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Chapter 3

Plurality Voting versus Proportional Representation in the Citizen-Candidate Model: The Role of Coalitions¹

3.1 Introduction

How does the electoral system influence the entry decision of candidates and parties? Specifically, how does the number of entrants and their polarization differ between plurality voting (i.e., first-past-the-post) and proportional representation? To address such questions this chapter offers a theoretical analysis comparing plurality voting and proportional representation while paying special attention to the coalition formation process typically associated with systems of proportional representation.

The research question underlying this chapter is at the core of one of the biggest questions in political science and political economy: How does the electoral system influence election outcomes both in terms of who wins and what policy results? This is a fundamental question since many political and economic outcomes are a function of election outcomes and the distribution of power they induce. In the end the electoral outcomes depend on the interaction of candidates, parties and voters. To comprehend the effects of the electoral system one needs first to understand the behavioral effects on each of these different groups. Subsequent research can aim at combining these effects and solving for an electoral equilibrium that includes all players.

While chapter 2 focuses on voter behavior, this chapter provides another such first step by focusing on candidates. The electoral system can influence their behavior in two important ways. First, it can influence which positions candidates adopt when campaigning. Second, the electoral rule can influence how many and what types of candidates (centrist vs. extreme) contest an election. A careful reading of the literature on the comparison of plurality voting (or first-past-the-post) and proportional representation reveals that it can be organized along these two dimensions. Starting with Duverger (1955) the number of candidates under the two electoral systems has received much attention and his hypothesis that proportional representation leads to more candidates running for office has been and

¹ This chapter is based on Kamm (2014).

continues to be studied extensively. Another hypothesis that has received a lot of attention is that proportional representation leads to more polarized parties than plurality voting (see for instance, Cox 1990). Up to now empirical work has not led to a consensus whether this is indeed the case, see for instance Ezrow (2008) for a null result and Dow (2011) for supportive evidence.²

Similar to the studies of voter behavior discussed in chapter 2, the possible interaction between a candidate's entry decision and the policy she adopts is mostly not taken account when studying the questions mentioned in the previous paragraph. The citizen-candidate paradigm (Osborne and Slivinsky, 1996, Besley and Coate, 1997) is an exception since it offers a tractable model that is able to address both decisions at the same time. For this reason, in this chapter I will employ this paradigm to analyze the effects of the electoral rule.

When comparing plurality voting and proportional representation a special focus will be on how to model the intricacies of proportional representation. One of the most important complicating factors when modeling proportional representation is the occurrence of coalitions, which is typically observed in this electoral system. Theoretical work in recent years shows an increased interest in integrating coalitions into models of voter behavior (see the discussion on coalitional voting in chapter 2) and candidate behavior in systems of proportional representation. With respect to party behavior most work has been concerned with modeling the coalition formation process (see Diermeier 2006 for a survey of this literature), i.e. the final decision parties have to make after an election. The effects of coalition formation on candidates' entry decision and policy choice when campaigning, on the other hand, has been studied much less (see Matakos et al 2013 for an example of theoretical work on this topic and Curini and Hino 2012 for an empirical paper that shows that coalitions are an important explanatory factor of party system polarization).

In this chapter I will contribute to this strand of the theoretical literature and compare candidates' entry decision and policy choice under plurality voting and proportional representation (PR), where PR is modeled first in the standard way that abstracts from coalition governments and subsequently in a novel way that explicitly takes coalitions into account. The main theoretical results are as follows. First, taking the coalitions associated with proportional representation into account allows for more centrist equilibria compared to proportional representation ignoring coalitions. Second, for policy-motivated candidates proportional representation with and without coalitions supports more polarized equilibria than plurality voting. And third, with Downsian –i.e. office-motivated– candidates, equilibria with many entrants are more likely under proportional representation with coalitions than with plurality voting or proportional representation without coalitions.

The remainder of this chapter is structured as follows. First, I present the general set-up of the citizen-candidate model for plurality voting and the two types of proportional representation. Next, I discuss the equilibria for the case of PR with coalitions and compare

² Another question that has received a lot of attention is how proportional representation and first-past-the-post differ in terms of public good provision. See Person and Tabellini (2004) for a survey of this literature.

the equilibria for the three different electoral rules. Section 3.4 concludes and discusses some avenues for possible future work.

3.2 The citizen-candidate model

The citizen-candidate model introduced by Besley and Coate (1997) and Osborne and Slivinsky (1996) is a model in the Downsian tradition of spatial voting (Downs, 1957). An electorate of citizens is distributed over a policy space where each citizen is described by her ideal point (i.e. position) in the policy space. The defining feature of the model –from which it derives its name– is that each citizen can run for office by paying a cost c . After simultaneous decisions on whether to run for office, all the candidates and their positions in the policy space are announced³ and an election takes place. This election determines a policy x^* that will be implemented as well as an allocation of office rents, denoted by b .⁴

The utility for a citizen with ideal point x_i is assumed to take the following form

$$U = -f(|x^* - x_i|) + b * W - c * R,$$

where $f \geq 0$; $f' > 0$; W is a dummy variable that is equal to 1 if the candidate obtains the office rents and R is a dummy variable that is equal to 1 if the candidate runs for election.

Contrasting the specific modeling assumptions made by Besley and Coate (1997, henceforth ‘B+C’), and Osborne and Slivinsky (1996, ‘O+S’), can help to clarify some of the important modeling decisions that have to be made. While O+S assume a continuous distribution of citizens along a one-dimensional policy space, B+C allow for a multi-dimensional policy space and focus on the finite case of N citizens. Regarding voter behavior, O+S impose sincere voting while B+C allow for strategic voting. Finally, O+S consider the case where candidates might care about office in itself (i.e. $b \geq 0$) while B+C assume that candidates are purely policy-motivated ($b = 0$).

My model is closer to O+S in the sense that I focus on a one-dimensional policy space where citizens are uniformly distributed on the interval $[0, 1]$ and I assume sincere behavior by the voters. Regarding the candidates’ motivation I analyze both the case of pure policy-motivation ($b = 0$) and pure-office motivation ($b = \infty$).⁵ Finally, I assume that a citizen’s utility function is linear in the distance between her ideal point x_i and implemented policy x^* . This gives rise to the following utility function:

$$U = -|x^* - x_i| + b * W - c * R$$

³ Großer and Palfrey (2014) demonstrate how important the assumption of perfect observability is by analyzing the opposite extreme where positions are not observable (or only whether they are to the left/right of the median) and show that this fundamentally changes the equilibria.

⁴ One can, for instance, think about these office rents as compensation for government work or perks from office but also as an improvement in job market opportunities upon leaving office.

⁵ More precisely, candidates have lexicographic preferences with office rents being the first priority. This follows from the observation that when multiple options lead to the same office rents, a candidate will choose the option that leads to the policy outcome she likes most.

The environment studied by B+C and O+S is one of plurality (or first-past-the-post) voting. In this case it is straightforward and intuitive to assume that the implemented policy x^* is the ideal point of the candidate receiving the most votes (with ties broken randomly) and that all the office rents are awarded to this candidate. I will adopt the same way of modelling plurality voting. An open question is what to assume for the case that no citizen enters the race. I follow O+S in assuming that each citizen then receives a payoff of $-\infty$ which can be interpreted as a large loss in utility due to a breakdown of democracy.^{6,7}

In the case of proportional representation it is less obvious how to model the mapping from votes to an implemented policy and the distribution of office rents. Hamlin and Hjortlund (2000) –who were the first to model candidate entry under proportional representation in the citizen-candidate paradigm– assume that the implemented policy is the vote-weighted average of all candidates’ positions and that the candidate with most votes is awarded the office rents (I will call this model ‘PR without coalitions’).⁸ While this way of modeling the implemented policy correctly takes into account that plurality is not needed in proportional representation to have an influence on the policy, as discussed in chapter 2, it cannot capture some other important features of proportional representation. For instance, the important discontinuity that arises when one candidate receives an absolute majority is not accounted for.⁹ But most importantly, it does not take account of one of the defining features of systems of proportional representation – coalition governments. Therefore I propose a different way of modeling proportional representation that takes coalitions explicitly into account.¹⁰

In my model, the implemented policy is assumed to be the vote-weighted average of the policy positions of the candidates that are part of the governing coalition. Furthermore, the office rents are allocated proportionally within the coalition. The proportional division within the coalition is motivated by Gamson’s Law (Gamson, 1961) which states that government portfolios are distributed proportionally within the coalition. Ansolabehere et al. (2005) offer supporting empirical evidence by investigating portfolio allocations in Western Europe from 1946 to 2001.

