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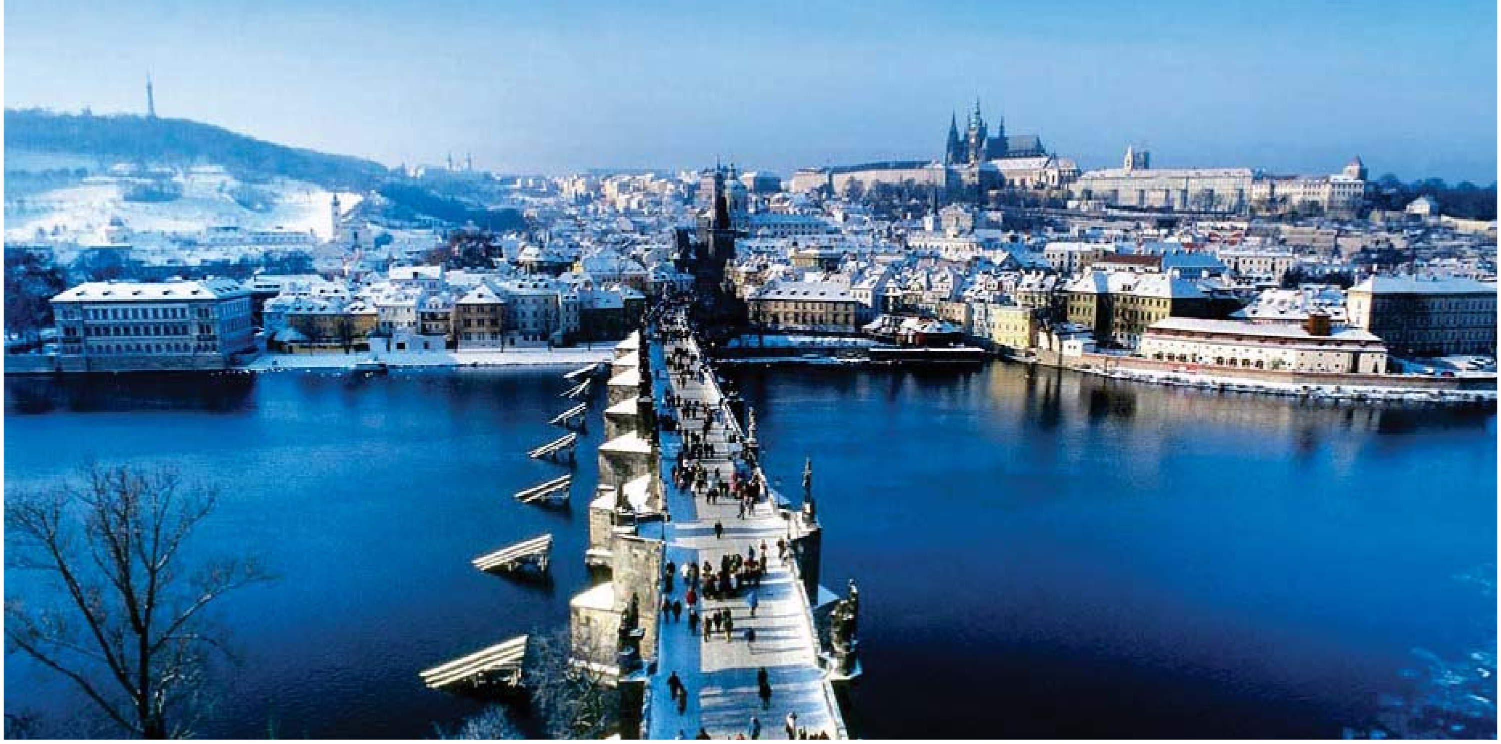

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The effect of high versus low guidance structured tasks on mathematical creativity

Sonia Palha, Jaap Schuitema, Carla van Boxtel and Thea Peetsma

University of Amsterdam, Research Institute of Child Development and Education, Amsterdam, The Netherlands, S.AbrantesGarcezPalha@uva.nl, J.A.Schuitema@uva.nl, C.A.M.vanBoxtel@uva.nl, T.T.D.Peetsma@uva.nl

