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Pension Fund Restoration Policy In General Equilibrium*

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Abstract

When the financial positions of pension funds worsen, regulations prescribe that pension funds reduce the gap between their assets (invested contributions) and their liabilities (accumulated pension promises). This paper quantifies the business cycle effects and distributional implications of various types of restoration policies. We extend a canonical New-Keynesian model with a tractable demographic structure and, as a novelty, a flexible pension fund framework. Fund participants accumulate real or nominal benefits and funding adequacy is restored by revaluing previously accumulated pension wealth (Defined Contribution) or changing the pension fund contribution rate on labour income (Defined Benefit). Generally, economies with Defined Contribution pension funds respond similarly to adverse capital quality shocks as economies without pension funds. Defined Benefit pension funds, however, distort labour supply decisions and exacerbate economic fluctuations. Retirees prefer Defined Benefit over Defined Contribution funds in case they face deficits, while the current and future working population prefers the opposite.

JEL classification: J32, E32, D91, E21

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1 Introduction

The financial positions of pension funds worsened worldwide during the financial crisis of 2008 and the ensuing sovereign debt crisis of 2009. Not only did these crises depress asset values, subsequently low interest rates inflated the discounted value of pension fund liabilities. Pension funds were left with a funding deficit since the present discounted value of accumulated pension promises of fund participants far exceed the value of invested contributions. Federal Reserve Flow of Funds data indicate that U.S. retirement fund assets were virtually cut in half between 2007 and 2009 as a result of the 2008 financial crisis (Treasury, 2012) and estimations by Novy-Marx and Rauh (2011) imply that the funding gap of U.S. state-sponsored pension plans in 2008 was as large as 3.23 trillion dollars. The experience in other countries has been similar. Laboul (2010) highlights that the estimated pension fund liabilities of 2100 exchange-listed companies from OECD countries were on average roughly 25% larger than their assets in 2008 and 2009.

If pension funds are to avoid exhausting their assets, funding deficits need to be covered through the implementation of suitable restoration policies. Regulations generally stipulate that pension funds should achieve funding adequacy in order to avoid shifting the costs to future generations. However, there are various ways in which this can be done. On the one hand, Defined Contribution systems write down the value of pension promises to fund participants in order to bring the liabilities of pension funds closer to assets. On the other hand, Defined Benefit systems increase the required contributions paid by current and future workers to bring the assets of pension funds closer to the liabilities. The 2013 Pensions at a Glance report of the OECD shows that there is little consensus amongst pension funds and regulators with regards to the preferable way of restoring the financial adequacy of pension funds: between 2009 and 2013 all OECD countries have reformed their pension systems, but the measures taken differ widely. This heterogeneity undoubtedly relates to the fact that different types of restoration policies have different distributional consequences and different implications for macroeconomic performance, which is especially relevant when the economy is in a state of crisis. Unfortunately, much of the pension economics literature has studied pension funds only from a long-term perspective (see for instance Gollier (2008) and Beetsma and Bovenberg (2009)) which inherently abstracts from effects materialising at business cycle frequencies. With the ongoing process of population ageing (which has motivated many countries to replace Pay-As-You-Go pension systems with funded systems) and the recently experienced sensitivity of pension funds to financial crises, insights about sound pension fund policy at a business cycle frequency are essential.

This paper aims to fill this gap in the literature and thus aims to provide an assessment of the business cycle effects and distributional consequences of pension fund restoration policy. To do so, we extend a canonical New-Keynesian, closed economy, dynamic general equilibrium model with a tractable demographic structure and a flexible pension fund framework. We build on the overlapping generations framework of Gertler (1999), who introduces lifecycle behaviour in a business cycle model. The production sectors of our model are inspired by Kara and von Thadden (2016) and incorporate investment adjustment costs, imperfect competition in the retail sector and nominal Calvo (1983)-pricing rigidities. As a novelty, we extend the pension fund framework of Romp (2013) and incorporate it into our model. This framework embeds various types of pension funds observed in reality, depending on the specific parametrisation.

The economy of our model is populated by two distinct groups of agents: workers and retirees. Workers face a constant probability of becoming retired and retirees face a constant probability of passing away. As in Gertler (1999), we invoke a special case of RINCE (Risk Neutral Constant Elasticity) preferences that restricts individuals to be risk neutral with respect to income risk, but that allows them to have any arbitrary intertemporal elasticity of substitution (Farmer, 1990). This class of preferences yields that all individuals that are in the same lifecycle stage consume an identical fraction of their total lifetime wealth, irrespective of their age or the amount of wealth they possess. This facilitates aggregation despite the heterogeneity of agents at the micro-level and allows us to derive closed-form expressions for aggregate variables.[1]

When supplying labour, individuals pay a mandatory contribution to the pension fund and in return accumulate pension wealth in the form of a pension annuity. This annuity changes when individuals pay additional pension fund contributions and when the pension fund writes down or marks up the value of previously accumulated pension wealth. The pension fund invests the contributions in the capital stock of the economy. The present discounted value of the promised pension payments to current fund participants represents the liabilities of the pension fund. The pension fund sets the contribution rate on labour income, the accumulation rate of the annuity and the revaluation instrument (with which it can write down or mark up previously accumulated pension wealth) depending on its financial position. If the funding rate (the ratio of assets to liabilities) is below target, the pension fund has to either increase the contribution rate, decrease the accrual rate or write down previously accumulated pension wealth.

The pension fund is non-Ricardian for three reasons. As in Gertler (1999), the finiteness of life drives a wedge between the market interest rate and the effective discount rate that individuals apply. Additionally, the pension fund annuity represents a new asset in the economy because it yields a return that is conditioned on the specific lifecycle stage of the individual. Finally, depending on the pension fund restoration policy, the accumulation of pension benefits acts as an effective tax or subsidy on labour.

In our analysis we consider a Gertler and Karadi (2011)-type unexpected capital quality shock which evaporates a fraction of the capital stock and leaves the pension fund with a funding gap that needs to be closed. We investigate how the macroeconomy responds when financial adequacy is restored by revaluing previously accumulated pension wealth (Defined Contribution) or changing the pension fund contribution rate on labour income (Defined Benefit) and compare the results to a Laissez-Faire economy without a pension fund. We find that when individuals accumulate real pension benefits a Defined Contribution economy behaves similarly to a Laissez-Faire economy, because the writing down of previously accumulated pension wealth has a similar effect on total lifetime wealth as losing private financial wealth. There are two counteracting forces at work in the Defined Benefit economy. On the one hand, the pension fund increases the contribution rates on labour income and distorts labour supply. On the other hand, the pension fund redistributes wealth towards the group of individuals that has a higher marginal propensity to consume out of wealth, which is important in a demand-driven model. We find that the former effect is the strongest and that the Defined Benefit pension fund exacerbates economic fluctuations. When individuals accumulate nominal pension benefits, the shock leaves the pension fund with a surplus due to ensuing inflation in the medium term as the

[1] More specifically, the assumed preference class ensures that we do not have to keep track of the period in which agents are born and in which period agents become retired. We can instead consider the groups of workers and retirees as stand-alone entities rather than comprised of a range of agents born in different periods. As such, the state-space of the model remains small and solving it is straightforward.

economy recovers. The Defined Benefit pension fund then subsidises labour supply, which dampens economic fluctuations.

The recovery from the unexpected capital quality shock effectively forces the pension fund to distribute welfare losses (or gains) to different groups of individuals and generations. We calculate equivalent variations to assess the welfare effects for three groups of individuals. Retirees are vulnerable to a loss of pension wealth and are insensitive to distortions on the labour market, and therefore prefer the pension system that maximises their pension wealth. The workers that already have accumulated pension wealth in the period the shock materialises dislike labour supply distortions, but also dislike losing their pension wealth because it is the only available asset that yields a return conditional on the lifecycle stage of the individual. The future generations prefer the pension system that brings about the most favourable labour market conditions. We find that there is no unanimous agreement between workers, retirees and future generations about optimal pension fund design. The sum of the equivalent variations indicates that in a real accounting framework a Defined Benefit pension fund is preferred and that in a nominal accounting framework a Defined Contribution pension fund is preferred. However, the sum is close to zero and depends on therate of time preference used to discount the equivalent variations of future generations and the welfare weights attached to different groups of individuals. Furthermore, sensitivity analyses indicate that the intertemporal elasticity of substitution, the size of the pension fund and the closure speed of the funding gap are crucial parameters for the estimated welfare effects.

The literature on Gertler (1999)-type models provides a tractable alternative to the large-scale overlapping generations models inspired by Auerbach and Kotlikoff (1987) and De Nardi et al. (1999).[2] Recent extensions of this model have primarily focussed on the long-term impact of demographic ageing on the interest rate and the transition towards new steady states. The implications for monetary policy are considered by Carvalho et al. (2016) and Kara and von Thadden (2016), while Katagiri (2012) focuses on output, deflation and unemployment. Fujiwara and Teranishi (2008) pay attention to the asymmetric effects of monetary policy on workers and retirees. Grafenhofer et al. (2006) construct a probabilistic ageing model which generalises Gertler (1999) by incorporating richer lifecycle dynamics. Our constructed pension fund framework contributes to this literature, since it thus far has only considered Pay-As-You-Go pension arrangements (such as in Kilponen and Ripatti (2006) and Kara and von Thadden (2016)).

This paper also adds to the literature on the design of pension fund systems and the possibilities for intergenerational risk sharing through them (see Beetsma and Romp (2016) for an overview). Beetsma and Bovenberg (2009) argue that Defined Benefit pension funds improve welfare compared to Defined Contribution pension funds through enhanced intergenerational risk sharing, but do not consider the distortionary effect of Defined Benefit pension funds on labour supply. Beetsma et al. (2013) highlight that, in the absence of a suitably arranged Pay-As-You-Go pension pillar, a Defined Benefit pension fund cannot implement the social optimum because of induced labour supply distortions. Bonenkamp and Westerhout (2014) and Draper et al. (2017) find for Defined Benefit pension funds that the welfare gain from intergenerational risk sharing dominates the cost of labour supply distortions. However, this conclusion is drawn on the basis of overlapping generations models that do not incorporate important market imperfections. Romp (2013) shows that a Defined Benefit pension fund leads to not only more unemployment but also a higher variation in the labour

[2] Examples with a focus on pension systems include Börsch-Supan et al. (2006) and Krueger and Ludwig (2007).

market pressure and unemployment in a search and matching model of the labour market. This distortion is not taken into account in the aforementioned papers on the intergenerational risk sharing properties of pension funds. This paper draws a similar conclusion on the basis of a canonical New-Keynesian model. When the pension fund faces a deficit, the labour supply distortions induced by a Defined Benefit pension fund imply that the total wealth of workers is depressed, causing aggregate demand to fall considerably due to the nominal rigidities. Lastly, this paper is linked to the empirical work of de Haan (2015), who finds that underfunded Dutch pension funds consider contribution increases first, not indexing previously accumulated pension wealth second and cuts to pensions only as a last resort. We show that the preferred restoration policies of Dutch pension funds exacerbate economic fluctuations and might not be optimal from a welfare perspective.

This paper is structured as follows. Section 2 describes the workings of the pension fund, the decision problems of retirees and workers, the supply side of the economy and the actions of fiscal and monetary authorities. Section 3 discusses the calibration of the model, analyses the effects of pension fund restoration policy on the rest of the economy, discusses the welfare implications after an unexpected shock to capital quality and presents several sensitivity analyses. Section 4 concludes. Technical issues are delegated to Appendix A and B, while Appendix C contains a summary of all equilibrium conditions.

2 The model

The timing of the model is such that, at the start of the period, an unexpected shock to capital quality (inspired by Gertler and Karadi (2011)) might materialise. The shock to capital quality depresses the value of the assets of the pension fund, potentially leaving it with a funding deficit that needs to be covered. The pension fund then announces its restoration policy. Afterwards individuals and firms optimise taking into account the capital quality shock and the policy of the pension fund.

2.1 Demographic structure

We consider a unit mass of individuals that is split up in two distinct groups. As in Gertler (1999), individuals have finite lives and flow through two consecutive stages of life: work and retirement. Each individual is born as a worker and conditional on being a worker in the current period, the probability of remaining one in the next period is ω and the probability of becoming retired in the next period is $1-\omega$. Upon reaching retirement, the probability of surviving until the next period is γ and the probability of death is $1-\gamma$. In order to facilitate aggregation within each group, we assume that the probabilities of retirement and death are independent of age. Furthermore, we assume that the number of individuals within each cohort is 'large'. Denote by N^w the stock of workers and by N^r the stock of retirees. We focus on the steady state of the demographics in which the stock of workers and retirees is stable. Since each period a share $1-\omega$ of workers retires, we assume that $(1-\omega)N^w$ workers are born each period. In order to keep the stock of retirees constant, we need that $N^r = (1-\omega)N^w + \gamma N^r$. This holds when we start out with the (old-age) dependency ratio $\psi = \frac{N^r}{N^w} = \frac{1-\omega}{1-\gamma}$.

2.2 Pension fund

In each period, workers and retirees decide how much to consume, how much labour to supply and how much to save.[3] When supplying labour, individuals pay a mandatory contribution to the pension fund and in return accumulate pension wealth in the form of an annuity. The annuity, also referred to as the per-period pension benefit, is paid out by the pension fund each period in which the individual is retired. The size of the annuity is not constant over time, but changes when individuals pay pension contributions or when the pension fund writes down or marks up the value of previously accumulated pension wealth. The pension fund collects the contributions paid by workers and retirees and invests them in the capital stock of the economy. This represents the assets of the pension fund. The present discounted value of the promised pension payments to current fund participants represents the liabilities of the pension fund. We consider two different pension fund accounting frameworks in which individuals accumulate either real or nominal pension benefits.

The pension fund sets the contribution rate on labour income, the accrual rate of the annuity and the revaluation instrument (with which it can write down or mark up previously accumulated pension wealth) depending on its financial position.[4] The fund aims to achieve a certain funding rate (the ratio of its assets to liabilities). If its funding rate is below target, the pension fund faces a deficit and it then has to restore the balance between its assets and its liabilities by either increasing the contribution rate, decreasing the accrual rate or writing down previously accumulated pension wealth. We refer to the specific menu of the chosen contribution rate, accumulation rate and revaluation instrument as the restoration policy of the pension fund.

We allow for flexibility in pension system design along various dimensions. Depending on the specific parametrisation, we fix the target funding rate, the pension fund accounting framework, the recovery speed when the funding rate is below or above the target funding rate and the instruments used to restore financial adequacy. We will show that the model set-up flexibly accommodates the behaviour of various types of existing pension funds.

2.2.1 Pension fund accounting

Since the restoration policy of the pension fund is determined at the start of the period, we use beginning-of-period notation for the state variables relevant to the finances of the pension fund (contrary to the end-of-period notation used later in the model for the savings of individuals and the capital stock of the economy).

Pension fund liabilities

[3] We allow retirees to supply labour and to accumulate additional pension benefits when retired, making the term 'retiree' a relatively poor descriptor. However, allowing retirees to continue to be active on the labour market makes the analysis of the decision problem of retirees conveniently similar to the decision problem of workers. Retirees will be less productive than workers and we will parametrise the productivity parameter such that the labour supply of retirees lies close to zero.

[4] We make the necessary assumptions with respect to the timing of decisions, size of groups of individuals and participation to guarantee that workers and retirees are not able to influence the restoration policy of the pension fund.

At the start of period t, the liabilities of the pension fund are given by the present discounted value of the previously accumulated pension wealth of currently alive workers and retirees.[5]

$$L_t^f = R_t^{r,f} B_t^r + R_t^{w,f} B_t^w, \tag{1}$$

which is the sum of the size of the accumulated annuity of the group of retirees B_t^r and workers B_t^w multiplied by the corresponding annuity factors $R_t^{r,f}$ and $R_t^{w,f}$. B_t^r and B_t^w denote the real number of per-period pension benefits that the group of retirees and workers receive each period in which they are retired (in the absence of accrual, revaluations or inflation from period t onwards). The annuity factors denote the real present discounted value of the expected lifetime payment by the pension fund to a fund participant per unit of accumulated per-period pension benefits. The pension fund liabilities are affected by the capital quality shock through the real interest rate.

We consider two accounting frameworks: participants either accumulate real or nominal pension wealth. For each framework we separately define the annuity factors $R_t^{r,f}$ and $R_t^{w,f}$ and the accumulated annuities B_t^r and B_t^w of the group of retirees and workers, respectively.

Real pension fund accounting framework

The evolutions of the annuities are given by:

$$B_t^r = \gamma \left(\mu_{t-1} B_{t-1}^{r,f} + \nu_{t-1} \xi w_{t-1} L_{t-1}^r \right) + (1-\omega) \left(\mu_{t-1} B_{t-1}^{w,f} + \nu_{t-1} w_{t-1} L_{t-1}^w \right), \tag{2}$$

$$B_t^w = \omega \left(\mu_{t-1} B_{t-1}^{w,f} + \nu_{t-1} w_{t-1} L_{t-1}^w \right), \tag{3}$$

where μ_t is the revaluation instrument, ν_t the accrual rate on labour income, $\xi \in (0,1]$ the relative productivity of retirees, w_t the wage rate, L_t^r the labour supply of the group of retirees and L_t^w the labour supply of the group of workers. Thanks to the assumption that the number of individuals within each cohort is 'large' we are certain that, since each period a fraction $1-\gamma$ of retirees deceases, B_t^r contains a γ share of the accumulated annuity of the group of retirees at the end of period $t-1$. Additionally, since each period a fraction $1-\omega$ of workers retires, B_t^r contains a $1-\omega$ share of the accumulated annuity of the group of workers at the end of period $t-1$. The remaining ω share is contained in B_t^w, while newborn workers in period t start out without any previously accumulated pension wealth.