Coalitions are formed according to the following procedure:¹¹

1. If a candidate receives an absolute majority of votes cast, she unilaterally forms a government and the implemented policy x^* is equal to this candidate's policy position.

⁶ B+C assume for this case that an exogenously given default policy will be implemented.

⁷ The results are robust to assuming that instead of breakdown of democracy one citizen will be randomly chosen to form a care-taker government by herself.

⁸ This can be interpreted as the office rent being the payoff associated with being the prime minister, a post often awarded to the largest party in parliament.

⁹ See Indridason (2011), who shows in a different context that it is sufficient to assume that a candidate with an absolute majority can implement her favorite policy to fundamentally change the equilibrium.

¹⁰ Bandyopadhyay and Oak (2004) also study the citizen-candidate model with coalitions. They use a similar set-up as mine but focus on the coalition formation stage and do not analyze the number and polarization of entrants.

¹¹ This is the same procedure for modelling coalition formation as in chapter 2.

2. If no candidate receives an absolute majority of votes cast, the candidate with the most votes is assigned the role of ‘government formateur’. This candidate then proposes a coalition to the candidate or candidates with whom she wants to cooperate; if everyone agrees, the coalition is formed.
3. If multiple candidates have the most votes, a fair random draw decides which of them is assigned the role of government formateur.
4. If the coalition is rejected, bargaining breaks down and every candidate receives a payoff of $-\infty$.

One can think about this as a simplified version of the bargaining approach used in Austen-Smith and Banks (1988), with bargaining breaking down after the first round. An alternative approach would be to have a random formateur (as used in Baron and Ferejohn, 1989) where recognition probabilities are proportional to vote shares. Which of the two assumptions about formateur choice has more external validity is an empirical question. Diermeier and Merlo (2004) analyze government formation in 14 Western European countries and find that the random formateur model fits the data better than recognition in order of seat shares but that the largest party has a disproportionately high probability of getting the first shot at forming a government. Additionally, Ansolabehere et al. (2005) find that controlling for vote shares the largest party is twice as likely to deliver the formateur. This provides some support for the simplifying assumption that the largest candidate forms the coalition.¹² Furthermore, while assuming bargaining to be take-it-or-leave-it is certainly restrictive, it greatly simplifies the analysis by avoiding the need to solve for a sub-game perfect equilibrium at the bargaining stage.¹³

Note that I do not impose that coalitions are minimal winning (Riker, 1962) or connected but let the formateur endogenously form the coalition that maximizes her utility (see Bandyopadhyay and Oak, 2008, for a model in a similar spirit).¹⁴

3.3 Equilibrium analysis and comparative statics

I offer an analysis for the two polar cases of purely policy-motivated ($b = 0$) candidates and the Downsian case of purely office-motivated ($b = \infty$) candidates.¹⁵ I focus on these two extreme cases since they are the ones used most frequently in the literature. Furthermore, while the case of candidates that care substantially for both office rents and policy is certainly very interesting and relevant, it complicates the equilibrium analysis tremendously. The interaction effect between the two motives is therefore left for future work. Furthermore, I restrict the equilibrium analysis to one-, two- and three-candidate equilibria since these three cases capture the important cases of uncontested elections, two-

¹² Moreover, in the case of purely policy-motivated candidates the equilibria do not change if I assume a random formateur rule instead.

¹³ Obviously, it would be interesting and important to investigate how robust the results are to the specifics of the bargaining protocol. I leave this for future work.

¹⁴ There is also an active empirical literature that tries to establish determinants for the type of coalition observed. Martin and Stephenson (2001), for instance, analyze 220 coalition formation processes in 14 established democracies. They find that both ideological alignment and office rents matter and that minimal winning coalitions are most frequently observed.

candidate elections and multi-candidate elections and they are therefore sufficient to make an argument regarding the number of entrants as well as their polarization across electoral rules. All proofs for the propositions presented here are relegated to appendix 3.A.

3.3.1 The case of policy-motivated candidates

Proposition 1 considers the case of plurality voting as analyzed by Osborne and Slivinsky (1996) and is applies their propositions 1, 2 and 4 under the assumptions made in this chapter.

Proposition 1 (Plurality voting and policy-motivated candidates)

- (a) A single entrant at position X is an equilibrium iff
 - a. $0 \leq X < 0.5$ and $1 - 2X \leq c$ **OR**
 - b. $X = 0.5$ **OR**
 - c. $0.5 < X \leq 1$ and $2X - 1 \leq c$
- (b) Two candidates entering at position L and R is an equilibrium iff:
 - a. $R = 1 - L$ and $\frac{1}{6} \leq L < \frac{1}{2}$ and $c < 0.5 - L$
- (c) No equilibrium with three candidates exists

The intuition for structure of these equilibria is the following: In a one-candidate equilibrium the costs of running for office have to be high enough so that it is not worthwhile for another, more centrist, candidate to enter. For the two-candidate equilibrium the symmetry is necessary since otherwise one of the candidates would not have any influence on the implemented policy and would therefore prefer not to enter the election. The reason why no three-candidate equilibrium exists is that for all possible candidate positions at least one of the candidates would prefer to stay out of the election thereby ensuring that the remaining candidate that is located closer to her position wins the election.

Proposition 2 describes the equilibria for the case of proportional representation without coalitions and is an application of propositions 1-3 in Hamlin and Hjortlund (2001).

Proposition 2 (PR without coalitions and policy-motivated candidates)

- (a) A single entrant at position X is an equilibrium iff
 - a. $0 \leq X < 0.5$ and $\frac{1}{2}(1 - X)^2 \leq c$ **OR**
 - b. $X = 0.5$ **OR**
 - c. $0.5 < X \leq 1$ and $\frac{1}{2}X^2 \leq c$

¹⁵ As mentioned above, the latter candidates do care about policy but only if multiple decisions give them the same office rents.

- (b) Two candidates entering at position L and R is an equilibrium iff:
- $0 \leq L < \sqrt{2} - 1$ and $\frac{L^2+1}{2} < R \leq 1 - L$ and $\frac{(1-R)^2}{2} < c < \frac{R^2-L^2}{2}$ **OR**
 - $0 \leq L \leq \sqrt{2} - 1$ and $1 - L \leq R \leq 1$ and $\frac{1}{2} L^2 < c < \frac{(2-R-L)*(R-L)}{2}$ **OR**
 - $\sqrt{2} - 1 < L < \frac{1}{2}$ and $1 - \sqrt{1-2L} < R \leq 1$ and $\frac{1}{2} L^2 < c < \frac{(2-R-L)*(R-L)}{2}$
- (c) No equilibrium with three candidates exists

The intuition for the one-candidate equilibrium is again that costs have to be high enough to deter entry by another entrant. In contrast to the case of plurality voting discussed above it is not entry by centrist but by extreme candidates that has to be deterred. This follows from the observation that they have the biggest influence on the policy and therefore the highest incentive to enter. For the two-candidate equilibrium two forces have to be traded off; on the one hand costs have to be high enough so that no additional candidates want to enter at the extremes of the policy space and on the other hand costs have to be low enough that both candidates want to enter. Finally, no three-candidate equilibrium exists because given the assumed uniform distribution of sincere voters the implemented policy would be the same whether the centrist candidate enters or not and therefore the centrist candidate will never enter.

Proposition 3 (PR with coalitions and policy-motivated candidates)

- (a) A single entrant at position X is an equilibrium iff
- $0 \leq X < 0.5$ and $1 - 2X \leq c$ **OR**
 - $X = 0.5$ **OR**
 - $0.5 < X \leq 1$ and $2X - 1 \leq c$
- (b) Two candidates entering at position L and R is an equilibrium iff:
- $R = 1 - L$ and $\frac{\sqrt{29}-1}{28} \leq L < \frac{1}{2}$ and $c < 0.5 - L$ **OR**
 - $R = 1 - L$ and $L < \frac{\sqrt{29}-1}{28}$ and $\frac{0.5-2L-14L^2}{1+2L} < c < 0.5 - L$
- (c) No equilibrium with three candidates exists

What is the intuition for these equilibria and their structure? In the case of a one-candidate equilibrium all entrants that are located closer to the median than the current entrant could implement their favorite policy and therefore the entry costs have to be high enough such that these citizens prefer accepting a sub-optimal policy over implementing their favorite policy by entering. The equilibrium where the candidate is located at the median position is a special case since the median candidate cannot be beaten by any candidate and therefore this equilibrium exists independent of the level of entry costs.