To engage in challenging tasks, students need to feel some autonomy and competence. Providing structure within the task can help to meet these needs. This study investigates the influence of structure within a modelling task on mathematical creativity among 79 eleventh-grade groups of students. Two versions of the task were developed and the groups were randomly assigned within their classroom to one of these. The analysis explored: (i) the level of mathematical creativity in groups solutions and (ii) if they were dependent on the amount of structure. The results were not statistically significant and, therefore, the question remains open. Additional results and implication of this study to mathematics education are further discussed.

Keywords: Integral calculus, creativity, modelling, collaborative learning, structure.

INTRODUCTION

Researchers express different views with regard to creativity and its connection with the learning environment. Some claim that creativity can be seen as a disposition towards mathematical activity and therefore it can be fostered through specific instruction, such as problem-solving (Silver, 1997). Others see creativity as characteristic of extraordinary individuals (Weisberg, 1988) and thus, not likely to be strongly influenced by the learning environment. Also, several researchers connect creativity to self-regulated learning (Feldhusen & Goh, 1995) and psychological characteristics such as task commitment and motivation (Renzulli, 1978). In our research we share the view that mathematical creativity can be fostered by adequate instruction and we study the relationship between aspects of the learning environment (e.g., task characteristics) and mathematical creativity.

This study is part of a longitudinal intervention research in which we investigate how aspects of the learning environment influences students' motivation, self-regulation and academic performance in mathematics. We developed a learning arrangement in which we used differentiated tasks with a deeper and broader content and method to create a more authentic and challenging learning context. The participants are 16/17 years old students in pre-university education in The Netherlands. Part of our research is to investigate which amount of structure is optimal for the students. We developed two versions of the same learning arrangement. One version consists of low-structured *(LS)* tasks and provides more open tasks, more choice and initiative for students. The other version contains more high-structured *(HS)* tasks, which still provide some choice but also hints, more sub-questions and guidance.

In this paper we discuss our findings with regard to a modelling-task: the parachute jump (Figure 1), which was used within the topic Introduction to Integral Calculus. Modelling-tasks as problem-posing tasks have been seen by several researchers as excellent opportunities for mathematical creativity (Kim & Kim, 2010; Chamberlin & Moon, 2005). The research questions that guided our study were:

- What can we say about the mathematical creativity of students' productions with regard to the parachute jump task?
- In which way does variation in the amount of structure in the parachute jump task influences students' mathematical creativity?

THEORETICAL FRAMEWORK

Mathematical reasoning and creativity

Mathematical creativity can be seen as the ability of students to create useful and original solutions in authentic problem-solving situations (Chamberlin & Moon, 2005). The core activity of the parachute task is to build a model that can be applied in the particular example and other situations. The students' products can then be evaluated in terms of mathematical creativity. In the literature, mathematical creativity is often defined in terms of three components: *flexibility*, *fluency* and *originality* (Silver, 1997; Yuan & Sriraman, 2001). Flexibility can be seen as the ability to generate multiple solutions to a given problem. Fluency can be seen as the ability to use several relevant ideas to solve the task and, in problem-situation tasks it is connected to many interpretations, methods, or answers Silver (1997). Originality concerns different solutions or innovative ways to approach a problem.

