)}{P_t^F} \right)^{-\epsilon_F} dj$ (more details on the price dispersion are in the section for importers). Then

$$M_t = S_t P_t^* D_t^F y^F_t \quad \text{(A.22)}$$

which in units of composite goods is given by

$$m_t \equiv \frac{M_t}{P_t} = rer_t D_t^F y^F_t \quad \text{(A.23)}$$

Second, we define nominal exports $EX_t$, expressed in units of domestic currency. Since exports are purchased at the price $P_t^{H*}$, expressed in foreign currency, nominal exports $EX_t$, expressed in units of
domestic currency, is given by

\[ EX_t = S_t P^H_t e x_t = P^H_t e x_t \]  \hspace{1cm} (A.24)

Finally, the trade balance \( TB_t \) evolves as

\[ TB_t = EX_t - M_t \]

Recall definitions for nominal exports and nominal imports in units of domestic currency (equations (A.24) and (A.22)). Then the trade balance in units of composite goods can be expressed as

\[ \begin{align*}
    tb_t &\equiv \frac{TB_t}{P_t} = \frac{P^H_t e x_t}{P_t} - \frac{S_t P^*_t D^F_t \gamma^F_t}{P_t} \\
    \Rightarrow \quad tb_t &\equiv P^H_t e x_t - \text{rer}_t D^F_t \gamma^F_t
\end{align*} \]

Since \( m_t \equiv \text{rer}_t D^F_t \gamma^F_t \),

\[ tb_t = P^H_t e x_t - m_t \]

A current account is given by the sum of nominal trade balance and nominal net income from abroad.

The domestic household owns banks that borrow from the foreign household, so, as a result, net income from abroad is negative and equal to minus payments on bank foreign debt:

\[ CA_t = TB_t - \left( R^*_t - 1 \right) S_t D^*_t \]

Further, we express the current account in units of composite goods as \( ca_t \) (\( ca_t \equiv CA_t / P_t \)):

\[ ca_t = tb_t - \left( R^*_t - 1 \right) S_t D^*_t \]

\[ \Rightarrow \quad ca_t = tb_t - \left( R^*_t - 1 \right) \text{rer}_t \frac{d^*_t}{\pi^*_t} \]  \hspace{1cm} (A.25)
In equilibrium the current account has to equal the capital account balance \( CP_t \). In our case the capital account balance is given by the change in stocks of bank foreign debt:

\[
CP_t = -(S_tD^*_t - S_tD^*_{t-1})
\]

We express the capital account balance in units of composite goods as \( cp_t \) (\( cp_t ≡ CP_t / P_t \)):

\[
cp_t = -\left( rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi_t^*}\right)
\]

Then, next to the current account definition (A.25), we impose an additional restriction that enters the set of equilibrium equations:

\[
ca_t = -\left( rer_t d^*_t - rer_t \frac{d^*_{t-1}}{\pi_t^*}\right)
\]  \(\text{(A.26)}\)

A.10 Equilibrium equations of the model with foreign currency debt and leverage-constrained banks

The model is described by 48 endogenous variables:

\[
\{\lambda_t, c_t, h_t, w_t, R_t, d_{1,t}, d_{2,t}, R^R_t, l_t, l^D_t, l^F_t, n_t, \pi_t, \Lambda_t, \Lambda_{t+1}, p^R_t, k_t, i_t, q_t, p^H_t, \bar{p}^H_t, D^H_t, \tilde{y}_t, \tilde{x}_t, F^H_t, F^C_t, p^F_t, y^F_t, x^F_t, m_t, e_t, F^F_t, F^F_{2,t}, R^L_t, d^*_t, d_t, n_t, v_{1,t}, v_{2,t}, b_t, rer_t, S_t, b_t, ca_t, \tilde{e}_t\}
\]

They are given by 48 equilibrium equations below.

**Households**

\[
\lambda_t = \left( c_t - \frac{\chi (h_t)^1+\varphi}{1 + \varphi}\right)^\gamma
\]  \(\text{(1)}\)

\[
w_t = \chi (h_t)^\varphi
\]  \(\text{(2)}\)

\[
\Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t}
\]  \(\text{(3)}\)

\[
E_t \beta \Lambda_{t,t+1} \frac{R_t}{\pi_{t+1}} = 1
\]  \(\text{(4)}\)

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\[ E_t \beta \Lambda_{t+1} \left\{ (1 - (1 - \Phi(d_{1,t}) \kappa) \left( \alpha p^R_{t+1} A_{t+1} k_t^{\alpha - 1} h_t^{1 - \alpha} + q_{t+1} (1 - \delta) \right) - (1 - \rho) \frac{q_t}{\pi_{t+1}} \right\} = \rho q_t \]  

(5)

\[ E_t \beta \Lambda_{t+1} \left\{ (1 - (1 - \Phi(d_{1,t}) \kappa) (1 - \alpha) p^R_{t+1} A_{t+1} k_t^{\alpha - 1} - (1 - \rho) \frac{w_t}{\pi_{t+1}} \right\} = \rho w_t \]  

(6)

\[ E_{t-1} \{ l_t \} + E_{t-1} \left\{ n^firms_t \right\} = E_{t-1} \{ \rho (q_t k_t + w_t h_t) \} \]  

(7)

\[ d_{2,t} = \frac{E_t \ln \left( \kappa \left( p^R_{t+1} y^R_{i,t+1} + q_{t+1} (1 - \delta) k_{t+1} \right) - R^R rer_{t+1} \frac{l^F_{i,t}}{\pi_{t+1}} \right) - E_t \ln \left( R^R \frac{i^D_{i,t}}{\pi_i} \right)}{\sigma_y} \]  

(8)

\[ d_{1,t} = d_{2,t} + \sigma_y \]  

(9)

\[ n^firms_t = \omega^firms \left( p^R_{t+1} y^R_{i,t+1} + q_{t+1} (1 - \delta) k_{t+1} - (1 - \rho) \frac{q_{t-1} k_{t-1} + w_{t-1} h_{t-1}}{\pi_t} \right) \]

\[ - \omega^firms \left( (1 - \Phi(d_{1,t-1}) \kappa) \left( p^R_{t} y^R_{i} + q_{t} (1 - \delta) k_{t-1} \right) + \Phi(d_{2,t-1}) R^R r^r_{t} \frac{l^F_{i}}{\pi_t} + \Phi(d_{1,t-1}) r^r_{t} \frac{l^D_{i}}{\pi_t} \right) \]

\[ + \iota^firms \cdot n^firms \]  

(10)

\[ l_t = l^D_t + rer_l t^F \]  

(11)

\[ l^D_t = (1 - \alpha^F) l_t \]  

(12)

Capital producers

\[ k_t = (1 - \delta) k_{t-1} + \left( 1 - \Gamma \left( \frac{i_t}{h_{t-1}} \right) \right) i_t \]  

(13)
\[
\frac{1}{q_t} = \rho \left( 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right) - \rho \gamma \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_{t+1}}{i_t} + \rho \gamma \beta E_t \Lambda_{t,t+1} \frac{q_{t+1}}{q_t} \left( \frac{i_{t+1}}{i_t} \right)^2 \\
+ (1 - \rho) \gamma \beta E_t \Lambda_{t,t+1} \frac{q_{t+1}}{q_t} \left( 1 - \frac{\gamma}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right)^2 - \rho \gamma \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t+1}} + (1 - \rho) \gamma \beta^2 E_t \Lambda_{t,t+2} \frac{q_{t+1}}{q_t} \left( \frac{i_{t+1}}{i_t} \right)^2
\]

Retail firms

\[
1 = (1 - \omega_H) \left( \tilde{p}_t^H \right)^{1-\epsilon_H} + \omega_H \left( \frac{p_{t-1}^H \left( \prod_{j=1}^{s} \pi_{t+j} \right)}{p_t^H \pi_t} \right)^{1-\epsilon_H} \tag{15}
\]

\[
D_t^H = (1 - \omega_H) \left( \tilde{p}_t^H \right)^{-\epsilon_H} + \omega_H \left( \frac{p_{t-1}^H \left( \prod_{j=1}^{s} \pi_{t+j} \right)}{p_t^H \pi_t} \right)^{-\epsilon_H} \tag{16}
\]

\[
p_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{F_{1,t}^H}{F_{2,t}^H} \tag{17}
\]

\[
F_{1,t}^H = p_t^R y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left( \frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left( \prod_{j=1}^{s} \pi_{t+j} \right)} \right)^{\epsilon_H} F_{1,t+1}^H \tag{18}
\]

\[
F_{2,t}^H = p_t^H y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left( \frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left( \prod_{j=1}^{s} \pi_{t+j} \right)} \right)^{\epsilon_H - 1} F_{2,t+1}^H \tag{19}
\]

\[
D_t^H y_t^H = A_t \theta_t F(k_{t-1}, n_{t-1}) \tag{20}
\]

Composite goods producer

\[
y_t^C \equiv (1 - \eta) \frac{1}{2} \left( x_t^H \right)^{\frac{\epsilon_H}{\epsilon}} + \eta \frac{1}{2} \left( x_t^F \right)^{\frac{\epsilon_F}{\epsilon}} \tag{21}
\]

\[
x_t^H = (1 - \eta) \left( p_t^H \right)^{-\epsilon} y_t^C \tag{22}
\]
\[ x_t^F = \eta \left( p_t^F \right)^{-\epsilon} y_t^C \]  

(23)

Exporters

\[ e_n = \eta^* \left( \frac{p_t^H}{rer_t} \right)^{-\epsilon} y_t^* \]  

(24)

Definition of the real exchange rate

\[ \frac{rer_t}{rer_{t-1}} = \frac{S_t}{S_{t-1}} \frac{\pi_t^*}{\pi_t} \]  

(25)

Importers

\[ 1 = (1 - \omega^F) \left( \bar{p}_t^F \right)^{1-\epsilon_F} + \omega^F \left( \frac{p_{t-1}^F \left( \prod_{j=s_{adj}}^{j=s} \pi_{t+j} \right)}{\bar{p}_t^F \pi_t} \right)^{1-\epsilon_F} \]  

(26)

\[ D_t^F = (1 - \omega^F) \left( \bar{p}_t^F \right)^{-\epsilon_F} + \omega^F \left( \frac{p_{t-1}^F \left( \prod_{j=s_{adj}}^{j=s} \pi_{t+j} \right)}{\pi_t \bar{p}_t^F} \right)^{-\epsilon_F} D_{t-1}^F \]  

(27)

\[ p_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{F_{2,t}^F} \]  

(28)

\[ F_{1,t}^F = rer_t y_t^F + \epsilon_t^F \beta \Lambda_{t+1} \left( \frac{p_{t+1}^F \pi_{t+1}^*}{p_t^F \left( \prod_{j=s_{adj}}^{j=s} \pi_{t+j} \right)} \right)^{\epsilon_F} \]  

(29)

\[ F_{2,t}^F = p_t^F y_t^F + \epsilon_t^F \beta \Lambda_{t+1} \left( \frac{p_{t+1}^F \pi_{t+1}^*}{p_t^F \left( \prod_{j=s_{adj}}^{j=s} \pi_{t+j} \right)} \right)^{-\epsilon_F^{-1}} \]  

(30)

\[ m_t = rer_t D_t^F y_t^F \]  

(31)

Banks

\[ E_t \left\{ \frac{R^L_t}{\pi_{t+1}} \right\} = E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p_{t+1}^F \pi_{t+1}^* + (1 - \delta) q_{t+1} k_t \right) + \Phi(d_{2,t}) R_t^R \frac{l_t^F}{\pi_{t+1}^*} + \Phi(d_{1,t}) \right\} \]  

(32)
\[(1 + \nu_{1,t})\beta E_t \Lambda_{t,t+1} \{(1 - \omega) + \omega \nu_{2,t+1}\} \left(\frac{R_{L}^t}{\pi_{t+1}}\right) = \lambda^L \nu_{1,t} + \nu_{2,t}\]  
\[(33)\]

\[(1 + \nu_{1,t})\beta E_t \Lambda_{t,t+1} \{(1 - \omega) + \omega \nu_{2,t+1}\} \left(\frac{R_{t}}{\pi_{t+1}}\right) = \nu_{2,t}\]  
\[(34)\]

\[(1 + \nu_{1,t})\beta E_t \Lambda_{t,t+1} \{(1 - \omega) + \omega \nu_{2,t+1}\} \left(\frac{R_{t}^* \xi_t \text{rer}_{t+1}}{\pi_{t+1}}\right) = \nu_{2,t}\]  
\[(35)\]

\[n_t = \omega \left(\frac{R_{L}^{t-1}}{\pi_t} l_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R_{t-1}^* \xi_{t-1}}{\pi_t^* \text{rer}_{t-1}} d_{t-1}^* \right) + \ln\]  
\[(36)\]

\[\nu_{2,t} n_t \geq \lambda^L l_t\]  
\[(37)\]

\[n_t + d_t + \text{rer}_t d_t^* = l_t\]  
\[(38)\]

**Monetary policy**

\[\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma^R} \left(\frac{\gamma_t^H}{\gamma^H}\right)^{(1-\gamma^R)\gamma} \left(\frac{p_t^H}{p_{t-1}^H \pi_t}\right)^{(1-\gamma^R)\gamma^H} \exp(mp_t)\]  
\[(39)\]

**Government**

\[g_t + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t\]  
\[(40)\]

\[t_t = \tilde{t} + \kappa_b (b_{t-1} - \tilde{b}) + \tau_t\]  
\[(41)\]

**Aggregate demand of domestic agents has to equal aggregate supply of composite goods**

\[y_t^C = c_t + i_t + g_t\]  
\[(42)\]
Aggregate demand for domestic aggregate goods and demand for exports clears with production of
domestic aggregate goods
\[ y_t^H = x_t^H + e x_t \] (43)

Aggregate domestic demand for foreign aggregate goods clears with imports
\[ y_t^F = x_t^F \] (44)

Trade balance
\[ tb_t = p_t^H e x_t - m_t \] (45)

Current account
\[ ca_t = tb_t - \left( R_t^* \xi_{t-1} - 1 \right) rer_t \frac{d_t^*}{\pi_t^*} \] (46)
\[ ca_t = - \left( rer_t d_t^* - rer_t \frac{d_{t-1}^*}{\pi_t^*} \right) \] (47)
\[ \xi_t = \exp \left( \phi \frac{rer_t d_t^* - rer \cdot d^*}{rer \cdot d^*} + \frac{\xi_t - \zeta}{\zeta} \right) \] (48)

There are 10 exogenous variables:
\[ \{ A_t, \theta_t, \pi_t^*, R_t^*, \xi_t, y_t^*, mp_t, g_t, \tau_t \} \]
Appendix B

B.1 Household debt contract

In the beginning of every period the net worth of defaulted households is pooled with the net worth of non-defaulted households which implies complete intergenerational consumption risk sharing. However, the dynasty is not liable for the unpaid debt. Given the pooled net worth, the dynasty of impatient household maximizes utility of her members with respect to consumption $c^I_t$, housing stock $h^I_t$, how much to borrow from the bank ($m_t$) and what is the optimal default threshold $\bar{\omega}_t$. The debt variables define the optimal debt contract as in Bernanke et al. (1999).