The two-candidate equilibria have to be symmetric since otherwise one of the two candidates obtains an absolute majority which means that the other candidate has no influence on the policy and prefers to stay out of the race. Furthermore, for very low entry costs the equilibrium cannot be too polarized. The reason is that such polarization would provide an opening for a moderate candidate to become formateur of the coalition and she would enter.

Finally, there can be no three-candidate equilibrium. The reason is that in all scenarios one of the extreme candidates would prefer an absolute majority of the center candidate over the situation where the other extreme candidate is part of the government.

Proposition 4 (comparative statics with policy-motivated candidates)

- (a) Proportional representation, with and without coalitions, yields higher polarization than plurality rule.
- (b) For proportional representation, an equilibrium with two centrist parties is more likely when coalitions are taken into account.

The intuition for part (a) of the proposition stems from the fact that with proportional representation an entrant has a different influence on the policy than under plurality voting. With a high degree of polarization an entrant can win a substantial fraction of the votes by entering in the center of the policy space. With plurality voting this entrant can implement her favorite policy and therefore has a strong incentive for entry. Under proportional representation she will have a smaller effect on the policy since at least one other candidate will also have an influence on the policy. This reduces the likelihood that polarization will be reduced due to entry at the center of the policy space.¹⁶

The intuition for the second part of the proposition is that with coalitions an extreme entrant (who would win many votes if the candidates are both centrist) will not be part of the coalition. The reason is that the formateur (which the entrant will never be) prefers a coalition with the closely aligned candidate and not the extreme entrant. Without coalitions on the other hand the extreme entrant has an influence on the policy and therefore a two-candidate equilibrium with little polarization is not sustainable.

3.3.2 The case of office-motivated candidates

Proposition 5 analyzes the case of plurality voting as modeled by Osborne and Slivinsky (1996) and is an application of their propositions 1-3.

Proposition 5 (Plurality voting and office-motivated candidates)

- (a) A single entrant at position X is an equilibrium iff
 - a. $X = 0.5$
- (b) Two candidates entering at position L and R is an equilibrium iff:
 - a. $R = 1 - L$ and $\frac{1}{6} \leq L < \frac{1}{2}$
- (c) Three entrants entering at position L , C and R is an equilibrium iff:
 - a. $L = \frac{2}{3} - C$ and $\frac{1}{3} < C < \frac{2}{3}$ and $R = \frac{4}{3} - C$

The reason that in the (only) one-candidate equilibrium the median citizen enters is that given the large office rents every citizen that can win against any candidate will enter. The only candidate that cannot be beaten is the median candidate. As in the case of policy-motivated candidates the two-candidate equilibrium has to be symmetric since otherwise

¹⁶ This is in line with theoretical work by Cox (1990) and recent empirical work by Matakos et al. (2013).

one of the candidate would neither get office rents nor have an influence on the implemented policy and would not enter. The structure of the three-candidate equilibrium is due to the fact that all candidates need to receive the same number of votes since otherwise the candidates with the least votes would prefer to stay out of the election.

Proposition 6 presents the equilibria for the case of proportional representation without coalitions and is based on claim 1, 2, 2* and 3 in Hamlin and Hjortlund (2001).

Proposition 6 (PR without coalitions and office-motivated candidates)

- (a) A single entrant at position X is an equilibrium iff
 - a. $X = 0.5$
- (b) Two candidates entering at position L and R is an equilibrium iff:
 - a. $c > \max\left\{\frac{1}{2}(1-R)^2; \frac{1}{2}L^2\right\}$ and $\max\left\{2-R-2L; \frac{R-L}{2}\right\} > L > R - \frac{2}{3}$ and $R > \min\left\{2-2R-L; \frac{2-R+L}{2}\right\}$ **AND**
 - b. $R+L \geq 1$ or $c < \frac{R^2-L^2}{2}$ **AND**
 - c. $R+L \leq 1$ or $c < \frac{2-R-L}{2}(R-L)$
- (c) Three entrants entering at position L , C and R is an equilibrium iff:
 - a. $R-L \geq \frac{2}{3}$ and $c > \max\left\{\frac{1}{2}(1-R)^2; \frac{1}{2}L^2\right\}$ **AND**
 - b. $C = R - 2L$ or $c < \frac{C^2-L^2}{2}$ **AND**
 - c. $C = 2 - 2R + L$ or $c < \frac{C^2-2C+2R-R^2}{2}$

The intuition for why the median candidate entering is the unique one-candidate equilibrium is the same as in the case of plurality voting: Any other candidate position would lead to entry by a more centrist candidate. The structure of the two-candidate equilibrium is determined by the trade-off between making sure that both candidates want to run for office (which implies costs cannot be too high and positions too close together) and deterring entry by other candidates (which means costs cannot be too low). The reason for the structure of the three-candidate equilibrium is that the center candidate needs to win (part of) the office rents since due to the uniform distribution of sincere voters she has no influence on the implemented policy. Furthermore, as in the two-candidate equilibrium costs can neither be too low nor too high as to ensure that the three candidates but no more want to enter.

Proposition 7 (PR with coalitions and office-motivated candidates):

- (a) A single entrant at position X is an equilibrium iff
 - a. $X = 0.5$
- (b) There is no equilibrium with two candidates entering.
- (c) Three entrants entering at position L , C and R is an equilibrium iff:
 - a. $L = \frac{1}{6}$ and $C = \frac{1}{2}$ and $R = \frac{5}{6}$
 - b. $C = \frac{1}{2}$ and $R = 1 - L$ and $L > \frac{1}{3}$

The reason that in the (only) one-candidate equilibrium the median citizen enters is the same as for the other two electoral rules: The only candidate that cannot be beaten is the median candidate. The reason for the non-existence of a two-candidate equilibrium is that there always exists an entrant that can join the coalition and which therefore would enter. Finally, it is noteworthy that in all three-candidate equilibria the extreme candidates have a weakly higher vote share than the center candidate. The intuition is that if the center candidate were to be the sole formateur there would always be an incentive for a moderate candidate to enter between the extreme and the center candidate to become part of the coalition.

Proposition 8 (comparative statics with office-motivated candidates):

- (a) Under proportional representation with coalitions, equilibria are more likely to involve multiple candidates.
- (b) Plurality rule leads to more polarization in three-party elections than PR with coalitions but PR without coalitions leads to even higher polarization.

Part (a) of the proposition follows from the result that with coalitions there are no equilibria with two candidates and therefore apart from the case where only the median citizen enters, the equilibrium is a multi-candidate outcome. Part (b) of the proposition highlights how taking coalitions into account changes the equilibrium structure. Coalitions allow a lower degree of polarization compared to plurality voting since under plurality voting the center candidate only enters when there is enough space between the extreme candidates for her to receive a third of the votes while with coalitions she is an attractive coalition partner and therefore will receive office rents even if she does not receive that many votes. If on the other hand one does not take coalitions into account, the result shows that that proportional representation leads to more polarized outcomes. The intuition is that in an equilibrium under plurality rule, all candidates receive office rents, which implies that the center candidate cannot win a strict plurality of the votes, which in turn is only possible if polarization is not too high. Under proportional representation the opposite is true and the center candidate needs to win office rents for her to enter which is only possible for a high degree of polarization. Finally, the reason that proportional representation is less polarized with coalitions than without is that with coalitions a large degree of polarization opens the door for moderate entrants that are very attractive coalition partners while if coalitions are not taken account, these moderates would only enter if they could win a plurality of the votes.

In conclusion, the comparison of equilibria across electoral rules leads to the following main findings: First, due to the majoritarian decision-making captured by coalition governments, proportional representation with coalitions allows for more centrist outcomes than in the absence of coalitions. Second, proportional representation without coalitions allows more polarized outcomes than plurality voting. Third, for office-motivated candidates proportional representation with coalitions is most conducive to multi-candidate outcomes.