Measurement of mathematical creativity remains critical. One reason is the absence of a universal definition applicable in different academic domains (Leikin & Lev, 2013; Kattou, Christou, & Pitta-Pantazi, 2015). Another reason is that one person's creativity can only be assessed indirectly (Piffer, 2012). The ability of posing problems given one mathematical scenario have been linked by several researchers to mathematical creativity (Silver, 1997; Yuan & Sriraman, 2001). Also, over the past years, researchers (Leikin & Lev, 2013) developed an analytical framework that can be used to evaluate creativity in students' productions using the components fluency, flexibility and originality. Mathematical creativity with regard to modelling activities often includes a fourth component: *usefulness*, which concerns the degree of relevance, adaptability and generality of solutions with regard to real world situations (Chamberlin & Moon, 2005). The criterion of usefulness has been contested by some authors. Sriraman (as cited in Yuan & Sriraman, 2001) argues that mathematics creative work might not be useful in terms of its applicability in the real world. Chamberlin and Moon (2005) propose the *Quality Assurance Guide* as a reliable instrument to evaluate creativity in students' products on modelling tasks. Each solution is scored within one of five levels. *Level 1*- requires redirection- the product is on the wrong track and working harder or longer will not improve it. At *level 2*, the product requires major extensions or refinements, the product is a good start towards meeting the goal of the task. At *level 3*, the product is nearly ready to be used; it is useful for the specific data or sharable or reusable. At *level 4*, no changes are needed and at *level 5*, others can use it as tool in similar situations.

High- and Low-structured tasks (HS and LS- tasks)

According to Silver (1997) problem-oriented instruction can assist students to develop more creative approaches to mathematics by increasing their capacity with respect to the core dimensions of creativity: fluency, flexibility, and originality. For instance, ill-structured problems require problem posing and conjecturing, which can foster the generation of novel conjectures. Silver (1997) stated: "It is in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity" (p. 76). However, engaging in problem-solving activity also requires certain ability and disposition to deal with uncertainty and challenge. Aspects of the learning environment that have been found to support the development of such disposition are autonomy support and structure provision (Deci & Ryan, 2000). According to these authors, in autonomy supportive environments students are allowed to make own decisions and are encouraged to solve problems. This can be achieved by providing authentic tasks and opportunities for taking initiative and minimize the use of controlling behaviour. Also, the provisions of structure contributes for students' feeling of competence and therefore is important for motivation. Providing structure involves communicating clear expectations, set limits to students' behaviour and provide help.

Task arrangement

We investigate the relationship between structure provision and mathematical creativity in a problem-oriented arrangement that consisted of the 'parachute jump' task (Figure 1) and small group work. Working together may enhance feelings of relatedness and a sense of autonomy (Schuitema, Peetsma, & Van der Veen, 2011). And, during students' collaboration there is an unpredictable flow of ideas and actions that emerge from the elements of the group while responding to each other. Levenson (2011) states: "Together, the group tries out various strategies and possibly produces solutions based on different mathematical properties or different representations" (p. 230). This is tied to mathematical creativity in the sense that participants must be flexible, establish

(both versions A and B)

Task 25 parachute jump

Dynamical processes, like a train ride, a traveling car and other speed-time processes can be described using a *mathematical model*. A mathematical model may include tables, graphs, formulas or any combination of these representations. These mathematical models can then be used to investigate (and solve) problems through calculations and reasoning or to invent better models to attack the stated problem. In a group of three students, you will create a mathematical model in which the distance travelled against time for a parachutist is described. You also prepare a demonstration (Powerpoint, poster or video clip) of your group's work as a homework task.

But first an example of a parachute jump is presented.

Example.
Imagine the following situation: A parachutist jumps from an airplane. The first five seconds she makes a free fall. Then she opens the parachute and because of that her fall velocity decreases linearly down until after 6 seconds she achieves a fall velocity of 4 meters per second. From this moment on the velocity remains constant during 70 seconds and she lands on the ground at this velocity.

(only version A)

In the example, time is called t (in seconds), with $t = 0$ at the jump from the plane. For the free fall the velocity is given by $v(t) = 9{,}8\ t$ with v in meter per second. The total jump, until reaching the ground, was 561.5 meters. A mathematical model for this example could be a formula (or some collection of formulas), a graph or table in which the falling process is described and that may help to solve the stated problem.

The process for another parachutist will be comparable although different in the three phases of the process.

