



## UvA-DARE (Digital Academic Repository)

### The effect of high versus low guidance structured tasks on mathematical creativity

Palha, S.; Schuitema, J.; van Boxtel, C.; Peetsma, T.

**Publication date**

2015

**Document Version**

Final published version

**Published in**

CERME9

[Link to publication](#)

**Citation for published version (APA):**

Palha, S., Schuitema, J., van Boxtel, C., & Peetsma, T. (2015). The effect of high versus low guidance structured tasks on mathematical creativity. In K. Krainer, & N. Vondrová (Eds.), *CERME9: proceedings of the Ninth Congress of the European Society for Research in Mathematics Education : 4-8 February 2015* (pp. 1039-1045). Charles University. [http://www.mathematik.uni-dortmund.de/~prediger/ERME/CERME9\\_Proceedings\\_2015.pdf](http://www.mathematik.uni-dortmund.de/~prediger/ERME/CERME9_Proceedings_2015.pdf)

**General rights**

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

**Disclaimer/Complaints regulations**

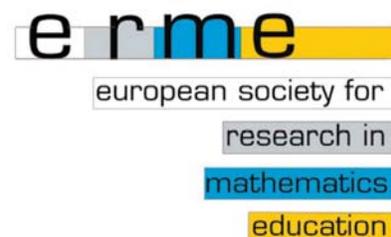
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.



# CERME9

## Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education

**Editors:** Konrad Krainer and Naďa Vondrová  
**Organized by:** Charles University in Prague,  
Faculty of Education  
**Year:** 2015



**Editors:** Konrad Krainer, Naďa Vondrová

**Editorial Board:** Paul Andrews, Samuele Antonini, Véronique Battie, Berta Barquero, Irene Biza, Lisa Björklund Boistrup, Marianna Bosch, Marc Bosse, Laurinda Brown, Orly Buchbinder, Mariolina Bartolini Bussi, Susana Carreira, Sona Ceretkova, Charalambos Charalambous, Aurélie Chesnais, Yves Chevallard, Renaud Chorlay, Anna Chronaki, Kathy Clark, Alison Clark-Wilson, Andreas Eichler, Lisser Rye Ejersbo, Ingvald Erfjord, Lourdes Figueiras, Inés María Gómez-Chacón, Alejandro González-Martín, Simon Goodchild, Ghislaine Gueudet, Corinne Hahn, Jeremy Hodgen, Alena Hospesova, Paola Iannone, Eva Jablonka, Arne Jakobsen, Uffe Thomas Jankvist, Darina , Jirotková, Gabriele Kaiser, Alexander Karp, Sibel Kazak, Ivy Kidron, Iveta Kohanová, Eugenia Koleza, Snezana Lawrence, Aisling Leavy, Roza Leikin, Esther Levenson, Peter Liljedahl, Thomas Lingefjord, Matija Lokar, Jan van Maanen, Božena Maj-Tatsis, Nicolina Malara, Mirko Maracci, Michela Maschietto, Pietro Di Martino, Heidi Strømskag Måsøval, Tamsin Meaney, Julia Meinke, Mônica Mesquita, Joris Mithalal, John Monaghan, Francesca Morselli, Reidar Mosvold, Elena Nardi, Reinhard Oldenburg, Hanna Palmér, Demetra Pitta Pantazi, Marilena Pantziara, Kirsten Pfeiffer, Núria Planas, Valentina Postelnicu, Despina Potari, Caterina Primi, Elisabeth Rathgeb-Schnierer, Sebastian Rezat, Miguel Ribeiro, Philippe R. Richard, Ornella Robutti, Bettina Rösken-Winter, Frode Rønning, Charalambos Sakonidis, Stanislaw Schukajlow, Marcus Schütte, Nathalie Sinclair, Florence Mihaela Singer, Jeppe Skott, Hauke Straehler-Pohl, Gabriel Stylianides, Jana Trgalova, Fatma Aslan Tutak, Olov Viirman, Geoff Wake, Hans-Georg Weigand, Carl Winsløw, Constantinos Xenofontos, Stefan Zehetmeier.

**Publisher:** Charles University in Prague, Faculty of Education and ERME

**Place:** Prague, Czech Republic

**Year:** 2015

**ISBN:** 978-80-7290-844-8

Copyright 2015 left to the authors.