3.4 Conclusions

In this chapter I employed the citizen-candidate paradigm to investigate the question of how distinct electoral systems influence the number of candidates running for office and the polarization of their policy positions. I introduce a way of modeling proportional representation that takes coalition governments explicitly into account and find that this leads –compared to ignoring coalitions– to candidate positions that are less polarized. This implies that the common criticism of proportional representation leading to high polarization has less bite once we take into account the incentives associated with coalition formation. For the case without coalition formation, I do find that plurality voting leads to more centrist outcomes than proportional representation. This is in line with the hypothesis put forth by Cox (1990) that proportional representation leads to more polarized parties. Furthermore, for the classical case of Downsian candidates, I find that proportional representation with coalitions is more conducive to multi-candidate equilibria than proportional representation without coalitions or plurality voting.

Overall, these theoretical results show that the dynamics associated with coalition formation have important implications for candidates' behavior under proportional representation. This analysis offers a first step to understanding how coalitions influence entry and candidates' policy choice and how these depend on the institutional environment, but a lot of work still remains to be done. A first important step would be to investigate how robust the results of this analysis are with respect to the specifics of the coalition formation process, for instance by employing a random formateur rule. Another avenue for future research is to allow for candidates that care about both policy and office rents. One could then investigate how the comparative statics across electoral rules change as candidates' relative concerns for office rents increase vis-à-vis their concerns for policy. An important question is also how the electoral equilibrium that results from the interplay between candidates and voters reacts to changes in the electoral system. To be able to answer this question one would need to allow for strategic voting¹⁷ or abstention. Especially allowing for abstention might be interesting since –as chapter 2 demonstrates– it has direct implications for the polarization of outcomes.

On a more general level the analysis reiterates the point made in the literature that ignoring the coalitions associated with proportional representation is not without effect on the equilibria and the comparative statics across electoral rules. Going forward we should therefore continue to integrate coalition governments into our analyses of proportional representation; not only in models that aim to understand election outcomes per se but also and especially in models that try to answer how electoral rules influence other outcomes such as taxation and redistribution.

¹⁷ De Sinopoli and Iannantuoni (2007) show that in the model of proportional representation without coalitions when strategic voting is allowed the voters engage in 'policy-balancing' (Kedar, 2009) which leads to only the most extreme candidates receiving votes.

Appendix 3.A: Proofs of propositions

Throughout the analysis x_A denotes the policy implemented in situation A . If A consist of one letter this implies that the player denoted by this letter determines the policy alone; if A consists of multiple letters the players denoted by these letters are part of the coalition determining the policy (i.e. x_{LC} denotes the policy implemented by a center-left coalition). Furthermore, U_B denotes a player's payoff in situation B and P_i is player i 's share of the votes.

3.A.1 Proof of proposition 1

This proposition depicts the equilibria for the case for plurality voting, with policy-motivated candidates. To prove the three parts of the proposition, we analyze all equilibria with up to three entrants.

Single party equilibrium with entry at position X

We first have to ensure that the player at position X wants to enter. If she stays out she will receive a payoff of $-\infty$ (since nobody enters). Therefore the player at position X will enter. Next, we have to ensure that no other player wants to enter. The potential entrant that has the most to gain from entering is located on the opposite site of the median with a distance that is slightly smaller than the distance between X and the median.

(a) $X < 0.5$

Denote the entrants position by $Z = 1 - X - \varepsilon$.

She prefers to stay out if the following condition holds:

$$-c < -(Z - X) \Leftrightarrow c > 1 - 2X - \varepsilon$$

(b) $X = 0.5$

No entrant can win the election

(c) $X > 0.5$

Denote the entrants position by $Z = 1 - X + \varepsilon$.

She prefers to stay out if the following condition holds:

$$-c < -(X - Z) \Leftrightarrow c > 2X - 1 - \varepsilon$$

Thus, entry by a candidate at position X is an equilibrium if:

(a) $0 \leq X < 0.5$ and $1 - 2X \leq c$

(b) $X = 0.5$ and $c < 0.25$

(c) $0.5 < X \leq 1$ and $2X - 1 \leq c$

Two party equilibrium with entry at L and R

The equilibrium has to be symmetric since otherwise one candidate loses for sure and will not enter. Therefore $R = 1 - L$.

The candidate at L enters if the following condition holds (by symmetry this is the same condition for the candidate at R):

$$-c - (0.5 - L) > -(R - L) \Leftrightarrow c < 0.5 - L$$

Given L and R, there are three types of potential entrants to consider:

(a) Entry by an extremist (wlog to the left of L)

Such an entrant can never win a plurality since player R does not lose any votes and still gets 50% of the votes. Therefore the effect of such an entrant is only that it makes party R the sole formateur. But given that she is to the left of candidate L the entrant dislikes this. Therefore this type of entrant has never an incentive to enter.

(b) Entry by a centrist voter (wlog to the left of $\frac{1}{2}$) not winning a plurality

By entering she takes away votes from L and R but given that she is closer to L this will lead to a plurality for candidate R. This is not in the interest of the entrant and therefore this type of entrant will not enter.

(c) Entry by a centrist voter that does win a plurality

Since the payoff is linear in the distance to the implemented policy all entrants have the same incentive to enter if they can obtain more votes than L and R. Furthermore all entrants between L and R will receive a vote share equal to $\frac{R-L}{2}$ while the vote share for candidates L and R is given by $\frac{0.5+L}{2}$. Therefore the entrant wins most votes if $L < \frac{1}{6}$. This implies that if $L \geq \frac{1}{6}$ no entrant can benefit from entry since she will never win.

Entry at positions L and 1-L is an equilibrium if:

$$\frac{1}{6} \leq L < \frac{1}{2} \text{ and } c < 0.5 - L$$

Three party equilibrium with entry at L, C and R

Both extreme candidates need to win a plurality of the votes since otherwise they prefer the center candidate to win which they can ensure by staying out of the race. The condition for this to be the case are:

$$(a) P_L = P_R \Leftrightarrow C + L = 2 - R - C \Rightarrow C = \frac{2-R-L}{2} \xrightarrow{L < C < R} 2 - 3R < L < \frac{2-R}{3}$$

$$(b) P_L > P_C \Leftrightarrow C + L > R - L \Rightarrow C > R - 2L \xrightarrow{\text{by (a)}} L > \frac{3R-2}{3}$$

There are now two cases to consider:

First, candidate C ties with the other candidates

This implies $L = \frac{2}{3} - C$, $R = \frac{4}{3} - C$ and $\frac{1}{3} < C < \frac{2}{3}$.

The candidate at position L enters if the following condition holds:

$$-c - \frac{1}{3}(C - L) - \frac{1}{3}(R - L) > -(C - L) \Leftrightarrow c < \frac{4}{3}C - \frac{2}{3}$$

Given that $c > 0$ this implies $C > \frac{1}{2}$.

The candidate at position R enters if the following condition holds:

$$-c - \frac{1}{3}(R - C) - \frac{1}{3}(R - L) > -(R - C) \Leftrightarrow c < \frac{2}{3} - \frac{4}{3}C$$

Given that $c > 0$ this implies $C < \frac{1}{2}$ and therefore this is not an equilibrium.

Second, the center candidate does not win a plurality of the votes

The candidate at position L enters if the following condition holds:

$$-c - \frac{1}{2}(R - L) > -(C - L) \Leftrightarrow c < 1 - R - L$$

The candidate at position R enters if the following condition holds:

$$-c - \frac{1}{2}(R - L) > -(R - C) \Leftrightarrow c < R + L - 1$$

Since the costs are positive these two conditions can never be satisfied at the same time.

There does not exist a three party equilibrium.

3.A.2 Proof of proposition 2

This proposition depicts the equilibria for the case for proportional representation without coalition formation, with policy-motivated candidates. To prove the three parts of the proposition, we analyze all equilibria with up to three entrants.

Single party equilibrium with entry at position X

We first have to ensure that the player at position X wants to enter. If she stays out she will receive a payoff of $-\infty$ (since nobody enters). Therefore the player at position X will enter. The potential entrant that has the most to gain from entering is the one located farthest away from X.