The household $i$ finds it optimal to default, if the collateral value is lower than the debt value. Given that a fixed fraction $\alpha^{FM}$ of the mortgage is in foreign currency, the household defaults if:

$$\omega_{i,t} \left( \zeta^h q^h_t (1 - \delta^h) h^I_{i,t-1} \right) \leq R^M_t \left( \frac{\alpha^{FM} r_{er,t}}{\pi^*_t} + \frac{1 - \alpha^{FM}}{\pi_t} \right) m_{i,t-1} \quad (B.1)$$

The left hand side of the inequality gives the value of the collateral (housing), which is affected by the housing price $q^h_t$, an idiosyncratic shock $\omega_t$ and the depreciation rate $\delta^h$. Parameter $\zeta^h$ is an exogenous loan-to-value ratio. The right hand side describes the value of debt. $R^M_t$ is the nominal gross mortgage interest rate, $r_{er,t}$ denote the real exchange rate and $\pi_t$ and $\pi^*_t$ denote consumer goods inflation in the domestic economy and the foreign inflation respectively. Thus, exchange rate depreciation boosts the debt value and has a positive effect on the default threshold increasing the default rate among indebted households.
The default threshold is given by the household-specific shock value \( \omega_t \) such that:

\[
\bar{\omega}_i, t \equiv \omega_i, t = \frac{R_M^{t-1} \left( \omega^{FM}_{rer} + \frac{1 - \alpha^{FM}}{\pi_t} \right) m_{i,t-1}}{\zeta^h q^h_t (1 - \delta^h) h^I_{l,t-1}} \tag{B.2}
\]

We can define the fraction of defaulted impatient households \( G_t \):

\[
G_t \equiv \int_0^{\bar{\omega}_t} \omega_t f (\omega_t) \, d\omega_t \tag{B.3}
\]

Let the fraction of household net worth attributed to the bank be \( \Gamma_t \equiv \int_0^{\bar{\omega}_t} \omega_t f (\omega_t) \, d\omega_t + \bar{\omega}_t \int_{\bar{\omega}_t}^{\infty} f (\omega_t) \, d\omega_t \tag{B.4} \]

Then the impatient dynasty maximizes utility by choosing consumption, housing, level of mortgage debt and leverage:

\[
\max_{\{c^I_t, h^I_t, \omega_{t+1}, m_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left( \beta^t \right)^t v_t \left( \log \left( c^I_t \right) + A_h \log \left( h^I_t \right) - A_n \frac{(n^I_t)^{1+\sigma_n}}{1 + \sigma_n} \right)
\]

subject to the budget constraint of the dynasty:

\[
c^I_t + q^h_t h^I_t \leq w_t n^I_t + (1 - \zeta^h \Gamma_t) q^h_t h^I_{t-1} + m_t \tag{B.5}
\]

and the participation constraint of the bank:

\[
E_t \left[ (\Gamma_{t+1} - \mu_H G_{t+1}) \zeta^h q^h_{t+1} h^I_t \right] = E_t \frac{\tilde{R}^M_t}{\pi_{t+1}} m_t \tag{B.6}
\]

where \( E_t \tilde{R}^M_t \) is the nominal expected return to the bank. Parameter \( \mu_H \) defines monitoring costs incurred by banks. The creditor has to bear monitoring costs so that a share of collateral gets lost in the spirit of the costly state verification literature starting with Townsend (1979) and Bernanke et al. (1999).

First-order conditions follow:

\[
c^I_t : \lambda^I_t = v_t \left( c^I_t \right)^{-\sigma_c} \tag{B.7}
\]

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\( h_t^l : v_t A_h \left(h_t^l\right)^{-\sigma_h} = \lambda_t^l q_t^h - \beta^l E_t A_{t+1}^l \left(1 - \xi_h \Gamma_{t+1}\right) q_{t+1}^h - \Omega_t E_t (\Gamma_{t+1} - \mu_H G_{t+1}) \xi_h q_{t+1}^h \)  
\( \tilde{\omega}_{t+1} : \beta^l E_t A_{t+1}^l \left(\xi_h q_{t+1}^h h_t^l\right) = \Omega_t E_t \left(\left(\Gamma_{t+1}\right)' - \mu_H (G_{t+1})'\right) \left(\xi_h q_{t+1}^h h_t^l\right) \)

\( m_t : \Omega_t E_t \frac{\tilde{R}_M}{\pi_{t+1}} = \lambda_t^l \)

The first-order conditions hold together with two slackness constraints:

\( \lambda_t^l \left(w_t n_t^l + (1 - \xi_h \Gamma_t) q_t^h h_{t-1}^l + m_t + t_t - c_t - q_t^h h_t^l\right) \geq 0, \quad \lambda_t^l \geq 0 \)

\( \Omega_t \left(\left(\Gamma_{t+1} - \mu_H G_{t+1}\right) \xi_h q_{t+1}^h h_t^l - E_t \frac{\tilde{R}_M}{\pi_{t+1}} m_t\right) \geq 0, \quad \Omega_t \geq 0 \)

\( \lambda_t^l \) and \( \Omega_t \) are the Lagrange multiplier to the budget constraint and the bank’s participation constraint respectively.

Given the extensive derivations for the optimal debt contract in Bernanke et al. (1999) provided in, for instance, the appendix of Devereux et al. (2006), it can shown that

\( \Gamma_t = \tilde{\omega}_t \left(1 - \Phi \left(d_t^M\right)\right) + (1 - \mu_F) \Phi \left(d_t^M - \sigma_t^M\right) \)

\( \Gamma_t' = \left(1 - \Phi \left(d_t^M\right)\right) - \mu_F \Phi' \left(d_t^M\right) \)

\( G_t = 1 - \Phi \left(d_t^M - \sigma_t^M\right) - \tilde{\omega}_t \left(1 - \Phi \left(d_t^M\right)\right) \)

\( G_t' = - \left(1 - \Phi \left(d_t^M - \sigma_t^M\right)\right) \)

where

\( d_t^M = \frac{\ln(\tilde{\omega}_t) + 1/2 \left(\sigma_t^M\right)^2}{\sigma_t^M} \)

Impatient households similarly to patient households have monopoly power in a labor market and can set wages. Similarly to patient households, we can show that optimal nominal wage \( W_t^{I*} \) for impatient households equals:

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\[
\left( \frac{W_{t+k}^i}{P_{t+k}} \right)^{1+\epsilon_W \sigma_n} = \frac{e_W}{e_W - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta^t \omega^W)^k \left( A_n \left( W_{t+k}^I \right)^{\epsilon_W (1+\sigma_n)} \right) n_{t+k}^I}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta^t \omega^W)^k \left( \left( W_{t+k}^I \right)^{\epsilon_W n_{t+k}^I} \right)^{-\sigma_c}}
\]

(B.18)

### B.2 Financially constrained firms

#### B.2.1 Solving the financially constrained firms’ profit maximization problem with FX loans

To pay in advance, a financially constrained firm \( i \) borrows from the bank an amount \( L_{i,t} \) that consists of both domestic currency funds \( L_{i,t}^D \) and foreign currency denominated funds \( L_{i,t}^F \) such that \( L_{i,t} = L_{i,t}^D + S_t L_{i,t}^F \) where \( S_t \) is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by \( \alpha^F F \), so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital. It follows that in the beginning of period \( t \) the following condition holds:

\[
E_{t-1} \{ L_{i,t} \} = E_{t-1} \{ (Q_t k_{i,t} + W_t n_{i,t}) \}
\]

Or, in units of composite goods associated with price \( P_t \),

\[
E_{t-1} \{ l_{i,t} \} = E_{t-1} \{ (q_t k_{i,t} + w_t n_{i,t}) \}
\]

\( q_t, w_t \) and \( rer_t \) denote the real price of capital, the real wage and the real exchange rate respectively. We express all three prices are expressed in units of composite goods. It follows that we define \( q_t \) as \( Q_t / P_t \), \( w_t \) as \( W_t / P_t \) and the real exchange rate as \( S_t P_t^* / P_t \) where \( S_t \) is the nominal exchange rate, \( P_t \) is the price of composite goods and \( P_t^* \) defines the price level of foreign composite goods. \( l_{i,t} \) stands for the size of the total
loan expressed in units of composite goods and is defined as $l_{i,t} \equiv L_{i,t}/P_t$. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm’s private incentives to invest in production inputs.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases households step in and transfer lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t}/P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on them as the main source of finance. These funds enter the domestic household’s budget constraint as a lump-sum transfer and have no effect on either the household’s or the firm’s incentives.

Let the matured loan in units of composite goods be $R_{i,t}^L \left( \frac{l_{i,t}^D \pi_{t+1}}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F \pi_{t+1}}{\pi_{t+1}} \right)$, where $R_{i,t}^L$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as $l_{i,t}^D \equiv L_{i,t}^D/P_t$ and $l_{i,t}^F \equiv L_{i,t}^F/P^*_t$. To borrow, the firm has to pledge a share $\kappa$ of future revenue as collateral where $0 < \kappa \leq 1$.

Then the contracted collateral is a fraction $\kappa$ of firms’ revenue from selling goods and depreciated capital in the next period, $p_{t+1}L_{i,t+1} + q_{t+1}(1 - \delta)k_{i,t}$. $p_{t+1}$ stands for the price of homogeneous goods, expressed in units of composite goods ($p_{t+1}^L \equiv P_{t+1}^L/P_{t+1}$). Then the decision of the financially constrained firm $i$ born in period $t$ whether to default or not is determined by the lower value:

$$
\min \left\{ R_{i,t}^L \left( L_{i,t}^D + S_{t+1}L_{i,t}^F \right), \quad \kappa \left( p_{t+1}L_{i,t+1} + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}
$$

(B.19)

Deflating by $P_{t+1}$ gives the expression in units of composite goods:

$$
\min \left\{ R_{i,t}^L \left( \frac{l_{i,t}^D \pi_{t+1}}{\pi_{t+1}} + rer_{t+1} \frac{l_{i,t}^F \pi_{t+1}}{\pi_{t+1}} \right), \quad \kappa \left( p_{t+1}^L L_{i,t+1} + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}
$$

(B.20)

where $p_{t+1}^L L_{i,t+1} = p_{t+1}A_{t+1}g_i, t+1 + \gamma_{i,t}^{\alpha} + \kappa_{i,t}^{\alpha}$. The firm maximizes expected profits given by future revenue from selling goods and depreciated capital minus the debt payment. Financial flows received in period $t$ also enter the maximization problem and can be summarized as the difference between the loan plus equity and working capital expenditure:
The resulting first-order conditions are:

\[
\max_{\{k_{i,t}, n_{i,t}\}} E_t \beta^P A_{t,t+1}^P \left\{ \frac{p_{t+1}^L y_{i,t+1}^L + Q_{t+1}(1-\delta)k_{i,t}}{P_{t+1}} \right\}
- E_t \beta^P A_{t,t+1}^P \min \left\{ \frac{R_{i,t}^L \left( L_{i,t}^D + S_{t+1} L_{i,t}^F \right)}{P_{t+1}}, \ \kappa \left( \frac{p_{t+1}^L y_{i,t+1}^L + Q_{t+1}(1-\delta)k_{i,t}}{P_{t+1}} \right) \right\}
+ \frac{L_{i,t} + Z_{i,t}}{P_t} - \frac{Q_t k_{i,t} + W_t n_{i,t}}{P_t}
\]

s.t.

\[
E_{t-1} \left\{ L_{i,t} \right\} = \frac{E_{t-1} \left\{ Q_t k_{i,t} + W_t n_{i,t} \right\}}{P_t}
\]

Using the previously introduced definitions yields

\[
\max_{\{k_{i,t}, n_{i,t}\}} E_t \beta^P A_{t,t+1}^P \left\{ \frac{p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1-\delta)k_{i,t}}{P_{t+1}} \right\}
- E_t \beta^P A_{t,t+1}^P \min \left\{ \frac{R_{i,t}^L \left( \frac{P_{t+1}}{P_{t+1}} + r_{t+1} \frac{L_{i,t}^F}{P_{t+1}} \right)}{P_{t+1}}, \ \kappa \left( \frac{p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1-\delta)k_{i,t}}{P_{t+1}} \right) \right\}
+ l_{i,t} + z_{i,t} - (q_t k_{i,t} + w_t n_{i,t})
\]

s.t.

\[
E_{t-1} \left\{ l_{i,t} \right\} = E_{t-1} \left\{ q_t k_{i,t} + w_t n_{i,t} \right\}
\]  

(B.21)

The resulting first-order conditions are:

\[
k_{i,t} : \ E_t \beta^P A_{t,t+1}^P \left\{ \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1-\delta) \right\}
- E_t \beta^P A_{t,t+1}^P \left\{ (1-\Phi(d_{i,t}))\kappa \left( \frac{p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1-\delta)}{P_{t+1}} \right) \right\}
\]

\[
= \partial \text{cov} \left( \beta^P A_{t,t+1}^P, \ \min \left\{ \frac{R_{i,t}^L \left( \frac{p_{t+1}^L}{P_{t+1}} + r_{t+1} \frac{L_{i,t}^F}{P_{t+1}} \right)}{P_{t+1}}, \ \kappa \left( \frac{p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1-\delta)k_{i,t}}{P_{t+1}} \right) \right\} \right)
\]

\[
= q_t
\]
\[ n_{i,t} : \quad E_t \beta^P \Lambda_{t+1} \left\{ p^{L}_{t+1} \frac{\partial y^L_{i,t+1}}{\partial n_{i,t}} \right\} \]

\[ - E_t \beta^P \Lambda_{t+1} \left\{ (1 - \Phi(d_{i,t})) \kappa \left( p^{L}_{t+1} \frac{\partial y^L_{i,t+1}}{\partial n_{i,t}} \right) \right\} \]

\[ \delta \text{cov} \left( \beta^P \Lambda_{t+1}, \quad \min \left\{ R^L_{t,i} \left( \frac{\nu^D_{t+1}}{\pi_{t+1}} + \text{rer}_{t+1} \frac{\nu^F_{t}}{\pi^*_{t+1}} \right), \quad \kappa \left( p^{L}_{i,t+1} y^L_{i,t+1} + q_{t+1}(1 - \delta)k_{i,t} \right) \right\} \right) \]

\[ \frac{\partial n_{i,t}}{\partial n_{i,t}} = w_t \]

where

\[ d_{2,t} \equiv \frac{E_t \ln \left( \kappa \left( p^{L}_{i,t+1} y^L_{i,t+1} + q_{t+1}(1 - \delta)k_{i,t} \right) - R^L_{i,t} \text{rer}_{t+1} \frac{\nu^F_{t}}{\pi^*_{t+1}} \right) - E_t \ln \left( R^L_{i,t} \frac{\nu^D_{t+1}}{\pi_{t+1}} \right)}{\sigma_F \cdot t}, \quad d_{1,t} = d_{2,t} + \sigma_F \cdot t \]

The derivation of \( d_{2,t} \) is given in the next section and results for the first-order conditions are given by equations (A2.1) and (A2.2).

The first-order conditions hold together with the ex-ante budget constraint provided in equation (B.21).

### B.2.2 Derivation of the default probability

We need to compute the expected value of the firm’s payment function (we abstract from indices \( i \) for the sake of brevity):

\[ E_t \min \left\{ R^L_{t,i} \left( \frac{\nu^D_{t+1}}{\pi_{t+1}} + \text{rer}_{t+1} \frac{\nu^F_{t}}{\pi^*_{t+1}} \right), \quad \kappa \left( p^{L}_{i,t+1} y^L_{i,t+1} + q_{t+1}(1 - \delta)k_{t} \right) \right\} \]

To simplify, we re-order the terms in the following way:

\[ E_t \min \left\{ R^L_{t,i} \frac{\nu^D_{t+1}}{\pi_{t+1}}, \quad \kappa \left( p^{L}_{i,t+1} y^L_{i,t+1} + q_{t+1}(1 - \delta)k_{t} \right) - R^L_{t,i} \text{rer}_{t+1} \frac{\nu^F_{t}}{\pi^*_{t+1}} \right\} + E_t R^L_{t,i} \text{rer}_{t+1} \frac{\nu^F_{t}}{\pi^*_{t+1}} \]

Further we focus on the first term only, since it defines the default decision and contains all necessary prices too:
Further, then the modified minimum function can be re-written as 

\[ E_t \min \left\{ R_t^{l, lD}, \bar{y}_{t+1} \right\} \]

Define \( \bar{y}_{t+1} = \pi_{t+1} \left( \kappa \left( p_t^L y_{t+1}^L + q_{t+1} (1 - \delta) k_t \right) - R_t^{l, rer_l}_{t+1} \right) \), where

\[ \bar{y}_{t+1} \sim \log-normal \left( \mu_{\bar{y}_{t+1}}, \sigma_{F,t}^2 \right) \]

Then the modified minimum function can be re-written as

\[ E_t \min \left\{ R_t^{l, lD}, \bar{y}_{t+1} \right\} \]

Further,

\[
E_t \min \left\{ R_t^{l, lD}, \bar{y}_{t+1} \right\} = R_t^{l, lD} \Pr \left( R_t^{l, lD} < \bar{y}_{t+1} \right) + \left( 1 - \Pr \left( R_t^{l, lD} < \bar{y}_{t+1} \right) \right) E_t \left( \bar{y}_{t+1} \mid \bar{y}_{t+1} < R_t^{l, lD} \right)
\]

\[ = R_t^{l, lD} \Pr \left( R_t^{l, lD} < \bar{y}_{t+1} \right) + \int_0^{R_t^{l, lD}} \bar{y}_{t+1} dF (\bar{y}_{t+1}) \]

\[ = R_t^{l, lD} \int_R^{l, lD} dF (\bar{y}_{t+1}) + \int_0^{R_t^{l, lD}} \bar{y}_{t+1} dF (\bar{y}_{t+1}) \]

\[ = R_t^{l, lD} \int \frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln(\bar{y}_{t+1}) - \mu_y)^2}{2\sigma_y^2}} d(\bar{y}_{t+1}) \]

\[ + \int_0^{R_t^{l, lD}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln(\bar{y}_{t+1}) - \mu_y)^2}{2\sigma_y^2}} d(\bar{y}_{t+1}) \]

\[ = R_t^{l, lD} \Phi \left( \frac{\ln(\bar{y}_{t+1}) - \mu_y}{\sigma_y} \right) + \int_0^{R_t^{l, lD}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(\ln(\bar{y}_{t+1}) - \mu_y + \sigma_y^2)^2}{2\sigma_y^2}} d(\bar{y}_{t+1}) \]

\[ = R_t^{l, lD} \Phi \left( \frac{\ln(\bar{y}_{t+1}) - \mu_y}{\sigma_y} \right) + \frac{1}{2} E_t(\bar{y}_{t+1}) \left( e^{-\frac{\ln(\bar{y}_{t+1}) - \mu_y - \sigma_y^2}{2\sigma_y^2}} + 1 \right) \]

\[ = R_t^{l, lD} \Phi \left( \frac{\mu_y - \ln(\bar{R_t^{l, lD}})}{\sigma_y} \right) + E_t(\bar{y}_{t+1}) \Phi \left( \frac{\ln(\bar{R_t^{l, lD}}) - \mu_y - \sigma_y^2}{\sigma_y^2} \right) \]

\[ = R_t^{l, lD} \Phi \left( \frac{\mu_y - \ln(\bar{R_t^{l, lD}})}{\sigma_y} \right) + E_t(\bar{y}_{t+1}) \left( 1 - \Phi \left( \frac{\mu_y - \ln(\bar{R_t^{l, lD}})}{\sigma_y + \sigma_y^2} \right) \right) \]