**Recommended citation:**

Author: Title of the contribution. In K. Krainer & N. Vondrová (Eds.) (2015). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education (CERME9, 4-8 February 2015)* (pp. xx-yy). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

# Table of Contents

<b>1</b>	<b>PREFACE</b>	121	Is the use of GeoGebra advantageous in the process of argumentation? <i>Özlem Erkek and Mine İşıksal-Bostan</i>
2	Preface <i>Viviane Durand-Guerrier and Susanne Prediger</i>	128	Constructing validity in classroom conversations <i>Manuel Goizueta and Maria Alessandra Mariotti</i>
4	Editorial information <i>Konrad Krainer and Nad'a Vondrová</i>	135	Pre-service teachers' perceptions of generic proofs in elementary number theory <i>Leander Kempen and Rolf Biehler</i>
<b>6</b>	<b>PLENARY LECTURES</b>	142	Disparate arguments in mathematics classrooms <i>Christine Knipping, Daniela Rott and David A Reid</i>
7	Cultural contexts for European research and design practices in Mathematics Education <i>Barbara Jaworski, Maria G. Bartolini Bussi, Susanne Prediger and Edyta Nowinska</i>	149	E-assessment of understanding of geometric proofs using interactive diagrams <i>Yael Luz and Michal Yerushalmy</i>
34	Understanding randomness: Challenges for research and teaching <i>Carmen Batanero</i>	156	Building stories in order to reason and prove in mathematics class in primary school <i>Marianne Moulin and Virginie Deloustal-Jorrand</i>
50	Research in teacher education and innovation at schools: Cooperation, competition or two separate worlds? <i>Jarmila Novotná</i>	164	The genesis of proof in ancient Greece: The pedagogical implications of a Husserlian reading <i>Andreas Moutsios-Rentzos and Panagiotis Spyrou</i>
<b>66</b>	<b>TWG01 ARGUMENTATION AND PROOF</b>	171	Discriminating proof abilities of secondary school students with different mathematical talent <i>Juan Antonio Moya, Angel Gutiérrez and Adela Jaime</i>
67	Introduction to the papers of TWG01: Argumentation and proof <i>Samuele Antonini, Orly Buchbinder, Kirsten Pfeiffer and Gabriel J. Stylianides</i>	178	Proof evaluation tasks as tools for teaching? <i>Kirsten Pfeiffer and Rachel Quinlan</i>
<b>71</b>	<b>Research papers</b>	185	Mathematical fit: A first approximation <i>Manya Raman-Sundström and Lars-Daniel Öhman</i>
72	Proof by reductio ad absurdum: An experience with university students <i>Angelina Alvarado Monroy and María Teresa González Astudillo</i>	192	Students of two-curriculum types Performance on a proof for congruent triangles <i>Ruthmae Sears and Óscar Chávez</i>
79	Proof writing at undergraduate level <i>Nadia Azrou</i>	198	A theoretical perspective for proof construction <i>Annie Selden and John Selden</i>
86	On a generality framework for proving tasks <i>Andreas Bergwall</i>	205	Textbook explanations: Modes of reasoning in 7 <sup>th</sup> grade Israeli mathematics textbooks <i>Boaz Silverman and Ruhama Even</i>
93	Analyzing the transition to epsilon-delta Calculus: A case study <i>Paolo Boero</i>	213	The role of mode of representation in students' argument constructions <i>Andreas J. Stylianides</i>
100	Pre-service teachers' construction of algebraic proof through exploration of math-tricks <i>Orly Buchbinder and Alice Cook</i>	<b>221</b>	<b>Posters</b>
107	A case study: How textbooks of a Spanish publisher justify results related to limits from the 70's until today <i>Laura Conejo, Matías Arce and Tomás Ortega</i>	222	Considerations on teaching methods to deepen student argumentation through problem solving activities <i>Tsutomu Ishii</i>
114	Argumentation below expectation: A double-threefold Habermas explanation <i>Jenny Christine Cramer</i>	224	Unpack and repack mathematical activity with pre-service teachers: A research project <i>Simon Modeste and Francisco Rojas</i>
		226	How is proving constituted in Cypriot classroom? <i>Maria Pericleous</i>