(a) $X \leq 0.5$

The relevant entrant is located at position 1. She does not enter if the following condition holds:

$$-c - \left(1 - \frac{1+X}{2}X - \frac{1-X}{2} * 1\right) < -(1-X) \Leftrightarrow c > \frac{1}{2}X^2 - X + 0.5$$

(b) $X \geq 0.5$

The relevant entrant is located at position 0. she does not enter if the following condition holds:

$$-c - \left(\frac{X}{2} * 0 - \frac{2+X}{2}X\right) < -X \Leftrightarrow c > \frac{1}{2}X^2$$

Entry at position X is an equilibrium if:

(a) $0 \leq X < \frac{1}{2}$ and $c > \frac{1}{2}X^2 - X + 0.5$

(b) $X = \frac{1}{2}$ and $0.125 < c < 0.25$

(c) $\frac{1}{2} < X \leq 1$ and $\frac{1}{2}X^2 < c$

Two party equilibrium with entry at L and R

The candidate at position L enters if:

$$-c - \left(\frac{R+L}{2}L + \frac{2-R-L}{2}R - L\right) > -(R-L) \Leftrightarrow c < \frac{R^2 - L^2}{2}$$

The candidate at position R enters if:

$$-c - \left(R - \frac{R+L}{2}L - \frac{2-R-L}{2}R\right) > -(R-L) \Leftrightarrow c < \frac{2-R-L}{2}(R-L)$$

The potential entrants with the most to gain are located at 0 and 1.

The entrant at position 0 stays out if:

$$-c - \left(\frac{R}{2} * 0 + \frac{2-R}{2} R - 0 \right) < - \left(\frac{R+L}{2} L + \frac{2-R-L}{2} R - 0 \right) \Leftrightarrow c > \frac{1}{2} L^2$$

The entrant at position 1 stays out if:

$$-c - \left(1 - \frac{1+L}{2} * L - \frac{1-L}{2} * 1 \right) < - \left(1 - \frac{R+L}{2} L - \frac{2-R-L}{2} R \right) \Leftrightarrow c > \frac{1}{2} (1-R)^2$$

Entry at positions L and R is an equilibrium if:

$$(a) 0 \leq L < \sqrt{2} - 1 ; \frac{L^2+1}{2} < R \leq 1 - L \text{ and } \frac{(1-R)^2}{2} < c < \frac{R^2-L^2}{2}$$

$$(b) 0 \leq L \leq \sqrt{2} - 1 ; 1 - L \leq R \leq 1 \text{ and } \frac{1}{2} L^2 < c < \frac{(2-R-L)*(R-L)}{2}$$

$$(c) \sqrt{2} - 1 < L < \frac{1}{2} ; 1 - \sqrt{1-2L} < R \leq 1 \text{ and } \frac{1}{2} L^2 < c < \frac{(2-R-L)*(R-L)}{2}$$

Three party equilibrium with entry at L, C and R

Since we assume sincere voting by uniformly distributed voters the implemented policy with candidates at positions L, C and R is:

$$\frac{C+L}{2} * L + \frac{R-L}{2} * C + \frac{2-R-C}{2} * R = \frac{L^2 + 2 * R - R^2}{2}$$

The implemented policy if C decides to not enter the race is:

$$\frac{L+R}{2} L + \frac{2-R-L}{2} * R = \frac{L^2 + 2 * R - R^2}{2}$$

This implies that candidate C has no influence on the implemented policy.

Therefore this situation cannot be an equilibrium.

3.A.3 Proof of proposition 3

To prove the proposition we analyze all equilibria with up to three entrants and thereby derive the three parts of proposition 1.

Single party equilibrium with entry at position X

We first have to ensure that the player at position X wants to enter. If she stays out she will receive a payoff of $-\infty$ (since nobody enters). Therefore the player at position X will enter. Next, we have to ensure that no other player wants to enter. The player that has the most to gain from entering is located on the opposite site of the median with a distance that is slightly smaller than the distance between X and the median.¹⁸ We can now consider three cases:

(a) $X < 0.5$

Denote the entrant's position by $Z = 1 - X - \varepsilon$. She prefers to stay out if the following condition holds:

$$-c < -(Z - X) \Leftrightarrow c > 1 - 2X - \varepsilon \xrightarrow{\varepsilon \rightarrow 0} c \geq 1 - 2X$$

(b) $X = 0.5$

No entrant can win the election.

(c) $X > 0.5$

Denote the entrant's position by $Z = 1 - X + \varepsilon$. She prefers to stay out if the following condition holds:

$$-c < -(X - Z) \Leftrightarrow c > 2X - 1 - \varepsilon \xrightarrow{\varepsilon \rightarrow 0} c \geq 2X - 1$$

Two party equilibrium with entry at positions L and R

The equilibrium has to be symmetric since otherwise one candidate loses for sure and does not enter. Therefore $R = 1 - L$. First, we need to make sure that both candidates prefer entry over staying out. The candidate at position L enters if the following condition holds (by symmetry this is the same condition for the candidate at R):

$$\begin{aligned} U_{enter} > U_{stay\ out} &\Leftrightarrow -c - (x_{LR} - L) > -(x_R - L) \Leftrightarrow \\ &-c - (0.5 - L) > -(R - L) \Leftrightarrow c < 0.5 - L \end{aligned}$$

Next, there are three types of entrants to consider.

First, an extreme entrant (without loss of generality located to the left of candidate L). Such an entrant never wants to enter. The reason is that the new coalition policy will be to the right of the old coalition policy. This follows from the observation that candidate R can

¹⁸ The reason is that this is the entrant that from the set of candidates that can win an absolute majority has most to lose from the policy that would result, if she does not enter. If this candidate has nothing to gain from entering, then no candidate does.

continue the coalition with candidate L and since R now has a higher weight the resulting policy lies to the right of the old policy. Since candidates only care about policy and given that candidate R can form a viable coalition with either of the two other candidates this implies that entry leads to a more right-wing policy than before. This is not in the interest of the extreme entrant and therefore she will not enter. The *second* type of entrant to consider, is located at position X between L and the median and does not become the formateur. By the same reasoning as above she also will not want to enter.¹⁹ The *third* type of entrant is located at position Y between candidate L and the median and wins a plurality of the votes. Since the entrants vote share is $\frac{R-L}{2}$ and candidate R's vote share is $\frac{2-R-Y}{2}$ this is the case if $Y > 3L$ (which given that Y is located to the left of the median implies that $L < \frac{1}{6}$). Furthermore, Y has to prefer a coalition with L over the grand coalition which is the case if $Y < \frac{\sqrt{36L^2-20L+17}+2L-1}{8}$ (which given $Y > 3L$ is possible if $L < \frac{\sqrt{8}-1}{14}$).²⁰ The reason is that the policy of the grand coalition is the same as the policy before entry and therefore entry is not in the interest of the entrant if she would form a grand coalition upon entering. Finally, the entrant prefers to stay out over entering and forming a coalition with L if the following condition holds:

$$\begin{aligned}
 U_{enter} < U_{stay\ out} &\Leftrightarrow -c - (Y - x_{LY}) > -(x_{LR} - Y) \Leftrightarrow \\
 &-c - \left(Y - \frac{(R-L)Y + (L+Y)L}{R+Y} \right) < -\left(\frac{1}{2} - Y \right) \\
 \xleftrightarrow{\text{using } R=1-L} c > &\frac{L^2 + LY - 0.5L - 0.5Y - 2Y^2 + 0.5}{1 - L + Y}
 \end{aligned}$$

The derivative of this expression with respect to Y is $\frac{1-2L}{(1-L+Y)^2} - 2 < 0$ and accordingly evaluating the expression at $Y = 3L$ (which is the position of the player most difficult to deter from entering) gives the relevant boundary for the cost of entry.

Therefore, entry at positions L and 1-L is an equilibrium if:

- (a) $0 < L < \frac{\sqrt{8}-1}{14}$ and $\frac{0.5-2L-14L^2}{1+2L} < c < 0.5 - L$ OR
 (b) $\frac{\sqrt{8}-1}{14} \leq L < \frac{1}{2}$ and $c < 0.5 - L$

Three party equilibrium with entry at L, C and R

To ensure that all candidates want to enter, no candidate can win an absolute majority since otherwise the losing candidates would prefer to stay out. Furthermore, an extreme candidate only enters if she becomes part of the coalition. The reason is that by staying out she can ensure that the center candidate wins which leads to a policy that is preferred to the

¹⁹ The same holds for an entrant located between the median and candidate R that does not win a plurality of the votes.

²⁰ Given that candidate L has fewer votes than candidate R and is also located closer to the entrant the entrant prefers a coalition with L over a coalition with R.

policy implemented by a coalition that she is not a part of. This leaves four cases to be considered.