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The expression can be simplified as

\[
E_t \min \{ R_t^L l_t^D, \ y_{t+1} \} = (1 - \Phi(d_{1,t})) E_t (\bar{y}_{t+1}) + \Phi(d_{2,t}) R_t^L l_t^D
\]

where

\[
d_{2,t} \equiv \frac{\mu_y - \ln (R_t^L l_t^D)}{\sigma_{F,t}}, \quad d_{1,t} \equiv d_{2,t} + \sigma_{F,t}
\]

where

\[
\mu_y \equiv E_t \ln (\bar{y}_{t+1})
\]

or

\[
d_{2,t} \equiv \frac{E_t \ln (\bar{y}_{t+1}/\pi_{t+1}) - \ln (R_t^L / \pi_{t+1})}{\sigma_{F,t}}, \quad d_{1,t} \equiv d_{2,t} + \sigma_{F,t}
\]

Recall that \( \bar{y}_{t+1} = \pi_{t+1} \left( \kappa (p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_t) - R_t^L rer_{t+1} \frac{l^F_{t+1}}{\pi_{t+1}} \right) \) so it can be substituted back to get complete expressions. Then \( \sigma_{F,t}^2 = \text{var} (\bar{y}_{t+1}) = \text{var} \left( \pi_{t+1} \left( \kappa (p_{t+1}^R y_{t+1}^R + q_{t+1}(1 - \delta)k_t) - R_t^L rer_{t+1} \frac{l^F_{t+1}}{\pi_{t+1}} \right) \right) \).

To solve for the first-order conditions, we differentiate the expected loan payment w.r.t. \( k_t \):

\[
\frac{\partial E_t \min \{ R_t^L l_t^D, \ y_{t+1} \}}{\partial k_t} = (1 - \Phi(d_{1,t})) \frac{\partial E_t \bar{y}_{t+1}}{\partial k_t} - E_t \bar{y}_{t+1} \left( \frac{\partial \Phi(d_{1,t})}{\partial d_{1,t}} \frac{\partial d_{1,t}}{\partial k_t} + R_t^L l_t^D \frac{\partial \Phi(d_{2,t})}{\partial d_{2,t}} \frac{\partial d_{2,t}}{\partial k_t} \right)
\]

\[
= (1 - \Phi(d_{1,t})) \frac{\partial E_t \bar{y}_{t+1}}{\partial k_t}
\]
where the proof of the last expression comes from by using \( \frac{\partial d_{1,t}}{\partial \kappa_t} = \frac{\partial d_{2,t}}{\partial \kappa_t} \) and computing the following:

\[
\begin{align*}
- E_t (\bar{y}_{t+1}) & \Phi'(d_{1,t}) + R_t^{L} D_t^D \Phi'(d_{2,t}) \\
= - & e^{\ln(E_t\bar{y}_{t+1})} \Phi'(d_{1,t}) + e^{\ln(R_t^{L} D_t^D)} \Phi'(d_{2,t}) \\
= - & e^{\ln(E_t\bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{1,t}^2} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
= - & e^{\ln(E_t\bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(d_{1,t}^2 + 2d_{2,t} \sigma_{F,t} + \frac{1}{2} \sigma_{F,t}^2\right)} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
= - & e^{\ln(E_t\bar{y}_{t+1})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{-\left(d_{2,t} \sigma_{F,t} + \frac{1}{2} \sigma_{F,t}^2\right)} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \\
= - & \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} \left[ e^{\ln(E_t\bar{y}_{t+1})} e^{-\left(E_t\ln(\bar{y}_{t+1}) - \ln(R_t^{L} D_t^D) + \frac{1}{2} \sigma_{F,t}^2\right)} + e^{\ln(R_t^{L} D_t^D)} \right] \\
= - & \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} d_{2,t}^2} e^{\ln(R_t^{L} D_t^D)} + e^{-\frac{1}{2} d_{2,t}^2} \\
= 0,
\end{align*}
\]

In this derivation we use the results for log-normal variables such as \( E_t \ln (\bar{y}_{t+1}) = \ln (E_t\bar{y}_{t+1}) - \frac{1}{2} \sigma_{F,t}^2 \) and the definition of \( d_{1,t} \). Substituting a definition for \( \bar{y}_{t+1} \) back gives

\[
\frac{\partial E_t \min \left\{ \frac{R_t^{L} D_t^D}{\pi_{t+1}^{L} D_t^D}, \ k \left( p_{t+1}^{L} y_{t+1}^{L} + q_{t+1}(1 - \delta) k_t \right) - R_t^{L} \text{rer}_{t+1} \right\} }{\partial k_t} = (1 - \Phi(d_{1,t})) \frac{\partial E_t k \left( p_{t+1}^{L} y_{t+1}^{L} + q_{t+1}(1 - \delta) k_t \right)}{\partial k_t}
\]

(A2.1)

Similarly it can be shown that

\[
\frac{\partial E_t \min \left\{ \frac{R_t^{L} D_t^D}{\pi_{t+1}^{L} D_t^D}, \ k \left( p_{t+1}^{L} y_{t+1}^{L} + q_{t+1}(1 - \delta) k_t \right) - R_t^{L} \text{rer}_{t+1} \right\} }{\partial n_t} = (1 - \Phi(d_{1,t})) \frac{\partial E_t k \left( p_{t+1}^{L} y_{t+1}^{L} \right)}{\partial n_t}
\]

(A2.2)
B.2.3 Solving the financially constrained firms’ profit maximization problem with domestic currency loans

Now the matured loan in units of composite goods is $R_{i,t}^L L_{i,t} \equiv R_{i,t}^L L_{i,t}^{t+1}$. The loan is denominated in domestic currency and $R_{i,t}^L$ is the nominal gross interest rate on the loan. The contracted collateral is a fraction $\kappa$ of firms’ revenue from selling goods and depreciated capital in the next period. In units of composite goods the contracted collateral can be expressed as $p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta)k_{i,t}$. Then the decision of the financially constrained firm $i$ born in period $t$ whether to default or not is determined by the lower value:

$$\min \left\{ R_{i,t}^L \frac{L_{i,t}}{P_{t+1}}, \kappa \left( p_{t+1}^L y_{i,t+1}^L + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}$$

As previously, $p_{t+1}^L y_{i,t+1}^L = p_{t+1}^L A_{t+1} \theta_{i,t+1} k_{i,t}^\theta n_{i,t}^{1-\theta}$, $P_{t+1}^L \equiv P_{t+1}^L / P_{t+1}$ and $q_{t+1} \equiv Q_{t+1} / P_{t+1}$.

Financial flows received in period $t$ also enter the maximization problem and can be summarized as the difference between the loan plus equity ($Z_{i,t}$) and working capital expenditure expressed in units of composite goods:

$$\max_{\{k_{i,t}, n_{i,t}\}} E_t \beta^{\lambda_{i,t+1}} \left\{ \frac{p_{t+1}^L y_{i,t+1}^L + Q_{t+1}(1 - \delta)k_{i,t}}{P_{t+1}} \right\} - E_t \beta^{\lambda_{i,t+1}} \min \left\{ R_{i,t}^L \frac{L_{i,t}}{P_{t+1}}, \kappa \left( p_{t+1}^L y_{i,t+1}^L + Q_{t+1}(1 - \delta)k_{i,t} \right) \right\}$$

$$+ \frac{L_{i,t} + Z_{i,t}}{P_t} - Q_i k_{i,t} + W_t n_{i,t}$$

s.t.

$$\frac{E_{t-1} \{ L_{i,t} \}}{P_t} = \frac{E_{t-1} \{ Q_i k_{i,t} + W_t n_{i,t} \}}{P_t}$$

Using the previously introduced definitions yields
$$\max_{\{k_{i,t}, n_{i,t}\}} \ E_t\beta^P \Lambda_{t+1}^P \left\{ \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \frac{k_{i,t}}{n_{i,t+1}} \right) \right\}$$

$$- E_t\beta^P \Lambda_{t+1}^P \min \left\{ R_{i,t}^L \frac{l_{i,t}}{n_{i,t+1}}, \ \kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta)k_{i,t} \right) \right\}$$

$$+ l_{i,t} + z_{i,t} - (q_t k_{i,t} + w_t n_{i,t})$$

s.t.

$$E_{t-1}\{l_{i,t}\} = E_{t-1}\{q_t k_{i,t} + w_t n_{i,t}\} \quad (B.22)$$

The resulting first-order conditions are:

$$k_{i,t} : \quad E_t\beta^P \Lambda_{t+1}^P \left\{ p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \right\}$$

$$- E_t\beta^P \Lambda_{t+1}^P \left\{ (1 - \Phi(d_{i,t}))\kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta) \right) \right\}$$

$$\partial \text{cov} \left( \beta^P \Lambda_{t+1}^P, \ \min \left\{ R_{i,t}^L \frac{l_{i,t}}{n_{i,t+1}}, \ \kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial k_{i,t}} + q_{t+1}(1 - \delta)k_{i,t} \right) \right\} \right)$$

$$= \frac{\partial}{\partial k_{i,t}}$$

$$+ q_t$$

$$n_{i,t} : \quad E_t\beta^P \Lambda_{t+1}^P \left\{ p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial n_{i,t}} \right\}$$

$$- E_t\beta^P \Lambda_{t+1}^P \left\{ (1 - \Phi(d_{i,t}))\kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial n_{i,t}} \right) \right\}$$

$$\partial \text{cov} \left( \beta^P \Lambda_{t+1}^P, \ \min \left\{ R_{i,t}^L \frac{l_{i,t}}{n_{i,t+1}}, \ \kappa \left( p_{t+1}^L \frac{\partial y_{i,t+1}^L}{\partial n_{i,t}} + q_{t+1}(1 - \delta)k_{i,t} \right) \right\} \right)$$

$$= \frac{\partial}{\partial n_{i,t}}$$

$$+ w_t$$

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where
\[
d_{2,t} \equiv \frac{E_t \ln \left( \kappa \left( p^L_{t+1,i,t+1} y^L_{i,t+1} + q_{i,t+1} (1 - \delta) k_{i,t} \right) \right) - E_t \ln \left( R^L_{i,t} \pi_{i+1} \right)}{\sigma_{F,t}}, \quad d_{1,t} = d_{2,t} + \sigma_{F,t}
\]

and \( \sigma^2_{F,t} = \text{var} \left( \pi_{i+1} \kappa \left( p^L_{t+1,i,t+1} y^L_{i,t+1} + q_{i,t+1} (1 - \delta) k_{i,t} \right) \right) \).

The first-order conditions hold together with the ex-ante budget constraint provided in equation (B.22).

### B.3 Production sector

#### B.3.1 Retail firms

Homogeneous goods produced by financially constrained firms are sold to domestic retail firms. A domestic retail firm \( j \) differentiates purchased inputs at no cost and sells at a monopolistic price \( p^H_t(j) \). We assume that only a fraction \((1 - \omega^H)\) of domestic retail firms can adjust prices every period as in Calvo (1983). The fraction \( \omega^H \) of remaining firms adjust past prices by the rate \( \pi^{adj}_t \). The aggregate price level that prevails in the retail sector is denoted by \( p^H_t \). Differentiated goods from the domestic retail sector, \( y^H_t(j), j \in (0, 1) \), are purchased by the composite goods producer.

#### B.3.2 Importers

Parallel to differentiated domestic goods produced in the domestic retail sector, there is another strand of differentiated goods in the economy that is used as an input for the production of domestic final goods. In particular, we assume a set of importers that buy foreign goods from abroad and differentiate them. Importers exercise market power and set prices in the staggered way as in Calvo (1983), which allows for the incomplete exchange rate pass-through. Thus, \((1 - \omega^F)\) of importers change their past prices to the optimal price at period \( t \). The fraction \( \omega^F \) of remaining firms adjust past prices by the rate \( \pi^{adj}_t \). The aggregate price level that prevails in the retail sector is denoted by \( p^F_t \).
B.3.3 Composite goods producer

We assume that the composite goods producer has access to an aggregation technology and can assemble differentiated goods at no cost. First, the composite goods producer assembles differentiated domestic goods \( y^H_t(j) \forall j \) into domestic aggregate goods \( y^H_t \) and differentiated imported goods \( y^F_t(j) \forall j \) into foreign aggregate goods \( y^F_t \). She uses the following assembling technologies:

\[
y^H_t = \left( \int_0^1 y^H_t(j)^{1 - \frac{1}{\eta^H_t}} dj \right)^{\frac{c^H}{\eta^H_t - 1}} , \\
y^F_t = \left( \int_0^1 y^F_t(j)^{1 - \frac{1}{\eta^F_t}} dj \right)^{\frac{c^F}{\eta^F_t - 1}}
\]

Aggregate goods can be used to produce consumption composite goods or investment composite goods. The producer combines domestic aggregate goods and foreign aggregate goods into consumption composite goods \( c_t \) with the aggregation technology that takes the taste parameter for foreign aggregate goods \( \eta^c \) as given:

\[
c_t = \left( \frac{1}{\eta^c} \left( c^H_t \right)^{\frac{c^H_t}{\eta^c}} + \frac{1}{\eta^c} \left( c^F_t \right)^{\frac{c^F_t}{\eta^c}} \right)^{\frac{\eta^c}{\eta^c - 1}} (B.23)
\]

Solving a perfectly competitive aggregator’s profit maximization problem \( c_t - p^H_t c^H_t - p^F_t c^F_t \) yields demand functions for domestic aggregate goods and foreign aggregate goods for consumption composites are as follows:

\[
c^H_t = \eta^c \left( p^H_t \right)^{-\epsilon^c} c_t , \quad (B.24) \\
c^F_t = (1 - \eta^c) \left( p^F_t \right)^{-\epsilon^c} c_t , \quad (B.25)
\]

\( \epsilon^c \) stands for elasticity of substitution between domestic aggregate goods and foreign aggregate goods. The composite consumption good has an associated price \( P_t \).

Similarly, investment composite goods require a perfectly competitive aggregation technology where

\[
i_t = \left( \frac{1}{\eta^i_t} \left( i^H_t \right)^{\frac{c^H_t}{c^H_t}} + \frac{1}{\eta^i_t} \left( i^F_t \right)^{\frac{c^F_t}{c^F_t}} \right)^{\frac{1}{\eta^i_t - 1}} (B.26)
\]
Assuming that the associated price of investment composite goods is $P^I_t$, the respective demand function can be derived straightforwardly and are given by:

\[ i^H_t = \eta_t \left( \frac{P^H_t}{P^I_t} \right)^{-\epsilon_t} i_t \]  

(B.27)

\[ i^F_t = (1 - \eta_t) \left( \frac{P^F_t}{P^I_t} \right)^{-\epsilon_t} i_t \]  

(B.28)

**B.3.4 Capital producers**

Capital producers sell capital to financially constrained firms at the real competitive price $q_t$ and buying the depreciated capital stock back next period. To restore the depreciated capital, capital producers add composite goods (investment) $i_t$ as additional inputs to the depreciated capital stock by using a technology subject to investment adjustment costs $\Gamma \left( \frac{i_t}{i_{t-1}} \right)$:

\[ k_t = (1 - \delta) k_{t-1} + \left( 1 - \Gamma \left( \frac{i_t}{i_{t-1}} \right) \right) u_t i_t \]  

(B.29)

where adjustment costs $\Gamma$ equal:

\[ \Gamma \left( \frac{i_t}{i_{t-1}} \right) = \gamma \left( \frac{i_t}{i_{t-1}} u_t - a \right)^2 \]

$u_t$ is an exogenous capital utilization shock.