The pension fund annuity factors are given by:

$$R_t^{r,f} = 1 + \frac{\gamma}{1+r_{t+1}} R_{t+1}^{r,f}, \tag{4}$$

$$R_t^{w,f} = \frac{1}{1+r_{t+1}} \left(\omega R_{t+1}^{w,f} + (1-\omega) R_{t+1}^{r,f} \right). \tag{5}$$

$R_t^{r,f}$ denotes the real present discounted value of the expected lifetime payment by the pension fund to a retiree per unit of accumulated real per-period pension benefits (similarly for $R_t^{w,f}$). The pension fund discounts future pension payments at the real interest rate as we consider the real accounting framework here. In computing $R_t^{r,f}$ and $R_t^{w,f}$, the pension fund sets the revaluation instrument $\mu_{t+i} = 1$, $i = 0, 1, 2....$

[5] Note that revaluations and accrual of pension benefits from period t onwards do not yet belong to the liabilities of the pension fund at the start of the period.

We can therefore interpret $R_t^{r,f}$ and $R_t^{w,f}$ as 'no policy' annuity factors, reflecting a 'normal' course of action in which the pension fund fully covers extended promises to retirees and workers. This is in accordance with Novy-Marx and Rauh (2011) who recognise the Accumulated Benefit Obligation (ABO) as a proper definition of the liabilities of a pension fund. Even if the pension fund would be completely frozen, the ABO would denote the current value of accrued pension benefits still contractually owed to pension fund participants.

Nominal pension fund accounting framework

The evolutions of the annuities are given by:

$$\Pi_t B_t^r = \gamma \left(\mu_{t-1} B_{t-1}^r + \nu_{t-1} \xi w_{t-1} L_{t-1}^r\right) + (1-\omega) \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w\right), \tag{6}$$

$$\Pi_t B_t^w = \omega \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w\right), \tag{7}$$

where Π_t denotes the gross inflation from period $t-1$ till t. Since individuals accumulate nominal pension wealth, we have to adjust the real value of the annuities B_t^r and B_t^w for the change in the price level from one period to the next.

The pension fund annuity factors are given by:

$$R_t^{r,f} = 1 + \frac{\gamma}{1+i_t} R_{t+1}^{r,f}, \tag{8}$$

$$R_t^{w,f} = \frac{1}{1+i_t} \left(\omega R_{t+1}^{w,f} + (1-\omega) R_{t+1}^{r,f}\right), \tag{9}$$

where we use the Fisher relation $1+i = \Pi_{t+1}(1+r_{t+1})$. $R_t^{r,f}$ denotes the real present discounted value of the expected lifetime payment by the pension fund to a retiree per unit of accumulated per-period pension benefits (similarly for $R_t^{w,f}$). The pension fund thus discounts the future payment of pension benefits at the nominal interest rate under the nominal accounting framework.

Pension fund assets

The assets of the pension fund are comprised of the paid contributions by workers and retirees, which are invested in the capital stock of the economy. Each period, the pension fund receives the pension contributions $\tau_t w_t L_t$, where τ_t denotes the contribution rate on labour income and L_t denotes the aggregate labour supply, and pays out $\mu_t B_t^r$ to the currently retired (where B_t^r is either given by (2) or (6) depending on the accounting framework). The pension fund starts out in period $t-1$ with A_{t-1}^f worth of assets and receives a return on its investment in the capital stock from period $t-1$ till t of $1+r_t$. The pension fund assets are affected by the capital quality shock through the real interest rate. This gives the following recursive formulation for the pension fund capital:

$$A_t^f = (1+r_t) \left(A_{t-1}^f + \tau_{t-1} w_{t-1} L_{t-1} - \mu_{t-1} B_{t-1}^r\right). \tag{10}$$

When discussing the pension fund policy in section 2.2.2 it will be useful to have a recursive definition of the liabilities of the pension fund. We can achieve this by substituting identities (2-5) or (6-9), depending on the accounting framework, in (1):

$$L_t^f = (1+r_t)\left(\mu_{t-1}L_{t-1}^f + (R_{t-1}^{r,f}-1)\nu_{t-1}\xi w_{t-1}L_{t-1}^r + R_{t-1}^{w,f}\nu_{t-1}w_{t-1}L_{t-1}^w - \mu_{t-1}B_{t-1}^r\right), \tag{11}$$

which states that the pension fund liabilities at the start of period t are equal to the current value of the revalued liabilities of the previous period $\mu_{t-1}L_{t-1}^f$ plus the real present discounted value of newly issued pension entitlements to retirees $(R_{t-1}^{r,f}-1)\nu_{t-1}\xi w_{t-1}L_{t-1}^r$ and workers $R_{t-1}^{w,f}\nu_{t-1}w_{t-1}L_{t-1}^w$, minus the fulfilled pension promises to retirees $\mu_{t-1}B_{t-1}^r$.

2.2.2 Pension fund restoration policy

As is typically the case in reality, the policy of the pension fund is determined on the basis of its financial position rather than on the basis of the maximisation of the welfare of the pension fund participants. Pension fund regulations generally stipulate that pension funds should attain a target funding rate $\bar{fr}$ in the long run, which is the ratio of the steady state value of the assets of the pension fund to its liabilities. Additionally, regulations prescribe that any funding surplus or deficit should be reduced over time to ensure that the pension fund does not run out of assets and that participation constraints are not an issue. To replicate such regulations in our model, we suppose that the policy of the pension fund is set to reduce the funding gap of the next period to a fraction υ of the current funding gap:

$$A_{t+1}^f - \bar{fr}L_{t+1}^f = \upsilon(A_t^f - \bar{fr}L_t^f), \tag{12}$$

where the funding gap is to be closed within one period if $\upsilon = 0$ and the funding gap is gradually closed over time if $0 < \upsilon < 1$. To get a better picture of how the contribution rate on labour income (τ), the accumulation rate of pension rights (ν) and the revaluation of previously accumulated pension wealth (μ) relate to the closure of the funding gap, we roll over (10) and (11) by one period and substitute them into (12) to obtain:

$$\underbrace{\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}}_{\text{closure fraction}}\underbrace{\left(A_t^f - \bar{fr}L_t^f\right)}_{\text{funding gap}} =$$

$$\bar{fr}\left(\underbrace{\frac{1-\bar{fr}}{\bar{fr}}\mu_t B_t^r + (\mu_t-1)L_t^f}_{\text{revaluation}} + \underbrace{\nu_t w_t\left(\left(R_t^{r,f}-1\right)\xi_t L_t^r + R_t^{w,f}L_t^w\right)}_{\text{accrual}}\right) - \underbrace{\tau_t w_t L_t}_{\text{contribution}}, \tag{13}$$

where the left-hand side denotes the 'gap to be filled' and the right-hand side specifies the ways in which the pension fund can do so. For instance, if $A_t^f < \bar{fr}L_t^f$ the pension fund can reduce the funding gap by writing off the value of previously accumulated pension rights ($\mu_t < 1$), hiking the contribution rate (increase τ_t) or lowering the accumulation of new pension benefits (decrease ν_t).[6] Note that when $0 < \bar{fr} < 1$ a Pay-As-You-Go element is introduced in our funded pension system. Since in the steady state the assets of the

[6]Note that in reasonable scenarios $\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}} > 0$.

pension fund are smaller than its liabilities the pension fund pays out a larger portion of the currently paid contributions directly to retirees. This is reflected in the term $\frac{1-\bar{f}r}{\bar{f}r}\mu_t B_t^r > 0$ when $0 < \bar{f}r < 1$.

Condition (13) characterises the specific restoration policy menu that the pension fund announces after the capital quality shock materialises. The other agents in the model will not be able to influence this choice, because the pension fund announces its policy before other agents make their decisions, participation in the pension scheme is mandatory for the retirees and workers and the number of individuals within each cohort is 'large'. More specifically, the assumption of a 'large' number of retirees and workers ensures that the contributions of a single agent have negligible effects on the financial position of the pension fund (and therefore its policies). Furthermore, mandatory participation ensures that the pension fund does not collapse in case of underfunding or overfunding. As highlighted by Beetsma et al. (2013), newly born workers would not want to participate in case the pension fund is underfunded as they would have to help restore funding adequacy. Additionally, van Bommel and Penalva (2012) highlight that older agents have an incentive to block newly born workers from participating in case the pension fund is overfunded so as to capture the funding surplus for themselves.

2.2.3 Various types of pension systems

The pension fund structure accommodates a range of different existing pension systems. In the simulations below, we will analyse the following three types of pension systems: Laissez-Faire, Defined Contribution and Defined Benefit. For each system we will discuss what type of restoration policy the pension fund implements when it faces a funding gap.

- **Laissez-Faire (also known as Individual Defined Contribution):** In this pension arrangement there effectively is no pension system. Agents save for their own retirement. The pension fund does not levy contributions ($\tau_t = 0, \forall t$), agents do not build up pension benefits ($\nu_t = 0, \forall t$) and $A_t^f = L_t^f = 0, \forall t$. The pension fund does not impact the rest of the economy and does not need to close a funding gap. This Laissez-Faire system can also be referred to as an Individual Defined Contribution pension system where agents save via a private account. Agents reap a private return on the capital market (contrary to the collective return that would be reaped through the pension fund) and are entirely exposed to any unanticipated changes to the value of their retirement savings. This pension arrangement will serve as a benchmark for the other two types of pension systems.
- **Defined Contribution (also known as Collective Defined Contribution):** In this pension arrangement, the contributions to the pension fund and the accrual of pension benefits are predetermined. The fund thus fixes the contribution rate ($\tau_t = \bar{\tau}$, $\forall t$) and accrual rate ($\nu = \bar{\nu}$, $\forall t$) on labour income, where $\bar{\tau}$ and $\bar{\nu}$ denotes the steady state values of the contribution and accrual rate, respectively. The revaluation instrument μ is used to close the funding gap in accordance with condition 13. If $\upsilon = 0$ the pension entitlements are written down immediately in order to restore funding adequacy. When $0 < \upsilon < 1$ this process is more gradual. Since retirees are most reliant on receiving pension benefits, they will be severely affected in case of an adverse shock to capital quality.
- **Defined Benefit:** In this pension arrangement, the fund fixes the revaluation instrument $\mu_t = 1$, $\forall t$ and the accrual rate on labour income $\nu_t = \bar{\nu}$, $\forall t$ so that it fully covers extended pension promises

to fund participants (either in a real or nominal sense, depending on the accounting framework). The contribution rate τ is used to close any funding gap in accordance with condition 13. When the pension fund guarantees the value of accumulated pension benefits, the retirees are relatively unaffected by an adverse capital quality shock. On the other hand, workers are made responsible for the closure of the funding gap through an increase in contribution payments, forcing them to contribute more than what they are expected to receive in return. Since the pension fund contributions are levied as a fraction of labour income, the Defined Benefit pension fund distorts labour supply decisions and therefore has substantial consequences for other macroeconomic variables such as output.

The pension systems described above are extreme cases: either there is effectively no pension fund, or if there is a pension fund, the funding gap is closed using one instrument exclusively. Of course it is possible to postulate that a fraction $0 < \phi^{\mu} < 1$ of the funding gap closure stems from the revaluation instrument, that a fraction $0 < \phi^{\tau} < 1$ stems from the contribution instrument and the remaining fraction $\phi^{\nu} = 1 - \phi^{\mu} - \phi^{\tau}$ stems from the accrual rate. However, any such convex combination will give impulse responses that lie between the extreme cases of Defined Contribution and Defined Benefit. To highlight the macroeconomic effects of various types of pension fund restoration policy we elect to focus on these extreme cases.

2.3 Decision problems of workers and retirees

As mentioned previously, the model does not incorporate aggregate risk. Individuals, however, face two types of idiosyncratic risk throughout their life cycle. Firstly, workers might become retired in the next period, which constitutes an income loss due to the assumed lower productivity of retirees. Secondly, retirees face the uncertainty about their time of death. As in Gertler (1999), we make specific assumptions about the insurability of idiosyncratic risk and the risk preferences of individuals so that aggregation of individual decision rules is possible.

Similar to Blanchard (1985), we introduce annuity markets that entirely shelter retirees from the risk of the timing of death. Upon retirement, individuals hand over their private financial savings to a perfectly competitive mutual fund that invests the proceeds in the market and promises a return $\frac{1+r}{\gamma}$ only to those who are lucky enough to survive until the next period. Since the return of the mutual fund dominates the return of the market (which is $1+r$), all retiring individuals decide to hand over their private savings. Additionally, the existence of the mutual fund ensures that there are no accidental bequests that need to be distributed over the surviving individuals.

While in principle it is possible to introduce an insurance market that mitigates the risk of income loss as a result of retirement, doing so would allow individuals to smooth their labour income over their life cycle and in turn would kill the lifecycle structure that we are aiming to impose. Instead, we specify that individuals are risk neutral with respect to income risk. Since the income risk in this model follows from the mechanical assumption of a constant transition probability $1-\omega$ into retirement, it appears natural to have risk neutral preferences so as to decrease the impact of income variation in the model.

A convenient utility class to invoke is that of RINCE (Risk Neutral Constant Elasticity) preferences. This has two reasons. Firstly, as shown by Farmer (1990), RINCE preferences restrict individuals to be risk neutral with respect to income risk, but allow them to have any arbitrary intertemporal elasticity of substitution. Since we motivate the presence of income risk on the mechanical grounds of generating meaningful lifecycle behaviour, it is favourable that this class of preferences allows for meaningful preferences with respect to smoothing income over time. Secondly, the specification of RINCE preferences allows us to aggregate the behaviour of workers and retirees.

Let $V^{z,i}(a_{t-1}^{z,i}, b_t^{z,i})$ be the value function of a particular individual i at period t, where $z = \{r, w\}$ indicates whether the individual is a retiree (r) or a worker (w) in that period, $a_{t-1}^{z,i}$ denotes the number of consumption goods saved and $b_t^{z,i}$ denotes the accumulated annuity at the pension fund at the start of period t.[7] Preferences are given by:

$$V^{r,i}(a_{t-1}^{r,i}, b_t^{r,i}) = \max_{c_t^{r,i}, l_t^{r,i}, a_t^{r,i}, b_{t+1}^{r,i}} \left(\left[(c_t^{z,i})^{v} (1 - l_t^{z,i})^{1-v} \right]^{\rho} + \gamma\beta \left[V^{r,i}(a_t^{r,i}, b_{t+1}^{r,i}) \right]^{\rho} \right)^{\frac{1}{\rho}}, \tag{14}$$

$$V^{w,i}(a_{t-1}^{w,i}, b_t^{w,i}) = \max_{c_t^{r,i}, l_t^{r,i}, a_t^{r,i}, b_{t+1}^{r,i}} \left(\left[(c_t^{w,i})^{v} (1 - l_t^{w,i})^{1-v} \right]^{\rho} + \beta \left[\omega V^{w,i}(a_t^{w,i}, b_{t+1}^{w,i}) + (1-\omega) V^{r,i}(a_t^{w,i}, b_{t+1}^{w,i}) \right]^{\rho} \right)^{\frac{1}{\rho}}, \tag{15}$$

where $c_t^{z,i}$ and $l_t^{z,i}$ denote consumption and labour supply, respectively. Each individual has one unit of time and enjoys $1 - l_t^{z,i}$ units of leisure. The curvature parameter ρ implies that individuals have a desire to smooth consumption over time. As shown by Farmer (1990), $\sigma = \frac{1}{1-\rho}$ is the familiar intertemporal elasticity of substitution.

2.3.1 Retiree decision problem

A retiree, who is indexed by i, maximises objective (14) in period t subject to:

$$a_t^{r,i} = \frac{1 + r_t}{\gamma} a_{t-1}^{r,i} + (1 - \tau_t)\xi w_t l_t^{r,i} + \mu_t b_t^{r,i} - c_t^{r,i},$$

$$b_{t+1}^{r,i} = \begin{cases} \mu_t b_t^{r,i} + \nu_t \xi w_t l_t^{r,i}, & \text{if real accounting framework} \\ \frac{\mu_t b_t^{r,i} + \nu_t \xi w_t l_t^{r,i}}{\Pi_{t+1}}, & \text{if nominal accounting framework} \end{cases}$$

where $a_t^{r,i}$ are the private savings of the retiree at period t, yielding a return of $\frac{1+r_{t+1}}{\gamma}$ in period $t+1$ through the mutual fund, and r_t is the real interest rate on savings from period $t-1$ till period t. The private financial wealth of the retiree is given by $\frac{1+r_t}{\gamma} a_{t-1}^{r,i}$. The effective wage rate of the retiree is given by ξw_t, where $\xi \in (0, 1]$ represents the productivity loss of retirees relative to workers. When working the retiree pays a mandatory contribution to the pension fund equal to a share τ_t, the contribution rate, of labour income. In return his annuity $b_{t+1}^{r,i}$ increases by a share ν_t, the accrual rate, of his earned labour income. The retiree receives his previously accumulated annuity $\mu_t b_t^{r,i}$ from the pension fund, which is corrected for the revaluation instrument μ_t (and the inflation Π_t in the nominal pension fund accounting framework). The

[7]The state variable $b_t^{z,t}$ contains the time subscript t as $b_t^{z,i}$ can depend on the inflation Π_t in the nominal accounting framework. Additionally, recall that $a_{t-1}^{z,i}$ is written in end-of-period notation, but that $b_t^{z,i}$ is written in beginning-of-period notation.

retiree, when deciding on his optimal amount of labour to supply and goods to consume, takes as given the financial position of the pension fund and thus the future path of its policy.[8]

As shown in Appendix A.1.1 and A.1.2, the decision problem of the retiree gives rise to the following two conditions:

$$c_{t+1}^{r,i} = \left(\beta(1+r_{t+1})\left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma} c_t^{r,i}, \tag{16}$$

$$1 - l_t^{r,i} = \frac{1-v}{v}\frac{c_t^{r,i}}{(1-\tau_t^r)\xi w_t}, \tag{17}$$

where (16) is the intertemporal Euler equation and (17) the optimal labour supply decision. The term $\tau_t^r = \tau_t - (R_t^r - 1)\nu_t$ is the effective labour income contribution rate that the retiree faces, where R_t^r is the retiree annuity factor which denotes the expected real present discounted value to a retiree of receiving a consumption good each period until death, corrected for the revaluation instrument (and inflation in the nominal accounting framework). If the retiree earns an additional unit of labour income in period t, he pays the mandatory contribution τ_t to the pension but also accumulates ν_t additional per-period pension benefits which he will receive from period $t+1$ onwards. Depending on the pension fund restoration policy (characterised by the current and future accrual ν, indexation μ, and contribution τ) the effective contribution rate τ^r acts as either an effective tax ($\tau^r > 0$) or subsidy ($\tau^r < 0$) on labour income. We define the retiree annuity factor as:

$$R_t^r = \begin{cases} 1 + \mu_{t+1}\frac{\gamma}{1+r_{t+1}}R_{t+1}^r, & \text{if real accounting framework} \\ 1 + \mu_{t+1}\frac{\gamma}{1+i_t}R_{t+1}^r, & \text{if nominal accounting framework} \end{cases}$$

The retiree annuity factor differs from $R_t^{r,f}$ (the retiree annuity factor used by the pension fund) due to the inclusion of the future path of the revaluation instrument μ (which was omitted from $R_t^{r,f}$ for supervision purposes). Whereas $R_t^{r.f}$ is to be interpreted as a 'no policy' annuity factor used by the pension fund to determine the restoration policy of the current period, the retiree takes into account the future path of the pension fund restoration policy in determining how much labour to supply and how much to consume.