I: The center party wins the plurality of votes and forms a grand coalition

If the coalition policy is to the left (right) of the center candidate's position candidate R (L) prefers to stay out to implement the center candidate's position as the policy. If the government's policy is equal to the center candidate's position the extreme candidates can save the entry costs without losing in terms of policy. Therefore, this cannot be an equilibrium.

II: The two extreme candidates are tied and each gets more votes than candidate C.

This situation arises if the following conditions hold:

$$(a) L + C = 2 - R - C \Leftrightarrow C = \frac{2-R-L}{2} \text{ (candidates L and R are tied)}$$

$$(b) L + C > R - L \Leftrightarrow C < R - 2L \xrightarrow{\text{using (a)}} R - L > \frac{2}{3} \text{ (candidate C receives fewest votes)}$$

First, we have to make sure that candidate L wants to enter. This is the case if:

$$\begin{aligned} U_{enter} > U_{stay\ out} &\Leftrightarrow -c - \left(\frac{1}{2}x_{LC} + \frac{1}{2}x_{CR} - L \right) > -(x_C - L) \Leftrightarrow \\ &-c - \left(\frac{(L+C)L + (R-L)C + (L+C)R + (R-L)C}{2R+2C} - L \right) > -(C-L) \\ \xrightarrow{c = \frac{2-R-L}{2}} &c < 1 - 0.5R - 1.5L - \left(\frac{(1-0.5R+0.5L)(R+L) + (R-L)(2-R-L)}{2+R-L} - L \right) \\ &\Leftrightarrow c < 3 + L + R - \frac{8R+4}{2-L+R} \end{aligned}$$

Candidate R enters if the following condition holds:

$$\begin{aligned} U_{enter} > U_{stay\ out} &\Leftrightarrow -c - \left(R - \frac{1}{2}x_{LC} - \frac{1}{2}x_{CR} \right) > -(R - x_C) \Leftrightarrow \\ &-c - \left(R - \frac{(L+C)L + (R-L)C + (L+C)R + (R-L)C}{2R+2C} \right) > -(R-C) \\ \xrightarrow{c = \frac{2-R-L}{2}} &c < 1.5R + 0.5L - 1 \\ &\quad - \left(R - \frac{(1-0.5R+0.5L)(R+L) + (R-L)(2-R-L)}{2+R-L} \right) \\ &\Leftrightarrow c < - \left[3 + L + R - \frac{8R+4}{2-L+R} \right] \end{aligned}$$

Combining these two conditions, we find that for both candidates R and L willing to enter we need that $c < \min \left\{ 3 + L + R - \frac{8R+4}{2-L+R}; - \left[3 + L + R - \frac{8R+4}{2-L+R} \right] \right\} \leq 0$ which is never satisfied. This implies that this is not an equilibrium.

III: All parties are tied

All parties are tied if:²¹

$$(a) R - L = L + C \Leftrightarrow C = R - 2L \text{ (C and L are tied)}$$

$$(b) R - L = 2 - R - C \Leftrightarrow C = 2 - 2R + L \stackrel{\text{by (a)}}{\Leftrightarrow} R = \frac{2}{3} + L \text{ (C and R are tied)}$$

We have to make sure that candidate L will enter. This is the case if:

$$\begin{aligned} U_{enter} > U_{stay\ out} &\Leftrightarrow -c - \left(\frac{1}{3}x_{LC} + \frac{1}{3}x_{LCR} + \frac{1}{3}x_{CR} - L \right) > -(x_C - L) \Leftrightarrow \\ -c - \frac{1}{3} \left(\frac{L+C}{2} - L \right) - \frac{1}{3} \left(\frac{R+C}{2} - L \right) - \frac{1}{3} \left(\frac{(R+L)L + (2-R-L)R}{2} - L \right) &> -(C-L) \\ \Leftrightarrow c < C - \frac{L+C+R+C+(R+L)L+(2-R-L)R}{6} &\Leftrightarrow \frac{4C-L-3R+R^2-L^2}{6} \\ \stackrel{C=R-2L}{\Leftrightarrow} c < \frac{R-9L+R^2-L^2}{6} \\ \stackrel{R=\frac{2}{3}+L}{\Leftrightarrow} c < \frac{10-60L}{54} \end{aligned}$$

Candidate R enters if:

$$\begin{aligned} U_{enter} > U_{stay\ out} &\Leftrightarrow -c - \left(R - \frac{1}{3}x_{LC} - \frac{1}{3}x_{LCR} - \frac{1}{3}x_{CR} \right) > -(R - x_C) \Leftrightarrow \\ -c + \frac{L+C}{6} + \frac{R+C}{6} + \frac{(R+L)L + (2-R-L)R}{6} &> C \\ \stackrel{C=R-2L}{\Leftrightarrow} c < \frac{9L-R-R^2+L^2}{6} \\ \stackrel{R=\frac{2}{3}+L}{\Leftrightarrow} c < \frac{60L-10}{54} \end{aligned}$$

Combining these two conditions, we find that for both candidates R and L willing to enter we need that $c < \min \left\{ \frac{10-60L}{54}; \frac{60L-10}{54} \right\} \leq 0$ which is never satisfied. This implies that this configuration is not an equilibrium.

²¹ Note, that the center candidate will form a consensus government instead of a two-party coalition.

IV: One extreme candidate (say candidate L) and the center candidate are tied and they each have more votes than the other extreme candidate.

This happens if:²²

(a) $L + C = R - L \Leftrightarrow C = R - 2L$ (candidates L and C are tied)

(b) $R - L > 2 - R - C \Leftrightarrow C > 2 - 2R + L \stackrel{\text{by (a)}}{\Leftrightarrow} R - L > \frac{2}{3}$ (candidate R gets fewer votes)

Whether candidate R enters depends on which coalition candidate L will form. We therefore have two situations to consider.

(a) L forms a center-left coalition

R enters if:

$$\begin{aligned}
 U_{enter} > U_{stay\ out} &\Leftrightarrow -c - \left(R - \frac{1}{2}x_{LC} - \frac{1}{2}x_{LCR} \right) > -(R - x_C) \Leftrightarrow \\
 -c - \frac{1}{2} \left(R - \frac{C+L}{2} \right) - \frac{1}{2} \left(R - \frac{(R+L)L + (2-R-L)R}{2} \right) &> -(R - C) \\
 \Leftrightarrow c < -\frac{R^2 - L^2 + 2R + 3C - L}{2} \\
 \stackrel{C=R-2L}{\Leftrightarrow} c < -\frac{R^2 - L^2 + 5R - 7L}{2} \\
 \stackrel{R-L > \frac{2}{3}}{\Leftrightarrow} c < -\frac{R^2 - L^2 + \frac{10}{3} - 2L}{2} < 0
 \end{aligned}$$

(b) L forms a coalition with candidate R

This situation arises if $\frac{C+L}{2} > \frac{(L+C)L + (2-R-C)R}{2-R+L}$, i.e. if the coalition with candidate R leads to a more left-wing policy than the coalition with candidate C. But in this case it is even less attractive than in case (a) for candidate R to enter instead of letting candidate C win an absolute majority. Therefore candidate R prefers to abstain.

Since for all four possible cases of three-candidate equilibria a violation of equilibrium conditions was detected we can conclude that no three-candidate equilibrium exists.

²² Note, that the center candidate will form a consensus government instead of a two-party coalition.

3.A.4 Proof of Proposition 4

Table 3A.1 shows the equilibria under the different electoral rules for the case of purely policy-motivated candidates.²³

Table 3A.1: Equilibria for purely policy-motivated candidates

	PL	PR	PR+C
One-candidate equilibrium	$0 \leq X < 0.5$ and $1 - 2X \leq c$ OR $X = 0.5$ OR $0.5 < X \leq 1$ and $2X - 1 \leq c$	$0 \leq X < 0.5$ and $\frac{1}{2}(1 - X)^2 < c$ OR $X = 0.5$ OR $0.5 < X \leq 1$ and $\frac{1}{2}X^2 < c$	$0 \leq X < 0.5$ and $1 - 2X \leq c$ OR $X = 0.5$ OR $0.5 < X \leq 1$ and $2X - 1 \leq c$
Two-candidate equilibrium	$R = 1 - L$ and $\frac{1}{6} < L < \frac{1}{2}$ and $c < 0.5 - L$	$0 \leq L < \sqrt{2} - 1$ and $\frac{L^2+1}{2} < R \leq 1 - L$ and $\frac{(1-R)^2}{2} < c < \frac{R^2-L^2}{2}$ OR $0 \leq L \leq \sqrt{2} - 1$ and $1 - L \leq R \leq 1$ and $\frac{L^2}{2} < c < \frac{(2-R-L)(R-L)}{2}$ OR $\sqrt{2} - 1 < L < \frac{1}{2}$ and $1 - \sqrt{1-2L} < R \leq 1$ and $\frac{L^2}{2} < c < \frac{(2-R-L)(R-L)}{2}$	$R = 1 - L$ and $0 < L < \frac{\sqrt{8}-1}{14}$ and $\frac{0.5-2L-14L^2}{1+2L} < c < \frac{1-2L}{2}$ OR $R = 1 - L$ and $\frac{\sqrt{8}-1}{14} < L < \frac{1}{2}$ and $c < 0.5 - L$

Notes. The table shows the equilibria for the three electoral rules. PL (PR; PR+C) denotes plurality voting (proportional representation without coalitions; proportional representation with coalitions). L, R and X denotes the candidates' positions in the policy space and c denotes the costs of entry.