**B.3.5 Exporters**

We assume that perfectly competitive exporters demand $y^H_{t*}$ units of the domestic aggregate good $y^H_t$, so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters. Exported goods consist of the domestic aggregate, so they do not use imported inputs.

Exports are sold at a price $p^H_t / rer_t$ which is the price of domestic aggregate goods expressed in units of foreign composite goods. The foreign demand for domestic aggregate goods is price-sensitive:

\[ y^H_{t*} = \eta^* \left( \frac{p^H_t}{rer_t} \right)^{-\epsilon^*} y^*_t \]  

(B.30)
Consistent with the small open economy assumption, $P_t^*$ and $y_t^*$ are assumed to evolve exogenously.

**B.4 Government**

We abstract from normative analysis of government policies and take government spending as exogenous. We assume that to finance a stochastic stream of real government expenditure, $g_t$, the government collects lump-sum taxes $t_t$ from the household and issues domestic bonds $b_t$. It has to satisfy the budget constraint:

$$g_t + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

Taxes in units of domestic final goods follow this tax rule:

$$t_t = t + \kappa^B (b_{t-1} - b) + e_t^T$$

**B.5 Aggregation of labor**

Financially constrained firms use labor supplied by both patient and impatient households. To aggregate over the labor supply, we assume that the following technology has to be applied:

$$n_t = \left( n_t^P \frac{\epsilon_{N-1}^{N-1}}{\epsilon_N} + n_t^I \frac{\epsilon_{N-1}^{N-1}}{\epsilon_N} \right)^{\frac{\epsilon_N}{\epsilon_{N-1}}}$$

(B.31)

Solving a perfectly competitive aggregator’s profit maximization problem $w_t n_t - w_t^P n_t^P - w_t^I n_t^I$ provides two demand functions for labor supplied by different types of households:

$$n_t^P = \left( \frac{w_t^P}{w_t} \right)^{-\epsilon_N} n_t$$

(B.32)

$$n_t^I = \left( \frac{w_t^I}{w_t} \right)^{-\epsilon_N} n_t$$

(B.33)
B.6 Solving the banks’ optimization problem

B.6.1 Lending in foreign currency with a fixed denomination structure

The domestic household owns all banks that operate in the domestic economy and lend to financially constrained firms and impatient households. We assume that there is a continuum of these banks and every period there is a probability $\omega_B$ that a bank continues operating. Otherwise, the net worth is transferred to the owner of the bank, the domestic household. We assume that banks give loans out of accumulated equity $N^B_t$, deposits $D_t$ and foreign debt $D^*_t$. The balance sheet constraint of a bank $j$, expressed in units of composite goods, is given by

$$\frac{N^B_{j,t} + D_{j,t} + S_t D^*_t}{P_t} = \frac{L_{j,t} + M_{j,t}}{P_t}$$

$L_{j,t}$ consists of both domestic currency funds $L^D_{j,t}$ and foreign currency denominated funds $L^F_{j,t}$ such that $L_{j,t} = L^D_{j,t} + S_t L^F_{j,t}$, where $S_t$ is the nominal exchange rate. $M_{j,t}$ consists of both domestic currency funds $M^D_{j,t}$ and foreign currency denominated funds $M^F_{j,t}$ such that $M_{j,t} = M^D_{j,t} + S_t M^F_{j,t}$.

Banks pay a nominal domestic interest rate $R_t$ on deposits and a nominal foreign interest rate $R^*_t \xi_t$ on foreign debt. $R^*_t$ follows a stationary AR(1) process. $\xi_t$ denotes a premium on bank foreign debt.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction $\lambda^B_t$ of assets, but if that happens the bank goes bankrupt (i.e. cannot continue).

The only asset on the banks’ balance sheet is loans to financially constrained firms, thus, the expected nominal return of the bank $j$ is defined as $R^L_{j,t}$ and given by:

$$E_t \left\{ R^L_{j,t+1} L_{j,t} \right\} \equiv E_t \min \left\{ R^L_{j,t} \left( L^D_{j,t} + S_{t+1} L^F_{j,t} \right), \kappa \left( P^{L}_{t+1} y^L_{j,t+1} + Q_{t+1} (1 - \delta) k_{j,t} \right) \right\}$$

Or, units of composite goods,

$$E_t \left\{ \tilde{R}^L_{j,t+1} \right\} \equiv E_t \min \left\{ R^L_{j,t} \left( \frac{L^D_{j,t}}{\pi_{t+1}} + r e_{t+1} \frac{L^F_{j,t}}{\pi^*_{t+1}} \right), \kappa \left( P^{L}_{t+1} y^L_{j,t+1} + q_{t+1} (1 - \delta) k_{j,t} \right) \right\}$$
\( \Rightarrow E_t \left\{ \tilde{R}^L_{j,t} \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p^L_{t+1} \frac{\eta_{j,t+1}}{\pi_{t+1}} + (1 - \delta)q_{t+1}k_{j,t} \right) + \Phi(d_{2,t}) \tilde{R}^L_{j,t} \frac{\eta_{j,t}}{\pi_{t+1}} + \Phi(d_{3,t}) \tilde{R}^F_{j,t} r_{F_{t+1}} \frac{\eta_{j,t}}{\pi_{t+1}} \right\} \) 

(B.34)

We know the expression for the expected return on mortgages \( \tilde{R}^M_{j,t} \) from the impatient households section. It is given by equation (B.6).

Then the optimization problem of the bank \( j \) can be written as:

$$V_{j,t} = \max_{D_{j,t,D^i_{j,t},L_{j,t},M_{j,t}}} \left[ E_t \left[ \beta^{P} \Lambda^{P}_{t,t+1} \left\{ (1 - \omega^{B}) \frac{N_{j,t+1}}{P_{t+1}} + \omega^{B}V_{j,t+1} \right\} \right] \right]$$

s.t.

$$V_{j,t} \geq \lambda^{B} \frac{L_{j,t} + M_{j,t}}{P_{t}},$$  \hspace{1cm} \text{(Incentive constraint)}

$$\frac{N^{B}_{j,t} + D_{j,t} + S_{j}d^{*}_{j,t}}{P_{t}} = \frac{L_{j,t} + M_{j,t}}{P_{t}},$$  \hspace{1cm} \text{(Balance sheet constraint)}

$$\frac{N^{B}_{j,t}}{P_{t}} = \frac{\tilde{R}^{L}_{j,t-1}}{P_{t}} \frac{L_{j,t-1}}{P_{t}} + \frac{\tilde{R}^{M}_{j,t-1}}{P_{t}} \frac{M_{j,t-1}}{P_{t}} - \frac{R_{t-1}}{P_{t}} D_{j,t-1} - \frac{R^{*}_{t-1} \xi_{t-1}}{P_{t}} S_{j}d^{*}_{j,t-1}$$ \hspace{1cm} \text{(LoM of net worth)}

We define \( r_{F_{t}} \equiv P^{*}_{t} S_{t} / P_{t} \), \( d^{*}_{j,t} \equiv D^{*}_{j,t} / P^{*}_{t} \), \( d_{j,t} \equiv D_{j,t} / P_{t} \), \( l_{j,t} \equiv L_{j,t} / P_{t} \), \( m_{j,t} \equiv M_{j,t} / P_{t} \) and \( n_{j,t} \equiv N^{B}_{j,t} / P_{t} \). It follows that

$$V_{j,t} = \max_{d_{j,t},D^i_{j,t},L_{j,t},M_{j,t}} \left[ E_t \left[ \beta^{P} \Lambda^{P}_{t,t+1} \left\{ (1 - \omega^{B}) n_{j,t+1} + \omega^{B}V_{j,t+1} \right\} \right] \right]$$

s.t.

$$V_{j,t} \geq \lambda^{B} \left( l_{j,t} + m_{j,t} \right),$$  \hspace{1cm} \text{(Incentive constraint)}

$$n^{B}_{j,t} + d_{j,t} + r_{F_{t}} d^{*}_{j,t} = l_{j,t} + m_{j,t},$$  \hspace{1cm} \text{(Balance sheet constraint)}

$$n^{B}_{j,t} = \frac{\tilde{R}^{L}_{j,t-1}}{\pi_{t}} l_{j,t-1} + \frac{\tilde{R}^{M}_{j,t-1}}{\pi_{t}} m_{j,t-1} - \frac{R_{t-1}}{\pi_{t}} d_{j,t-1} - \frac{R^{*}_{t-1} \xi_{t-1}}{\pi_{t}} r_{F_{t}} d^{*}_{j,t-1}$$ \hspace{1cm} \text{(LoM of net worth)}
Lagrangean of the problem can be formulated as:

\[ L = (1 + \nu_{1,t})E_t \beta^P \Lambda^P_{t,t+1} \left\{ (1 - \omega^B) \left( \frac{\tilde{R}_{j,t}^L}{\pi_{t+1}} l_{j,t} + \frac{\tilde{R}_{j,t}^M}{\pi_{t+1}} m_{j,t} - \frac{R_t}{\pi_{t+1}} d_{j,t} - \frac{R_t^* \xi_t}{\pi_{t+1}} \right) + \omega^B V_{j,t+1} \right\} \\
- \nu_{1,t} \lambda^L (l_{j,t} + l_{j,t}) \\
+ \nu_{2,t} \left( \frac{\tilde{R}_{j,t-1}^L}{\pi_t} l_{j,t-1} + \frac{\tilde{R}_{j,t-1}^M}{\pi_t} m_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R_{t-1}^* \xi_{t-1}}{\pi_t} \right) - l_{j,t} - m_{j,t} + d_{j,t} + \nu_{2,t} \]

This gives the first-order conditions:

\[ l_{j,t} : (1 + \nu_{1,t})\beta^P E_t \Lambda^P_{t,t+1} \left\{ (1 - \omega^B) \left( \frac{\tilde{R}_{j,t}^L}{\pi_{t+1}} \right) + \omega^B \frac{\partial V(.)}{\partial l_{j,t}} \right\} = \lambda^L v_{1,t} + \nu_{2,t} \]

\[ m_{j,t} : (1 + \nu_{1,t})\beta^P E_t \Lambda^P_{t,t+1} \left\{ (1 - \omega^B) \left( \frac{\tilde{R}_{j,t}^M}{\pi_{t+1}} \right) + \omega^B \frac{\partial V(.)}{\partial l_{j,t}} \right\} = \lambda^P v_{1,t} + \nu_{2,t} \]

\[ d_{j,t} : (1 + \nu_{1,t})\beta^P E_t \Lambda^P_{t,t+1} \left\{ (1 - \omega^B) \left( \frac{R_t}{\pi_{t+1}} \right) - \omega^B \frac{\partial V(.)}{\partial d_{j,t}} \right\} = \nu_{2,t} \]

\[ d_{j,t}^* : (1 + \nu_{1,t})\beta^P E_t \Lambda^P_{t,t+1} \left\{ (1 - \omega^B) \left( \frac{R_t^* \xi_t}{\pi_{t+1}} \right) - \omega^B \frac{\partial V(.)}{\partial d_{j,t}^*} \right\} = \nu_{2,t} \]

with complementary slackness conditions:

\[ \nu_{1,t} : \nu_{1,t} \left( V_{j,t} - \lambda^B (l_{j,t} + m_{j,t}) \right) = 0 \]

\[ \nu_{2,t} \left( \frac{\tilde{R}_{j,t-1}^L}{\pi_t} l_{j,t-1} + \frac{\tilde{R}_{j,t-1}^M}{\pi_t} m_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R_{t-1}^* \xi_{t-1}}{\pi_t} \right) - l_{j,t} - m_{j,t} + d_{j,t} + \nu_{2,t} \right\) = 0 \]

Further, the first-order conditions can be expressed as

\[ l_{j,t} : (1 + \nu_{1,t})\beta^P E_t \Lambda^P_{t,t+1} \left\{ (1 - \omega^B) + \omega^B \nu_{2,t+1} \right\} \left( \frac{R_{j,t}^L}{\pi_{t+1}} \right) = \lambda^B v_{1,t} + \nu_{2,t} \]
\[ m_{j,t} : \quad (1 + v_{1,t})\beta^P E_t \Lambda_{t,t+1}^P \{(1 - \omega^B) + \omega^B v_{2,t+1}\} \left( \frac{\hat{R}_t}{\pi_{t+1}} \right) = \Lambda_t^B v_{1,t} + v_{2,t} \]

\[ d_{j,t} : \quad (1 + v_{1,t})\beta^P E_t \Lambda_{t,t+1}^P \{(1 - \omega^B) + \omega^B v_{2,t+1}\} \left( \frac{R_t}{\pi_{t+1}} \right) = v_{2,t} \]

\[ d^*_j : \quad (1 + v_{1,t})\beta^P E_t \Lambda_{t,t+1}^P \{(1 - \omega^B) + \omega^B v_{2,t+1}\} \left( \frac{R^*_t \xi \rho_{t+1}^{*}}{\pi_{t+1} \rho_{t+1}^{*}} \right) = v_{2,t} \]

Besides these first-order conditions, the set of equilibrium conditions includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. Aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. Every period a fraction \((1 - \omega^B)\) of banks bankrupt exogenously and are replaced by the same number of new banks. The new equity is injected by domestic households and is assumed to be of the size \(t^B n^B \Lambda_{t-1}^B \). Then

\[ n^B_t = \omega^B \left( \frac{R^L_{j,t-1} l_{t-1} - R_{t-1} d_{t-1} - R^*_t \xi \rho_{t-1}^{*} \rho_{t-1}^{*} \rho_{t-1}^{*}}{\pi_t} \right) + t^B n^B \Lambda_{t-1}^B \]

To include the incentive constraint in the equilibrium conditions, we have to redefine it by using the value of marginal utility from increasing assets by one unit and the value of marginal disutility from increasing debt by one unit. It follows from the previously derived results that the value of the bank \( j \) can also be defined as:

\[ V_{j,t} = \left( \Lambda_t^B \frac{v_{1,t} + v_{2,t}}{1 + v_{1,t}} \right) (l_{j,t} + m_{j,t}) - \frac{v_{2,t}}{1 + v_{1,t}} d_{j,t} - \frac{v_{2,t} \rho_{t}^{*}}{1 + v_{1,t}} \rho_{t}^{*} d^*_j \]

\[ = \frac{v_{2,t}}{1 + v_{1,t}} (l_{j,t} + m_{j,t} - d_{j,t} - \rho_{t}^{*} d^*_j) + \Lambda_t^B \frac{v_{1,t}}{1 + v_{1,t}} (l_{j,t} + m_{j,t}) \]

\[ \Rightarrow V_{j,t} = \frac{v_{2,t}}{1 + v_{1,t}} n_{j,t} + \Lambda_t^B \frac{v_{1,t}}{1 + v_{1,t}} (l_{j,t} + m_{j,t}) \]

Then we can modify the incentive constraint as

\[ \frac{v_{2,t}}{1 + v_{1,t}} n_{j,t} + \Lambda_t^B \frac{v_{1,t}}{1 + v_{1,t}} (l_{j,t} + m_{j,t}) \geq \Lambda_t^B (l_{j,t} + m_{j,t}) \]
⇒ \( \nu_{2,t} n_{j,t} \geq \lambda^B_t (l_{j,t} + m_{j,t}) \) (B.35)

## B.6.2 Lending in domestic currency only

Now the only asset on the banks’ balance sheet is domestic currency loans extended to financially constrained firms, thus, the expected nominal return of the bank \( j \) is defined as \( \tilde{R}^L_{j,t} \) and given by:

\[
E_t \{\tilde{R}^L_{j,t} L_{j,t}\} \equiv E_t \min \left\{ R^L_{j,t} L_{j,t}, \kappa \left( p^{L_{t+1}} y^{L_{t+1}} + Q_{t+1} (1 - \delta) k_{j,t} \right) \right\}
\]

Or, in units of composite goods,

\[
E_t \left\{ \frac{\tilde{R}^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \min \left\{ R^L_{j,t} \frac{l_{j,t}}{\pi_{t+1}}, \kappa \left( p^{L_{t+1}} y^{L_{t+1}} + q_{t+1} (1 - \delta) k_{j,t} \right) \right\}
\]

⇒ \( E_t \left\{ \frac{\tilde{R}^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t})) \kappa \left( p^{L_{t+1}} y^{L_{t+1}} + (1 - \delta) q_{t+1} k_{j,t} \right) + \Phi(d_{2,t}) R^L_{j,t} \frac{l_{j,t}}{\pi_{t+1}} \right\} \) (B.36)

The rest of derivations for the bank’s optimization problem remain the same.