(only version B)

The process for another parachutist will be comparable, although different in the three phases of the process. That process may be described using a mathematical model. It is usual to start such a model with a concrete example and after that you try to design a more general model or representation. In this class period you will develop a mathematical model that describes the distance traveled against elapsed time for a parachutist. There are guiding questions describing an example that will help you to understand what is going on (questions a-d) and after that you are asked to design your own model (question e).

In the example, time is called t (in seconds), with $t = 0$ at the jump from the plane. For the free fall the velocity is given by $v(t) = 9{,}8\ t$ with v in meter per second.

a. What distance does the parachutist cover during the free fall?

b. What is the total distance covered from start to landing??

a. *(version A)* c. *(version B)*

Watch the Youtube video of a parachutist jump: http://www.youtube.com/watch?v=STDlEFhIPrw. Which similarities and differences do you notice, compared to the situation of the example?

d. Re-watch the video and try to collect data to design the model that describes the parachute jump (describe the data with use of tables/graphs or both).

e. Can you find a relationship between distance covered and time, based on the data you collected fom the video?

b. *(version A)* e. *(version B)*

Create a mathematical model that describes the distance covered during the total parachute jump against time.
After that, you preare a group presentation (powerpoint, poster or video for a 2-5 minutes presentation) in which:
- the mathematical model is presented;
- you give a justification of the choices made;
- show some examples of situations in which the chosen model will work;
- a critical reflection on the model.

Figure 1: Parachute jump task

mathematical relations and approach the task in distinct or novel ways.

The ‘parachute jump’ task was entailed to provide challenge and authentic experiences, as these are important elements of autonomy supportive tasks. It was designed according to the following four criteria.

Appealing and accessible to all students. The context of a parachute jump and the YouTube video make the task interesting to the students. And, the task becomes more accessible by providing an initial example with concrete values and asking to compare it with the one in the video. The pre-knowledge needed to start working on the task was known from previous year (functions, graphs and derivatives).

Authentic. By providing students with an authentic task, and enough freedom of choice we expect that students will be willing to spend thinking effort on it.

Foster mathematical reasoning and creativity. The accomplishment of the task requires the use of mathematical understanding and high-level reasoning. The students must produce at least one representation of the integral function (table, formula, graph, words) and describe its variation at the different instances of the jump. This involves high-level reasoning, as the students must imagine the total accumulating distance varying over time (Thompson & Silverman, 2008).

Suitable for collaborative learning. The task is complex and it can be approached at several levels of understanding. Moreover, the students were encouraged to discuss their ideas and communicate their findings within the group.

Solving the task takes about two lessons of 50 minutes each and some homework time. We agreed with the teachers that the students would work in small groups during one lesson on the task and that they should finish it in their own time (not more than one week). The final product would have the format of a Power Point or a short video-film and would be delivered to the teacher, who would send it to us.

METHOD

Participants and data collection

Seventy-nine groups of 3 students (16/17 years old) from 10 classrooms in 5 schools participated in the study. The data was collected in the spring 2014 and consists of delivered groups products and lesson observations. The groups were formed based on a cognitive ability test. The 40 groups in the LS condition and the 39 groups at the HS condition were, in each classroom, random assigned to one of the conditions.

Instrument used for the evaluation of mathematical creativity

The instrument that we used to evaluate the students' solutions to the parachute jump is based on three of the four components discussed in the theoretical section (we excluded originality because of the difficulty on assessing it in our data).

Usefulness regards the creation of a model that is useful to describe a parachute jump. For each written solution, we decided whether the model was incorrect (level 1), was in the good way but needed major improvements (level 2) or it was ready to be used but needed editing (level 3). Levels 4 and 5 were not observed in our data.

Fluency was seen as the ability to use several mathematical relevant ideas to solve the task. In the context of the parachute task it should be connected to the mathematical concept of the integral function, which is here treated as the total accumulating distance. Based on our theoretical framework, we define mathematical fluency as the ability to (i) link integration and differentiation as inverse processes; (ii) represent the total accumulated distance as a process (operational concept) and as an object (object oriented concept) within at least one functional representation (analytical, graphical, by words or numerical in a table); (iii) Indicate parameters that influence the model and to explain choices made.