We define the polarization of the equilibria as the distance between the left- and right-wing candidate's position, i.e. $R - L$.

²³ The equilibria for proportional representation without coalitions and plurality voting are derived in appendix B.

Table 3A.2, shows the possible levels of polarization for the different treatments. We find that the maximal polarization for plurality voting is $\frac{2}{3}$ while with proportional representation polarization can almost be maximal, i.e. 1. This proves part (a) of proposition 4, that proportional representation, with and without coalitions, yields higher polarization than plurality rule.

Part (b) of proposition 4 states that for proportional representation, an equilibrium with two centrist parties is more likely when coalitions are taken into account. Table 3A.2 shows this, since with coalitions the two candidates' positions can converge to the same position while without coalitions a polarization lower than $3 - 2\sqrt{2}$ is not sustainable.

Table 3A.2: Polarization of two party equilibria

PL	PR	PR+C
$R - L \in \left(0; \frac{2}{3}\right)$	$R - L \in \left(3 - 2\sqrt{2}; 1\right)$	$R - L \in (0; 1)$

Notes. The table shows the possible range of polarization of the two-candidate equilibria for the three electoral rules. PL (PR; PR+C) denotes plurality voting (proportional representation without coalitions; proportional representation with coalitions).

3.A.5 Proof of proposition 5

This proposition depicts the equilibria for the case for plurality voting, with office-motivated candidates. To prove the three parts of the proposition, we analyze all equilibria with up to three entrants.

Single party equilibrium with entry at position X

Any entrant that can win a majority will do so just to win the office rents.

Entry by a candidate at position X is an equilibrium if:

$$X = 0.5$$

Two party equilibrium with entry at L and R

The equilibrium has to be symmetric since otherwise one candidate loses for sure and will not enter. Therefore $R = 1 - L$. Since they both win office rents they will enter. There are three types of entrants to consider:

(a) Entry by an extremist (wlog to the left of L)

Such an entrant can never win a simple majority since player R does not lose any votes and still gets 50% of the votes. Therefore the effect of such an entrant is only that it makes party R the sole formateur. But given that she is to the left of candidate L the entrant dislikes this. Therefore this type of entrant has never an incentive to enter.

(b) Entry by a centrist voter (wlog to the left of $\frac{1}{2}$) not winning a plurality

By entering she takes away votes from L and R but given that she is closer to L this will lead to a majority for candidate R. This is not in the interest of the entrant and therefore this type of entrant will not enter.

(c) Entry by a centrist voter that does win a plurality

Any entrant that can win a plurality of the votes will enter. Given that the vote share for the center entrant $\frac{R-L}{2}$ while the vote share for candidates L and R is given by $\frac{0.5+L}{2}$ the entrant wins a plurality of the votes if $L \leq \frac{1}{6}$. This implies that if $L > \frac{1}{6}$ no entrant can benefit from entry since she will never win most votes.

Entry at positions L and 1-L is an equilibrium if:

$$\frac{1}{6} < L < \frac{1}{2}$$

Three party equilibrium with entry at L, C and R

Both extreme candidates need to tie and win a plurality of the votes since otherwise they prefer the center candidate to win which they can ensure by staying out of the race. The condition for this to be the case are:

$$(a) P_L = P_R \Leftrightarrow C + L = 2 - R - C \Rightarrow C = \frac{2-R-L}{2} \xrightarrow{L < C < R} 2 - 3R < L < \frac{2-R}{3}$$

$$(b) P_L \geq P_C \Leftrightarrow C + L > R - L \Rightarrow C \geq R - 2L \xrightarrow{\text{by (a)}} L \geq \frac{3R-2}{3}$$

There are now two cases to consider:

First, candidate C ties with the other candidates

This implies $L = \frac{2}{3} - C$, $R = \frac{4}{3} - C$ and $\frac{1}{3} \leq C \leq \frac{2}{3}$. Since all the entrants win office rents they will for sure enter. No extreme entrant wants to enter since this will result in a tie between the extreme candidate on the other side of the policy space and the center candidate which increases the expected distance to the implemented policy. An entrant that enters in the space between an extreme and the center candidate makes the extreme party on the other side of the policy space the formateur and therefore she does not want to enter.

Second, the center candidate does not win a plurality of the votes

Again the argument against entry by additional candidates from above holds. Furthermore the extreme candidates will enter for sure since they win office rents. We therefore only have to check under which conditions candidate C enters. We have three cases to consider:

- (a) If she does not enter L wins the election, which is the case for $R + L > 1$

Candidate C enters if:

$$-c - \frac{1}{2}(R - C) - \frac{1}{2}(C - L) > -(C - L) \xrightarrow{C = \frac{2-R-L}{2}} c < 1 - R - L < 0$$

- (b) If she does not enter L and R tie, which is the case for $R + L = 1$:

In this case C has no influence on the implemented policy and will therefore only enter if:

$$c < 0$$

- (c) If she does not enter R wins the election, which is the case for $R + L < 1$

Candidate C enters if:

$$-c - \frac{1}{2}(R - C) - \frac{1}{2}(C - L) > -(R - C) \xrightarrow{C = \frac{2-R-L}{2}} c < R + L - 1 < 0$$

Hence, this second case does not constitute an equilibrium.

Entry at positions L, C and R is an equilibrium if:

$$L = \frac{2}{3} - C, \frac{1}{3} \leq C \leq \frac{2}{3} \text{ and } R = \frac{4}{3} - C$$

3.A.6 Proof of proposition 6

This proposition depicts the equilibria for the case for proportional representation without coalition formation, with office-motivated candidates. To prove the three parts of the proposition, we analyze all equilibria with up to three entrants.

Single party equilibrium with entry at position X

Any entrant that can win a majority will do so just to win the office rents.

Entry by a candidate at position X is an equilibrium if:

$$X = 0.5$$

Two party equilibrium with entry at L and R

The candidate at position L enters if:

$$(a) R + L > 1$$

The entrant wins the office rents and therefore will enter for sure

$$(b) R + L = 1$$

The entrant shares office rents and therefore will enter for sure.

$$(c) R + L < 1$$

The entrant does not win the office rents and therefore only enters if the influence on the implemented policy is worth the costs of entry. This is the case if:

$$-c - \left(\frac{R+L}{2}L + \frac{2-R-L}{2}R - L \right) > -(R-L) \Leftrightarrow c < \frac{R^2 - L^2}{2}$$

The candidate at position R enters if:

$$(a) R + L > 1$$

The entrant does not win the office rents and therefore only enters if the influence on the implemented policy is worth the costs of entry. This is the case if:

$$-c - \left(R - \frac{R+L}{2}L - \frac{2-R-L}{2}R \right) > -(R-L) \Leftrightarrow c < \frac{2-R-L}{2}(R-L)$$

$$(b) R + L = 1$$

The entrant shares the office rents and therefore will enter for sure.

$$(c) R + L < 1$$

The entrant wins the office rents and therefore will enter for sure.

There cannot be any further entrants that would win a plurality of the votes since such an entrant would enter with certainty because the office rents go to the largest party..

An entrant at $X \leq L$ can win a plurality if:

$$X + L \geq R - X \text{ and } X + L \geq 2 - R - L \Leftrightarrow \max \left\{ 2 - R - 2L; \frac{R-L}{2} \right\} \leq X$$

Therefore to avoid entry we need $\max \left\{ 2 - R - 2L; \frac{R-L}{2} \right\} > L$.