Let Δ_t^r denote the inverse of the marginal propensity to consume out of wealth of a retiree and let $x_t^{r,i} \equiv c_t^{r,i} + (1-\tau_t^r)\xi w_t(1-l_t^{r,i}) = \frac{c_t^{r,i}}{v}$ denote retiree full consumption. Additionally, let retiree full income $d_t^{r,i}$ and retiree human wealth $h_t^{r,i}$ be defined as:

$$d_t^{r,i} = (1-\tau_t^r)\xi w_t, \tag{18}$$

$$h_t^{r,i} = d_t^{r,i} + \frac{\gamma}{1+r_{t+1}}h_{t+1}^{r,i}. \tag{19}$$

[8]This specification of the budget constraint assumes that the retiree was retired already in the previous period. Kara and von Thadden (2016) show that this characterisation is sufficient to derive the aggregate behaviour of retirees and workers.

Appendix A.1.3 shows that full consumption and inverse marginal propensity to consume out of wealth of a retiree satisfy the following two conditions:

$$x_t^{r,i} = \frac{1}{\Delta_t^r}\left(\frac{1+r_t}{\gamma}a_{t-1}^{r,i} + h_t^{r,i} + \mu_t b_t^{r,i} R_t^r\right), \tag{20}$$

$$\Delta_t^r = 1 + \gamma\beta^\sigma \Delta_{t+1}^r \left((1+r_{t+1})\left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}}\right)^{1-v}\right)^{\sigma-1}. \tag{21}$$

Retirees spend a fraction $\frac{1}{\Delta_t^r}$ of their total lifetime wealth on consumption goods and leisure. Retiree total lifetime wealth consists of the sum of private financial wealth $\frac{1+r_t}{\gamma}a_{t-1}^{r,i}$, human wealth $h_t^{r,i}$ (which contains the expected value of pension wealth to be accumulated in the future) and previously accumulated pension wealth $\mu_t b_t^{r,i} R_t^r$. Since the inverse marginal propensity to consume out of wealth of a retiree is the same for all retirees, irrespective of age and total lifetime wealth, aggregation over retirees will be straightforward. Appendix A.1.3 shows that (20) and (21) can be used to derive an analytical expression for the indirect retiree value function:

$$V_t^{r,i} = \left(\Delta_t^r\right)^{\frac{1}{\rho}} v x_t^{r,i}\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}.$$

2.3.2 Worker decision problem

A worker, who is indexed by j, maximises objective (15) in period t subject to:

$$a_t^{w,j} = (1+r_t)\,a_{t-1}^{w,j} + (1-\tau_t)w_t l_t^{w,j} + f_t^{w,j} - c_t^{w,j},$$

$$b_{t+1}^{w,j} = \begin{cases} \mu_t b_t^{w,j} + \nu_t w_t l_t^{w,j}, & \text{if real accounting framework} \\ \frac{\mu_t b_t^{w,j} + \nu_t w_t l_t^{w,j}}{\Pi_{t+1}}, & \text{if nominal accounting framework} \end{cases}$$

where $a_t^{w,j}$ are the private savings of the worker at the end of period t and $b_{t+1}^{w.j}$ is the size of the worker annuity at the start of period $t+1$. The private financial wealth of the worker is given by $(1+r_t)\,a_{t-1}^{w,j}$ and the worker receives profits $f_t^{w,j}$ from the intermediate and capital good producing firms. As shown in Appendix A.2.1 and A.2.2 , the decision problem of the worker gives rise to the following two conditions:

$$\omega c_{t+1}^{w,j} + (1-\omega)c_{t+1}^{r,j}\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_{t+1}^w}{\Delta_{t+1}^r}\right)^{\frac{\sigma}{1-\sigma}} = \left(\beta(1+r_{t+1})\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma} c_t^{w,j}, \tag{22}$$

$$1 - l_t^{w,j} = \frac{1-v}{v}\frac{c_t^{w,j}}{(1-\tau_t^w)w_t}, \tag{23}$$

where we define:

$$\Omega_t = \omega + (1-\omega)\left(\frac{1-\tau_t^w}{1-\tau_t^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_t^w}{\Delta_t^r}\right)^{\frac{1}{1-\sigma}}. \tag{24}$$

The worker Euler equation (22) shows that the worker takes into account that he might become retired in period $t+1$. The term Ω_t reflects that a worker, when switching into retirement, reaches the next (and irreversible) stage in his life cycle. The retirement stage is characterised by a different effective wage rate (captured by ξ), marginal propensity to consume out of wealth (captured by Δ_t^w and Δ_t^r) and effective pension fund contribution rate on labour income (captured by τ^r and τ^w). The effective worker contribution rate is given by $\tau_t^w = \tau_t - R_t^w \nu_t$ and, similarly to the retiree effective contribution rate, reflects the balance between the costs (τ_t) and the benefits ($R_t^w \nu_t$) of the mandatory pension fund participation to the worker. We define the worker annuity factor as:

$$R_t^w = \begin{cases} \frac{\mu_{t+1}}{1+r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} R_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) R_{t+1}^r \right), & \text{if real accounting framework} \\ \frac{\mu_{t+1}}{1+i_t} \left(\frac{\omega}{\Omega_{t+1}} R_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) R_{t+1}^r \right), & \text{if nominal accounting framework} \end{cases}$$

The worker annuity factor denotes the expected real present discounted value to a worker of receiving one consumption good each period when retired until death, corrected for the revaluation instrument (and inflation in the nominal accounting framework). The definition of R_t^w shows that the term Ω_t can be interpreted as a subjective reweighting of transition probabilities. The irreversible event of transitioning into retirement entails an income shock for the individual and implies that the worker attaches more importance to receiving income when retired compared to remaining a worker in future periods. This is reflected in the fact that the worker attaches a subjective transition probability of $\frac{\omega}{\Omega_{t+1}}$ (compared to the objective probability ω) to income received when remaining a worker in period $t+1$ and a subjective transition probability of $1 - \frac{\omega}{\Omega_{t+1}}$ (compared to the objective probability $1-\omega$) when becoming a retiree in period $t+1$.[9] The worker annuity factor R_t^w thus does not only differ from $R_t^{w,f}$ (the worker annuity factor used by the pension fund) due to the inclusion of the future path of the revaluation instrument μ, but also due to the subjective reweighting of transition probabilities of the worker. The pension fund is an ongoing concern, which does not have a lifecycle motive like workers do, and uses the objective transition probabilities for the annuity factor $R_t^{w,f}$.

Let Δ_t^w denote the inverse of the marginal propensity to consume out of wealth of a worker and let $x_t^{w,j} \equiv c_t^{w,j} + (1-\tau_t^w) w_t (1 - l_t^{w,j}) = \frac{c_t^{w,j}}{v}$ denote worker full consumption. Additionally, let worker full income $d_t^{w,j}$ and worker human wealth $h_t^{w,j}$ be defined as:

$$d_t^{w,j} = (1-\tau_t^w) w_t + f_t^{w,j}, \tag{25}$$

$$h_t^{w,j} = d_t^{w,j} + \frac{1}{1+r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} h_{t+1}^{w,j} + (1 - \frac{\omega}{\Omega_{t+1}}) h_{t+1}^{r,j} \right). \tag{26}$$

Appendix A.2.2 and A.2.3 show that the full consumption function and inverse marginal propensity to consume out of wealth of a worker satisfy the following two conditions:

$$x_t^{w,j} = \frac{1}{\Delta_t^w} \left((1+r_t)\, a_{t-1}^{w,j} + h_t^{w,j} + \mu_t b_t^{w,j} R_t^w \right), \tag{27}$$

$$\Delta_t^w = 1 + \beta^\sigma \Delta_{t+1}^w \left((1+r_{t+1})\, \Omega_{t+1} \left(\frac{(1-\tau_t^w) w_t}{(1-\tau_{t+1}^w) w_{t+1}} \right)^{1-v} \right)^{\sigma - 1}. \tag{28}$$

[9]In our calibration it will hold that $\Omega_t > 1$, $\forall t$.

Workers spend a fraction $\frac{1}{\Delta_t^w}$ of their total lifetime wealth on consumption goods and leisure. Worker total lifetime wealth is comprised of the sum of private financial wealth $(1 + r_t)\, a_{t-1}^{w,j}$, human wealth $h_t^{w,j}$ (which contains the expected value of pension wealth to be accumulated in the future) and previously accumulated pension wealth $\mu_t b_t^{w,j} R_t^w$. Since the inverse marginal propensity to consume out of wealth of a worker is the same for all workers, irrespective of age and total lifetime wealth, aggregation over workers will be straightforward. Appendix A.2.2 and A.2.3 show that that (27) and (28) can be used to derive an analytical expression for the indirect worker value function:

$$V_t^{w,j} = (\Delta_t^w)^{\frac{1}{\rho}}\, v x_t^{w,j} \left(\frac{1-v}{v} \frac{1}{(1-\tau_t^w) w_t} \right)^{1-v}.$$

It is important to appreciate the implication that the subjective reweighting of transition probabilities has for the model. As shown by Gertler (1999), the Ricardian equivalence breaks down in this type of model and therefore the path of government debt influences macroeconomic outcomes. In this model, the subjective reweighting of transition probabilities entails that the pension fund is non-Ricardian as well and that its restoration policy influences macroeconomic outcomes. When an adverse capital quality shock materialises, the pension fund closes its funding gap by implementing a restoration policy and effectively decides which group of individuals closes the gap. It therefore implicitly redistributes income between different groups of individuals (i.e. the current group of workers and retirees and future generations) that have different marginal propensities to consume out of wealth and in turn influences macroeconomic outcomes.

The pension fund influences macroeconomic outcomes through a second avenue and that is by effectively creating a new asset in the economy. When workers invest their private financial wealth in the capital stock of the economy they obtain the same return regardless of their lifecycle stage in the next period. In the absence of the pension fund, workers thus cannot invest in an asset that yields a different return depending on whether they are a retiree or a worker in the next period. The pension fund introduces such an asset: it only pays out the accumulated pension benefits when the worker is actually retired and the mandatory investment in the pension fund yields a return that is conditioned on the specific lifecycle stage of the individual. Workers cannot replicate this when they invest their private financial wealth in the capital stock.

2.3.3 Aggregation over retirees and workers

Aggregate variables will be identified by the lack of a superscript i and j and are written in capital letters. We start by aggregating human wealth and private financial wealth and afterwards aggregate the consumption and labour supply functions. Recall that the aggregate annuities of the retirees B_t^r and workers B_t^w are defined recursively by conditions (2) and (3) or (6) and (7). Aggregate full income of retirees and workers satisfies:

$$D_t^r = N^r (1 - \tau_t^r) \xi w_t,$$
$$D_t^w = N^w (1 - \tau_t^w) w_t + F_t,$$

where F_t denotes the aggregate profits of the intermediate and capital good producers and is specified by condition (B.14). Note that we do not have to specify how firm profits are distributed over individual workers due to the structure of the derived worker consumption function. Aggregate human wealth of retirees and workers satisfies:

$$
\begin{aligned}
H_t^r &= D_t^r + \frac{\gamma}{1+r_{t+1}} H_{t+1}^r, \\
H_t^w &= D_t^w + \frac{1}{1+r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} H_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) \frac{1}{\psi} H_{t+1}^r \right),
\end{aligned}
$$

Aggregate private financial wealth of retirees and workers can be defined recursively:

$$
A_t^r = (1+r_t)A_{t-1}^r + \mu_t B_t^r + (1-\tau_t)\xi w_t L_t^r - C_t^r + \frac{1-\omega}{\omega} A_t^w, \tag{29}
$$

$$
A_t^w = \omega \left((1+r_t)A_{t-1}^w + (1-\tau_t) w_t L_t^w + F_t - C_t^w \right). \tag{30}
$$

Condition (29) shows that the aggregate private savings brought into period $t+1$ by those who are retired in period $t+1$ consists of two parts. Firstly, it consists of the sum of income that was not spent by retirees in period t. The lack of a multiplication by γ reflects that all savings by retirees in period t are transferred to the surviving retirees in period $t+1$. On the level of the group of retirees, private financial wealth invested in the capital stock of the economy yields a return of $1+r_t$. Secondly, it consists of the sum of income not spent in period t by those workers who become retired in period $t+1$. The remainder of the sum of income not spent in period t by workers is given by (30), since newly born workers start out without private financial wealth. Having specified retiree and worker private financial wealth, human wealth and pension wealth, we arrive at the aggregate full consumption functions:

$$
X_t^z = \frac{1}{\Delta_t^z} \left((1+r_t)A_{t-1}^z + H_t^z + \mu_t B_t^z R_t^z \right), \quad z \in \{w, r\}. \tag{31}
$$

Aggregate consumption of retirees, workers and total population satisfies:

$$
\begin{aligned}
C_t^z &= v X_t^z, \quad z \in \{w, r\}, \\
C_t &= C_t^r + C_t^w.
\end{aligned}
$$

Aggregate labour supply of retirees, workers and total population satisfies, where $w_t^r = \xi w_t$ and $w_t^w = w_t$:

$$
\begin{aligned}
L_t^z &= N^z - \frac{(1-v)X_t^z}{(1-\tau_t^w)w_t^z}, \quad z \in \{w, r\}, \\
L_t &= L_t^w + \xi L_t^r.
\end{aligned}
$$

Aggregate welfare of retirees and workers satisfies:

$$
V_t^z = (\Delta_t^z)^{\frac{1}{\rho}} v X_t^z \left(\frac{1-v}{v} \frac{1}{(1-\tau_t^z)w_t^z} \right)^{1-v}, \quad z \in \{w, r\}. \tag{32}
$$

2.4 Firms and government

The supply-side of the economy is modelled in a familiar New-Keynesian fashion. Intermediate good producing firms borrow from the households and the pension fund to purchase the capital necessary for production. The revenue generated from the sale of the output to retail firms and of the capital after it has been used is spent on the wages of households and used to pay back the loans from households and the pension fund. Capital producing firms buy the used capital and transform it, together with goods purchased from final good producing firms, into new capital. This new capital is sold to intermediate good producing firms who will use it for production in the next period. While intermediate good producing firms do not face investment adjustment costs at the firm level, the capital producing sector is subject to investment adjustment costs à la Fernandez-Villaverde (2006) and Christiano et al. (2005). The retail firms repackage the purchased output from intermediate good producing firms in order to produce a unique and differentiated retail product. The output of retail firms is sold to final good producing firms, but at a markup due to the differentiated nature of the retail product. In effect, each retail firm has 'local' monopoly power. Retail firms face Calvo (1983)-type pricing frictions. The final good producers convert the output of retail firms into final goods, which are then sold to households and capital producers. This splits up the economy in four production sectors. The capital producing sector isolates the investment adjustment costs. The retail goods sector isolates the Calvo pricing and imperfect competition. The intermediate goods sector isolates the pricing of capital and labour. The final goods sector aggregates. There are no government purchases and the central bank sets its monetary policy according to a Taylor rule. Since the decision making of firms and government is standard in the New-Keynesian literature, we delegate the derivations to Appendix B.

3 Model analysis

We calibrate the model in section 3.1, assess the macroeconomic effects of an unexpected adverse capital quality shock that urges the pension fund to close a funding gap in section 3.2, consider the welfare implications in section 3.3 and present sensitivity analyses in section 3.4.

3.1 Baseline calibration

Since the restoration policy of a pension fund is a relatively short-term phenomenon, we elect to calibrate the model at a quarterly frequency. Table 1 summarises the chosen values for each model parameter. The demographic parameters are set such that the implied average working period is 45 years and the average retirement period is 15 years. This is consistent with agents entering the labour force at the age of 20, working until 65 and passing away at 80. The old-age dependency ratio $\frac{1-\omega}{1-\gamma}$ is therefore equal to $\frac{1}{3}$. These values are close to empirical estimates for the Euro area in 2008 reported in the statistical annex of the 2009 Ageing Report by the European Commission, who report a life expectancy at birth of 79.5 years and an old-age dependency ratio of 0.27.

The intertemporal elasticity of substitution is a crucial parameter in our analysis. In Gertler (1999)-type models the chosen values range from $\frac{1}{4}$ to $\frac{1}{2}$. In the baseline calibration we set it to the intermediate $\frac{1}{3}$

Table 1: Model parameters (excluding those of the pension fund)

Demographics		
Retirement probability of workers	$1-\omega$	$\frac{1}{180}$
Death probability of retirees	$1-\gamma$	$\frac{1}{60}$
Households		
Intertemporal elasticity of substitution	σ	$\frac{1}{3}$
Discount factor	β	$1.07^{-\frac{1}{4}}$
Consumption preference	v	0.6
Relative productivity of retirees	ξ	0.2
Intermediate good producing firms		
Cobb-Douglas share of capital	α	$\frac{1}{3}$
Depreciation rate of capital	δ	$1.1^{-\frac{1}{4}}-1$
AR(1)-coefficient of capital quality shock	ρ_ζ	$\frac{2}{3}$
Capital good producing firms		
Investment adjustment costs parameter	κ	1.728
Retail good producing firms		
Elasticity of demand for intermediate goods	ϵ	4.167
Central bank		
Inertial parameter in Taylor rule	η_i	0
Inflation coefficient in Taylor rule	γ_π	1.5
Output coefficient in Taylor rule	γ_y	$\frac{1}{8}$
Target inflation rate	$\bar{\Pi}$	1.0025

(implying that $\rho = -2$), but in section 3.4 we perform sensitivity analyses with respect to this parameter. The relative productivity of retirees ξ is set to a smaller value than in other papers in this literature to ensure that retiree labour force participation remains low. We set the discount factor β to achieve a yearly real interest rate of roughly 2% in the steady state. As we implement the capital quality shock of Gertler and Karadi (2011), we calibrate the production sectors and central bank in precisely the same fashion. However, we deviate by setting $\bar{\Pi} = 1.0025$ which implies a yearly steady state net inflation rate of 1%. This gives a meaningful difference between the real and nominal pension fund accounting framework in the steady state.