Flexibility refers to the ability to set up a model and to use values that go beyond the information provided in the examples.

Analysis

To investigate the first research question we operationalized mathematical creativity in terms of the three components and explored the frequencies found in the students solutions. To investigate the second research question we gave scores to the 3 components and sub-components. Each student solution was then scored within 1–3 for usefulness, 0–2 for each subcategories of fluency, 0–2 for flexibility. We used the Mann-Whitney test, which is indicated for data at ordinal level of measurement, to explore whether the products of the two conditions differed from each other.

RESULTS

Fifty-two of the 79 groups that worked on the task in classroom handed in their final product to the teacher. In the following of this section we report on these products.

Students' creativity in terms of usefulness, fluency and flexibility

The first research question concerned the mathematical creativity of student productions. Table 1 shows that the majority of the groups solutions (36) were at level 1 and therefore, not useful to model the parachute jump. Only 16 groups produced models that could be used.

Usefulness	Groups solutions (N=52)
Level 1	36 (45,6%)
Level 2	15 (19%)
Level 3	1 (1,3%)

Table 1: Results on usefulness

The results on fluency are shown in Table 2. Almost half of the groups (22) explicitly established the link between integration and differentiation. For instance, one group draw both graphs, with the text differentiation and integration and two arrows pointing opposite directions. Most of the solutions (37) presented traces of an operational- oriented conception of total distance. This means that students can draw a total distance graph, use formulas to calculate single values but have difficulty to conceptualize the total distance as a mathematical object on which operations can be performed (Sfard, 1991). Very few groups (7) showed to have an object-oriented conception of total distance. An example of a student explanation that we consider exemplary of object-oriented conception is: "The distance increases at the beginning very fast, during the free fall. After 36 second, when the parachute opens the velocity becomes more or less constant and the distance increases linearly (...)". In contrast, students who would have no functional concept would not refer to distance in their explanations but describe the changes along the jump in terms of velocity, slope of line graphs (the line goes up or down) or in phenomenological terms. Most of the groups (34) did not consider parameters or provided choices.

The results on flexibility are summarized in Table 3. The majority of the groups (35) used only the values from the example. Few groups (14) refer to the values of the video and only 3 groups went beyond the information given in the task setting. Figure 2 contrast one of these solutions (right column) with a solution of the major group.

Influence of HS and LS task on mathematical creativity

The second research question investigates whether the amount of structure in the task has effect on

Fluency	Criteria	Groups solutions (N=52)
link between integration and differentiation	Not visible Unclear Explicit	24 (30,4%) 6 (7,6%) 22 (27,8%)
Conceptions of accumulating distance function	No functional concept Operational concept Object oriented concept	8 (10,1%) 37 (46,8%) 7 (8,9%)
Parameters and choices	No parameters nor choices Parameters or choices Both	34 (43%) 11 (13,9%) 7 (8,9%)

Table 2: Results on fluency

Flexibility	Groups solutions (N=52)
Confined to example or undefined	35 (44,3%)
Beyond example and confined to film	14 (17,7%)
Beyond video and example	3 (3,8%)

Table 3: Results on flexibility

Confined to the example

interval	afstandsgrafiek	invullen
(0-5)	s(x)=4,9x^2	s(5)=122,5
(5-11)	s(x)=-3,75x^2 + 86,5x – 338,75	s(11)=159
(11-81)	s(x)=4x-44	s(81)=280
		561,5m

Beyond the example and the film

"Imagine that you want to make a parachute jump. You want to make a free fall of 7 seconds. After opening the parachute you have a constant velocity of 3 m/s. Opening the parachute takes 4 seconds. After opening it you want to stay 3 minutes in the sky . How high must be the jump?"