## B.7 The monitoring costs friction for corporates

The model with monitoring costs for corporates is different in two main ways. First, we introduce two layers, intermediate firms and entrepreneurs, that replace the sector of financially-constrained firms. Entrepreneurs have access to technology to transform new capital produced by capital goods producers into productive capital that can be used by intermediate firms. Entrepreneurs borrow to acquire capital and may default. We assume a costly-state-verification problem, as in Bernanke et al. (1999). Banks cannot observe idiosyncratic shocks unless they pay deadweight monitoring costs, so they charge a default premium to all loans to entrepreneurs, leading to the external finance premium. Intermediate firms buy capital from entrepreneurs and combine it with labor. They sell the new homogeneous product to retail firms.

We first describe the optimization problem of entrepreneurs. It solves for optimal debt contract as in Bernanke et al. (1999). The technology that entrepreneurs use to transform capital works in the following
way. $k_t$ units of new capital become $\omega^F_{t+1} k_t$ units of capital available to rent to intermediate firms. The variable $\omega^F_{t+1}$ is log-normally distributed such that $E(\omega^F_{t+1}) = 1$. The variable $\sigma_{F,t}$ denotes the time-varying cross-sectional standard deviation of entrepreneurs’ productivity. We assume that purchases of new capital $(q_t, k_t)$ have to be financed by either loans from banks $l_t$ or accumulated net worth $n^F_t$.

The gross interest rate on loans is denoted by $R^L_t$. The optimal debt contract specifies a cut-off value $\tilde{\omega}^F_{t+1}$. If $\omega^F_{t+1} \geq \tilde{\omega}^F_{t+1}$, the entrepreneur repays: he pays $\tilde{\omega}^F_{t+1} (r^K_{t+1} + (1 - \delta)q_{t+1}) k_t$ to the bank and keeps $(\omega^F_{t+1} - \tilde{\omega}^F_{t+1}) (r^K_{t+1} + (1 - \delta)q_{t+1}) k_t$. If $\omega^F_{t+1} < \tilde{\omega}^F_{t+1}$, the entrepreneur defaults. The bank gets total revenues from the project minus monitoring costs that is equal to a fraction $\mu_F$ of the total revenues. Thus, the bank obtains $(1 - \mu_F) \omega^F_{t+1} (r^K_{t+1} + (1 - \delta)q_{t+1}) k_t$. We assume that a fixed share of the loan $\alpha^{FF}$ is denominated in foreign currency. Then the loan repayment is given by $R^L_t \left( \frac{\alpha^{FF} \rho_{er} t + 1}{\pi^*_t} + \frac{1 - \alpha^{FF}}{\pi^*_t} \right) l_t$.

It follows that the interest rate can be expressed as:

$$R^L_t \left( \frac{\alpha^{FF} \rho_{er} t + 1}{\pi^*_t} + \frac{1 - \alpha^{FF}}{\pi^*_t} \right) = \tilde{\omega}^F_{t+1} (r^K_{t+1} + (1 - \delta)q_{t+1}) \frac{k_t}{l_t}$$ (B.37)

Therefore, for the bank to be willing to lend it must be the case that the return on loans to entrepreneurs satisfies:

$$\tilde{R}^L_t l_t \leq g \left( \tilde{\omega}^F_{t+1} \right) \left( r^K_{t+1} + (1 - \delta)q_{t+1} \right) k_t$$ (B.38)

where

$$g \left( \tilde{\omega}^F_{t+1} \right) = \tilde{\omega}^F_{t+1} \left( 1 - F \left( \tilde{\omega}^F_{t+1} \right) \right) + (1 - \mu_F) \int_{0}^{\tilde{\omega}^F_{t+1}} \omega^F_{t+1} f \left( \omega^F_{t+1} \right) d\omega^F_{t+1}$$ (B.39)

The first term in the right-hand side of equation B.39 is the share of total revenues that the bank obtains from non-defaulted entrepreneurs and the second term is returns from defaulted entrepreneurs.

Entrepreneurs’ expected profits equal

$$E_t \left( h \left( \tilde{\omega}^F_{t+1} \right) \left( r^K_{t+1} + (1 - \delta)q_{t+1} \right) k_t \right)$$ (B.40)

where

$$h \left( \tilde{\omega}^F_{t+1} \right) = \int_{\tilde{\omega}^F_{t+1}}^{\infty} \omega^F_{t+1} f \left( \omega^F_{t+1} \right) d\omega^F_{t+1} - \tilde{\omega}^F_{t+1} \left( 1 - F \left( \tilde{\omega}^F_{t+1} \right) \right)$$ (B.41)
Equation B.41 represents the average share of total returns that is attributed to non-defaulting entrepreneurs. The first term in the right-hand side of equation B.41 is the expected share of average revenue that entrepreneurs obtain. The second term is the expected loan repayment.

We can solve for the optimal debt contract from:

$$E_t \left( \frac{r_t K_t + (1 - \delta) q_{t+1}}{q_t} \left( \frac{h'(\bar{\omega}_{t+1}) g(\bar{\omega}_{t+1})}{g'(\bar{\omega}_{t+1})} - h(\bar{\omega}_{t+1}) \right) \right) = E_t \left( \frac{\bar{R} L_t}{g'(\bar{\omega}_{t+1})} \right) \quad (B.42)$$

The ratio $E_t \left( \frac{r_t K_t + (1 - \delta) q_{t+1}}{q_t} \right) / E_t \bar{R} L_t$ is known as the external finance premium.

We assume that a fraction $\omega^F$ of entrepreneurs survives every period$^1$, and an equal number of new entrepreneurs enters the market. New entrepreneurs get an equity injection from patient households $k_{t-1}$. Given that the average share of total returns that is attributed to entrepreneurs at time $t$ is defined by $h(\bar{\omega}_t^F)$, we get:

$$n_t^F = \omega^F \left( h\left( \bar{\omega}_t^F \right) \left( r_t^K + (1 - \delta) q_t \right) k_{t-1} \right) + t^F n^F A_{t-1} \quad (B.43)$$

Devereux et al. (2006) show in the appendix that

$$g\left( \bar{\omega}_t^F \right) = \bar{\omega}_t^F \left( 1 - \Phi\left( d_t^F \right) \right) + (1 - \mu_F) \Phi\left( d_t^F - \sigma_t^F \right) \quad (B.44)$$

$$g'\left( \bar{\omega}_t^F \right) = \left( 1 - \Phi\left( d_t^F \right) \right) - \mu_F \Phi'\left( d_t^F \right) \quad (B.45)$$

$$h\left( \bar{\omega}_t^F \right) = 1 - \Phi\left( d_t^F - \sigma_t^F \right) - \bar{\omega}_t^F \left( 1 - \Phi\left( d_t^F \right) \right) \quad (B.46)$$

$$h'\left( \bar{\omega}_t^F \right) = -\left( 1 - \Phi\left( d_t^F - \sigma_t^F \right) \right) \quad (B.47)$$

where

$$d_t^F = \frac{\ln(\bar{\omega}_t^F) + 1/2 \left( \sigma_t^F \right)^2}{\sigma_t^F} \quad (B.48)$$

$^1$ This assumption prevents entrepreneurs from accumulating infinite net worth.
B.8 Equilibrium equations of the model with foreign currency debt and leverage-constrained banks

The model is described by 72 endogenous variables:

\[ \{ \lambda_t^P, \lambda_t^I, c_t^P, c_t^I, h_t^P, h_t^I, w_t^P, w_t^I, n_t^P, n_t^I, b_t, d_t, R_t, m_t, m^F_t, m^H_t, q_t^h, R_t^M, \hat{R}_t^M, \hat{\omega}_t, \Omega_t, d_{1,t}, d_{2,t}, R_t^L, \hat{R}_t^L, l_t, l_t^D, l_t^F, \pi_t, p_t^L, k_t, n_t, q_t, w_t, p_t \} \]

\[ p_t^H, \hat{p}_t^H, D_t^H, y_t^H, F_{1,t}^H, F_{2,t}^H, p_t^F, y_t^F, c_t^H, c_t^F, i_t^H, i_t^F, \hat{p}_t^F, D_t^F, F_{1,t}^F, F_{2,t}^F \]

\[ \tilde{\omega}_t^P, D_t^{WP}, \tilde{\Omega}_t^{WP}, \tau_t^{WP}, \tilde{w}_t, D_t^{WI}, \tilde{\Omega}_t^{WI}, \tau_t^{WI}, y_t^H, \tilde{y}_t^H, r, \tilde{r}_t, d_t^e, n_t, \nu_{1,t}, \nu_{2,t}, g_t, t, S_t, t_b, \xi_t \}

The data fitting exercise requires accounting for non-stationary trends in the data, therefore, the model equations are expressed not only in real domestic currency terms, but also transformed into a stationary form with explicitly specified non-stationary growth components. This introduces non-stationary productivity growth variable \( a_t \) that occurs throughout the model description. It is defined as \( a_t \equiv A_t / A_{t-1} \) where \( A_t \) is non-stationary productivity shock. Further, variables with hats denote non-stationary variables divided by \( A_{t-1} \) to obtain their stationary values, e.g. \( \hat{c}_t = c_t / A_{t-1} \). There are a few exceptions: \( \hat{\lambda}_t^P = \lambda_t^P A_{t-1} \), \( \hat{\lambda}_t^I = \lambda_t^I A_{t-1} \), \( \hat{\Omega}_t = \Omega_t A_{t-1} \), \( \hat{\Omega}_t^{WP} = \Omega_t^{WP} A_{t-1}^{-1} \), \( \hat{\Omega}_t^{WI} = \Omega_t^{WI} A_{t-1}^{-1} \). Variables without hats are stationary.

They are given by 72 equilibrium equations below.

**Patient households**

\[ \hat{\lambda}_t^P = v_t \left( \hat{c}_t^P \right)^{-\sigma_c} \tag{1} \]

\[ v_t A_h \left( \hat{h}_t^P \right)^{-\sigma_h} = \hat{\lambda}_t^P q_t^h - \beta^P E_t \hat{\lambda}_{t+1}^P \frac{q_{t+1}^h}{a_t} \tag{2} \]

\[ E_t \beta^P \frac{\hat{\lambda}_{t+1}^P}{\lambda_t^P a_t} \frac{R_t}{\pi_t+1} = 1 \tag{3} \]

**Impatient households**

\[ \hat{\lambda}_t^I = v_t \left( \hat{c}_t^I \right)^{-\sigma_c} \tag{4} \]
\[
v_t A_h \left( \hat{h}_t^h \right)^{\sigma_h} = \hat{\lambda}_t^I q_t^h - \beta^I E_t \hat{\lambda}_{t+1}^I (1 - \zeta^h \Gamma_{t+1}) \frac{q_{t+1}^h}{a_t} - \Omega_t E_t (\Gamma_{t+1} - \mu_H G_{t+1}) \hat{c}_t^h q_t^h
\]

(5)

\[
\frac{\beta^I}{a_t} E_t \hat{\lambda}_{t+1}^I (\Gamma_{t+1})' = \Omega_t E_t (\Gamma_{t+1})' - \mu_H (G_{t+1})'
\]

(6)

\[
\hat{\Omega}_t E_t \hat{R}^M_{t+1} = \hat{\lambda}_t^I
\]

(7)

\[
\hat{w}_t n_t^I + (1 - \zeta^h \Gamma_t) q_t^h \hat{h}_t^I \frac{1}{a_{t-1}} + \hat{m}_t - \hat{c}_t^I - q_t^h \hat{h}_t^I = 0
\]

(8)

\[
(\Gamma_{t+1} - \mu_H G_{t+1}) \hat{c}_t^h q_{t+1}^h \hat{h}_t^I = E_t \hat{R}^M_{t+1} \hat{m}_t
\]

(9)

\[
\hat{m}_t^H + rer_t \hat{m}_t^F = \hat{m}_t
\]

(10)

\[
\hat{m}_t^H + rer_t \hat{m}_t^F = \alpha^{FM} \hat{m}_t
\]

(11)

**Wage setting**

\[
\left( \hat{w}_t^p \right)^{1+\sigma_n e_W} = \frac{\epsilon_W \hat{\Omega}_t^{WP}}{\epsilon_W - 1 \hat{\gamma}_t^{WP}}
\]

(13)

\[
\hat{\Omega}_t^{WP} = v_t A_n \left( \hat{w}_t^p \right)^{(1+\sigma_n)e_W} \left( n_t^p \right)^{(1+\sigma_n)} + \beta^P \omega^W \hat{\lambda}_{t+1}^P \frac{\pi}{\ell_{t+1}^P} \hat{\lambda}_t^P a_t \left( \frac{\pi}{\ell_{t+1}^P} \right)^{(1+\sigma_n)e_W} \hat{\Omega}_{t+1}^{WP} a_t^{e_W-1}
\]

(14)

\[
\hat{\gamma}_t^{WP} = \hat{\lambda}_t^P \left( \hat{w}_t^p \right)^{e_W} n_t^P + \beta^P \omega^W \hat{\lambda}_{t+1}^P \frac{\pi}{\ell_{t+1}^P} \hat{\lambda}_t^P a_t \left( \frac{\pi}{\ell_{t+1}^P} \right)^{e_W} \hat{\gamma}_{t+1}^{WP} a_t^{e_W-1}
\]

(15)

\[
\frac{1}{\hat{w}_t^p} = (1 - \omega^W) \left( \frac{\hat{w}_t^p}{\ell_{t-1}^P} \right)^{\frac{1}{\ell_{t-1}^P}} + \omega^W \left( \frac{\hat{w}_t^p}{\ell_{t-1}^P} \pi \right)^{\frac{1}{\ell_{t-1}^P}}
\]

(16)
Financially constrained firms

\[ D_{t}^{WP} = (1 - \omega^{W}) \left( \frac{\hat{w}_{t}^{P}}{\hat{w}_{t}^{P}} \right)^{(1+\sigma_{n})} n_{t}^{P} + \omega^{W} \left( \frac{\hat{w}_{t-1}^{P} \pi}{\hat{w}_{t}^{P} \pi_{t}} \right)^{(1+\sigma_{n})} D_{t-1}^{WP} \] (17)

\[ n_{t}^{P} = \left( \frac{\hat{w}_{t}^{P}}{\hat{w}_{t}^{P}} \right)^{-\epsilon^{W}} n_{t} \] (18)

\[ \left( \frac{\hat{w}_{t}^{I}}{\hat{w}_{t}^{P}} \right)^{1+\sigma_{n}\epsilon^{W}} = \frac{\epsilon^{W}}{\epsilon^{W} - 1} \hat{\Omega}_{t}^{WI} \] (19)

\[ \hat{\Omega}_{t}^{WI} = v_{t} A_{I} \left( \hat{w}_{t}^{I} \right)^{(1+\sigma_{n})} e_{W}^{n_{t}} \left( \frac{\hat{b}^{I}}{\hat{b}^{I}} \right) + \beta^{I} \omega^{W} \hat{\lambda}_{t+1}^{I} \left( \frac{\pi}{\pi_{t+1}} \right)^{(1+\sigma_{n})} \hat{\Omega}_{t+1}^{WI} e_{W}^{-1} \] (20)

\[ \tilde{\Phi}_{t}^{WI} = \hat{\lambda}_{t}^{I} \left( \hat{w}_{t}^{I} \right)^{e_{W}^{n_{t}}} + \beta^{I} \omega^{W} \hat{\lambda}_{t+1}^{I} \left( \frac{\pi}{\pi_{t+1}} \right)^{e_{W}} \tilde{\Phi}_{t+1}^{WI} e_{W}^{-1} \] (21)

\[ \left( \hat{w}_{t}^{I} \right)^{e_{W}^{n_{t}}} = (1 - \omega^{W}) \left( \hat{w}_{t}^{I} \right)^{e_{W}^{n_{t}}} + \omega^{W} \left( \frac{\hat{w}_{t-1}^{I} \pi}{\hat{w}_{t}^{I} \pi_{t}} \right) \] (22)

\[ D_{t}^{WI} = (1 - \omega^{W}) \left( \frac{\hat{w}_{t}^{I}}{\hat{w}_{t}^{P}} \right)^{(1+\sigma_{n})} n_{t}^{I} + \omega^{W} \left( \frac{\hat{w}_{t-1}^{I} \pi}{\hat{w}_{t}^{I} \pi_{t}} \right)^{(1+\sigma_{n})} D_{t-1}^{WI} \] (23)

\[ n_{t}^{I} = \left( \frac{\hat{w}_{t}^{I}}{\hat{w}_{t}^{I}} \right)^{-\epsilon^{W}} n_{t} \] (24)

\[ \hat{w}_{t} = \left( \left( \frac{\hat{w}_{t}^{P}}{\hat{w}_{t}^{P}} \right)^{1-\epsilon^{W}} + \left( \frac{\hat{w}_{t}^{I}}{\hat{w}_{t}^{I}} \right)^{1-\epsilon^{W}} \right) \] (25)