While the OECD (2017) Pension Markets In Focus report highlights that pension funds have been gaining importance with pension fund assets growing faster than GDP in most countries from 2006-2016, there is still a wide disparity between countries in terms of the size of the pension fund market. For instance, pension fund assets in Denmark, The Netherlands, Canada and Iceland are larger than 150% of GDP, while pension fund assets in Spain, Portugal, Norway, France, Italy and Germany are smaller than 15% of GDP. In our baseline calibration, we set the pension fund parameters such that the assets of the pension fund are roughly equal to 88% of yearly output, which is in between the average of 50% and weighted average of 125% of OECD countries in 2016 as reported by the Pension Markets In Focus report. Since the calibration of the pension fund is such a delicate issue, we perform sensitivity analyses with respect to the size of the pension fund in section 3.4.

Table 2 summarises the set pension fund parameters and several implied indicators of pension fund size in the steady state. In the steady state we postulate that the pension fund covers its extended promises to retirees by setting the revaluation $\bar{\mu} = 1$. Fixing the accrual rate $\bar{\nu}$ then determines the size of the balance sheet of the pension fund and implies a steady state contribution rate $\bar{\tau}$. We specify that in the steady state the pension fund should achieve a nominal funding rate of 100%. Together with a yearly net inflation rate of 1% in the steady state this implies a real target funding rate of 78.27% in the real accounting framework. The resulting contributions to output ratios of roughly 2% are smaller than the OECD average in 2016 of 2.11% and weighted average of 4.15%, while the benefits to output ratios of roughly 3.5% and 4% lie between the OECD average in 2016 of 1.67% and weighted average of 5.30%.[10]

Our pension fund system gives relatively high benefits to output ratios compared to the contributions to output ratios for two reasons. First, in our model the only investment opportunity for the pension fund is the capital stock, which yields a return akin to an equity investment. In reality, in 2016 pension funds in OECD countries invested roughly 40% of contributions in bonds according to the Pension Markets In Focus report. The same report states that because of this investment portfolio the geometric average annual real returns of pension funds in OECD countries from 2006-2016 was 1.7%, while our steady state annual real interest rates are roughly 2.0%. Second, condition (13) shows that underfunded pension funds (where assets are smaller than liabilities) contain a Pay-As-You-Go component. The more underfunded the pension fund, the more contributions are directly transferred to retirees instead of invested. In the wake of the financial crisis of 2008, many pension funds faced funding deficits, explaining the empirically observed low benefits to output ratios relative to the contributions to output ratios. Lastly, the closure speed υ is set such that the half-life of the funding gap is equal to 1 year, but we will perform sensitivity analyses in section 3.4.

Table 3 provides an overview of the steady state values of important endogenous variables. The marginal propensity to consume out of wealth is considerably higher for retirees than for workers, which is in line with the calibrations of Gertler (1999)-type models and the empirical estimations by Harrison et al. (2002). The subjective reweighting of transition probabilities $\Omega > 1$ drives a substantial wedge between the worker annuity factor R^w and the annuity factor applied by the pension fund $R^{w,f}$. Because saving through the pension fund allows workers to condition their future return on their future lifecycle stage, the effective contribution rate of workers τ^w is negative or close to zero. Especially the effective contribution rate of retirees τ^r is negative. This is a well-known feature of uniform policy pension systems in which contribution and accrual rates are equal for all participants irrespective of the participant's age at the payment time of the contribution. Chen and van Wijnbergen (2017) document that this is the case in many public sector pension plans in OECD countries. In our model, workers face the same contribution and accrual rate as retirees despite the fact that the contributions of the workers are expected to be invested for a longer period of time. As a consequence of the sizeable effective subsidy on labour income, the labour force participation of retirees is higher compared to the findings of other papers in this literature and OECD data.[11]

3.2 Restoring pension funding adequacy after an adverse capital quality shock

In this section we describe the restoration policy implemented by Defined Contribution and Defined Benefit pension funds and the implications this policy has for the rest of the economy after an unexpected adverse

[10] Calculated using data gathered from the OECD.Stat database.

[11] The OECD.Stat database reports that the average labour force participation rate amongst retirees aged 65 or above in OECD countries was 0.145 in 2016.

Table 2: Pension fund parameters and implied pension fund size in steady state

		Real accounting framework	Nominal accounting framework
Set parameters			
Accrual rate	$\bar{\nu}$	0.13%	0.19%
Steady state funding rate	$\bar{fr}$	78.27%	100%
Funding gap closure speed	υ	0.8409	0.8409
Implied steady state values			
Contribution rate	$\bar{\tau}$	4.14%	3.56%
Pension fund assets to yearly output ratio*	$\frac{A^f}{4Y}$	88%	88%
Contributions to output ratio	$\frac{\tau w L}{Y}$	2.10%	1.81%
Benefits to output ratio	$\frac{B^r}{Y}$	3.96%	3.49%
Pension fund capital to aggregate capital ratio	$\frac{A^f}{K}$	40.78%	40.04%
Fraction of pension wealth owned by retirees	$\frac{R^{r,f}B^r}{L^f}$	40.13%	32.26%

* targeted value

Table 3: Steady state values of selected endogenous variables

		Real accounting framework	Nominal accounting framework
Inverse marginal propensity to consume out of wealth of workers	Δ^w	56.87	57.91
Inverse marginal propensity to consume out of wealth of retirees	Δ^r	39.02	39.57
Yearly real interest rate	$(1+r)^4-1$	2.13%	1.91%
Subjective reweighting of transition probabilities	Ω	1.01	1.01
Worker annuity factor	R^w	35.51	30.18
Worker annuity factor of pension fund	$R^{w,f}$	23.45	18.27
Effective contribution rate of workers	τ^w	−0.48%	−2.21%
Effective contribution rate of retirees	τ^r	−1.69%	−4.19%
Labour force participation rate of workers	$\frac{L^w}{N^w}$	0.51	0.52
Labour force participation rate of retirees	$\frac{L^r}{N^r}$	0.19	0.22
Capital to output ratio	$\frac{K}{Y}$	8.62	8.79
Worker consumption to output ratio	$\frac{C^w}{Y}$	0.72	0.71
Retiree consumption to output ratio	$\frac{C^r}{Y}$	0.08	0.08
Investment to output ratio	$\frac{I}{Y}$	0.20	0.21

capital quality shock materialises. With the adverse shock to capital quality we aim to mimic the dynamics of a financial crisis such as the one of 2008, but with a specific interest in the financial situation of pension funds. We consider an adverse shock of 1% to capital quality.[12]

3.2.1 Real pension fund accounting framework

Figure 1 provides a plot of pension fund accounting variables and the implemented restoration policy for the real accounting framework. The unexpected adverse capital quality shock depresses the value of the pension fund assets by roughly 2% on impact. Despite the fact that the pension fund issues real promises to participants in a Defined Benefit system, the value of its liabilities is depressed by roughly 1% on impact due to the response of the real interest rate. Both types of pension funds face a funding deficit of roughly 1% as a result of the adverse capital quality shock. The Defined Benefit pension fund responds by significantly increasing the contribution rate on labour income, while the Defined Contribution pension fund gradually writes down the value of previously accumulated pension wealth. In the Defined Benefit pension system retirees are comparatively well off since the value of their pension wealth is guaranteed. However, the workers are comparatively worse off as they rely on their labour income. This is reflected in the plots of the effective contribution rates of workers and retirees. Figure 1 highlights that the effective contribution rate of workers turns positive, while the effective contribution rate of retirees stays negative. The costs to workers of participating in the mandatory pension fund are higher than the benefits and thus the workers subsidise the retirees to guarantee their pension wealth. Even though in the steady state the two pension funds are of equal size, in the recovery they are significantly different because the Defined Benefit fund implements a restoration policy of amassing assets and the Defined Contribution fund implements a restoration policy of cutting liabilities.

Figure 2 presents a plot of various important macroeconomic variables in the Defined Benefit, Defined Contribution and Laissez-Faire economies. When assessing the impacts of the restoration policy of the pension fund on the rest of the economy, it is important to recall that aggregate demand plays a crucial role in our New-Keynesian set-up. When the pension fund implements a Defined Benefit policy, there are two forces counteracting each other. On the one hand, since the pension fund contributions are levied as a fraction of labour income, the Defined Benefit restoration policy distorts labour supply. On the other hand, since retirees have a higher marginal propensity to consume out of wealth, guaranteeing the value of previously accumulated pension wealth ensures that wealth is allocated to the group of individuals that, in the margin, exercises a stronger demand for consumption goods. The numerical simulations indicate that the former effect is stronger than the latter effect. The labour supply distortions imply that the total wealth of workers is depressed, causing aggregate demand to fall. This process is exacerbated by the nominal rigidities which prevent the retail sector from adjusting the price of output appropriately. Since the retirees are outnumbered by workers, the effect of their higher proclivity to consume is quantitatively unimportant

[12] We estimate the model using Dynare. Since we consider a perfect foresight model, the solution does not require linearisation and instead has a fully nonlinear solution. Dynare numerically simulates in order to find the exact paths of the endogenous variables that meet the equilibrium conditions and the paths of exogenously specified shocks. Dynare makes use of the Newton method of simultaneously solving all the equations for every period and makes the simplifying assumption that our system of equations is back in equilibrium at the end of the simulation period.

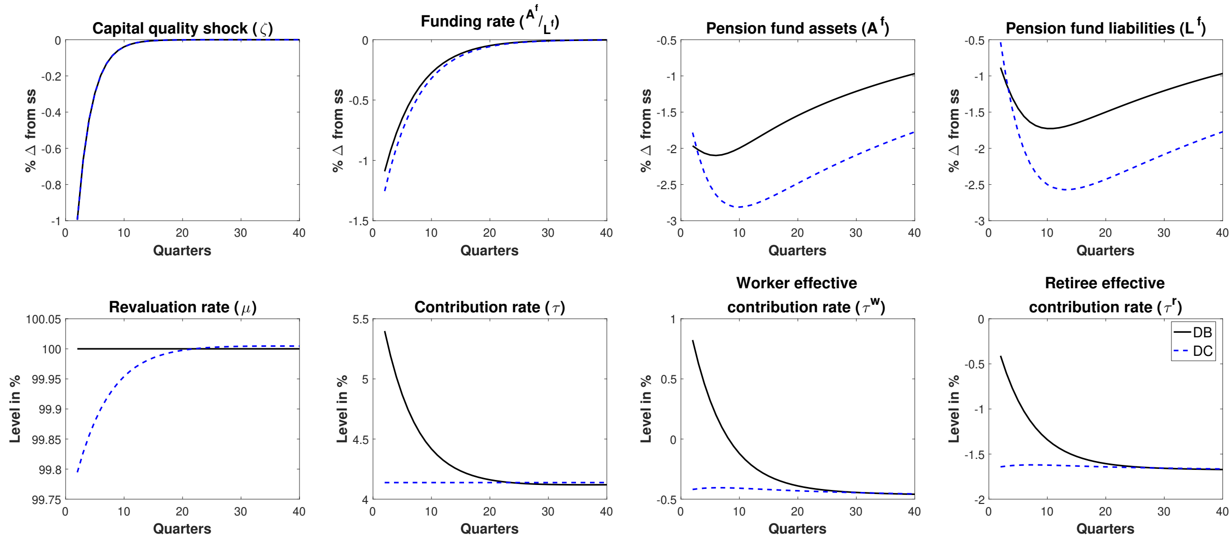

Figure 1: Pension fund restoration policy after a 1% capital quality shock in a New-Keynesian model with a real pension fund framework. Defined Benefit is denoted by the solid black line, while Defined Contribution is denoted by the striped blue line.

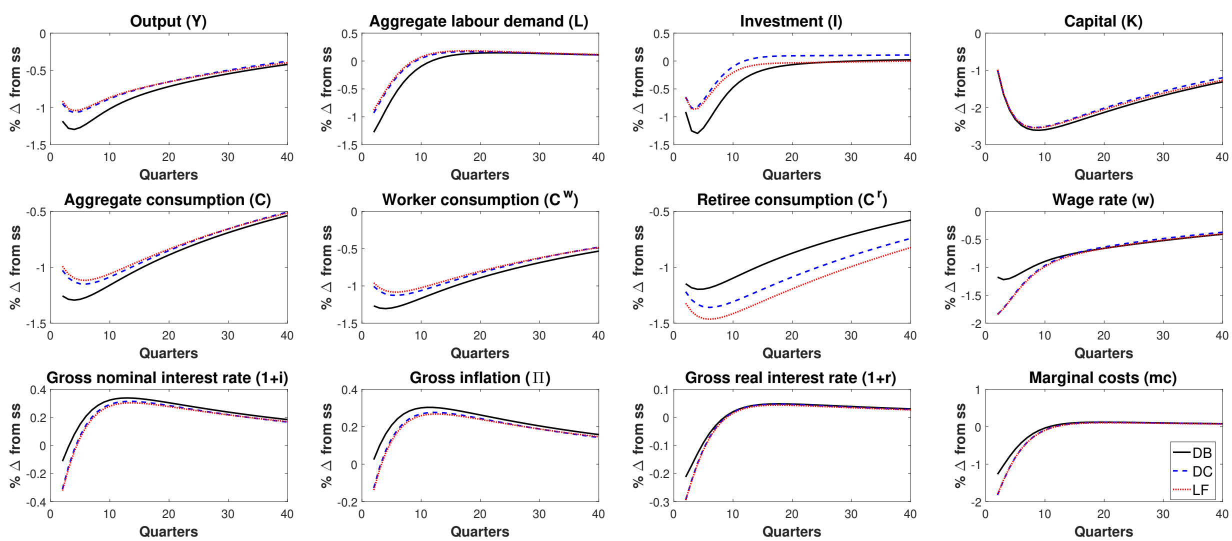

Figure 2: Effect of pension fund restoration policy after a 1% capital quality shock on macroeconomic variables in a New-Keynesian model with a real pension fund framework. Defined Benefit is denoted by the solid black line, while Defined Contribution is denoted by the striped blue line and Laissez-Faire is denoted by the dotted red line.

for the determination of macroeconomic aggregates. As a result aggregate output, consumption, investment and capital are all lower compared to the Defined Contribution and Laissez-Faire economies.

Lastly, figure 2 indicates that the Defined Contribution economy behaves similarly to an economy without a pension fund. This is intuitive: in a Laissez-Faire economy agents save for retirement through their private financial wealth which evaporates due to the adverse capital quality shock in a similar fashion as the writing off of previously accumulated pension wealth under the Defined Contribution pension fund. However, since the accumulated pension wealth is written down gradually over time, retiree consumption is higher, coming at the expense of worker consumption, in the Defined Contribution economy than in the Laissez-Faire economy.

3.2.2 Nominal pension fund accounting framework

Figure 3 highlights that the adverse capital quality shock actually leads to a funding surplus for the pension fund in the nominal accounting framework. This is predominantly explained by the movement of the nominal interest rate in response to the unexpected shock and its effects on the liabilities of the pension fund. As in the real pension fund framework, the value of the assets of the fund are depressed by roughly 2% on impact. However, the value of the liabilities drop roughly 4% and 9% in the Defined Benefit and Defined Contribution economy, respectively. While in the short run the shock causes the price level to decrease, inflation picks up in the medium term as the economy recovers. Since the pension fund issues nominal promises to fund participants under this accounting framework, the ensuing inflation drives down the value of the fund liabilities substantially.[13] This holds especially for the Defined Contribution economy which is characterised by a higher inflation rate and a higher nominal interest rate compared to the Defined Benefit economy. Since the pension fund now faces a funding surplus, it implements a restoration policy which distributes welfare gains over different groups of individuals and cohorts. The Defined Contribution pension fund increases the revaluation rate which offsets the loss of previously accumulated pension wealth resulting from the ensuing inflation. While the liabilities of the Defined Benefit pension fund are decreasing in the short run due to the increasing path of the nominal interest rate, the liabilities of the Defined Contribution pension fund recover quickly due to the marking up of previously accumulated pension wealth. The Defined Benefit pension fund instead lowers the contribution rate and thus makes the accrual of new pension wealth relatively cheap.[14] The plots of the effective contribution rates highlight this.

Figure 4 presents a plot of various important macroeconomic variables in the Defined Benefit, Defined Contribution and Laissez-Faire economies. The cheap accrual of new pension wealth under the Defined Benefit pension system implies that labour supply is subsidised. As a result, the economic downturn is mitigated compared to the Defined Contribution and Laissez-Faire economies. The comparatively high labour supply leads to a lower wage rate and marginal cost, meaning that the retail firms that can change their prices set a lower reset price. This in turn leads to a lower inflation rate and nominal interest rate along the adjustment path and explains why the liabilities of the pension fund do not collapse as much with a Defined Benefit pension fund as with a Defined Contribution fund.[15] Since retirees have a lower productivity

[13]Note that the inflation is not caused by a jump in the risk premium since we consider a model without aggregate risk.

[14]This explains why the assets of the pension fund drop by roughly 5.5% in the Defined Benefit case. The fund draws down its assets because it collects less contributions while it continues to pay the pensions of retirees.

[15]Figure 2 shows that the opposite is the case under the real accounting framework. However, the pension fund finances are unaffected by the inflation rate in the real accounting framework and thus the higher inflation rate in the Defined Benefit case does not affect the restoration policy of the pension fund.