Figure 2: Examples of two levels of flexibility

student's mathematical creativity. Table 4 shows the results on usefulness, fluency, and flexibility in both conditions. A Mann-Whitney test indicated that there was no statistically significant difference between the two conditions for all components and sub-components of mathematical creativity.

DISCUSSION

In this paper we explored the influence of task structure on the mathematical creativity in students' productions in the context of a modelling task. Next we discuss our results in the light of the two research questions.

What can we say about the mathematical creativity of students' productions with regard to the parachute jump task? Overall the student solutions attained low scores with regard to the three components of mathematical creativity. Only 52 out 79 groups delivered their final product, none of the groups created a general and reusable solution (levels 4 and 5) and only 16 out of 52 groups have created a model with level 2 or 3. Most students' use of mathematical functions involved thinking in operational views rather than object-oriented. Also, most groups failed in considering relevant side conditions (wind, gravity, etc.) and parameters that are necessary to present a realistic model for the parachute jump. These difficulties suggest that the task as we presented to the students was too challenging for most of them. Several researchers (Silver, 1997; Lithner, 2008) suggest that relationships between creativity and problem solving might be the product of previous instructional patterns. Therefore it is possible that students' previous experiences with mathematical tasks (note that the students are not used to problem-oriented instruction) may have limited their searching process. For instance, only few students tried to go beyond the given examples, as it can be seen by the low levels of usefulness and flexibility. Or, they have tried to explain their choices and present different parameters, as most of the students scored very low on these subcomponents of fluency. Therefore, one suggestion to improve the task is to provide additional information on side conditions that are not part of the mandatory curriculum or provide explicitly directions to look for them. Other suggestion involve the improvement of students' problem-solving activity. The teacher should encourage more the students during the solving process, e.g., to explore different paths, to look for other examples and not to give up too easily. Other aspects that we did not discuss here but also should be taken into consideration are the amount of time available to solve the task in the classroom, the specific directions to be provided by the teachers and assessment practices.

In which way does variation in the amount of structure in the parachute jump task influences students' mathematical creativity? The products created by the groups of students in the two conditions are not statistically significant different with regard to mathematical creativity. Therefore, the question whether providing more/less guidance in the mathematical

	HS-task Median	(N=27) Range	LS-task Median	(N=25) Range	Mann-Whitney U test (two tailed)
Usefulness (scores 1–3)	1	2	1	1	U=257.000, p=.066
Fluency (scores 0–2)					
integration-differentiation	1	2	1	2	U=308.500, p=.559
concept accumulating distance	1	2	1	2	U=304.000, p=.441
parameters and choices	0	2	0	2	U=333.000, p=.992
Flexibility (scores 0–2)	0	2	0	1	U=272.000, p=.144

Table 4: Results on mathematical creativity within high- and low-structured tasks

tasks have impact on students' mathematical creativity remains open. In this paper we studied the effect of task structure on the groups products without refer to the solution process. However the way students approach the tasks and reasoning processes might reveal mathematical creativity aspects of the students not revealed in the final product (Karakok, Milos, Tang, & El Turkey, 2015). This is one question that deserves further investigation. Another interesting question to be further investigated regards the collective creativity process. In our research the students work in small groups, thus the intrapersonal creativity of one student produces a creative product which is then appropriated by others. In this case it is difficult to determine to what extend the final creative ideas and solutions are the product of particular students or from the collective (Levenson, 2011). An interesting question therefore is: in what extend this collective process is mediated by the amount of structure provision in the task?

Concluding, although our study could not provide a conclusive answer to the question whether the amount of structure in the task influences students' mathematical creativity, it contributes to the field of research and teacher education in two ways. It extends previous research on mathematical creativity by accounting the relationship between the learning environment and creativity and, by providing a way to operationalize fluency and flexibility in conceptual mathematical terms. And it provides a practical example (the parachute task) with potential to engage students in problem-solving and concrete suggestions for its implementation. The use of this kind of tasks in the classroom and in teacher education can help teachers to recognize mathematical creativity in their lessons and therefore to better support it.

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