228	Four steps on the way to create argumentation competence supported by technology <i>Evelyn Süss-Stepancik and Stefan Götz</i>	324	The rules for the order of operations: The case of an inservice teacher <i>Ioannis Papadopoulos</i>
230	An example of Proof-Based Teaching: 3 <sup>rd</sup> graders constructing knowledge by proving <i>Estela Vallejo Vargas and Candy Ordoñez Montañez</i>	331	Cracking percent problems in different formats: The role of texts and visual models for students with low and high language proficiency <i>Birte Pöhler, Susanne Prediger and Henrike Weinert</i>
<b>232</b>	<b>TWG02 ARITHMETIC AND NUMBER SYSTEMS</b>	339	Cognitive flexibility and reasoning patterns in American and German elementary students when sorting addition and subtraction problems <i>Elisabeth Rathgeb-Schnierer and Michael Green</i>
233	Introduction to the papers of TWG02: Arithmetic and number systems <i>Sebastian Rezat, Lisser Rye Ejersbo, Darina Jirotkova and Elisabeth Rathgeb-Schnierer</i>	346	A network of notions, concepts and processes for fractions and rational numbers as an interpretation of Didactical Phenomenology <i>Rubí Realand Olimpia Figueras</i>
<b>237</b>	<b>Research papers</b>	354	Flexible mental calculation and “Zahlenblickschulung” <i>Charlotte Rechtsteiner-Merz and Elisabeth Rathgeb-Schnierer</i>
238	A study on the changes in the use of number sense in secondary students <i>Rut Almeida and Alicia Bruno</i>	361	Foundational number sense: Summarising the development of an analytical framework <i>Judy Sayers and Paul Andrews</i>
245	Student strategies and errors in mental computation with rational numbers in open number sentences <i>Renata Carvalho and João Pedro da Ponte</i>	368	Additive adaptive thinking in 1st and 2nd grades pupils <i>Lurdes Serrazina and Margarida Rodrigues</i>
252	Spatial structuring, enumeration and errors of S.E.N. students working with 3D arrays <i>Carla Finesilver</i>	375	Improving classroom assessment in primary mathematics education in the Netherlands <i>Michiel Veldhuis and Marja van den Heuvel-Panhuizen</i>
259	Computing by counting in first grade: It ain't necessarily so <i>Michael Gaidoschik, Anne Fellmann and Silvia Guggenbichler</i>	<b>382</b>	<b>Poster</b>
266	What is a better buy? Rationale and empirical analysis of unequal ratios tasks in commercial offers contexts <i>Bernardo Gómez and Amparo García</i>	383	A socially built understanding of rational numbers <i>Helena Gil Guerreiro and Lurdes Serrazina</i>
274	Replacing persistent counting strategies with cooperative learning <i>Uta Häsel-Weide</i>	<b>385</b>	<b>TWG03 ALGEBRAIC THINKING</b>
281	Students' argumentation schemes in terms of solving tasks with negative numbers <i>Mathias Hattermann and Rudolf vom Hofe</i>	386	Introduction to the papers of TWG03: Algebraic thinking <i>Jeremy Hodgen, Reinhard Oldenburg, Valentina Postelnicu and Heidi Strømskag</i>
288	Reversible and irreversible desemantization <i>Milan Hejny, Darina Jirotková and Jana Slezáková</i>	<b>390</b>	<b>Research papers</b>
295	Sixth grade students' explanations and justifications of distributivity <i>Kerstin Larsson</i>	391	Flexible algebraic action on quadratic equations <i>Jan Block</i>
302	Investigating fourth graders' conceptual understanding of computational estimation using indirect estimation questions <i>Sabrina Lübke</i>	398	Connections between algebraic thinking and reasoning processes <i>Maria Chimoni and Demetra Pitta-Pantazi</i>
309	The impact of a teaching intervention on sixth grade students' fraction understanding and their performance in seven abilities that constitute fraction understanding <i>Aristoklis A. Nicolaou and Demetra Pitta-Pantazi</i>	405	Which algebraic learning can a teacher promote when her teaching does not focus on interpretative processes? <i>Annalisa Cusi and Nicolina A. Malara</i>
316	Processes of mathematical reasoning of equations in primary mathematics lessons <i>Marcus Nührenböcker and Ralph Schwarzkopf</i>	412	Does bodily movement enhance mathematical problem solving? Behavioral and neurophysiological evidence <i>Diana Henz, Reinhard Oldenburg and Wolfgang I. Schöllhorn</i>
		419	Solving equations: Gestures, (un)allowable hints, and the unsayable matter <i>Thomas Janßen and Luis Radford</i>

# The effect of high versus low guidance structured tasks on mathematical creativity

Sonia Palha, Jaap Schuitema, Carla van Boxtel and Thea Peetsma

University of Amsterdam, Research Institute of Child Development and Education, Amsterdam, The Netherlands,

[S.AbrantesGarcezPalha@uva.nl](mailto:S.AbrantesGarcezPalha@uva.nl), [J.A.Schuitema@uva.nl](mailto:J.A.Schuitema@uva.nl), [C.A.M.vanBoxtel@uva.nl](mailto:C.A.M.vanBoxtel@uva.nl), [T.T.D.Peetsma@uva.nl](mailto:T.T.D.Peetsma@uva.nl)