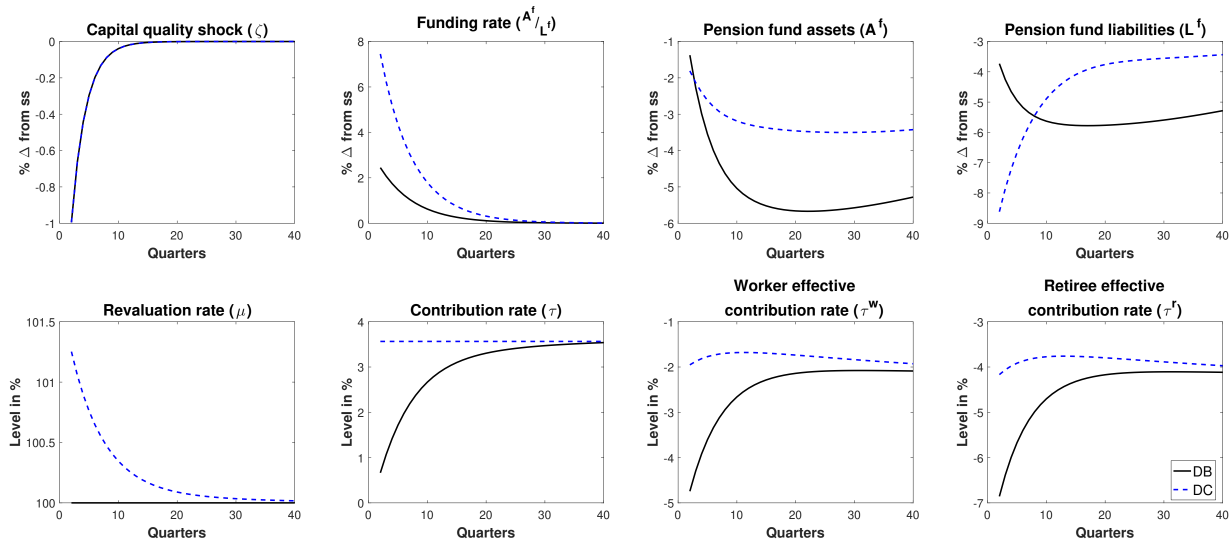

Figure 3: Pension fund restoration policy after a 1% capital quality shock in a New-Keynesian model with a nominal pension fund framework. Defined Benefit is denoted by the solid black line, while Defined Contribution is denoted by the striped blue line.

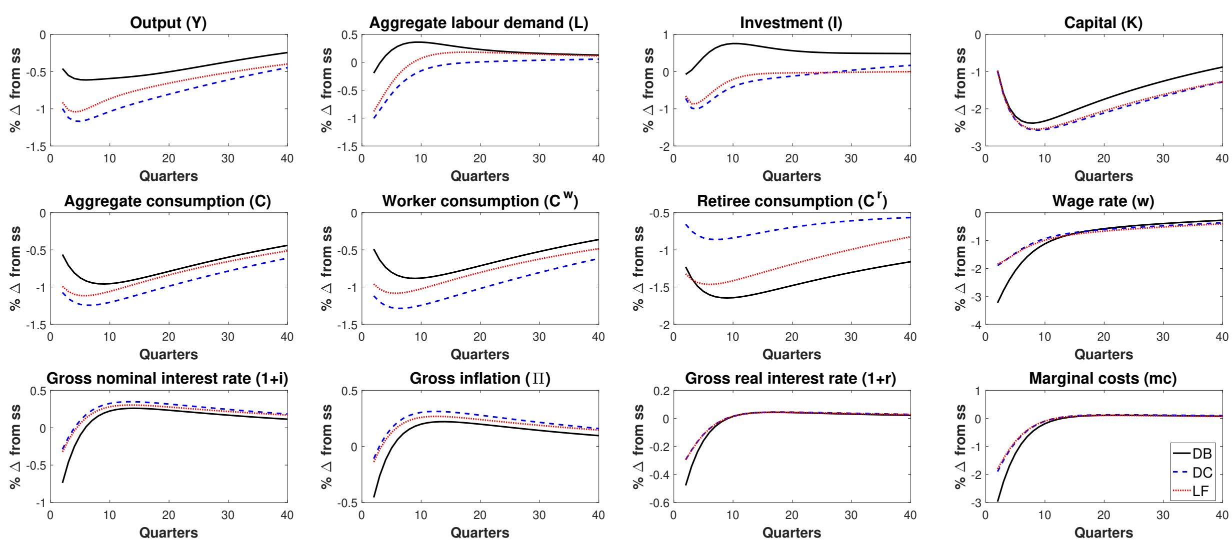

Figure 4: Effect of pension fund restoration policy after a 1% capital quality shock on macroeconomic variables in a New-Keynesian model with a nominal pension fund framework. Defined Benefit is denoted by the solid black line, while Defined Contribution is denoted by the striped blue line and Laissez-Faire is denoted by the dotted red line.

compared to workers, it is difficult for them to accumulate sufficient additional pension wealth to offset the evaporation of their previously accumulated pension wealth. Therefore, retirees consume less under a Defined Benefit system compared to a Defined Contribution system, while the opposite is the case for workers. Again it turns out that the effect of the labour supply distortion of the Defined Benefit pension fund outweighs the effect of the higher marginal propensity to consume out of wealth of retirees. Furthermore, it is important to appreciate that a rather small adverse capital quality shock of 1% can have sizeable effects on the finances of a nominally defined pension fund, especially when the pension fund introduces an implicit subsidy or tax on labour supply which influences the pricing decisions of retail firms and in turn the financial position of the pension fund.

While the Defined Contribution and the Laissez-Faire economy behave similarly under the real accounting framework, we observe considerable differences between the two under the nominal accounting framework. Figure 3 shows that the effective contribution rate of workers increases with the Defined Contribution pension fund, meaning that the labour supply of workers is distorted downwards. This stems from the fact that accumulating additional pension wealth is less attractive due to the relatively high level of the inflation rate in response to the adverse capital quality shock. Workers are affected negatively not only by the implicit tax on labour supply, but also by the fact that their previously accumulated pension wealth is marked up in the first periods after the shock and afterwards, as inflation picks up, written down again. Retirees on the other hand are less reliant on their labour income and, due to their short remaining lifetime, benefit from receiving more pension benefits in the initial periods after the adverse capital quality shock.

3.3 Welfare effects of pension fund restoration policy

We now turn to an assessment of the welfare effects of the various forms of pension fund restoration policy to see which pension fund system each group of individuals prefers. We conduct a welfare analysis that is similar to Jaag et al. (2010) who compute equivalent variations when comparing the desirability of policies. The equivalent variation EV^z measures the lump-sum transfer a group of individuals with labour market status $z \in \{w, r\}$, initial financial assets A^z_{t-1} and pension entitlements B^z_t must receive under scenario 1 to obtain the same utility as in scenario 0. That is, the equivalent variation between scenario 0 and 1 is defined by:

$$V^z(A^{z,1}_{t-1} + EV^z, B^{z,1}_t, \Gamma^1_t) = V^z(A^{z,0}_{t-1}, B^{z,0}_t, \Gamma^0_t),\ z \in \{w, r\},$$

where Γ^i_t, a scenario i at period t, denotes all relevant aggregate information on factor prices and pension fund restoration policy from period t onwards.[16] Condition (31) highlights that total consumption is linear in total lifetime wealth and condition (32) highlights that the indirect lifetime utility is linear in total consumption. We use this to calculate the equivalent variation:

$$EV^z\left(A^{z,0}_{t-1}, A^{z,1}_{t-1}, B^{z,0}_t, B^{z,1}_t, \Gamma^0_t, \Gamma^1_t\right) = \frac{V^z\left(A^{z,0}_{t-1}, B^{z,0}_t, \Gamma^0_t\right) - V^z\left(A^{z,1}_{t-1}, B^{z,1}_t, \Gamma^1_t\right)}{\frac{\partial V^z\left(A^{z,1}_{t-1}, B^{z,1}_t, \Gamma^1_t\right)}{\partial A^{z,1}_{t-1}(1+r^1_t)}},\ z \in \{w, r\}.$$

[16] Note that the equivalent variation is not necessarily symmetric in the environments, while differences are minor. Also note that we do not actually implement the wealth transfers, but consider the equivalent variations to be useful hypotheticals to asses the relative attractiveness of pension fund arrangements.

Let time period 0 denote the steady state period and period 1 denote the period in which the adverse capital quality shock materialises. Additionally, let DC denote the scenario of the Defined Contribution economy and DB denote the scenario of the Defined Benefit scenario. We then consider the equivalent variations of the following three groups of individuals.

Group of individuals	**Equivalent Variation**
Retirees alive at $t = 1$	$EV^r\left(A_0^r, A_0^r, B_1^{r,DC}, B_1^{r,DB}, \Gamma_1^{DC}, \Gamma_1^{DB}\right)$
Workers alive at $t = 1$	$EV^w\left(A_0^w, A_0^w, B_1^{w,DC}, B_1^{w,DB}, \Gamma_1^{DC}, \Gamma_1^{DB}\right)$
Workers born after $t = 1$	$\sum_{i=2}^{\infty}\prod_{j=2}^{i}\left(\frac{1}{1+r_j^{DB}}\right)(1-\omega)\,EV^w\left(0,0,0,0,\Gamma_i^{DC}, \Gamma_i^{DB}\right)$

For ease of interpretation, we express the equivalent variations as a share of yearly steady state output. Table 4 depicts the welfare effects of switching from a Defined Benefit pension fund to a Defined Contribution pension fund in the period in which the adverse capital quality shock materialises for the baseline calibration and various model set-ups.[17] In the real business cycle model all individuals alive at period $t = 1$ prefer a Defined Benefit pension fund over a Defined Contribution fund, while the future generations prefer the opposite. However, the desirability of a Defined Benefit pension fund arrangement diminishes in a New-Keynesian environment where aggregate demand becomes important. While it is unsurprising that the group of retirees prefers a Defined Benefit pension fund in the real accounting framework, all workers now prefer the Defined Contribution pension fund. To workers, the adverse labour supply distortions in the Defined Benefit pension fund outweigh the positive effect of allocating more wealth to the group of individuals with the highest proclivity to consume in the margin. It is important to relate this finding to the pension fund literature. Without taking nominal rigidities into consideration, Bonenkamp and Westerhout (2014) and Draper et al. (2017) conclude for Defined Benefit pension funds that the welfare gain from intergenerational risk sharing dominates the cost of labour supply distortions, which is consistent with our findings in a real business cycle model. However, the nominal rigidities in the New-Keynesian model exacerbate the labour supply distortions, making the Defined Contribution pension fund more desirable to all groups of individuals.

Under the nominal accounting framework, the adverse capital quality shock depresses the value of accumulated pension wealth so much that retirees prefer the pension funding surplus to be paid out through increases in the valuation of previously accumulated pension wealth rather than through discounts on the accumulation of new pension wealth. Conversely, since workers are still active on the labour market and have relatively less dependence on accumulated pension wealth, workers prefer the pension funding surplus to be distributed through lower contribution rates.

Table 4 highlights that there is no preferred pension fund arrangement. Each system distributes welfare losses or gains over different groups of individuals and therefore there is no unanimous agreement between workers, retirees and future generations about optimal pension fund design. The sum of the equivalent variations indicates that in a real accounting framework a Defined Benefit pension fund is preferred and in a nominal accounting framework a Defined Contribution pension fund is preferred. However, the sum is close to zero

[17] The welfare effects of the real business cycle model are obtained by switching off the New-Keynesian elements described in the model section.

Group of individuals	**Real business cycle**	**New-Keynesian Real framework**	**New-Keynesian Nominal Framework**
Retirees alive at $t = 1$	-0.44%	-0.41%	$+1.45\%$
Workers alive at $t = 1$	-0.14%	$+0.11\%$	-0.36%
Workers born after $t = 1$	$+0.07\%$	$+0.13\%$	-0.36%
Total	-0.51%	-0.17%	$+0.73\%$

Table 4: Welfare effects of switching from a Defined Benefit pension fund to a Defined Contribution pension fund in various model environments after an adverse shock to capital quality of 1%. Measured as an equivalent variation showing the transfer of wealth as a percentage of steady state yearly output necessary for indifference between the DC and DB pension fund.

and furthermore depends on the rate used to discount the equivalent variations of future generations and the welfare weights attached to different groups of individuals. For simplicity we weigh each group equally and discount with the real interest rate, but one could make sensible arguments for different welfare weights and discount factors. Nevertheless, the welfare effects allow us to draw a consistent conclusion: when the pension fund faces a deficit, retirees prefer the labour market to be distorted and the value of their pension wealth to be guaranteed while workers prefer the opposite. When the pension fund faces a surplus, retirees prefer that the value of their pension wealth is marked up and that the accrual of new pension wealth is relatively expensive while workers prefer the opposite.

It is important to consider the limitations of our model framework when interpreting the reported welfare effects. Typically, a Defined Benefit pension fund would reduce the volatility of retiree income compared to a Defined Contribution arrangement, which in a set-up containing aggregate risk and risk aversion could lead workers to favour a Defined Benefit system. However, our perfect foresight economy with risk neutral preferences does not take this characteristic of a Defined Benefit pension fund into account. Workers only appreciate the Defined Benefit pension fund in the sense that it guarantees the value of their previously accumulated pension wealth, which is an asset that pays out conditional on the specific lifecycle stage of the individual and mitigates the implications of the idiosyncratic risk of becoming retired.

3.4 Sensitivity analyses

To test the robustness of our findings, we vary the values of the intertemporal elasticity of substitution σ, the size of the pension fund and the closure speed of the funding gap and assess how the reported welfare effects from the previous section are affected. The baseline calibration is characterised by $\sigma = \frac{1}{3}$, $\frac{A^f}{4Y} = 0.88$ and a half-life of one year.

Within the literature of adapted Gertler (1999)-models the calibrated values range range between $\frac{1}{4}$ and $\frac{1}{2}$, and we report the welfare effects for these two values in table 5. We adjust the accrual and contribution rates such that the size of the pension fund remains $\frac{A^f}{4Y} = 0.88$ in the steady state. In the real accounting framework retirees more strongly prefer a Defined Benefit pension fund for higher levels of σ because the funding gap is larger after the adverse capital quality shock materialises. The workers who are alive at $t = 1$ also more strongly prefer a Defined Benefit pension fund, because at a higher level of σ the subjective

Equivalent variations	Real business cycle			
	Retirees alive at $t = 1$	Workers alive at $t = 1$	Workers born after $t = 1$	**Total**
$\sigma = \frac{1}{4}$	-0.35%	-0.08%	$+0.06\%$	-0.37%
$\sigma = \frac{1}{3}$	-0.44%	-0.14%	$+0.07\%$	-0.51%
$\sigma = \frac{1}{2}$	-0.50%	-0.28%	$+0.08\%$	-0.70%
$\frac{A^f}{4Y} = 0.50$	-0.22%	-0.07%	$+0.04\%$	-0.25%
$\frac{A^f}{4Y} = 0.88$	-0.44%	-0.14%	$+0.07\%$	-0.51%
$\frac{A^f}{4Y} = 1.25$	-0.72%	-0.22%	$+0.10\%$	-0.84%
Half-life = 1 year	-0.44%	-0.14%	$+0.07\%$	-0.51%
Half-life = 2 years	-0.41%	-0.25%	$+0.09\%$	-0.57%
Half-life = 4 years	-0.37%	-0.39%	$+0.12\%$	-0.64%

Equivalent variations	New-Keynesian, real framework			
	Retirees alive at $t = 1$	Workers alive at $t = 1$	Workers born after $t = 1$	**Total**
$\sigma = \frac{1}{4}$	-0.31%	$+0.16\%$	$+0.08\%$	-0.07%
$\sigma = \frac{1}{3}$	-0.41%	$+0.11\%$	$+0.13\%$	-0.17%
$\sigma = \frac{1}{2}$	-0.47%	$+0.00\%$	$+0.16\%$	-0.31%
$\frac{A^f}{4Y} = 0.50$	-0.21%	$+0.08\%$	$+0.07\%$	-0.06%
$\frac{A^f}{4Y} = 0.88$	-0.41%	$+0.11\%$	$+0.13\%$	-0.17%
$\frac{A^f}{4Y} = 1.25$	-0.67%	$+0.11\%$	$+0.21\%$	-0.35%
Half-life = 1 year	-0.41%	$+0.11\%$	$+0.13\%$	-0.17%
Half-life = 2 years	-0.37%	$+0.01\%$	$+0.17\%$	-0.19%
Half-life = 4 years	-0.32%	-0.14%	$+0.22\%$	-0.23%

Equivalent variations	New-Keynesian, nominal framework			
	Retirees alive at $t = 1$	Workers alive at $t = 1$	Workers born after $t = 1$	**Total**
$\sigma = \frac{1}{4}$	$+1.90\%$	-0.52%	-0.56%	$+0.82\%$
$\sigma = \frac{1}{3}$	$+1.45\%$	-0.36%	-0.36%	$+0.73\%$
$\sigma = \frac{1}{2}$	$+0.97\%$	-0.06%	-0.21%	$+0.70\%$
$\frac{A^f}{4Y} = 0.50$	$+1.11\%$	-0.01%	-0.43%	$+0.67\%$
$\frac{A^f}{4Y} = 0.88$	$+1.45\%$	-0.36%	-0.36%	$+0.73\%$
$\frac{A^f}{4Y} = 1.25$	$+1.52\%$	-0.66%	-0.24%	$+0.62\%$
Half-life = 1 year	$+1.45\%$	-0.36%	-0.36%	$+0.73\%$
Half-life = 2 years	$+1.31\%$	-0.01%	-0.46%	$+0.84\%$
Half-life = 4 years	$+1.13\%$	$+0.46\%$	-0.58%	$+1.01\%$

Table 5: Welfare effects of switching from a Defined Benefit pension fund to a Defined Contribution pension fund for various changes to the baseline calibration. Measured as an equivalent variation showing the transfer of wealth as a percentage of steady state yearly output necessary for indifference between the DC and DB pension fund. The baseline calibration is characterised by $\sigma = \frac{1}{3}$, $\frac{A^f}{4Y} = 0.88$ and half-life = 1 year.

reweighting of transition probabilities variable Ω is higher, implying that they are more eager to guarantee the value of their previously accumulated pension wealth. The workers born after $t = 1$, on the other hand, do not have previously accumulated pension wealth and are negatively affected by their distorted labour supply for higher levels of σ. In the nominal accounting framework the effects are the opposite. For $\sigma = \frac{1}{4}$ the funding surplus is larger due to a higher inflation path. Retirees then more strongly prefer a Defined Contribution pension fund for lower levels of σ, while the opposite holds for all workers who more strongly prefer the cheap accrual of new pension wealth to a revaluation of previously accumulated pension wealth.