Financially constrained firms

\[ E_{t}^{P} \hat{p}_{t+1}^{I} \frac{\hat{p}_{t+1}^{P}}{\hat{a}_{t}} \left( 1 - (1 - \Phi(d_{1,t})) \kappa \right) \left( \alpha p_{t+1}^{I} z_{t+1} \left( a_{t+1} \hat{a}_{t} \right)^{1-\alpha} \hat{k}_{t+1} \hat{n}_{t+1}^{1-\alpha} + q_{t+1} \left( 1 - \delta \right) \right) = q_{t} \] (26)

\[ E_{t}^{P} \hat{p}_{t+1}^{I} \frac{\hat{p}_{t+1}^{P}}{\hat{a}_{t}} \left( 1 - (1 - \Phi(d_{1,t})) \kappa \right) \left( 1 - \alpha p_{t+1}^{I} z_{t+1} \left( a_{t+1} \hat{a}_{t} \right)^{1-\alpha} \hat{k}_{t+1} \hat{n}_{t+1}^{1-\alpha} \right) = \hat{w}_{t} \] (27)
\[ E_{t-1} \left\{ \hat{l}_t \right\} = E_{t-1} \left\{ q_t \hat{k}_t + \hat{w}_t n_t \right\} \]  

(28)

\[
d_{2,t} \equiv \frac{E_t \ln \left( \kappa \left( p_{t+1}^{R} \hat{\lambda}^R_{t+1} + q_{t+1} (1 - \delta) \hat{h}_{t+1} \right) - R_t^{L} \hat{r}_{t+1} \hat{\pi}_{t+1}^F \right) - E_t \ln \left( R_t^{L} \hat{\pi}_{t+1}^F \right)}{\sigma_{F,t}} \]  

(29)

\[ d_{1,t} \equiv d_{2,t} + \sigma_{F,t} \]  

(30)

\[ \hat{l}_t = \hat{i}_t^P + \hat{r}_{t} \hat{\pi}_t^F \]  

(31)

\[ \hat{i}_t^P = (1 - \alpha^F) \hat{l}_t \]  

(32)

Capital producers

\[ \hat{k}_t = (1 - \delta) \frac{\hat{k}_{t-1}}{a_{t-1}} + \left( 1 - \Gamma \left( \frac{\hat{i}_t}{\hat{i}_{t-1}} \right) \right) \hat{i}_t \]  

(33)

\[ \frac{p_t^L}{q_t} = \left( 1 - \gamma \left( \frac{\hat{i}_t - a_{t-1} - a}{\hat{i}_{t-1}} \right)^2 \right) u_t - \gamma \left( \frac{\hat{i}_t - a_{t-1} - a}{\hat{i}_{t-1}} \right) \hat{i}_t - a_{t-1} \]  

\[ + \gamma \beta^P E_t \lambda_{t+1}^{p} q_{t+1} + \frac{\hat{\pi}_t^F}{\hat{\pi}_t^R} \left( \frac{\hat{i}_{t+1} - a_t - a}{\hat{i}_t - a_t} \right)^2 u_{t+1} \]  

(34)

Retail firms

\[ 1 = (1 - \omega^H) \left( \hat{p}_t^H \right)^{1-\varepsilon_H} + \omega^H \left( \frac{\hat{p}_{t-1}^{H} \prod_{j=1}^{s} \pi_{a_{t+j}}^{adj}}{p_t^H \pi_t} \right)^{1-\varepsilon_H} \]  

(35)

\[ D_t^H = (1 - \omega^H) \left( \hat{p}_t^H \right)^{-\varepsilon_H} + \omega^H \left( \frac{\hat{p}_{t-1}^{H} \prod_{j=1}^{s} \pi_{a_{t+j}}^{adj}}{p_t^H \pi_t} \right)^{-\varepsilon_H} D_{t-1}^H \]  

(36)
\[ p_t^H = \frac{e_H}{(e_H - 1)} \hat{F}_{1,t}^H \]

\[ \hat{F}_{1,t}^H = p_t^H \hat{y}_t^H + E_t \omega^H \beta^P \frac{\hat{\lambda}_t^{P+1}}{\hat{\lambda}_t^P} \left( \frac{p_t^{H+1} \hat{\pi}_{t+1}}{p_t^H \left( \prod_{j=1}^{J^s} \hat{\pi}_{t+j}^{adj} \right)} \right)^{e_H} \hat{F}_{1,t+1}^H \]

\[ \hat{F}_{2,t}^H = p_t^H \hat{y}_t^H + E_t \omega^H \beta^P \frac{\hat{\lambda}_t^{P+1}}{\hat{\lambda}_t^P} \left( \frac{p_t^{H+1} \hat{\pi}_{t+1}}{p_t^H \left( \prod_{j=1}^{J^s} \hat{\pi}_{t+j}^{adj} \right)} \right)^{e_H-1} \hat{F}_{2,t+1}^H \]

**Exporters**

\[ \hat{y}_t^{H*} = \eta^* \left( \frac{p_t^H}{\text{rer}_t} \right)^{-e_*} y_t^* \]

**Composite goods producer**

\[ \hat{c}_t = \left( \frac{1}{\eta_c} \left( \hat{c}_t^H \right)^{\frac{e_c-1}{e_c}} + \frac{1}{\eta_c} \left( \hat{c}_t^F \right)^{\frac{e_c-1}{e_c}} \right)^{\frac{e_c}{e_c-1}} \]

\[ \hat{c}_t^H = \eta_c \left( \frac{p_t^H}{c_t^*} \right)^{-e_c} \hat{c}_t \]

\[ \hat{c}_t^F = \left( 1 - \eta_c \right) \left( \frac{p_t^F}{p_t^H} \right)^{-e_c} \hat{c}_t \]

\[ \hat{i}_t = \left( \frac{1}{\eta_I} \left( \hat{i}_t^H \right)^{\frac{e_I-1}{e_I}} + \frac{1}{\eta_I} \left( \hat{i}_t^F \right)^{\frac{e_I-1}{e_I}} \right)^{\frac{e_I}{e_I-1}} \]

\[ \hat{i}_t^H = \eta_I \left( \frac{p_t^H}{i_t^*} \right)^{-e_I} \hat{i}_t \]

\[ \hat{i}_t^F = \left( 1 - \eta_I \right) \left( \frac{p_t^F}{p_t^I} \right)^{-e_I} \hat{i}_t \]

**Definition of the real exchange rate**

\[ \frac{\text{rer}_t}{\text{rer}_{t-1}} = \frac{S_t \pi_t^*}{S_{t-1} \pi_t} \]

**Importers**

212
\[
1 = (1 - \omega^F) \left( \hat{p}_t^F \right)^{1-\epsilon_F} + \omega^F \left( \frac{p_{t-1}^F \left( \prod_{j=1}^{t} \pi_{t+j}^{adj} \right)}{p_t^F \pi_t} \right)^{1-\epsilon_F} \\
\]

(48)

\[
D_t^F = (1 - \omega^F) \left( \hat{p}_t^F \right)^{-\epsilon_F} + \omega^F \left( \frac{p_{t-1}^F \left( \prod_{j=1}^{t} \pi_{t+j}^{adj} \right)}{\pi_t p_t^F} \right)^{-\epsilon_F} \\
\]

(49)

\[
\hat{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \hat{F}_{1,t}^F \\
\]

(50)

\[
\hat{F}_{1,t}^F = \text{rer} \gamma_t^F + E_t \omega^F \beta^P \hat{\lambda}_{t+1}^P \left( \frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left( \prod_{j=1}^{t} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F} \hat{F}_{1,t+1}^F \\
\]

(51)

\[
\hat{F}_{2,t}^F = p_t^F \gamma_t^F + E_t \omega^F \beta^P \hat{\lambda}_{t+1}^P \left( \frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left( \prod_{j=1}^{t} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F-1} \hat{F}_{2,t+1}^F \\
\]

(52)

Banks

\[
E_t \left\{ \tilde{R}^R_{t+1} l_t \right\} \equiv E_t \left( 1 - \Phi(d_{1,t}) \right) \kappa \left( p_{t+1}^L \gamma_{t+1}^R + (1 - \delta) q_{t+1} \hat{k}_{t} \right) + \Phi(d_{2,t}) R_t^L \frac{i_D^t}{\pi_t} + \Phi(d_{1,t}) R_t^L \text{rer}_{t+1} \frac{i_F^t}{\pi_t} \right\} \\
\]

(53)

\[
(1 + v_{1,t}) E_t \beta^P \hat{\lambda}_{t+1}^P \left( \frac{1}{\lambda_t^P a_t} \right) \{ (1 - \omega) + \omega v_{2,t+1} \} \tilde{R}^L_{t+1} = \lambda^R v_{1,t} + v_{2,t} \\
\]

(54)

\[
(1 + v_{1,t}) E_t \beta^P \hat{\lambda}_{t+1}^P \left( \frac{1}{\lambda_t^P a_t} \right) \{ (1 - \omega) + \omega v_{2,t+1} \} \tilde{R}^M_{t+1} = \lambda^R v_{1,t} + v_{2,t} \\
\]

(55)

\[
(1 + v_{1,t}) E_t \beta^P \hat{\lambda}_{t+1}^P \left( \frac{1}{\lambda_t^P a_t} \right) \{ (1 - \omega) + \omega v_{2,t+1} \} \left( \frac{R_t}{\pi_t} \right) = v_{2,t} \\
\]

(56)
\[
(1 + \nu_{1,t}) E_t \beta^P \frac{\hat{x}_{t+1}^{\lambda^P}}{\lambda^P a_t} \left\{ (1 - \omega) + \omega v_{2,t+1} \right\} \left( \frac{R_t^t \xi_t \text{rer}_{t+1}}{\pi^*_t \text{rer}_t} \right) = v_{2,t}
\]

(57)

\[
\hat{n}_t^B = \omega^B \frac{\mu_t^B}{a_{t-1}} \left( \bar{R}_t \hat{\eta}_{t-1} + \bar{R}_t \hat{m}_{t-1} - \frac{R_t - 1}{\pi_t} \hat{a}_{t-1} - \frac{R_t^* \xi_{t-1}}{\pi^*_t \text{rer}_t} \hat{d}_{t-1} \right) + \lambda^B \hat{n}_t^B
\]

(58)

\[
v_{2,t} \hat{n}_t^B \geq \lambda^B \left( \hat{\mu}_t + \hat{m}_t \right)
\]

(59)

\[
\hat{n}_t^B + \hat{d}_t + \text{rer}_t \hat{d}_t^* = \hat{\mu}_t + \hat{m}_t
\]

(60)

**Monetary policy**

\[
\frac{R_t}{\bar{R}} = \left( \frac{R_t - 1}{\bar{R}} \right)^{\gamma R} \left( \frac{\gamma H^H}{\gamma H} \right)^{(1-\gamma R)\gamma Y} \left( \frac{p_t^H / p_{t-1}^H \pi_t}{\bar{\pi}} \right)^{(1-\gamma R)\gamma Y} \exp(mp_t)
\]

(61)

**Government**

\[
\hat{g}_t + \frac{R_t - 1}{\pi_t} \hat{b}_{t-1} = \hat{\mu}_t + \hat{b}_t
\]

(62)

\[
\hat{\mu}_t = \bar{\mu} + \kappa_b \left( \frac{\hat{b}_{t-1}}{a_{t-1}} - \bar{b} \right) + \tau_t
\]

(63)

**Market clearing**

\[
\hat{c}_t = \hat{c}_t^P + \hat{c}_t^l
\]

(64)

\[
h = \hat{h}_t^P + \hat{h}_t^l
\]

(65)

\[
\hat{y}_t = z_t \theta_t \left( \frac{k_{t-1}}{a_{t-1}} \right)^\alpha (a_t \nu_{t-1})^{1-\alpha}
\]

(66)
\[
\hat{y}_t^H = \hat{c}_t^H + i_t^H + \hat{g}_t
\]  
(67)

\[
\hat{y}_t^F = \hat{c}_t^F + i_t^F
\]  
(68)

\[
\hat{y}_t = D_t^H \hat{y}_t^H + \hat{y}_t^H
\]  
(69)

**Trade balance**

\[
\hat{t}b_t = p_t^H \hat{y}_t^H - rer_t D_t^F \hat{y}_t^F
\]  
(70)

**Current account**

\[
\hat{t}b_t - (R_{t-1}^r \xi_{t-1} - 1) \frac{\hat{d}_{t-1}^r}{\pi_t a_{t-1}} = \left( rer_t \hat{d}_t^r - rer_t \frac{\hat{d}_{t-1}^r}{\pi_t a_{t-1}} \right)
\]  
(71)

\[
\xi_t = \exp \left( \phi \frac{rer_t \hat{d}_t^r - rer \cdot d^r}{rer \cdot d^r} \right) \frac{\xi_t - \zeta}{\zeta}
\]  
(72)

There are 15 exogenous variables:

\[\{ z_t, a_t, \theta_t, \pi_t^*, R_t^r, \xi_t, y_t^*, mp_t, g_t, \tau_t, u_t, v_t, \sigma_{F,t}, \sigma_{M,t}, \lambda_t^B \}\]

**B.9 Tables and figures**
Table B.1: Calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_P$</td>
<td>Patient HH discount factor</td>
<td>0.99</td>
<td>Iacoviello (2005)</td>
</tr>
<tr>
<td>$\beta_I$</td>
<td>Impatient HH discount factor</td>
<td>0.95</td>
<td>Iacoviello (2005)</td>
</tr>
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<td>$\sigma_m$</td>
<td>Inverse of Frisch elasticity</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$A_n$</td>
<td>Disutility weight of labor</td>
<td>varies</td>
<td>matches $n = 0.33$</td>
</tr>
<tr>
<td>$A_h$</td>
<td>Utility weight of housing</td>
<td>0.1</td>
<td>Iacoviello (2005)</td>
</tr>
<tr>
<td>$a$</td>
<td>Non-stationary technology growth in SS</td>
<td>1.0141</td>
<td>GDP growth - population growth</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
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<td>-</td>
</tr>
<tr>
<td>$\theta^W$</td>
<td>Calvo parameter, wages</td>
<td>0.62</td>
<td>Jakab and Kónya (2016)</td>
</tr>
<tr>
<td>$\epsilon_W$</td>
<td>Labor supply elasticity among different types of labor</td>
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<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\epsilon^*$</td>
<td>E.o.S. for exports</td>
<td>1.5</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\theta^H$</td>
<td>Calvo parameter, domestic goods</td>
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<td>Jakab and Kónya (2016)</td>
</tr>
<tr>
<td>$\theta^F$</td>
<td>Calvo parameter, imported goods</td>
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<td>-</td>
</tr>
<tr>
<td>$\epsilon_H$</td>
<td>E.o.S. between domestic varieties</td>
<td>6</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\epsilon_F$</td>
<td>E.o.S. between imported varieties</td>
<td>6</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Investment adjustment cost parameter</td>
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<td>Jakab and Kónya (2016)</td>
</tr>
<tr>
<td>$\kappa_\xi$</td>
<td>Elasticity of country risk to net asset position</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>Interest rate smoothing</td>
<td>0.766</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\gamma_\pi$</td>
<td>Interest policy rule (inflation)</td>
<td>1.375</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\gamma_\psi$</td>
<td>Interest policy rule (output)</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon_c$</td>
<td>E.o.S. between domestic and imported consumption goods</td>
<td>3</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\epsilon_I$</td>
<td>E.o.S. between domestic and imported investment goods</td>
<td>3</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Share of domestic goods in consumption basket</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>$\eta_I$</td>
<td>Share of domestic goods in investment basket</td>
<td>0.6</td>
<td>-</td>
</tr>
<tr>
<td>$z$</td>
<td>Technology in SS</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation in SS</td>
<td>1.017</td>
<td>avg. in data</td>
</tr>
<tr>
<td>$p^H$</td>
<td>Relative price of $x^H$ in SS</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$S$</td>
<td>Nominal exchange rate in SS</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Housing stock</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$R$</td>
<td>Risk-free rate in SS</td>
<td>1.0304</td>
<td>matches $\pi = 1.017$</td>
</tr>
<tr>
<td>$R^*$</td>
<td>Foreign interest rate in SS</td>
<td>1.0072</td>
<td>matches $\pi^* = 1.004$</td>
</tr>
<tr>
<td>$s^g$</td>
<td>Gov. consumption/ GDP in SS</td>
<td>0.10</td>
<td>Jakab and Világi (2008)</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>Foreign inflation rate</td>
<td>1.004</td>
<td>from RER definition</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Risk premium on international bonds in SS</td>
<td>1.01</td>
<td>-</td>
</tr>
<tr>
<td>$\zeta_h$</td>
<td>LTV</td>
<td>0.8</td>
<td>calibrated</td>
</tr>
<tr>
<td>$\omega^H$</td>
<td>HH leverage in SS</td>
<td>varies</td>
<td>matches HH spread of 1 p.p.</td>
</tr>
<tr>
<td>$\mu_H$</td>
<td>Monitoring costs</td>
<td>0.18</td>
<td>endog. determined</td>
</tr>
<tr>
<td>$\alpha_{HF}$</td>
<td>FX share in HH debt</td>
<td>0.8</td>
<td>avg. in data</td>
</tr>
<tr>
<td>$\sigma_M$</td>
<td>Volatility of housing shocks in SS</td>
<td>0.11</td>
<td>Clerc et al. (2011) as guidance</td>
</tr>
<tr>
<td>$1 - \Phi(d_z)$</td>
<td>Corporate default rate in SS</td>
<td>varies</td>
<td>matches corp. spread of 0.6 p.p.</td>
</tr>
<tr>
<td>$\alpha_{FF}$</td>
<td>Share of FX loans</td>
<td>0.6</td>
<td>avg. in data</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Share of production revenue to be seized by creditors</td>
<td>0.08</td>
<td>endog. determined</td>
</tr>
<tr>
<td>$\sigma_F$</td>
<td>Volatility of firms’ profits in SS</td>
<td>0.08</td>
<td>Clerc et al. (2011)</td>
</tr>
<tr>
<td>$\lambda^H$</td>
<td>Fraction of capital that can be diverted</td>
<td>0.43</td>
<td>matches bank spread 0.3 p.p.</td>
</tr>
<tr>
<td>$\omega^B$</td>
<td>Probability of bankers’ survival</td>
<td>0.97</td>
<td>Gertler and Karadi (2011)</td>
</tr>
<tr>
<td>$\lambda^B$</td>
<td>Proportional transfer to the entering bankers</td>
<td>0.002</td>
<td>Gertler and Karadi (2011)</td>
</tr>
</tbody>
</table>