*To engage in challenging tasks, students need to feel some autonomy and competence. Providing structure within the task can help to meet these needs. This study investigates the influence of structure within a modelling task on mathematical creativity among 79 eleventh-grade groups of students. Two versions of the task were developed and the groups were randomly assigned within their classroom to one of these. The analysis explored: (i) the level of mathematical creativity in groups solutions and (ii) if they were dependent on the amount of structure. The results were not statistically significant and, therefore, the question remains open. Additional results and implication of this study to mathematics education are further discussed.*

**Keywords:** Integral calculus, creativity, modelling, collaborative learning, structure.

## INTRODUCTION

Researchers express different views with regard to creativity and its connection with the learning environment. Some claim that creativity can be seen as a disposition towards mathematical activity and therefore it can be fostered through specific instruction, such as problem-solving (Silver, 1997). Others see creativity as characteristic of extraordinary individuals (Weisberg, 1988) and thus, not likely to be strongly influenced by the learning environment. Also, several researchers connect creativity to self-regulated learning (Feldhusen & Goh, 1995) and psychological characteristics such as task commitment and motivation (Renzulli, 1978). In our research we share the view that mathematical creativity can be fostered by adequate instruction and we study the relationship between aspects of the learning environment (e.g., task characteristics) and mathematical creativity.

This study is part of a longitudinal intervention research in which we investigate how aspects of the learning environment influences students' motivation, self-regulation and academic performance in mathematics. We developed a learning arrangement in which we used differentiated tasks with a deeper and broader content and method to create a more authentic and challenging learning context. The participants are 16/17 years old students in pre-university education in The Netherlands. Part of our research is to investigate which amount of structure is optimal for the students. We developed two versions of the same learning arrangement. One version consists of low-structured (LS) tasks and provides more open tasks, more choice and initiative for students. The other version contains more high-structured (HS) tasks, which still provide some choice but also hints, more sub-questions and guidance.

In this paper we discuss our findings with regard to a modelling-task: the parachute jump (Figure 1), which was used within the topic Introduction to Integral Calculus. Modelling-tasks as problem-posing tasks have been seen by several researchers as excellent opportunities for mathematical creativity (Kim & Kim, 2010; Chamberlin & Moon, 2005). The research questions that guided our study were:

- What can we say about the mathematical creativity of students' productions with regard to the parachute jump task?
- In which way does variation in the amount of structure in the parachute jump task influences students' mathematical creativity?

## THEORETICAL FRAMEWORK

### Mathematical reasoning and creativity

Mathematical creativity can be seen as the ability of students to create useful and original solutions in authentic problem-solving situations (Chamberlin & Moon, 2005). The core activity of the parachute task is to build a model that can be applied in the particular example and other situations. The students' products can then be evaluated in terms of mathematical creativity. In the literature, mathematical creativity is often defined in terms of three components: *flexibility*, *fluency* and *originality* (Silver, 1997; Yuan & Sriraman, 2001). Flexibility can be seen as the ability to generate multiple solutions to a given problem. Fluency can be seen as the ability to use several relevant ideas to solve the task and, in problem-situation tasks it is connected to many interpretations, methods, or answers Silver (1997). Originality concerns different solutions or innovative ways to approach a problem.

Measurement of mathematical creativity remains critical. One reason is the absence of a universal definition applicable in different academic domains (Leikin & Lev, 2013; Kattou, Christou, & Pitta-Pantazi, 2015). Another reason is that one person's creativity can only be assessed indirectly (Piffer, 2012). The ability of posing problems given one mathematical scenario have been linked by several researchers to mathematical creativity (Silver, 1997; Yuan & Sriraman, 2001). Also, over the past years, researchers (Leikin & Lev, 2013) developed an analytical framework that can be used to evaluate creativity in students' productions using the components fluency, flexibility and originality. Mathematical creativity with regard to modelling activities often includes a fourth component: *usefulness*, which concerns the degree of relevance, adaptability and generality of solutions with regard to real world situations (Chamberlin & Moon, 2005). The criterion of usefulness has been contested by some authors. Sriraman (as cited in Yuan & Sriraman, 2001) argues that mathematics creative work might not be useful in terms of its applicability in the real world. Chamberlin and Moon (2005) propose the *Quality Assurance Guide* as a reliable instrument to evaluate creativity in students' products on modelling tasks. Each solution is scored within one of five levels. *Level 1*- requires redirection- the product is on the wrong track and working harder or longer will not improve it. At *level 2*, the product requires major extensions or refinements, the product is a good start towards meeting the goal

of the task. At *level 3*, the product is nearly ready to be used; it is useful for the specific data or sharable or reusable. At *level 4*, no changes are needed and at *level 5*, others can use it as tool in similar situations.