We consider both a smaller pension fund (with pension fund assets equal to 50% of yearly output, the OECD average in 2016) and a larger one (with pension fund assets equal to 125% of yearly output, the weighted OECD average in 2016). Table 5 indicates that qualitatively the reported results for the default calibration are maintained and that the stakes are simply scaled up. The only exception comes from the welfare of the future generations in a nominal accounting framework, who have a less pronounced preference for the Defined Benefit pension fund as it manages more assets. This stems from the fact that the funding gap is larger for the smaller pension fund due to a higher path for inflation. In the Defined Benefit system the effective contribution rate on labour income is therefore relatively lower (in terms of relative deviation from its steady state value) for the smaller pension fund compared to the larger pension fund.

Lastly, we consider slower recoveries with a half-life of eight and twelve quarters. When the pension fund postpones the closure of its funding gap in the real accounting framework, retirees in the meantime receive a pension that more closely matches what was promised to them before the adverse capital quality shock materialised. As such, the retiree preference for either type of pension fund diminishes. The workers alive at $t = 1$ have a similar preference, because with a longer half-life labour supply is distorted comparatively less in the first periods after the adverse capital quality shock and more in future periods. The workers born after $t = 1$ are on the receiving end of these distortions and therefore more strongly prefer a Defined Contribution pension fund as the closure speed becomes lower. In the nominal accounting framework, the individuals alive in period $t = 1$ have a stronger preference for their preferred pension system when the recovery speed is higher because then the funding surplus is distributed more quickly. The future generations, however, more strongly prefer a Defined Benefit pension fund with a longer recovery as they then capture a larger portion of the cheap accrual of new pension wealth.

4 Conclusions

This paper provides an assessment of the business cycle effects and distributional implications of pension fund restoration policy by extending a canonical New-Keynesian dynamic general equilibrium model with a tractable demographic structure and a flexible pension fund framework. We build on the overlapping generations framework of Gertler (1999), who introduces lifecycle behaviour in a business cycle model, while the production sectors are inspired by Kara and von Thadden (2016) and incorporate investment adjustment costs, imperfect competition in the retail sector and nominal Calvo (1983)-pricing rigidities. As a novelty, the pension fund framework of Romp (2013) is expanded to allow for a choice of the target funding rate, whether individuals accumulate real or nominal pension benefits, the recovery speed in case of a funding gap and the instruments used to restore financial adequacy.

This model is used to investigate how the macroeconomy responds to an unexpected Gertler and Karadi (2011)-type capital quality shock when financial adequacy is restored by revaluing previously accumulated pension wealth (Defined Contribution) or changing the pension fund contribution rate on labour income (Defined Benefit). Economies with Defined Contribution pension funds generally respond similarly to adverse capital quality shocks as economies without pension funds, while Defined Benefit pension funds distort labour supply decisions and exacerbate economic fluctuations. Unsurprisingly, retirees prefer Defined Benefit over Defined Contribution funds in case they face deficits, while the current and future working population prefers the opposite. More importantly, the model indicates that the induced labour supply distortions by deficit-restoring pension funds are quantitatively important, especially for the working population. The intergenerational risk sharing literature, which has abstracted from nominal rigidities and distortions that materialise at a business cycle frequency, thus overstates the welfare improvement of Defined Benefit pension funds by understating their potential for distorting labour supply.

An important shortcoming of this paper is the lack of aggregate risk and risk aversion. While the literature on the risk sharing possibilities of pension funds neglects the important labour supply distortions of Defined Benefit pension funds, this paper ignores the risk sharing properties of different pension fund systems. The incorporated risks in the model and the preferences of individuals towards them facilitates aggregation despite the heterogeneity at the micro-level. However, for a holistic welfare perspective on pension fund system design the labour market distortions and risk sharing properties of pension fund systems have to be considered jointly. Other possible extensions are to introduce more states to the life cycle as in Grafenhofer et al. (2006) to allow for richer lifecycle dynamic and to generalise the pension fund framework to allow for varying contribution, accrual and revaluation rates for different groups of individuals and generations such that individual pension fund systems can be contrasted to the collective pension schemes discussed here.

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A Decision problems of retirees and workers

We introduce some notation in order to make the derivations more readable. While we still solve the decision problems of individual retirees and workers, we drop the superscripts i and j. Furthermore, denote with $V_2^r(a_t^r, b_{t+1}^r)$ the derivative of the value function of a retiree in period $t+1$ with respect to per-period pension benefits b_{t+1}^r (i.e. the second state variable). We only show the derivations for the real accounting framework since those for the nominal accounting framework are analogous.

A.1 Retiree decision problem

A retiree maximises the following objective in period t:

$$V^r(a_{t-1}^r, b_t^r) = \max_{c_t^r, l_t^r, a_t^r, b_{t+1}^r} \left(\left((c_t^r)^v (1 - l_t^r)^{1-v} \right)^\rho + \gamma\beta \left(V^r(a_t^r, b_{t+1}^r) \right)^\rho \right)^{\frac{1}{\rho}}$$

subject to:

$$\begin{aligned} a_t^r &= \frac{1 + r_t}{\gamma} a_{t-1}^r + (1 - \tau_t)\xi w_t l_t^r + \mu_t b_t^r - c_t^r, \\ b_{t+1}^r &= \mu_t b_t^r + \nu_t \xi w_t l_t^r. \end{aligned}$$

Substituting the constraints:

$$V^r(a_{t-1}^r, b_t^r) =$$
$$\max_{c_t^r, l_t^r} \left(\left((c_t^r)^v (1 - l_t^r)^{1-v} \right)^\rho + \gamma\beta \left(V^r\left(\frac{1 + r_t}{\gamma} a_{t-1}^r + (1 - \tau_t)\xi w_t l_t^r + \mu_t b_t^r - c_t^r, \mu_t b_t^r + \nu_t \xi w_t l_t^r\right) \right)^\rho \right)^{\frac{1}{\rho}}.$$

A.1.1 First-order conditions

The first-order condition with respect to c_t^r:

$$v \left(c_t^r\right)^{v\rho-1} \left(1 - l_t^r\right)^{(1-v)\rho} = \beta\gamma \left(V^r \left(a_t^r, b_{t+1}^r\right)\right)^{\rho-1} V_1^r \left(a_t^r, b_{t+1}^r\right). \tag{A.1}$$

Using the envelope theorem:

$$V_1^r \left(a_{t-1}^r, b_t^r\right) = \left(V^r \left(a_{t-1}^r, b_t^r\right)\right)^{1-\rho} v \frac{1 + r_t}{\gamma} \left(c_t^r\right)^{v\rho-1} \left(1 - l_t^r\right)^{(1-v)\rho}. \tag{A.2}$$

Shifting (A.2) one period forward and combining with (A.1) gives the Euler equation:

$$\frac{c_{t+1}^r}{c_t^r} = \beta(1 + r_{t+1}) \frac{\left((c_{t+1}^r)^v (1 - l_{t+1}^r)^{1-v}\right)^\rho}{\left((c_t^r)^v (1 - l_t^r)^{1-v}\right)^\rho}. \tag{A.3}$$

The first-order condition with respect to l_t^r:

$$(1-v)\left(c_t^r\right)^{v\rho}\left(1-l_t^r\right)^{(1-v)(\rho-1)} = \beta\gamma\left(V^r\left(a_t^r,b_{t+1}^r\right)\right)^{\rho-1}\left(V_1^r\left(a_t^r,b_{t+1}^r\right)(1-\tau_t)\xi w_t + V_2^r\left(a_t^r,b_{t+1}^r\right)\mu_{t+1}\nu_t\xi w_t\right) \Leftrightarrow$$

$$(1-v)\left(c_t^r\right)^{v\rho}\left(1-l_t^r\right)^{(1-v)(\rho-1)} = \beta\gamma\left(V^r\left(a_t^r,b_{t+1}^r\right)\right)^{\rho-1}V_1^r\left(a_t^r,b_{t+1}^r\right)\left(1-\tau_t^r\right)\xi w_t, \tag{A.4}$$

where we use the linearity of the consumption function in total lifetime wealth to determine that $V_2^r\left(a_t^r,b_{t+1}^r\right) = R_{t+1}^r\frac{\gamma}{1+r_{t+1}}V_1^r\left(a_t^r,b_{t+1}^r\right)$ and define $\tau_t^r = \tau_t - (R_t^r-1)\nu_t$. Working one extra unit of time in period t gives $\mu_{t+1}\nu_t\xi w_t$ additional per-period pension benefits from period $t+1$ onwards. $V_2^r\left(a_t^r,b_{t+1}^r\right)$ denotes the proper valuation of one additional accrued unit of per-period pension benefits. Recall that the annuity factor $R_{t+1}^r = 1+\mu_{t+2}\frac{\gamma}{1+r_{t+1}}R_{t+2}^r$ represents the present discounted value to a retiree in period $t+1$ of receiving one consumption good each period from period $t+1$ until death (corrected for future revaluation). One additional accrued unit of per-period pension benefits from period $t+1$ onwards is therefore equally valuable to a retiree as having $R_{t+1}^r\frac{\gamma}{1+r_{t+1}}$ additional units of a_t^r. Combining (A.4) with (A.1):

$$1-l_t^r = \frac{1-v}{v}\frac{c_t^r}{(1-\tau_t^r)\xi w_t}. \tag{A.5}$$

A.1.2 Writing the Euler equation solely in terms of consumption

Substituting (A.5) into (A.3):

$$\frac{c_{t+1}^r}{c_t^r} = \left(\beta(1+r_{t+1})\left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma}, \tag{A.6}$$

where we have used that $\sigma = \frac{1}{1-\rho}$. We define retiree full consumption as $x_t^r \equiv c_t^r + \left(1-l_t^r\right)\left(1-\tau_t^r\right)\xi w_t = \frac{c_t^r}{v}$, which follows the same Euler equation as c_t^r:

$$x_\tau^r = x_t^r\prod_{s=t}^{\tau-1}\left(\beta(1+r_{s+1})\left(\frac{(1-\tau_s^r)w_s}{(1-\tau_{s+1}^r)w_{s+1}}\right)^{(1-v)\rho}\right)^{\sigma} \quad \forall\ \tau = t, t+1, \ldots$$

A.1.3 Deriving the full consumption function and indirect value function

Let retiree full income d_t^r and retiree human wealth h_t^r be defined as:

$$d_t^r = (1-\tau_t^r)\xi w_t,$$
$$h_t^r = d_t^r + \frac{\gamma}{1+r_{t+1}}h_{t+1}^r.$$

Iterating the budget constraint forwards and imposing a transversality condition gives the lifetime budget constraint and full consumption function:

$$\sum_{\tau=t}^{\infty}\left(\prod_{s=t}^{\tau-1}\frac{\gamma}{1+r_{s+1}}\right)x_\tau^r=\frac{1+r_t}{\gamma}a_{t-1}^r+h_t^r+\mu_t b_t^r R_t^r \Leftrightarrow$$
$$x_t^r=\frac{1}{\Delta_t^r}\left(\frac{1+r_t}{\gamma}a_{t-1}^r+h_t^r+\mu_t b_t^r R_t^r\right),$$

with Δ_t^r the inverse marginal propensity to consume (of full consumption) out of total wealth (using that $\sigma=\frac{1}{1-\rho}$ and $\sigma\rho=\sigma-1$):

$$\Delta_t^r=1+\gamma\beta^\sigma\Delta_{t+1}^r\left((1+r_{t+1})\left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}}\right)^{1-v}\right)^{\sigma-1}.$$

Writing out the indirect retiree value function:

$$(V_t^r)^\rho=\sum_{s=t}^{\infty}\left((\beta\gamma)^{s-t}c_s^r\left(\frac{1-v}{v}\frac{1}{(1-\tau_s^r)\xi w_s}\right)^{1-v}\right)^\rho \Leftrightarrow$$
$$V_t^r=(\Delta_t^r)^{\frac{1}{\rho}}vx_t^r\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}.$$

A.2 Worker decision problem

A worker maximises the following objective in period t:

$$V^w(a_{t-1}^w,b_t^w)=\max_{c_t^w,l_t^w,a_t^w,b_{t+1}^w}\left(\left((c_t^w)^v(1-l_t^w)^{1-v}\right)^\rho+\beta\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^\rho\right)^{\frac{1}{\rho}},$$

subject to the constraints that become operative once he retires and subject to:

$$a_t^w=(1+r_t)\,a_{t-1}^w+(1-\tau_t)w_t l_t^w+f_t^w-c_t^w,$$
$$b_{t+1}^w=\mu_t b_t^w+\nu_t w_t l_t^w.$$

Substituting the constraints:

$$V^w(a_{t-1}^w,b_t^w)=\max_{c_t^w,l_t^w}\Bigg(\left((c_t^w)^v(1-l_t^w)^{1-v}\right)^\rho+\beta\omega\Big(V^w\left((1+r_t)\,a_{t-1}^w+(1-\tau_t)w_t l_t^w+f_t^w-c_t^w,\mu_t b_t^w+\nu_t w_t l_t^w\right)+$$
$$(1-\omega)V^r\left((1+r_t)\,a_{t-1}^w+(1-\tau_t)w_t l_t^w+f_t^w-c_t^w,\mu_t b_t^w+\nu_t w_t l_t^w\right)\Big)^\rho\Bigg)^{\frac{1}{\rho}}.$$

A.2.1 First-order conditions

The first-order condition with respect to c_t^r:

$$v\left(c_t^w\right)^{v\rho-1}\left(1-l_t^w\right)^{(1-v)\rho}= \beta\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right), \quad (A.7)$$

where we can find $V_1^w\left(a_t^w,b_{t+1}^w\right)$ and $V_1^r\left(a_t^w,b_{t+1}^w\right)$ using the envelope theorem and shifting the conditions one period forward:

$$V_1^w\left(a_t^w,b_{t+1}^w\right)=\left(V^w\left(a_t^w,b_{t+1}^w\right)\right)^{1-\rho}v\left(1+r_{t+1}\right)\left(c_{t+1}^w\right)^{v\rho-1}\left(1-l_{t+1}^w\right)^{(1-v)\rho}, \quad (A.8)$$

$$V_1^r\left(a_t^w,b_{t+1}^w\right)=\left(V^r\left(a_t^w,b_{t+1}^w\right)\right)^{1-\rho}v\left(1+r_{t+1}\right)\left(c_{t+1}^r\right)^{v\rho-1}\left(1-l_{t+1}^r\right)^{(1-v)\rho}. \quad (A.9)$$

The first-order condition with respect to l_t^r:

$$\begin{aligned}(1-v)\left(c_t^w\right)^{v\rho}\left(1-l_t^w\right)^{(1-v)(\rho-1)}&= \\ &\beta(1-\tau_t)w_t\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right)+ \\ &\beta\mu_{t+1}\nu_t w_t\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega V_2^w(a_t^w,b_{t+1}^w)+(1-\omega)V_2^r\left(a_t^w,b_{t+1}^w\right)\right). \quad (A.10)\end{aligned}$$

As in the case of the retiree, it is required to determine the proper valuation of obtaining an additional unit of accrued per-period pension benefits in case the worker remains a worker in period $t+1$, $V_2^w(a_t^w,b_{t+1}^w)$, and in case the worker retires in period $t+1$, $V_2^r\left(a_t^w,b_{t+1}^w\right)$. As in section A.1.1 it holds that $V_2^r\left(a_t^w,b_{t+1}^w\right)=R_{t+1}^r\frac{1}{1+r_{t+1}}V_1^r\left(a_t^w,b_{t+1}^w\right)$, where γ is omitted since an individual who is a worker in period t and retired in period $t+1$ reaps a return on his private financial wealth of $1+r_{t+1}$. Anticipating that the worker consumption function is linear in perceived total lifetime wealth, it holds that $V_2^w\left(a_t^w,b_{t+1}^w\right)=R_{t+1}^w\frac{1}{1+r_{t+1}}V_1^w\left(a_t^w,b_{t+1}^w\right)$. Recall that the annuity factor $R_{t+1}^w=\frac{\mu_{t+2}}{1+r_{t+2}}\left(\frac{\omega}{\Omega_{t+2}}R_{t+2}^w+(1-\frac{\omega}{\Omega_{t+2}})R_{t+2}^r\right)$ represents the present discounted value to a worker in period $t+1$ of receiving one consumption good each period in which he is retired in the future (corrected for future revaluation μ and the subjective reweighting of transition probabilities Ω). Using this in (A.10):

$$\begin{aligned}(1-v)\left(c_t^w\right)^{v\rho}\left(1-l_t^w\right)^{(1-v)(\rho-1)}&= \\ &\beta(1-\tau_t)w_t\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right)+ \\ \beta\frac{\mu_{t+1}}{1+r_{t+1}}\nu_t w_t&\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega R_{t+1}^w V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)R_{t+1}^r V_1^r\left(a_t^w,b_{t+1}^w\right)\right).\end{aligned}$$

We conjecture that the following equivalency holds:

$$\frac{\mu_{t+1}}{1+r_{t+1}}\left(\omega R_{t+1}^w V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)R_{t+1}^r V_1^r\left(a_t^w,b_{t+1}^w\right)\right)= R_t^w\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right). \quad (A.11)$$

After deriving the consumption and indirect value function of the worker, we will verify that the above equivalency indeed holds. This will ensure that all conjectures add up to consistent solutions across all equations characterising the optimal decisions of retirees and workers. Defining $\tau_t^w = \tau_t - R_t^w \nu_t$ then gives:

$$\begin{aligned}
&(1-v)\left(c_t^w\right)^{v\rho}\left(1-l_t^w\right)^{(1-v)(\rho-1)} = \\
&\beta(1-\tau_t^w)w_t\left(\omega V^w(a_t^w, b_{t+1}^w) + (1-\omega)V^r(a_t^w, b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)V_1^r\left(a_t^w, b_{t+1}^w\right)\right). \quad \text{(A.12)}
\end{aligned}$$