Note: a sign ‘-’ in the ‘Source’ column means that we chose the value. Motivations for most of such cases are provided in the parameter section.
Table B.2: Calibrated parameters for shock processes and measurement errors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Computed from data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shock processes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Gov. spending shock autoregr. coeff.</td>
<td>0.68</td>
<td>No; Jakab and Kónya (2016)</td>
</tr>
<tr>
<td>$\rho_y^*$</td>
<td>World demand shock autoregr. coeff.</td>
<td>0.32</td>
<td>yes</td>
</tr>
<tr>
<td>$\rho_{\pi}^*$</td>
<td>Foreign inflation shock autoregr. coeff.</td>
<td>0.77</td>
<td>yes</td>
</tr>
<tr>
<td>$\rho_R^*$</td>
<td>Foreign inflation shock autoregr. coeff.</td>
<td>0.68</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_y^*$</td>
<td>World demand shock s.d.</td>
<td>1.44</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{\pi}^*$</td>
<td>Foreign inflation shock s.d.</td>
<td>0.17</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_R^*$</td>
<td>Foreign inflation shock s.d.</td>
<td>0.08</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>Measurement errors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{F,t}$</td>
<td>ME of GDP growth</td>
<td>0.90</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>ME of consumption growth</td>
<td>1.19</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>ME of investment growth</td>
<td>2.35</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{rer}$</td>
<td>ME of real exchange rate growth</td>
<td>2.20</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>ME of inflation</td>
<td>0.18</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{tbgdp}$</td>
<td>ME of TB/GDP</td>
<td>0.02</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{ys}$</td>
<td>ME of foreign output growth</td>
<td>0.62583</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{\pi}^*$</td>
<td>ME of foreign inflation</td>
<td>7.8530e-04</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{spr,F}$</td>
<td>ME of investment growth</td>
<td>0.001</td>
<td>yes</td>
</tr>
<tr>
<td>$\sigma_{spr,H}$</td>
<td>ME of investment growth</td>
<td>4.6232e-04</td>
<td>yes</td>
</tr>
</tbody>
</table>

Note: Autoregressive coefficients and standard deviations of shock processes are computed by HP-filtering variables first. Measurement errors are computed as 10% of variance of observed variables. Observed variables are expressed in deviations from their means. Means are computed over the period 2000:Q1-2016:Q3.
Table B.3: Data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition in the used data</th>
<th>Data source, data series number (if available)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP growth*</td>
<td>Total Gross Domestic Product, National Currency, Quarterly, Seasonally Adjusted</td>
<td>FRED, NAEXKP01HUQ189S</td>
</tr>
<tr>
<td>Consumption growth*</td>
<td>Private Final Consumption Expenditure, National Currency, Quarterly, Seasonally Adjusted</td>
<td>FRED, NAEXKP02HUQ189S</td>
</tr>
<tr>
<td>Investment growth*</td>
<td>Gross Fixed Capital Formation, National Currency, Quarterly, Seasonally Adjusted</td>
<td>FRED, NAEXKP04HUQ189S</td>
</tr>
<tr>
<td>CPI inflation</td>
<td>Consumer Price Index: OECD Groups: All Items Non-Food and Non-Energy, Growth Rate</td>
<td>FRED, CPGRLE01HUQ659N</td>
</tr>
<tr>
<td></td>
<td>Same Period Previous Year, Quarterly, Not Seasonally Adjusted</td>
<td></td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>3-Month or 90-day Rates and Yields: Treasury Securities for Hungary, Percent, Quarterly, Not Seasonally Adjusted</td>
<td>FRED, IR3TIB01EZQ156N</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>Real Broad Effective Exchange Rate, Index 2010=100, Monthly, Not Seasonally Adjusted</td>
<td>FRED, RBHUBIS</td>
</tr>
<tr>
<td>Trade balance to GDP</td>
<td>Exports of Goods and Services for Hungary</td>
<td>NAEXKP06HUQ652S, NAEXKP07HUQ652S</td>
</tr>
<tr>
<td></td>
<td>- Imports of Goods and Services for Hungary</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chained 2000 National Currency Units, Quarterly, Seasonally Adjusted</td>
<td></td>
</tr>
<tr>
<td>Foreign interest rate</td>
<td>Interest Rates, Government Securities, Government Bonds for Euro Area, Percent per Annum, Quarterly, Not Seasonally Adjusted</td>
<td>FRED, INTGSBEZQ193N</td>
</tr>
<tr>
<td>Foreign CPI inflation</td>
<td>HICP: All Items for Euro area</td>
<td>FRED, CP0000EZCCM086NEST</td>
</tr>
<tr>
<td></td>
<td>All Items for Euro area</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Index 2015=100, Monthly, Not Seasonably Adjusted</td>
<td></td>
</tr>
<tr>
<td>Foreign GDP growth</td>
<td>Real Gross Domestic Product for Euro area, Millions of Chained 2010 Euros, Quarterly, Seasonably Adjusted</td>
<td>FRED, CLVMEURSCAB1GQEA19</td>
</tr>
<tr>
<td>Corporate loan spread</td>
<td>Diff. between monthly average agreed interest rate of HUF loans and HUF deposits to households up to 1 year to non-financial corporations</td>
<td>MNB</td>
</tr>
<tr>
<td>Household loan spread</td>
<td>Diff. between monthly average agreed interest rate of HUF loans for house purchase and HUF deposits up to 1 year to households</td>
<td>MNB</td>
</tr>
<tr>
<td>Bank spread</td>
<td>Diff. between monthly average interest rates of unsecured HUF interbank lending transactions and HUF deposits up to 1 year to households</td>
<td>MNB</td>
</tr>
</tbody>
</table>

* - Variables are expressed per capita by dividing with 'Working Age Population: Aged 15 and Over: All Persons for Hungary, Persons, Quarterly, Seasonally Adjusted', FRED, series no. HOHWMN03HUQ065N.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shock description</th>
<th>Distr.</th>
<th>Prior mean</th>
<th>Prior s.d.</th>
<th>Posterior mean</th>
<th>Posterior mean</th>
<th>Posterior mean</th>
<th>Posterior mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\alpha}$</td>
<td>Non-stationary prod.</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.0768</td>
<td>0.2350</td>
<td>0.4297</td>
<td>0.9150</td>
</tr>
<tr>
<td>$\rho_{\zeta}$</td>
<td>Stationary prod.</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2053</td>
<td>0.2593</td>
<td>0.4550</td>
<td>0.9654</td>
</tr>
<tr>
<td>$\rho_{u}$</td>
<td>Capital utilization</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9739</td>
<td>0.9793</td>
<td>0.9580</td>
<td>0.9941</td>
</tr>
<tr>
<td>$\rho_{\xi}$</td>
<td>Risk premium</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9977</td>
<td>0.9920</td>
<td>0.6744</td>
<td>0.9674</td>
</tr>
<tr>
<td>$\rho_{\sigma M}$</td>
<td>HH vol.</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9439</td>
<td>0.9483</td>
<td>0.9616</td>
<td>0.9941</td>
</tr>
<tr>
<td>$\rho_{\sigma F}$</td>
<td>Corp. revenue vol.</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9777</td>
<td>0.9594</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{AB}$</td>
<td>Bank asset diversion</td>
<td>beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.0288</td>
<td>0.0233</td>
<td>0.0208</td>
<td>0.0074</td>
</tr>
<tr>
<td>$\sigma_{\mu}$</td>
<td>Non-stationary prod.</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.0102</td>
<td>0.0096</td>
<td>0.0153</td>
<td>0.0218</td>
</tr>
<tr>
<td>$\sigma_{\zeta}$</td>
<td>Stationary prod.</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.0995</td>
<td>0.0911</td>
<td>0.1107</td>
<td>0.4629</td>
</tr>
<tr>
<td>$\sigma_{u}$</td>
<td>Capital utilization</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.0031</td>
<td>0.0029</td>
<td>0.0032</td>
<td>0.0037</td>
</tr>
<tr>
<td>$\sigma_{\xi}$</td>
<td>Risk premium</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.0031</td>
<td>0.0025</td>
<td>0.0023</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\sigma_{\sigma M}$</td>
<td>HH volatility</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.0851</td>
<td>0.0098</td>
<td>0.2188</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\sigma F}$</td>
<td>Corp. revenue vol.</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.1827</td>
<td>0.1632</td>
<td>0.0803</td>
<td>0.1386</td>
</tr>
<tr>
<td>$\sigma_{\lambda B}$</td>
<td>Bank asset diversion</td>
<td>beta</td>
<td>0.01</td>
<td>$\infty$</td>
<td>0.2533</td>
<td>0.0447</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Marginal likelihood was computed using the Laplace approximation at the posterior mode. All estimations use macro data. Macro data includes: real GDP growth, consumption growth, investment growth, CPI inflation, nominal gross interest rate, real exchange rate, foreign interest rate, foreign inflation, foreign real GDP growth.

**Abbreviations:** 'HH' stands for household and 'Fin.' for financial. 'S' as in the 'HH type' column stands for savers. S&B means savers and borrowers. 'Corp.' means corporate. DO means debt overhang, BGG means monitoring frictions as implemented in Bernanke et al. (1999). GK means the endogenous bank leverage constraint as implemented in Gertler and Karadi (2011).
Figure B.1: Country’s premium shock and currency mismatch for households.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mixed denomination mortgages mean that 80% of mortgages is dominated in foreign currency. Corporate loans in both cases are issued in domestic currency only.
Figure B.2: Country’s premium shock and currency mismatch for corporates.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mortgages in both cases are issued in domestic currency only.
Figure B.3: Country’s premium shock and currency mismatch for households and corporates.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mixed denomination mortgages and loans mean that 80% of mortgages and 60% of loans is dominated in foreign currency. In the domestic currency case, 5% corporate loans and zero of mortgages are denominated in foreign currency.
Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mixed denomination mortgages mean that 80% of mortgages is dominated in foreign currency. Corporate loans in both cases are issued in domestic currency only.
Figure B.5: Country’s premium shock and currency mismatch for corporates with leveraged banks.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mortgages in both cases are issued in domestic currency only.
Figure B.6: Country’s premium shock and currency mismatch for households and corporates with leveraged banks.

Note: The figure plots IRFs to an unexpected increase in the country’s premium by 3 p.p. Mixed denomination mortgages and loans mean that 80% of mortgages and 60% of loans is dominated in foreign currency. In the domestic currency case, 10% corporate loans and zero of mortgages are denominated in foreign currency.
Appendix C

C.1 Parameters

Model parametrization is presented in Table C.1. I take parameters for the utility function from Mitman (2016). Discount factors are common in the business cycles literature which distinguishes between savers and borrowers, e.g. the discussion on this in Iacoviello (2005). Endowment values, stock values are ad-hoc. The rest of the parameters are standard except for the parametrization of shocks which I describe below.

The housing value shock $\zeta$ can take two values: $\zeta \in \{\zeta^L, \zeta^H\}$. The probability of a bad state $\zeta^L$ occurring is denoted by $p_{\zeta}^L$ and is commonly known. The number of the states is arbitrary but comes without a loss of generality. The lower value of the housing value shock is chosen low enough such that household default would sometimes occur in bad states even in the absence of earnings shocks. Corbae and Quintin (2015) set it higher (to 0.7) to model the drop in house prices in the US, however, given my model setup, this value is too high to make household default occur in the absence of earnings shocks.

The earnings shock takes five states with commonly known probabilities. One can think of it as an idiosyncratic shock that affects a continuum of households in period 2 when all households are identical in period 1. Assuming a continuum of households, the probability of a particular value of earnings shock occurring would correspond to a share of households that get hit with the earnings shock of this size.

My measure of the variance of earnings value shock comes from Singh and Stoltenberg (2017). Singh and Stoltenberg (2017) use Consumer Expenditure Interview Survey (CEX) data 1999-2003 to compute within group inequality. They follow Krueger and Perri (2006) and Blundell et al. (2008), and regress the logs of household consumption and income on a cubic function of age and a set of dummies that include region, marital status, race, education, experience, occupation and sex. The unobserved risk amounts to idiosyncratic
income risk used to compute the income risk. Further they calibrate an ergodic earnings distribution for an incomplete market model by specifying permanent and transitory shocks to yield a transition matrix. I follow this methodology closely as I choose a permanent income shock with an autoregressive coefficient of 0.9899 to compute the earnings states and the corresponding probabilities. The number of the states is ad hoc.

I assume that earnings shocks and housing value shocks are independent.

I use two grids, for savings and housing. Both have the size of 60.

Table C.1: Baseline parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saver’s discount factor</td>
<td>β_S</td>
<td>0.99</td>
</tr>
<tr>
<td>Borrower’s discount factor</td>
<td>β_B</td>
<td>0.98</td>
</tr>
<tr>
<td>CRRA parameter</td>
<td>σ</td>
<td>2</td>
</tr>
<tr>
<td>Housing parameter in the utility f-n</td>
<td>σ_h</td>
<td>0.14</td>
</tr>
<tr>
<td>Endowment for the saver</td>
<td>y^S</td>
<td>1</td>
</tr>
<tr>
<td>Endowment for the borrower</td>
<td>y^B</td>
<td>1</td>
</tr>
<tr>
<td>Housing stock</td>
<td>h</td>
<td>1</td>
</tr>
<tr>
<td>Capital depreciation</td>
<td>δ</td>
<td>1</td>
</tr>
<tr>
<td>Capital share of output</td>
<td>α</td>
<td>0.30</td>
</tr>
<tr>
<td>Number of housing value states</td>
<td>s^h</td>
<td>2</td>
</tr>
<tr>
<td>Housing value shock: probabilities</td>
<td>(p_L^ζ; 1 − p_L^ζ)</td>
<td>(0.05;0.95)</td>
</tr>
<tr>
<td>Housing value shock: states</td>
<td>(ζ_L^L; ζ_H^H)</td>
<td>(0.5;1.0263)</td>
</tr>
<tr>
<td>Housing value shock: expected value</td>
<td>Eζ</td>
<td>1</td>
</tr>
<tr>
<td>Housing value shock: variance</td>
<td>var(ζ)</td>
<td>0.002</td>
</tr>
<tr>
<td>Number of earnings states</td>
<td>s^z</td>
<td>5</td>
</tr>
<tr>
<td>Earnings value shock: probabilities</td>
<td></td>
<td>(0.2039; 0.2000; 0.1921; 0.2000; 0.2039)</td>
</tr>
<tr>
<td>Earnings value shock: states</td>
<td></td>
<td>(0.3591; 0.5488; 0.8387; 1.2818; 1.9590)</td>
</tr>
<tr>
<td>Earnings shock: expected value</td>
<td>Ez</td>
<td>1</td>
</tr>
<tr>
<td>Earnings shock: variance</td>
<td>var(z)</td>
<td>0.3655</td>
</tr>
<tr>
<td>LTV ratio</td>
<td>ρ</td>
<td>1</td>
</tr>
<tr>
<td>Rent</td>
<td>h^r</td>
<td>0.1</td>
</tr>
<tr>
<td>Minimum consumption level</td>
<td>c_{min}</td>
<td>0.1</td>
</tr>
</tbody>
</table>
C.2 Solution algorithm

In the description of the algorithm we introduce uncertainty in the form of the housing quality shock $\zeta$ and earnings shock $z$ and allow the borrowing household to default. We employ the backward induction method.