### High- and Low-structured tasks (HS and LS- tasks)

According to Silver (1997) problem-oriented instruction can assist students to develop more creative approaches to mathematics by increasing their capacity with respect to the core dimensions of creativity: fluency, flexibility, and originality. For instance, ill-structured problems require problem posing and conjecturing, which can foster the generation of novel conjectures. Silver (1997) stated: "It is in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity" (p. 76). However, engaging in problem-solving activity also requires certain ability and disposition to deal with uncertainty and challenge. Aspects of the learning environment that have been found to support the development of such disposition are autonomy support and structure provision (Deci & Ryan, 2000). According to these authors, in autonomy supportive environments students are allowed to make own decisions and are encouraged to solve problems. This can be achieved by providing authentic tasks and opportunities for taking initiative and minimize the use of controlling behaviour. Also, the provisions of structure contributes for students' feeling of competence and therefore is important for motivation. Providing structure involves communicating clear expectations, set limits to students' behaviour and provide help.

### Task arrangement

We investigate the relationship between structure provision and mathematical creativity in a problem-oriented arrangement that consisted of the 'parachute jump' task (Figure 1) and small group work. Working together may enhance feelings of relatedness and a sense of autonomy (Schuitema, Peetsma, & Van der Veen, 2011). And, during students' collaboration there is an unpredictable flow of ideas and actions that emerge from the elements of the group while responding to each other. Levenson (2011) states: "Together, the group tries out various strategies and possibly produces solutions based on different mathematical properties or different representations" (p. 230). This is tied to mathematical creativity in the sense that participants must be flexible, establish

(both versions A and B)	
<p><b>Task 25 parachute jump</b></p> <p>Dynamical processes, like a train ride, a traveling car and other speed-time processes can be described using a <i>mathematical model</i>. A mathematical model may include tables, graphs, formulas or any combination of these representations. These mathematical models can then be used to investigate (and solve) problems through calculations and reasoning or to invent better models to attack the stated problem. In a group of three students, you will create a mathematical model in which the distance travelled against time for a parachutist is described. You also prepare a demonstration (Powerpoint, poster or video clip) of your group's work as a homework task.</p> <p>But first an example of a parachute jump is presented.</p> <p><b>Example.</b> Imagine the following situation: A parachutist jumps from an airplane. The first five seconds she makes a free fall. Then she opens the parachute and because of that her fall velocity decreases linearly down until after 6 seconds she achieves a fall velocity of 4 meters per second. From this moment on the velocity remains constant during 70 seconds and she lands on the ground at this velocity.</p>	
	
	
(only version A)	(only version B)
<p>In the example, time is called <math>t</math> (in seconds), with <math>t = 0</math> at the jump from the plane. For the free fall the velocity is given by <math>v(t) = 9,8 t</math> with <math>v</math> in meter per second. The total jump, until reaching the ground, was 561.5 meters. A mathematical model for this example could be a formula (or some collection of formulas), a graph or table in which the falling process is described and that may help to solve the stated problem.</p> <p>The process for another parachutist will be comparable although different in the three phases of the process.</p>	<p>The process for another parachutist will be comparable, although different in the three phases of the process. That process may be described using a mathematical model. It is usual to start such a model with a concrete example and after that you try to design a more general model or representation. In this class period you will develop a mathematical model that describes the distance traveled against elapsed time for a parachutist. There are guiding questions describing an example that will help you to understand what is going on (questions a-d) and after that you are asked to design your own model (question e).</p> <p>In the example, time is called <math>t</math> (in seconds), with <math>t = 0</math> at the jump from the plane. For the free fall the velocity is given by <math>v(t) = 9,8 t</math> with <math>v</math> in meter per second.</p> <p>a. What distance does the parachutist cover during the free fall?</p> <p>b. What is the total distance covered from start to landing??</p>
<p>a. (version A) c. (version B)</p> <p>Watch the Youtube video of a parachutist jump: <a href="http://www.youtube.com/watch?v=STDIEFhIPrw">http://www.youtube.com/watch?v=STDIEFhIPrw</a>. Which similarities and differences do you notice, compared to the situation of the example?</p>	
	<p>d. Re-watch the video and try to collect data to design the model that describes the parachute jump (describe the data with use of tables/graphs or both).</p> <p>e. Can you find a relationship between distance covered and time, based on the data you collected from the video?</p>
<p>b. (version A) e. (version B)</p> <p>Create a mathematical model that describes the distance covered during the total parachute jump against time. After that, you prepare a group presentation (powerpoint, poster or video for a 2-5 minutes presentation) in which:</p> <ul style="list-style-type: none"> <li>- the mathematical model is presented;</li> <li>- you give a justification of the choices made;</li> <li>- show some examples of situations in which the chosen model will work;</li> <li>- a critical reflection on the model.</li> </ul>	

Figure 1: Parachute jump task

mathematical relations and approach the task in distinct or novel ways.