Combining (A.12) with (A.7):

$$1 - l_t^w = \frac{1-v}{v}\frac{c_t^w}{(1-\tau_t^w)w_t}. \tag{A.13}$$

A.2.2 Writing the Euler equation solely in terms of consumption

We define worker full consumption as $x_t^w \equiv c_t^w + (1-l_t^w)(1-\tau_t^w)w_t = \frac{c_t^w}{v}$. Substituting this, the optimal labour supply decisions (A.5) and (A.13), and the envelope conditions (A.8) and (A.9) into (A.7), the first-order condition with respect to c_t^w, gives the worker Euler equation:

$$\begin{aligned}
(x_t^w)^{\rho-1} = \beta(1+r_{t+1})&\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{(1-v)\rho}\left(\omega V^w\left(a_t^w, b_{t+1}^w\right) + (1-\omega)V^r\left(a_t^w, b_{t+1}^w\right)\right)^{\rho-1} \\
&\left(\omega\left(V^w\left(a_t^w, b_{t+1}^w\right)\right)^{1-\rho}\left(x_{t+1}^w\right)^{\rho-1} + (1-\omega)\left(V^r\left(a_t^w, b_{t+1}^w\right)\right)^{1-\rho}\left(x_{t+1}^r\right)^{\rho-1}\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{(1-v)\rho}\right).
\end{aligned}$$

In section A.1.3 we have shown that $V_t^r = (\Delta_t^r)^{\frac{1}{\rho}} v x_t^r \left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}$. Conjecture similarly that $V_t^w = (\Delta_t^w)^{\frac{1}{\rho}} v x_t^w \left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}$. Denote with $\Omega_t = \omega + (1-\omega)\left(\frac{1-\tau_t^w}{1-\tau_t^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_t^w}{\Delta_t^r}\right)^{\frac{1}{1-\sigma}}$. Plugging these in the above condition and cancelling out terms:

$$\begin{aligned}
&\omega x_{t+1}^w + (1-\omega)x_{t+1}^r\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_{t+1}^w}{\Delta_{t+1}^r}\right)^{\frac{\sigma}{1-\sigma}} = \\
&\qquad\left(\beta(1+r_{t+1})\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma} x_t^w. \quad \text{(A.14)}
\end{aligned}$$

We can now show that, using (A.14), our conjecture for the value function implies the following difference equation for Δ^w:

$$V^w(a^w_{t-1}, b^w_t) = \max_{c^w_t, l^w_t, a^w_t, b^w_{t+1}} \left(\left((c^w_t)^v (1-l^w_t)^{1-v} \right)^\rho + \beta \left(\omega V^w(a^w_t, b^w_{t+1}) + (1-\omega) V^r(a^w_t, b^w_{t+1}) \right)^\rho \right)^{\frac{1}{\rho}} \Leftrightarrow$$

$$\left((\Delta^w_t)^{\frac{1}{\rho}} v x^w_t \left(\frac{1-v}{v} \frac{1}{(1-\tau^w_t) w_t} \right)^{1-v} \right)^\rho = \left(v x^w_t \left(\frac{1-v}{v} \frac{1}{(1-\tau^w_t) w_t} \right)^{1-v} \right)^\rho +$$

$$\beta \left(\omega \left(\Delta^w_{t+1} \right)^{\frac{1}{\rho}} v x^w_{t+1} \left(\frac{1-v}{v} \frac{1}{(1-\tau^w_{t+1}) w_{t+1}} \right)^{1-v} + (1-\omega) \left(\Delta^r_{t+1} \right)^{\frac{1}{\rho}} v x^r_{t+1} \left(\frac{1-v}{v} \frac{1}{(1-\tau^r_{t+1}) \xi w_{t+1}} \right)^{1-v} \right)^\rho \Leftrightarrow$$

$$\Delta^w_t = 1 + \beta^\sigma \Delta^w_{t+1} \left((1+r_{t+1}) \, \Omega_{t+1} \left(\frac{(1-\tau^w_t) w_t}{(1-\tau^w_{t+1}) w_{t+1}} \right)^{1-v} \right)^{\sigma-1}. \quad \text{(A.15)}$$

A.2.3 Deriving the full consumption function

Using (A.14) we can show that the difference equation for Δ^wgiven by (A.15) is consistent with the following full consumption function:

$$\begin{aligned}
x^w_t &= \frac{1}{\Delta^w_t} \left((1+r_t) \, a^w_{t-1} + h^w_t + \mu_t b^w_t R^w_t \right), \\
d^w_t &= (1-\tau^w_t) w_t + f^w_t, \\
h^w_t &= d^w_t + \frac{1}{1+r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} h^w_{t+1} + (1 - \frac{\omega}{\Omega_{t+1}}) h^r_{t+1} \right),
\end{aligned}$$

where h^w_t is the perceived human wealth of a worker and d^w_t worker full income. Substituting the above full consumption function in (A.14) indeed gives the same difference equation for Δ^w:

$$\begin{aligned}
&\omega \frac{1}{\Delta^w_{t+1}} \left((1+r_{t+1}) \, a^w_t + h^w_{t+1} + \mu_{t+1} b^w_{t+1} R^w_{t+1} \right) + \\
&\quad (1-\omega) \left(\frac{1-\tau^w_{t+1}}{1-\tau^r_{t+1}} \frac{1}{\xi} \right)^{1-v} \left(\frac{\Delta^w_{t+1}}{\Delta^r_{t+1}} \right)^{\frac{\sigma}{1-\sigma}} \frac{1}{\Delta^r_{t+1}} \left((1+r_{t+1}) \, a^w_t + h^r_{t+1} + \mu_{t+1} b^w_{t+1} R^r_{t+1} \right) = \\
&\quad \left(\beta (1+r_{t+1}) \Omega_{t+1} \left(\frac{(1-\tau^w_t) w_t}{(1-\tau^w_{t+1}) w_{t+1}} \right)^{(1-v)\rho} \right)^\sigma \frac{1}{\Delta^w_t} \left((1+r_t) \, a^w_{t-1} + h^w_t + \mu_t b^w_t R^w_t \right) \Leftrightarrow
\end{aligned}$$

$$\Delta^w_t \frac{a^w_t + h^w_t - d^w_t + b^w_{t+1} R^w_t}{(1+r_t) \, a^w_{t-1} + h^w_t + \mu_t b^w_t R^w_t} = \beta^\sigma \Delta^w_{t+1} \left((1+r_{t+1}) \Omega_{t+1} \left(\frac{(1-\tau^w_t) w_t}{(1-\tau^w_{t+1}) w_{t+1}} \right)^{(1-v)} \right)^{\sigma-1} \Leftrightarrow$$

$$\Delta^w_t = 1 + \beta^\sigma \Delta^w_{t+1} \left((1+r_{t+1}) \, \Omega_{t+1} \left(\frac{(1-\tau^w_t) w_t}{(1-\tau^w_{t+1}) w_{t+1}} \right)^{1-v} \right)^{\sigma-1}.$$

Since it holds that $1-\frac{1}{\Delta_t^w}=\frac{a_t^w+h_t^w-d_t^w+b_{t+1}^w R_t^w}{(1+r_t)a_{t-1}^w+h_t^w+\mu_t b_t^w R_t^w}$, which can be shown using the worker budget constraint:

$$\begin{aligned}
a_t^w &= (1+r_t)\,a_{t-1}^w+(1-\tau_t)w_t l_t^w+f_t^w-c_t^w \Leftrightarrow\\
a_t^w+h_t^w &= (1+r_t)a_{t-1}^w+h_t^w+d_t^w-x_t^w+(\tau^w-\tau)w_t l_t^w \Leftrightarrow\\
a_t^w+h_t^w-d_t^w &= (1+r_t)a_{t-1}^w+h_t^w-R_t^w\left(b_{t+1}^w-\mu_t b_t^w\right)-x_t^w \Leftrightarrow\\
a_t^w+h_t^w-d_t^w+b_{t+1}^w R_{t+1}^w &= (1+r_t)a_{t-1}^w+h_t^w+\mu_t b_t^w R_t^w-\frac{1}{\Delta_t^w}\left((1+r_t)\,a_{t-1}^w+h_t^w+\mu_t b_t^w R_t^w\right) \Leftrightarrow\\
1-\frac{1}{\Delta_t^w} &= \frac{a_t^w+h_t^w-d_t^w+b_{t+1}^w R_{t+1}^w}{(1+r_t)a_{t-1}^w+h_t^w+\mu_t b_t^w R_t^w}.
\end{aligned}$$

This confirms that our conjectures of the worker full consumption function and the worker indirect value function are mutually consistent and are similar to those of the retiree.

A.2.4 Coming back to the worker first-order condition for labour

Now that we have derived the expressions for the subjective reweighting of transition probabilities Ω_t and the indirect value functions of the worker V_t^w and retiree V_t^r, we show that the assumed equivalency (A.11) indeed holds.

$$\begin{aligned}
&\frac{\mu_{t+1}}{1+r_{t+1}}\left(\omega R_{t+1}^w V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)R_{t+1}^r V_1^r\left(a_t^w,b_{t+1}^w\right)\right)=\\
&\qquad\qquad R_t^w\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right)\Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
&\omega R_{t+1}^w V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)R_{t+1}^r V_1^r\left(a_t^w,b_{t+1}^w\right)=\\
&\qquad\qquad\left(\frac{\omega}{\Omega_{t+1}}R_{t+1}^w+(1-\frac{\omega}{\Omega_{t+1}})R_{t+1}^r\right)\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right)\Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
\Omega_{t+1} &= \omega+(1-\omega)\frac{V_1^r\left(a_t^w,b_{t+1}^w\right)}{V_1^w\left(a_t^w,b_{t+1}^w\right)}\Leftrightarrow\\
\Omega_{t+1} &= \omega+(1-\omega)\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_{t+1}^w}{\Delta_{t+1}^r}\right)^{\frac{1}{1-\sigma}},
\end{aligned}$$

where in the last line we use that, for an individual who is working in period t and retires in period $t+1$, $V_1^r\left(a_t^w,b_{t+1}^w\right)=(1+r_{t+1})\left(\Delta_t^r\right)^{\frac{1}{\sigma-1}}\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}$, while $V_1^w\left(a_t^w,b_{t+1}^w\right)=(1+r_{t+1})\left(\Delta_t^w\right)^{\frac{1}{\sigma-1}}\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}$. This expression for Ω_{t+1} is identical to how it is defined in section (A.2.2), therefore confirming our conjecture.

B Decision problems of firms and government

B.1 Final goods sector

There is a continuum of retail firms, indexed by $z \in [0,1]$. The perfectly competitive final goods sector assembles the differentiated retail goods according to:

$$Y_t = \left(\int_0^1 (Y_{z,t})^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{B.1}$$

where $\epsilon > 1$ is the elasticity of demand for the intermediate goods purchased from different retail firms. Each retail good $Y_{z,t}$ is produced by one retail firm (which is also indexed by z) and sold at the nominal price $P_{z,t}$. The final goods producing sector maximises profits taking all prices (P_t, the nominal price of the final good, and $P_{z,t}$, $\forall z \in [0,1]$) as given:

$$\max_{Y_{z,t}} P_t Y_t - \int_0^1 P_{z,t} Y_{z,t} dz.$$

Using (B.1) and differentiating with respect to a particular $Y_{z,t}$ gives rise to the following demand function for the output of a particular retail good z producing firm:

$$Y_{z,t} = Y_t \left(\frac{P_{z,t}}{P_t} \right)^{-\epsilon}. \tag{B.2}$$

Imposing zero profits in the final goods sector maximisation problem yields that the price of the final good can be understood as an average of the retail firm prices:

$$P_t = \left(\int_0^1 (P_{z,t})^{1-\epsilon} dz \right)^{\frac{1}{1-\epsilon}}. \tag{B.3}$$

B.2 Capital producing sector

At the end of period t, the competitive capital producing sector purchases the remaining stock of capital $(1-\delta)\zeta_t K_{t-1}$ from the intermediate goods producing firms at the real price q_t. This capital is combined with I_t units of investment (in the form of output purchased from final goods producers) to produce next period's beginning of period stock of capital K_t. This stock of capital is then sold to the intermediate goods producing firms at the real price q_t. The capital producing sector faces convex adjustment costs when transforming final goods into capital. Capital evolves as follows:

$$K_t = (1-\delta)\,\zeta_t K_{t-1} + \left(1 - S[\frac{I_t}{I_{t-1}}] \right) I_t, \tag{B.4}$$

with $S[\frac{I_t}{I_{t-1}}] = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$. This capital evolution specification contains investment adjustment costs in the sense that investing I_t final goods in period t will only increase tomorrow's capital stock by $\left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t$. This specification is similar to Fernandez-Villaverde (2006) and Christiano et al. (2005), and κ (the second derivative of $S[\frac{I_t}{I_{t-1}}]$) represents the severity of the investment adjustment costs. In period t the profits of the capital producing sector are given by $\Pi_t^c = q_t K_t - q_t(1-\delta)\zeta_t K_{t-1} - I_t$. The capital producing sector maximises the present discounted value of profits, where we substitute (B.4) in Π_t^c:

$$\max_{\{I_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \prod_{s=1}^{i} \frac{1}{1+r_{t+s}} \left(q_{t+i} \left(1 - S[\frac{I_{t+i}}{I_{t+i-1}}] \right) I_{t+i} - I_{t+i} \right).$$

Profits (which can arise outside of the steady state) are redistributed lump sum to the group of workers. Differentiating with respect to investment I_t gives the following condition for the investment path:

$$1 = q_t \left(1 - S[\frac{I_t}{I_{t-1}}] + \frac{I_t}{I_{t-1}} S'[\frac{I_t}{I_{t-1}}] \right) + \frac{q_{t+1}}{1+r_{t+1}} \left(\frac{I_{t+1}}{I_t} \right)^2 S[\frac{I_{t+1}}{I_t}].$$

B.3 Intermediate goods sector

There is a continuum of competitive intermediate good producing firms indexed by $j \in [0,1]$. The intermediate good j is produced by the intermediate good j producer according to:

$$Y_{j,t} = \left(\zeta_t K_{j,t-1}\right)^{\alpha} \left(L_{j,t}\right)^{1-\alpha}, \tag{B.5}$$
$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \varepsilon_t.$$

Capital quality is denoted by ζ_t, follows an AR(1)-process and is subject to the unanticipated shock ε_t. $L_{j,t}$ and $K_{j,t-1}$ denote the employed labour and capital by the intermediate good j producing firm. As previously mentioned, the intermediate good producing firms purchase their employed capital for period $t+1$ from the capital producing sector in period t and therefore capital used for production in period t is indexed by $t-1$. A negative realisation of ε_t decreases the quality of the capital stock such that the effective capital used in production in period t is $\zeta_t K_{j,t-1}$. The intermediate good producing firms produce output $Y_{j,t}$ and hire labour $L_{j,t}$ at a unit cost of w_t. The markets for labour and capital are perfectly competitive and so the intermediate good j producing firm takes their prices as given. The intermediate good producers sell their output to the retail firms at the real price mc_t. After production, the remaining effective capital stock is sold back to the capital producing sector at the real price q_t. The intermediate good producing firms finance their capital purchases each period by obtaining funds from the households and the pension fund. We assume that there are no frictions in the process of obtaining these funds. The intermediate good producing firms offer the households and the pension fund a perfectly state-contingent security, which is best interpreted as equity.

The period t profits of the intermediate good j producing firm are given by:

$$\Pi_{j,t}^i = mc_t \left(\zeta_t K_{j,t-1}\right)^{\alpha} \left(L_{j,t}\right)^{1-\alpha} + q_t(1-\delta)\zeta_t K_{j,t-1} - w_t L_{j,t} - (1+r_t) q_{t-1} K_{j,t-1},$$

which consists of the sale of output to retail firms $mc_t \left(\zeta_t K_{j,t-1}\right)^{\alpha} \left(L_{j,t}\right)^{1-\alpha}$, the sale of the remaining capital stock to the capital producing sector $q_t(1-\delta)\zeta_t K_{j,t-1}$, the hiring of labour $w_t L_{j,t}$ and the repayment of previous period's borrowed funds $(1+r_t)q_{t-1}K_{j,t-1}$. The intermediate good j producing firm maximises the present discounted value of profits taking all prices as given:

$$\max_{\{K_{j,t+i},L_{j,t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \prod_{s=1}^{i} \frac{1}{1+r_{t+s}} \Pi^{i}_{j,t+i}.$$

Differentiating with respect to $L_{j,t}$ and $K_{j,t}$ gives the following first-order conditions for labour and capital, respectively:

$$w_t = (1-\alpha)\, mc_t \frac{Y_{j,t}}{L_{j,t}}, \tag{B.6}$$

$$q_t = \frac{1}{1+r_{t+1}} \left(\alpha mc_{t+1} \frac{Y_{j,t+1}}{K_{j,t}} + q_{t+1}(1-\delta)\zeta_{t+1} \right).$$

Since the intermediate goods sector is perfectly competitive, per-period profits are zero state-by-state. Using (B.6) in $\Pi^{i}_{j,t} = 0$ gives the required ex post return on capital the intermediate good producing firms pay out to the households and pension fund, confirming the perfectly state-contingent nature of the traded security:

$$1 + r_t = \frac{\alpha mc_t \frac{Y_{j,t}}{K_{j,t-1}} + q_t(1-\delta)\zeta_t}{q_{t-1}}. \tag{B.7}$$

Rewriting (B.6) and (B.7) gives the factor demands:

$$L_{j,t} = (1-\alpha)\, mc_t \frac{Y_{j,t}}{w_t}, \tag{B.8}$$

$$K_{j,t-1} = \frac{\alpha mc_t Y_{j,t}}{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}. \tag{B.9}$$

From this it follows that all intermediate good producing firms employ the same capital-labour ratio:

$$\frac{K_{j,t-1}}{L_{j,t}} = \frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}.$$

Substituting the factor demands into the production function of the intermediate good j producer, we obtain the real intermediate good price mc_t:

$$mc_t = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}{\zeta_t \alpha} \right)^{\alpha}.$$

B.4 Retail sector

Recall that there is a continuum of retail firms, indexed by $z \in [0,1]$. After purchasing output from the intermediate good producing firms at the real price mc_t, the retail firms convert the intermediate goods sector output into retail goods which are sold to the final goods sector at the nominal price $P_{z,t}$. The intermediate goods are converted one-to-one into retail goods, which entails that the retailers simply repackage the intermediate goods. We assume that each retail firm produces a differentiated retail good $Y_{z,t}$ such that it operates in a monopolistically competitive market and charges a markup over the input price mc_t.