The housing value shock occurs in period 2 and has two ($s^h = 2$) realization values $\{\zeta^L, \zeta^H\}$. The low value occurs with a probability $p^L_\zeta$. The productivity shock occurs in period 2 as well and has five ($s^z = 5$) realization values with commonly known probabilities. Parameters that concern the solution algorithm are presented in Table C.1.

1. Generate two grids: one for savings and one for housing. The housing grid size is $g^h_d$ and the savings grid size is $g^d_d$.

2. Take $y^B, y^S$ and $h$ as given.

3. Make a guess for a risk-free interest rate $r_1$, housing price $q$, a mortgage interest rate $r_1^m$ and a corporate interest rate $r_1^l$. Since firms do not default, $r_1^l$ always will be set equal to $r_1$.

4. $w_1$ and $k_1 / n_2$ follow.

5. Compute wages $w_1$ and capital stock $k_1$ by using the firm’s first order conditions (equations (4.3.2)-(4.3.2)).

6. Express the saver’s value function in period 2 $V^S_2$ as a function of instantaneous utility in period 2, given the realizations of shocks ($s^h \cdot s^z$ possible states) and all possible combinations of savings values $d_1$ and housing values $h^S_1$ from the respective grids. This yields $V^S_2 (d_1, h^S_1; \zeta, z)$.

7. Express the borrower’s value functions in period 2 $V^{BP}_2$ and $V^{BD}_2$ as a function of instantaneous utility in period 2, given the realizations of shocks ($s^h \cdot s^z$ possible states) and all possible housing values $h^B_1$ from the grid for housing. To impute mortgages make use of $m_1 = \rho \zeta h_1 / (1 + r_1^m)$. This yields $V^{BP}_2 (h_1; \zeta, z)$ and $V^{BD}_2 (h_1; \zeta, z)$.

8. Determine the optimal default choice for the borrowing household by computing $V^B_2 (h^B_1; \zeta, z) = \max \{V^{BP}_2 (h^B_1; \zeta, z), V^{BD}_2 (h^B_1; \zeta, z)\}$. This yields decision rules for every possible $h^B_1$ the agent brings to period 2, given the materialization of shocks in period 2.
9. Express $V^S_1$ as a function of instantaneous utility in period 1 for every combination of savings $s_1$ and housing $h^S_1$ and the respective value in period 2. To find the corresponding optimal value in period 2, the realizations of $V^S_2(d_1, \zeta, z)$ have to be not only discounted but also weighted with respective probabilities of each shock occurring. Do an analogical exercise for $V^B_1$.

10. Compute the value of the saver in period 1 ($V^S_1$) for every possible combination of $d_1$ and $h^S_1$. Compute the value of the borrower in period 1 ($V^B_1$) for every possible $h^B_1$.

11. Find the maximum value of $V^S_1(d_1, h^S_1)$ and $V^B_1(h^B_1)$. This yields optimal savings $d_1$ and optimal housing choices $h^S_1$ and $h^B_1$. Given these variables, mortgages can be inputed and borrower’s default decisions can be determined for all possible combinations of shocks in period 2.

12. Compute expected returns on the mortgage $\tilde{r}_2^m$ by weighting returns under different combinations of shock with the respective probabilities. variance of returns can also be computed accordingly.

13. Check if $\tilde{r}_2^m - r_1$ is different from zero.

14. If yes, change $r_1^m$ and repeat steps 7-13 until $\tilde{r}_2^m \approx r_1$ under the set tolerance level ($fminsearch$).

15. Check if $h^S_1 + h^B_1 - h$ is different from zero.

16. If yes, change $q$ and repeat steps 6-15 until the housing market clears: $h^S_1 + h^B_1 \approx h$ under the set tolerance level ($fminsearch$).

17. Check if $k_1 + m_1 - d_1$ is different from zero.

18. If yes, change $r_1$ and repeat steps 3-17 until the deviation from the savings market clearing condition disappears ($fminsearch$).
C.3 Model extensions

C.4 Banking sector with a mean-variance utility function for the bank manager

A risk neutral banking sector in the baseline model is extended to account for the risk aversion of a bank manager. For simplicity, it is done by assuming the mean-variance utility for the manager. In this extension a representative competitive bank is operated by the risk averse manager and owned by the saving household. I assume that in the beginning of every period the realized profits/losses from the previous period are transferred to the saving household which allows me to abstract from the dynamic properties of the bank problem and focus on the credit allocation in the current period only. The uncertainty of returns associated with different assets on their balance sheet plays a role in this problem, because the assumed risk-aversion affects the bank manager decision how much to lend to the borrowing household and how much to the firm, i.e. the portfolio choice. Thus interest rates on different types of credit are set not only to make the bank break even in expectation, but also to account for the relative risk the bank manager is taking given the return on the mortgage and the return on the corporate loan. Thus the bank would ask a higher interest rate on mortgages to compensate for otherwise lower expected returns compared to the net deposit rate but also to compensate for the uncertainty.

In period 1 the bank manager expects a net return $\tilde{r}^m_2$ on the mortgage and a return $\tilde{r}^l_2$ on the loan, where the indicator $I\left(repaid\right)$ denotes the borrower’s (either household’s or firm’s) decision to repay debt. The expected return on the mortgage reflects that in case of default the bank will seize the non-exempt earnings together with housing:

$$E_1 \left(1 + \tilde{r}^m_2\right) m_1 = E_1 \left\{I\left(repaid\right) \left(1 + r^m_1\right) m_1\right\} + E_1 \left(\left(1 - I\left(repaid\right)\right) \left(\zeta h^B + \max\left\{z w^B - \kappa, 0\right\}\right)\right)$$

The expected return on the corporate loan reflects that the firm always repays:

$$E_1 \left(1 + \tilde{r}^l_2\right) l_1 = \left(1 + r^l_1\right) l_1$$
To determine the optimal asset allocation, I assume that the bank manager discounts expected returns with the scaled variance of the returns. The associated risk is by assumption scaled by a parameter $A$ which reflects the degree of bank manager’s risk aversion and can be varied to capture different levels of risk aversion. I assume a utility function for the bank manager that takes this into account and use it to solve the bank profit maximization problem:

$$\max_{m_1,l_1,d_1} E_1 \tilde{r}_2^m m_1 + E_1 \tilde{r}_2^l l_1 - r_1 d_1 - \frac{A}{2} \text{var} \left( E_1 \tilde{r}_2^m m_1 + E_1 \tilde{r}_2^l l_1 - r_1 d_1 \right)$$

s.t.

$$m_1 + l_1 = d_1$$

Note that the stochastic properties of the housing shock and the productivity shock (more precisely, independence of the two shocks) imply that $\text{cov}(\tilde{r}_2^l, \tilde{r}_2^m) = 0$. Given this I can simplify the problem:

$$\max_{m_1,l_1} E_1 \tilde{r}_2^m m_1 + E_1 \tilde{r}_2^l l_1 - r_1 (m_1 + l_1) - \frac{A}{2} \left( \text{var} \left( \tilde{r}_2^m \right) m_1^2 - \text{var} \left( \tilde{r}_2^l \right) l_1^2 \right)$$

It follows that in equilibrium returns on credit should account for both the bank funding costs $r_1$ and the associated risk:

$$E_1 \tilde{r}_2^m = r_1 + A \cdot \text{var} \left( \tilde{r}_2^m \right) m_1 \quad \text{(C.1)}$$

$$E_1 \tilde{r}_2^l = r_1 + A \cdot \text{var} \left( \tilde{r}_2^l \right) l_1 \quad \text{(C.2)}$$

The derived equilibrium conditions show that, the expected variance of returns increases, credit conditions would tighten for that particular type of credit proportionally to the risk aversion parameter $A$. When $A=0$, the bank profit maximization problem features that of a risk-neutral perfectly competitive banking sector.
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Samenvatting (Dutch Summary)

Dit proefschrift richt zich op de interactie tussen de macro-economie, buitensporige private schuld en de onderliggende faillissemenswetgeving. Ten eerste, onderzoekt dit proefschrift hoe opkomende landen in Europa omgingen met schulden in vreemde valuta, toen lokale valuta’s sterl om waarde verminderden tijdens de kredietcrisis ten opzichte van de euro en de Zwitserse frank. De herverdeling van waardevermindering-ingsverliezen van kredietnemers naar banken kan verschillende macro-economische gevolgen hebben. Deze gevolgen zijn niet alleen afhankelijk van de omvang van de schuld, maar ook van het type kredietnemer. De analyse concentreert zich dus op de verschillende beleidsmaatregelen die na de onverwachte waarde-vermindering van de valuta zijn genomen: (i) de verliezen van bedrijven door een valutamismatch naar banken verschuiven (ii) de verliezen van consumenten door een valutamismatch naar banken verschuiven. Het vervolg van dit proefschrift onderzoekt de effecten van de wetgeving ter bescherming van kredietnemers, met name regreswetten, op de macro-economie en welvaart, ongeacht de waarde van de schuld.

In hoofdstuk 2 analyseer ik de beleidsmaatregelen die de verliezen van bedrijven, die zijn veroorzaakt door valutamismatches, doorschuift naar banken. Het belangrijkste nieuwe aspect in het model is de expliciete modellering van de schuldenlast van bedrijven in lijn is met de financiële literatuur (Merton (1974)). Ik pas het model toe op de Hongaarse economie, die begin 2009 met een plotselinge waardevermindering werd geconfronteerd. Ik laat zien dat het betere macro-economische resultaten oplevert wanneer lenende bedrijven gevrijwaard zijn van wisselkoersrisico’s en verliezen niet aan banken hoeven te betalen. De reden hiervoor is dat het lenen in een vreemde valuta het bankwisselkoersrisico vermindert maar het kredietrisico verhoogt, waardoor banken eigen vermogen verliezen. Daarom kunnen banken verliezen niet volledig vermijden, maar de omvang van deze verliezen hangt af van de mate van financiële fricties voor bedrijven. De schuldenlast van bedrijven versterkt de totale invloed van geagregeerde schokken aanzienlijk. Het verminderen van verliezen voor bedrijven met te hoge schulden leidt tot betere macro-economische resultaten ondanks het feit dat banken verlies maken op een open valutapositie. De resultaten wijzen erop dat het beleid waarbij de verliezen door een valutamismatch op lenende bedrijven werd afgeschoven een mogelijke reden is voor een traag herstel van de Hongaarse economie zes jaar na de crisis. Dit heeft het tegenovergestelde resultaat opgeleverd. Niet alleen omdat de stimulansen van bedrijven om neerwaarts te beleggen werden verstoord, maar ook omdat

1 Dit is gebaseerd op gezamenlijke werk met prof. Sweder van Wijnbergen.
banken te maken kregen met een hoger kredietrisico, waardoor het beleid er niet in slaagde om banken te beschermen.

In de analyse laat ik ook zien dat de overheid ervoor kan kiezen om banken opnieuw van kapitaal te voorzien om verliezen in de banksector door valutamismatches te beperken. Indien het beleid was gevoerd om banken in reactie hierop opnieuw van kapitaal te voorzien, waren de totale verliezen lager geweest en hadden de effecten van de verliezen door valutamismatches op de kredietvoorziening kunnen worden verminderd.

In hoofdstuk 3\(^2\) behandel ik opnieuw de macro-economische gevolgen van mislukte carry trades, maar nu neem ik ook de gevolgen van verliezen door valutamismatches binnen huishoudens mee in het onderzoek. Verliezen door valutamismatches binnen huishoudens waren relevant in verschillende opkomende Europese landen, vooral in Hongarije en Polen, waar ongeveer de helft van de hypotheeklen leningen in Zwitserse franken waren uitgegeven. Hoewel ik de financiële fricties kies gebaseerd op de ervaringen van Hongarije tijdens de recessie, pas ik het model ook toe op Hongaarse data om de relevantie van deze gekozen fricties te evalueren. De op Bernanke et al. (1999) gebaseerde dynamiek van het vermogen van de kredietnemer en de daaraan gerelateerde financieringskosten kunnen zowel de geaggregeerde fluctuaties in de Hongaarse economie als ook de beperking in zakelijke kredietverstrekking niet verklaren. Dit laatste zorgde ervoor dat bedrijven niet werden gestimuleerd om te investeren en personeel in dienst te nemen. Bovendien presteert het model aanzienlijk beter als de schulden van huishoudens worden toegevoegd aan de schulden van bedrijven. Dit suggereert dat de schuldenlast van huishoudens ook een belangrijke rol heeft gespeeld bij de verklaren van de totale fluctuaties. Met behulp van het gekalibreerde/geschatte model onderzoek ik het probleem met betrekking tot de toewijzing van verliezen. Als lenende bedrijven valutarisico’s lopen, resulteert dit in slechtere macro-economische resultaten dan wanneer verliezen door valutamismatches terug naar de banken worden verschoven. Dit resultaat bevestigt de bevindingen van hoofdstuk 2. Hypotheken in vreemde valuta voor huishoudens hebben echter minder ernstige gevolgen dan valutamismatches in de banksector. Dit komt omdat de schuldenlast van huishoudens de kredietvoorziening niet rechtstreeks beïnvloedt. Ook heeft consumptieverlies niet rechtstreeks invloed op de binnenlandse producenten maar hangt dit af van de importstructuur. De resultaten suggereren dat het verschuiven van bedrijfsverliezen naar banken (en indien nodig het herkapitaliseren van banken) de gevolgen van de recessie meer zou hebben beperkt dan het

\(^2\) Dit is gebaseerd op gezamenlijke werk met prof. Sweder van Wijnbergen.

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verschuiven van de verliezen van huishoudens naar banken. Paradoxaal genoeg deed de Hongaarse regering het tegenovergestelde.

In hoofdstuk 4 onderzoek ik de argumenten die volgen uit het algemene evenwichtsmodel voor het verhogen van de bescherming voor kredietnemers. Ik concentreer me op een bepaald type bescherming, namelijk regreswetten, die op woninghypotheken zijn gericht. Een hoger beschermingsniveau betekent in dit geval een mindere regreswet. Dan kan een kredietverstrekker in geval van wanbetaling of een negatief eigen vermogen een kleiner deel van het arbeidsinkomen van de kredietnemer opeisen. Vanwege de algemene evenwichtseffecten, zorgt een hoger niveau van bescherming voor de kredietnemer in de meeste gevallen voor meer vermogen. Echter de omvang van het effect op het vermogen hangt af van het startniveau van de bescherming en het risiconiveau binnen de economie. Hieruit volgt het nieuwe inzicht dat, in gevallen waarbij het niveau van faillissementsbescherming oorspronkelijk laag was, zowel het hypothecaire krediet als het bedrijfskrediet stijgt als reactie op een betere bescherming van de kredietnemer. Hierdoor wordt er een groter vermogen gecreëerd en groeit de welvaart. Het hypothecaire krediet stijgt sterk omdat hypotheken vanuit het perspectief van de kredietnemer minder risicovol worden. De toename van de vraag naar hypotheken zorgt, vanwege de huizenprijzen, voor sterke algemene evenwichtseffecten. Namelijk, hogere huizenprijzen stimuleren het inkomen en spaartegoeden van spaarders, zodat banken aan zowel huishoudens als bedrijven meer krediet kunnen verstrekken. Het algemene evenwichtseffect vermindert dus de verdringing van kapitaal en een hoger kapitaal verhoogt het evenwichtsinkomen voor zowel spaarders als kredietnemers. Het effect op het vermogen hangt ook af van het risiconiveau binnen de economie. Hoe hoger het niveau van het totale risico, hoe hoger het positieve effect op vermogens door op alle niveaus van inkomstenvrijstellingen minder strenge maatregelen te nemen.

De gerelateerde algemene evenwichtseffecten dragen dankzij een betere bescherming van de kredietnemer aanzienlijk bij aan de totale welvaarts groei. Het hoogste welvaartsniveau wordt bereikt wanneer voor mensen met een hypotheek meer dan de helft van hun inkomen is vrijgesteld in het geval van faillissement.

De resultaten ondersteunen de optie voor het verbeteren van de bescherming van kredietnemers omdat minder strenge regreswetten er niet alleen voor zorgen dat er meer geconsumeerd wordt, maar een betere bescherming van kredietnemers leidt ook tot positieve algemene evenwichtseffecten. Deze bevinding is ook belangrijk voor opkomende landen in Europa; hoewel de regio de afgelopen twintig jaar een snelle financiële ontwikkeling heeft doorgemaakt, bleef de ontwikkeling van relevante instellingen daarbij achter.
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