The 'parachute jump' task was entailed to provide challenge and authentic experiences, as these are

important elements of autonomy supportive tasks. It was designed according to the following four criteria.

*Appealing and accessible to all students.* The context of a parachute jump and the YouTube video make the task interesting to the students. And, the task becomes

more accessible by providing an initial example with concrete values and asking to compare it with the one in the video. The pre-knowledge needed to start working on the task was known from previous year (functions, graphs and derivatives).

*Authentic.* By providing students with an authentic task, and enough freedom of choice we expect that students will be willing to spend thinking effort on it.

*Foster mathematical reasoning and creativity.* The accomplishment of the task requires the use of mathematical understanding and high-level reasoning. The students must produce at least one representation of the integral function (table, formula, graph, words) and describe its variation at the different instances of the jump. This involves high-level reasoning, as the students must imagine the total accumulating distance varying over time (Thompson & Silverman, 2008).

*Suitable for collaborative learning.* The task is complex and it can be approached at several levels of understanding. Moreover, the students were encouraged to discuss their ideas and communicate their findings within the group.

Solving the task takes about two lessons of 50 minutes each and some homework time. We agreed with the teachers that the students would work in small groups during one lesson on the task and that they should finish it in their own time (not more than one week). The final product would have the format of a Power Point or a short video-film and would be delivered to the teacher, who would send it to us.

## METHOD

### Participants and data collection

Seventy-nine groups of 3 students (16/17 years old) from 10 classrooms in 5 schools participated in the study. The data was collected in the spring 2014 and consists of delivered groups products and lesson observations. The groups were formed based on a cognitive ability test. The 40 groups in the LS condition and the 39 groups at the HS condition were, in each classroom, random assigned to one of the conditions.

### Instrument used for the evaluation of mathematical creativity

The instrument that we used to evaluate the students' solutions to the parachute jump is based on three of the four components discussed in the theoretical section (we excluded originality because of the difficulty on assessing it in our data).

*Usefulness* regards the creation of a model that is useful to describe a parachute jump. For each written solution, we decided whether the model was incorrect (level 1), was in the good way but needed major improvements (level 2) or it was ready to be used but needed editing (level 3). Levels 4 and 5 were not observed in our data.

*Fluency* was seen as the ability to use several mathematical relevant ideas to solve the task. In the context of the parachute task it should be connected to the mathematical concept of the integral function, which is here treated as the total accumulating distance. Based on our theoretical framework, we define mathematical fluency as the ability to (i) link integration and differentiation as inverse processes; (ii) represent the total accumulated distance as a process (operational concept) and as an object (object oriented concept) within at least one functional representation (analytical, graphical, by words or numerical in a table); (iii) Indicate parameters that influence the model and to explain choices made.

*Flexibility* refers to the ability to set up a model and to use values that go beyond the information provided in the examples.

### Analysis

To investigate the first research question we operationalized mathematical creativity in terms of the three components and explored the frequencies found in the students solutions. To investigate the second research question we gave scores to the 3 components and sub-components. Each student solution was then scored within 1–3 for usefulness, 0–2 for each subcategories of fluency, 0–2 for flexibility. We used the Mann-Whitney test, which is indicated for data at ordinal level of measurement, to explore whether the products of the two conditions differed from each other.

## RESULTS

Fifty-two of the 79 groups that worked on the task in classroom handed in their final product to the teacher. In the following of this section we report on these products.

### Students' creativity in terms of usefulness, fluency and flexibility

The first research question concerned the mathematical creativity of student productions. Table 1 shows that the majority of the groups solutions (36) were at level 1 and therefore, not useful to model the parachute jump. Only 16 groups produced models that could be used.