Additionally, we introduce nominal rigidities by means of Calvo (1983)-type pricing frictions. By construction, each period a fraction $1-\theta$ of retail firms can adjust its price (which it will do so in an optimal fashion, taking into account the probability that it might not be able to change its price in future periods) and a fraction θ of firms cannot adjust its price. Denote with $P^*_{z,t}$ the nominal optimal reset price in period t of retail firm z that can change its price. Since the group of workers are assumed to receive the profits of the retail firms, the appropriate pricing kernel used to value profits received in i periods is $\beta^i \frac{\Lambda_{t+i}}{\Lambda_t}$ with $\Lambda_t = v\left(\Delta_t^w\right)^{\frac{\rho+1}{\rho}} \left(\frac{1-v}{v} \frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}$ being the marginal value to a worker of receiving one additional unit of lifetime wealth in period t. Indeed, the pricing kernel is the same for all workers because the marginal propensity to consume out of wealth is the same for all workers.

When retail firm z is allowed to change its price in period t, it solves the following optimisation problem:

$$\max_{P^*_{z,t}} \sum_{i=0}^{\infty} (\beta\theta)^i \frac{\Lambda_{t+i}}{\Lambda_t} \left(\frac{P^*_{z,t}}{P_{t+i}} - mc_{t+i}\right) Y_{z,t+i} \text{ s.t. } Y_{z,t+i} = Y_{t+i} \left(\frac{P^*_{z,t}}{P_{t+i}}\right)^{-\epsilon}.$$

Profit maximisation yields the following first-order condition:

$$\sum_{i=0}^{\infty} (\beta\theta)^i \Lambda_{t+i} \left((1-\epsilon) \frac{P^*_{z,t}}{P_t} \left(\prod_{s=1}^{i} \frac{1}{\Pi_{t+s}}\right)^{1-\epsilon} + \epsilon mc_{t+i} \left(\prod_{s=1}^{i} \frac{1}{\Pi_{t+s}}\right)^{-\epsilon} \right) Y_{t+i} = 0,$$

where $\Pi_{t+s} = \frac{P_{t+s}}{P_{t+s-1}}$. Reorganising and realising that the symmetric nature of the economic environment implies that all price adjusting firms will choose the same price, i.e. $P^*_t = P^*_{z,t}\ \forall z$, yields the following condition characterising the optimal real reset price $\Pi^*_t = \frac{P^*_t}{P_t}$:

$$\Pi^*_t = \frac{\epsilon}{\epsilon-1} \frac{\sum_{i=0}^{\infty} (\beta\theta)^i \Lambda_{t+i} mc_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}}{\sum_{i=0}^{\infty} (\beta\theta)^i \Lambda_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}}. \tag{B.10}$$

To express the first-order condition (B.10) recursively, we write it as $\Pi^*_t = \frac{\epsilon}{\epsilon-1} \frac{g^1_t}{g^2_t}$ with:

$$g^1_t = \Lambda_t mc_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon} g^1_{t+1},$$
$$g^2_t = \Lambda_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon-1} g^2_{t+1}.$$

Because of the Calvo-pricing rigidity a share $1-\theta$ of retail firms can adjust its price and sets it to $P_{z,t} = P^*_t$ and a share θ of retail firms cannot adjust its price and has to set it to $P_{z,t} = P_{z,t-1}$. This gives in (B.3) the evolution of the aggregate price level as a geometric average of the past aggregate price level and the current optimal price:

$$1 = \theta \left(\Pi_t\right)^{\epsilon-1} + (1-\theta) \left(\Pi^*_t\right)^{1-\epsilon}.$$

B.5 Government and central bank

Since the government is non-Ricardian in this model, we elect to minimise the role of the fiscal authority so as to not distort our research findings regarding the macroeconomic and welfarian implications of various types of pension fund restoration policy. As such, we rule out government purchases. We suppose that the central bank follows a Taylor rule with interest rate smoothing. The monetary authority responds to deviations of inflation from the target inflation rate $\bar{\Pi}$ and to deviations of output from steady state output $\bar{Y}$:

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\eta_i} \left(\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\eta_\Pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\eta_Y}\right)^{1-\eta_i},$$

where $\bar{i}$ is the steady-state nominal interest rate, $\eta_i \in (0,1)$ the interest rate smoothing parameter, η_Π the inflation coefficient and η_Y the output coefficient. Additionally, the Fisher relation holds:

$$1+i_t = \Pi_{t+1}\left(1+r_{t+1}\right).$$

B.6 Aggregation

For the output markets to clear it is required that $\int_0^1 Y_{z,t}dz = \int_0^1 Y_{j,t}dj = Y_t \int_0^1 \left(\frac{P_{z,t}}{P_t}\right)^{-\epsilon} dz$, for the labour market to clear it is required that $\int_0^1 L_{j,t}dj = L_t$ and for the capital market to clear it is required that $\int_0^1 K_{j,t}dj = K_t$. Integrating the factor demand conditions (B.8) and (B.9) over j gives the aggregate factor demand conditions:

$$L_t = (1-\alpha)mc_t \frac{Y_t v_t^p}{w_t}, \tag{B.11}$$

$$K_{t-1} = \frac{\alpha mc_t Y_t v_t^p}{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}, \tag{B.12}$$

where $v_t^p = \int_0^1 \left(\frac{P_{z,t}}{P_t}\right)^{-\epsilon} dz$ is a measure of price dispersion. Because of the Calvo-pricing rigidity a share $1-\theta$ of retail firms can adjust its price and sets it to $P_{z,t} = P_t^*$ and a share θ of retail firms cannot adjust its price and has to set it to $P_{z,t} = P_{z,t-1}$. This allows us to express v_t^p recursively:

$$v_t^p = (1-\theta)\left(\Pi_t^*\right)^{-\epsilon} + \theta\left(\Pi_t\right)^{\epsilon} v_{t-1}^p. \tag{B.13}$$

Aggregate supply is obtained through integrating (B.5) over j and using that $\frac{K_{j,t-1}}{L_{j,t}} = \frac{K_{t-1}}{L_t}$, $\forall j$ and that $\int_0^1 L_{j,t}dj = L_t$:

$$Y_t v_t^p = \left(\zeta_t K_{t-1}\right)^{\alpha}\left(L_t\right)^{1-\alpha}.$$

Aggregate demand satisfies:

$$Y_t = C_t + I_t.$$

Savings market clearing requires that the total value of savings (which is the sum of the private financial wealth of workers and retirees and the end-of-period assets of the pension fund) equates the total value of the capital stock:

$$A_t^w + A_t^r + \frac{A_{t+1}^f}{1 + r_{t+1}} = q_t K_t.$$

Aggregate profits (comprised of those of the retail sector and the capital goods sector) are given by:

$$F_t = (1 - mc_t v_t^p) \, Y_t + q_t \left(1 - S\left[\frac{I_t}{I_{t-1}}\right]\right) I_t - I_t. \tag{B.14}$$

C Equilibrium conditions

C.1 Pension fund

C.1.1 Real pension fund framework

Private annuity factors of retirees and workers:

$$R_t^r = 1 + \gamma \frac{\mu_{t+1}}{1 + r_{t+1}} R_{t+1}^r \tag{C.1}$$

$$R_t^w = \frac{\mu_{t+1}}{1 + r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} R_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) R_{t+1}^r \right) \tag{C.2}$$

Pension fund annuity factors of retirees and workers:

$$R_t^{r,f} = 1 + \frac{\gamma}{1 + r_{t+1}} R_{t+1}^{r,f} \tag{C.3}$$

$$R_t^{w,f} = \frac{1}{1 + r_{t+1}} \left(\omega R_{t+1}^{w,f} + (1 - \omega) R_{t+1}^{r,f} \right) \tag{C.4}$$

Aggregate per-period pension benefits of retirees and workers:

$$B_t^r = \gamma \left(\mu_{t-1} B_{t-1}^r + \nu_{t-1} \xi w_{t-1} L_{t-1}^r \right) + (1 - \omega) \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w \right) \tag{C.5}$$

$$B_t^w = \omega \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w \right) \tag{C.6}$$

Pension fund assets and liabilities:

$$A_t^f = (1 + r_t) \left(A_{t-1}^f + \tau_{t-1} w_{t-1} L_{t-1} - \mu_{t-1} B_{t-1}^r \right) \tag{C.7}$$

$$L_t^f = R_t^{r,f} B_t^r + R_t^{w,f} B_t^w \tag{C.8}$$

C.1.2 Nominal pension fund framework

Private annuity factors of retirees and workers:

$$R_t^r = 1 + \gamma \frac{\mu_{t+1}}{1 + i_t} R_{t+1}^r \tag{C.9}$$

$$R_t^w = \frac{\mu_{t+1}}{1 + i_t} \left(\frac{\omega}{\Omega_{t+1}} R_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) R_{t+1}^r \right) \tag{C.10}$$

Pension fund annuity factors of retirees and workers:

$$R_t^{r,f} = 1 + \frac{\gamma}{1 + i_t} R_{t+1}^{r,f} \tag{C.11}$$

$$R_t^{w,f} = \frac{1}{1 + i_t} \left(\omega R_{t+1}^{w,f} + (1 - \omega) R_{t+1}^{r,f} \right) \tag{C.12}$$

Aggregate per-period pension benefits of retirees and workers:

$$\Pi_t B_t^r = \gamma \left(\mu_{t-1} B_{t-1}^r + \nu_{t-1} \xi w_{t-1} L_{t-1}^r\right) + (1-\omega)\left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w\right) \tag{C.13}$$

$$\Pi_t B_t^w = \omega \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w\right) \tag{C.14}$$

Pension fund assets and liabilities:

$$A_t^f = (1+r_t)\left(A_{t-1}^f + \tau_{t-1} w_{t-1} L_{t-1} - \mu_{t-1} B_{t-1}^r\right) \tag{C.15}$$

$$L_t^f = R_t^{r,f} B_t^r + R_t^{w,f} B_t^w \tag{C.16}$$

C.1.3 Restoration policy

Pension fund restoration policy is set such that the following condition is satisfied:

$$\underbrace{\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}}_{\text{closure fraction}} \underbrace{\left(A_t^f - \bar{f}r L_t^f\right)}_{\text{funding gap}} = \bar{f}r\left(\underbrace{\frac{1-\bar{f}r}{\bar{f}r}\mu_t B_t^r + (\mu_t - 1)L_t^f}_{\text{revaluation}} + \underbrace{\nu_t w_t \left(\left(R_t^{r,f} - 1\right)\xi_t L_t^r + R_t^{w,f} L_t^w\right)}_{\text{accrual}}\right) - \underbrace{\tau_t w_t L_t}_{\text{contribution}}$$

This gives the following pension fund policy in the Defined Benefit case (with $\bar{\nu}$ exogenously given):

$$\mu_t = 1 \tag{C.17}$$

$$\nu_t = \bar{\nu} \tag{C.18}$$

$$\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}\left(A_t^f - \bar{f}r L_t^f\right) = \bar{f}r\left(\frac{1-\bar{f}r}{\bar{f}r} B_t^r + \bar{\nu} w_t \left(\left(R_t^{r,f} - 1\right)\xi_t L_t^r + R_t^{w,f} L_t^w\right)\right) - \tau_t w_t L_t \tag{C.19}$$

This gives the following pension fund policy in the Defined Contribution case (with $\bar{\tau}$ and $\bar{\nu}$ exogenously given):

$$\tau_t = \bar{\tau} \tag{C.20}$$

$$\nu_t = \bar{\nu} \tag{C.21}$$

$$\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}\left(A_t^f - \bar{f}r L_t^f\right) = \bar{f}r\left(\frac{1-\bar{f}r}{\bar{f}r}\mu_t B_t^r + (\mu_t - 1)L_t^f + \bar{\nu} w_t \left(\left(R_t^{r,f} - 1\right)\xi_t L_t^r + R_t^{w,f} L_t^w\right)\right) - \bar{\tau} w_t L_t \tag{C.22}$$

C.2 Workers and retirees

Inverse marginal propensity to consume out of wealth for retirees and workers:

$$\Delta_t^r = 1 + \gamma\beta^\sigma \Delta_{t+1}^r \left((1 + r_{t+1}) \left(\frac{(1 - \tau_t^r) w_t}{(1 - \tau_{t+1}^r) w_{t+1}} \right)^{1-v} \right)^{\sigma - 1} \tag{C.23}$$

$$\Delta_t^w = 1 + \beta^\sigma \Delta_{t+1}^w \left((1 + r_{t+1}) \, \Omega_{t+1} \left(\frac{(1 - \tau_t^w) w_t}{(1 - \tau_{t+1}^w) w_{t+1}} \right)^{1-v} \right)^{\sigma - 1} \tag{C.24}$$

Subjective reweighting of transition probabilities:

$$\Omega_t = \omega + (1 - \omega) \left(\frac{1 - \tau_t^w}{1 - \tau_t^r} \frac{1}{\xi} \right)^{1-v} \left(\frac{\Delta_t^w}{\Delta_t^r} \right)^{\frac{1}{1-\sigma}} \tag{C.25}$$

Effective contribution rates on labour:

$$\tau_t^r = \tau_t - (R_t^r - 1) \, \nu_t \tag{C.26}$$

$$\tau_t^w = \tau_t - R_t^w \nu_t \tag{C.27}$$

Aggregate full consumption of retirees and workers:

$$X_t^z = \frac{1}{\Delta_t^z} \left((1 + r_t) A_{t-1}^z + H_t^z + \mu_t B_t^z R_t^z \right), \quad z \in \{w, r\}. \tag{C.28}$$

Aggregate human wealth of retirees and workers:

$$H_t^r = D_t^r + \frac{\gamma}{1 + r_{t+1}} H_{t+1}^r \tag{C.29}$$

$$H_t^w = D_t^w + \frac{1}{1 + r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} H_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) \frac{1}{\psi} H_{t+1}^r \right) \tag{C.30}$$

Aggregate full income of retirees and workers:

$$D_t^r = N^r (1 - \tau_t^r) \xi w_t \tag{C.31}$$

$$D_t^w = N^w (1 - \tau_t^w) w_t + F_t \tag{C.32}$$

Aggregate consumption of retirees, workers and total population:

$$C_t^z = v X_t^z, \quad z \in \{w, r\}, \tag{C.33}$$

$$C_t = C_t^r + C_t^w \tag{C.34}$$

Aggregate labour supply of retirees, workers and total population, where $w_t^r = \xi w_t$ and $w_t^w = w_t$:

$$L_t^z = N^z - \frac{(1-v)X_t^z}{(1-\tau_t^w)w_t^z}, \quad z \in \{w, r\}, \tag{C.35}$$

$$L_t = L_t^w + \xi L_t^r. \tag{C.36}$$

Aggregate private financial wealth of retirees and workers:

$$A_t^r = (1+r_t)A_{t-1}^r + \mu_t B_t^r + (1-\tau_t)\xi w_t L_t^r - C_t^r + \frac{1-\omega}{\omega} A_t^w \tag{C.37}$$

$$A_t^w = \omega\left((1+r_t)A_{t-1}^w + (1-\tau_t)w_t L_t^w + F_t - C_t^w\right) \tag{C.38}$$

C.3 Firms and government

Production function:

$$Y_t v_t^p = (\zeta_t K_{t-1})^{\alpha} (L_t)^{1-\alpha} \tag{C.39}$$

Aggregate resource constraint:

$$Y_t = C_t + I_t \tag{C.40}$$

Marginal cost:

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}{\zeta_t \alpha}\right)^{\alpha} \tag{C.41}$$

Real interest rate:

$$1 + r_t = \frac{\alpha mc_t v_t^p \frac{Y_t}{K_{t-1}} + q_t(1-\delta)\zeta_t}{q_{t-1}}$$

Capital stock law of motion:

$$K_t = (1-\delta)\zeta_t K_{t-1} + \left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t \tag{C.42}$$

Adjustment costs percentage:

$$S[\frac{I_t}{I_{t-1}}] = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{C.43}$$

Investment:

$$1 = q_t \left(1 - S[\frac{I_t}{I_{t-1}}] + \frac{I_t}{I_{t-1}} S'[\frac{I_t}{I_{t-1}}]\right) + \frac{q_{t+1}}{1+r_{t+1}} \left(\frac{I_{t+1}}{I_t}\right)^2 S[\frac{I_{t+1}}{I_t}] \tag{C.44}$$

Market clearing for savings:

$$A_t^w + A_t^r + \frac{A_{t+1}^f}{1+r_{t+1}} = q_t K_t \tag{C.45}$$

Optimal real reset price:

$$\Pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{g_t^1}{g_t^2} \tag{C.46}$$

$$g_t^1 = \Lambda_t mc_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon} g_{t+1}^1 \tag{C.47}$$

$$g_t^2 = \Lambda_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon-1} g_{t+1}^2 \tag{C.48}$$

Pricing kernel of intermediate goods producing firms:

$$\Lambda_t = v \left(\Delta_t^w\right)^{\frac{\rho+1}{\rho}} \left(\frac{1-v}{v} \frac{1}{(1-\tau_t^w) w_t}\right)^{1-v} \tag{C.49}$$

Evolution of aggregate price level:

$$1 = \theta (\Pi_t)^{\epsilon-1} + (1-\theta)(\Pi_t^*)^{1-\epsilon} \tag{C.50}$$

Price dispersion:

$$v_t^p = (1-\theta) \left(\Pi_t^*\right)^{-\epsilon} + \theta \left(\Pi_t\right)^{\epsilon} v_{t-1}^p \tag{C.51}$$

Profits:

$$F_t = (1 - mc_t v_t^p)\, Y_t + q_t \left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t - I_t \tag{C.52}$$

Fisher relation:

$$1 + i_t = \Pi_{t+1} \left(1 + r_{t+1}\right) \tag{C.53}$$

Monetary policy rule:

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\eta_i} \left(\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\eta_\Pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\eta_Y}\right)^{1-\eta_i} \tag{C.54}$$

Capital quality shock:

$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \varepsilon_t \tag{C.55}$$