Usefulness	Groups solutions (N=52)
Level 1	36 (45,6%)
Level 2	15 (19%)
Level 3	1 (1,3%)

Table 1: Results on usefulness

The results on fluency are shown in Table 2. Almost half of the groups (22) explicitly established the link between integration and differentiation. For instance, one group draw both graphs, with the text differentiation and integration and two arrows pointing opposite directions. Most of the solutions (37) presented traces of an operational- oriented conception of total distance. This means that students can draw a total

distance graph, use formulas to calculate single values but have difficulty to conceptualize the total distance as a mathematical object on which operations can be performed (Sfard, 1991). Very few groups (7) showed to have an object-oriented conception of total distance. An example of a student explanation that we consider exemplary of object-oriented conception is: "The distance increases at the beginning very fast, during the free fall. After 36 second, when the parachute opens the velocity becomes more or less constant and the distance increases linearly (...)". In contrast, students who would have no functional concept would not refer to distance in their explanations but describe the changes along the jump in terms of velocity, slope of line graphs (the line goes up or down) or in phenomenological terms. Most of the groups (34) did not consider parameters or provided choices.

The results on flexibility are summarized in Table 3. The majority of the groups (35) used only the values from the example. Few groups (14) refer to the values of the video and only 3 groups went beyond the information given in the task setting. Figure 2 contrast one of these solutions (right column) with a solution of the major group.

### Influence of HS and LS task on mathematical creativity

The second research question investigates whether the amount of structure in the task has effect on

Fluency	Criteria	Groups solutions (N=52)
link between integration and differentiation	Not visible	24 (30,4%)
	Unclear	6 (7,6%)
	Explicit	22 (27,8%)
Conceptions of accumulating distance function	No functional concept	8 (10,1%)
	Operational concept	37 (46,8%)
	Object oriented concept	7 (8,9%)
Parameters and choices	No parameters nor choices	34 (43%)
	Parameters or choices	11 (13,9%)
	Both	7 (8,9%)

Table 2: Results on fluency

Flexibility	Groups solutions (N=52)
Confined to example or undefined	35 (44,3%)
Beyond example and confined to film	14 (17,7%)
Beyond video and example	3 (3,8%)

Table 3: Results on flexibility

Confined to the example		Beyond the example and the film	
interval	afstandsgrafiek	invullen	
(0-5)	$s(x)=4,9x^2$	$s(5)=122,5$	"Imagine that you want to make a parachute jump. You want to make a free fall of 7 seconds. After opening the parachute you have a constant velocity of 3 m/s. Opening the parachute takes 4 seconds. After opening it you want to stay 3 minutes in the sky . How high must be the jump?"
(5-11)	$s(x)=-3,75x^2 + 86,5x - 338,75$	$s(11)=159$	
(11-81)	$s(x)=4x-44$	$s(81)=280$	
		561,5m	

Figure 2: Examples of two levels of flexibility

student’s mathematical creativity. Table 4 shows the results on usefulness, fluency, and flexibility in both conditions. A Mann-Whitney test indicated that there was no statistically significant difference between the two conditions for all components and sub-components of mathematical creativity.

**DISCUSSION**

In this paper we explored the influence of task structure on the mathematical creativity in students’ productions in the context of a modelling task. Next we discuss our results in the light of the two research questions.

*What can we say about the mathematical creativity of students’ productions with regard to the parachute jump task?* Overall the student solutions attained low scores with regard to the three components of mathematical creativity. Only 52 out of 79 groups delivered their final product, none of the groups created a general and reusable solution (levels 4 and 5) and only 16 out of 52 groups have created a model with level 2 or 3. Most students’ use of mathematical functions involved thinking in operational views rather than object-oriented. Also, most groups failed in considering relevant side conditions (wind, gravity, etc.) and parameters that are necessary to present a realistic model for the parachute jump. These difficulties suggest that the task as we presented to the students was too challenging for most of them. Several researchers (Silver, 1997; Lithner, 2008) suggest that relationships

between creativity and problem solving might be the product of previous instructional patterns. Therefore it is possible that students’ previous experiences with mathematical tasks (note that the students are not used to problem-oriented instruction) may have limited their searching process. For instance, only few students tried to go beyond the given examples, as it can be seen by the low levels of usefulness and flexibility. Or, they have tried to explain their choices and present different parameters, as most of the students scored very low on these subcomponents of fluency. Therefore, one suggestion to improve the task is to provide additional information on side conditions that are not part of the mandatory curriculum or provide explicitly directions to look for them. Other suggestion involve the improvement of students’ problem-solving activity. The teacher should encourage more the students during the solving process, e.g., to explore different paths, to look for other examples and not to give up too easily. Other aspects that we did not discuss here but also should be taken into consideration are the amount of time available to solve the task in the classroom, the specific directions to be provided by the teachers and assessment practices.

*In which way does variation in the amount of structure in the parachute jump task influences students’ mathematical creativity?* The products created by the groups of students in the two conditions are not statistically significant different with regard to mathematical creativity. Therefore, the question whether providing more/less guidance in the mathematical

	HS-task Median	(N=27) Range	LS-task Median	(N=25) Range	Mann-Whitney U test (two tailed)
Usefulness (scores 1-3)	1	2	1	1	U=257.000, p=.066
Fluency (scores 0-2)					
integration-differentiation	1	2	1	2	U=308.500, p=.559
concept accumulating distance	1	2	1	2	U=304.000, p=.441
parameters and choices	0	2	0	2	U=333.000, p=.992
Flexibility (scores 0-2)	0	2	0	1	U=272.000, p=.144

Table 4: Results on mathematical creativity within high- and low-structured tasks

tasks have impact on students' mathematical creativity remains open. In this paper we studied the effect of task structure on the groups products without refer to the solution process. However the way students approach the tasks and reasoning processes might reveal mathematical creativity aspects of the students not revealed in the final product (Karakok, Milos, Tang, & El Turkey, 2015). This is one question that deserves further investigation. Another interesting question to be further investigated regards the collective creativity process. In our research the students work in small groups, thus the intrapersonal creativity of one student produces a creative product which is then appropriated by others. In this case it is difficult to determine to what extend the final creative ideas and solutions are the product of particular students or from the collective (Levenson, 2011). An interesting question therefore is: in what extend this collective process is mediated by the amount of structure provision in the task?

Concluding, although our study could not provide a conclusive answer to the question whether the amount of structure in the task influences students' mathematical creativity, it contributes to the field of research and teacher education in two ways. It extends previous research on mathematical creativity by accounting the relationship between the learning environment and creativity and, by providing a way to operationalize fluency and flexibility in conceptual mathematical terms. And it provides a practical example (the parachute task) with potential to engage students in problem-solving and concrete suggestions for its implementation. The use of this kind of tasks in the classroom and in teacher education can help teachers to recognize mathematical creativity in their lessons and therefore to better support it.

## REFERENCES

- Chamberlin, S., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Prufrock Journal*, 17(1), 37–47.
- Deci, E. L., & Ryan, R. M. (2000). The 'what' and 'why' of goal pursuits: Human needs and the self-determination of behaviour. *Psychological Inquiry*, 11, 227–268.
- Feldhusen, J. F., & Goh, B. E. (1995). Assessing and accessing creativity: An integrative review of theory, research, and development. *Creativity Research Journal*, 8(3), 231–247.
- Karakok, G, Milos, S., Tang, G., & El Turkey, H. (2015). Mathematicians' views on undergraduate students' creativity. In K. Krainer & N. Vondrová (Eds.), *Proceedings of CERME9* (this volume).
- Kattou, M., Christou, C., & Pitta-Pantazi, D. (2015). Mathematical creativity or general creativity? In K. Krainer & N. Vondrová (Eds.), *Proceedings of CERME9* (this volume).
- Kim, S. H., & Kim, S. (2010). The effects of mathematical modeling on creative production ability and self-directed learning attitude. *Asia Pacific Education Review*, 11(2), 109–120.
- Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference? *ZDM*, 45(2), 183–197.
- Levenson, E. (2011). Exploring collective mathematical creativity in elementary school. *The Journal of Creative Behavior*, 45(3), 215–234.
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, 67(3), 255–276.
- Piffer, D. (2012). Can creativity be measured? An attempt to clarify the notion of creativity and general directions for future research. *Thinking Skills and Creativity*, 7(3), 258–264.
- Renzulli, J. S. (1978). What makes giftedness? Reexamining a definition. *Phi Delta Kappan*, 60(3), 180.
- Schuitema, J., Peetsma, T., & Van der Veen, I. (2012). Self-regulated learning and students' perceptions of innovative and traditional learning environments: A longitudinal study in secondary education. *Educational Studies*, 38(4), 397–413.
- Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM*, 29(3), 75–80.
- Thompson, P. W., & Silverman, J. (2008). The concept of accumulation in calculus. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics* (pp. 43–52). Washington, DC: Mathematical Association of America.
- Weisberg, R.W. (1988). Problem solving and creativity. In R. J. Sternberg (Ed.), *The nature of creativity* (pp. 148–176). New York: Cambridge University Press.
- Yuan, X., & Sriraman, B. (2011). An exploratory study of relationships between students' creativity and mathematical problem-posing abilities. In B. Sriraman & K. H. Lee (Eds.), *The elements of creativity and giftedness in mathematics* (pp. 5–28). Rotterdam, The Netherlands: Sense Publishers.