An entrant at $L \leq X \leq R$ can win a plurality if:

$$R - L \geq L + X \text{ and } R - L \geq 2 - R - X \Leftrightarrow 2 - 2R + L \leq X \leq R - 2L$$

Therefore to avoid entry we need $R - L < \frac{2}{3}$.

An entrant at $X \geq R$ can win a plurality if:

$$2 - X - R \geq L + R \text{ and } 2 - X - R \geq X - L \Leftrightarrow X \leq \min \left\{ 2 - 2R - L; \frac{2-R+L}{2} \right\}$$

Therefore to avoid entry we need $R > \min \left\{ 2 - 2R - L; \frac{2-R+L}{2} \right\}$.

Next, consider entrants that do not obtain a plurality. They obtain no office rents but may be interested in affecting policies. The potential entrants with the most to gain in terms of policy are located at 0 and 1.

The entrant at position 0 stays out if:

$$-c - \left(\frac{R}{2} * 0 + \frac{2-R}{2} R - 0 \right) < - \left(\frac{R+L}{2} L + \frac{2-R-L}{2} R - 0 \right) \Leftrightarrow c > \frac{1}{2} L^2$$

The entrant at position 1 stays out if:

$$-c - \left(1 - \frac{1+L}{2} * L - \frac{1-L}{2} * 1 \right) < - \left(1 - \frac{R+L}{2} L - \frac{2-R-L}{2} R \right) \Leftrightarrow c > \frac{1}{2} (1-R)^2$$

All in all, entry at positions L and R is an equilibrium if any of the following sets of conditions holds:

$$(a) 1 - L < R < \frac{3L+2}{3}; \frac{1}{6} \leq L \leq \frac{1}{3} \text{ and } \frac{L^2}{2} < c < \frac{2-R-L}{2} (R-L)$$

$$(b) 1 - L < R < 2 - 3L; \frac{1}{3} < L \leq \sqrt{2} - 1 \text{ and } \frac{L^2}{2} < c < \frac{2-R-L}{2} (R-L)$$

$$(c) 1 - \sqrt{1-2L} < R < 2 - 3L; \sqrt{2} - 1 < L < \frac{4}{9} \text{ and } \frac{L^2}{2} < c < \frac{2-R-L}{2} (R-L)$$

$$(d) R = 1 - L; \frac{1}{6} < L < \frac{1}{2} \text{ and } c > \frac{1}{2} L^2$$

$$(e) \frac{2-L}{3} < R < \frac{3L+2}{3}; 0 \leq L \leq \frac{1}{6} \text{ and } (1-R)^2 < c < \frac{R^2-L^2}{2}$$

$$(f) \frac{2-L}{3} < R < 1-L; \frac{1}{6} < L \leq \frac{1}{3} \text{ and } (1-R)^2 < c < \frac{R^2-L^2}{2}$$

$$(g) \frac{1+L^2}{2} < R < 1-L; \frac{1}{3} < L < \sqrt{2}-1 \text{ and } (1-R)^2 < c < \frac{R^2-L^2}{2}$$

Three party equilibrium with entry at L, C and R

Since candidate C has no impact on the implemented policy she has to win (part of) the office rents to be willing to enter. This implies:

$$R-L \geq L+C \text{ and } R-L \geq 2-R-C \Leftrightarrow 2-2R+L \leq C \leq R-2L \Rightarrow R-L \geq \frac{2}{3}$$

Candidate L enters if:

$$(a) R-L = L+C \text{ (she wins office rents) OR}$$

$$(b) -c - \left(\frac{(L+R)L + (2-L-R)R}{2} - L \right) > - \left(\frac{(C+R)C + (2-C-R)R}{2} - L \right) \Leftrightarrow c < \frac{C^2-L^2}{2} \quad (\text{the influence on the implemented policy makes entry worthwhile})$$

Candidate R enters if:

$$(a) R-L = 2-R-C \text{ (she wins office rents) OR}$$

$$(b) -c - \left(R - \frac{(L+R)L + (2-L-R)R}{2} \right) > - \left(R - \frac{(C+L)L + (2-C-L)C}{2} \right) \Leftrightarrow c < \frac{C^2-2C+2R-R^2}{2} \quad (\text{the influence on the implemented policy makes entry worthwhile})$$

There do not exist any potential entrants that can win a plurality of the votes. Therefore only entry for policy reasons needs to be considered. The largest incentives have the voters at 0 and 1.

The entrant at position 0 stays out if:

$$-c - \left(\frac{R}{2} * 0 + \frac{2-R}{2} R - 0 \right) < - \left(\frac{R+L}{2} L + \frac{2-R-L}{2} R - 0 \right) \Leftrightarrow c > \frac{1}{2} L^2$$

The entrant at position 1 stays out if:

$$-c - \left(1 - \frac{1+L}{2} * L - \frac{1-L}{2} * 1 \right) < - \left(1 - \frac{R+L}{2} L - \frac{2-R-L}{2} R \right) \Leftrightarrow c > \frac{1}{2} (1-R)^2$$

All in all, entry at positions L, C and R is an equilibrium if any of the following sets of conditions holds:

$$(a) C = \frac{2}{3} - L; R = \frac{2}{3} + L; L < \frac{1}{3} \text{ and } c > \max \left\{ \frac{1}{2} L^2; \frac{1}{2} (1-R)^2 \right\}$$

$$(b) C = R - 2L; R > \frac{2}{3} + L; L < \frac{1}{3} \text{ and } \max \left\{ \frac{1}{2} L^2; \frac{1}{2} (1-R)^2 \right\} < c < 2[L^2 - RL + L]$$

$$(c) C = 2 - 2R + L; \frac{2 - (\sqrt{2} - 1)L}{2} > R > \frac{2}{3} + L; L < \frac{2(\sqrt{2} - 1)}{3} \text{ and } \max\left\{\frac{1}{2}L^2; \frac{1}{2}(1 - R)^2\right\} < c < 2[1 - 2R + L - RL + R^2]$$

$$(d) 2 - 2R + L < C < R - 2L; R > \frac{2}{3} + L; L < \frac{1}{3} \text{ and } \max\left\{\frac{1}{2}L^2; \frac{1}{2}(1 - R)^2\right\} < c < \min\left\{\frac{C^2 - 2C + 2R - R^2}{2}; \frac{C^2 - L^2}{2}\right\}$$

3.A.7 Proof of proposition 7

This proposition depicts the equilibria for the case for proportional representation with coalition formation, with office-motivated candidates. To prove the three parts of the proposition, we analyze all equilibria with up to three entrants.

Single party equilibrium with entry at position X

The candidate at position X prefers entering over staying out since she wins the office rents. Furthermore, any entrant that can win a majority will enter to win the office rents. Therefore the only one-candidate equilibrium is the median player entering since she is the only candidate who cannot be defeated.

Two party equilibrium with entry at L and R

The equilibrium has to be symmetric since otherwise one candidate loses for sure and will not enter. Therefore $R = 1 - L$. Given that in this case both candidates receive half the office rents, they will enter.

An entrant at the extremes of the policy space can always ensure getting some of the office rents since her vote share will be strictly lower than the vote share of the candidate on her side of the policy space and therefore she would be the preferred coalition partner. Therefore only $L = 0$ could be an equilibrium. But in this case an entrant located at the center of the policy space can win a plurality of the votes, thereby guaranteeing herself a part of the office rents. Ergo, no two-candidate equilibrium exists.

Three party equilibrium with entry at L, C and R

To ensure that all candidates want to enter, no candidate can win an absolute majority since otherwise the losing candidates would prefer to stay out. This implies $L + C \leq 1$ (candidate L does not win an absolute majority) and $R + C \geq 1$ (candidate R does not win an absolute majority), which implies $L < \frac{1}{2} < R$.²⁴ The extreme candidates only enter if they are part of the coalition. The reason is that by staying out they can ensure that the center candidate wins, which leads to a policy that is preferred to the policy implemented by a coalition that she is not part of. This leaves five cases to be considered as potential equilibria:

I: The center party wins the plurality of votes and is indifferent between a center-left and center-right coalition

This is the case if:

- (a) $L + C = 2 - R - C \Leftrightarrow C = \frac{2-R-L}{2}$ (candidate L and R have the same vote shares)
- (b) $R - L > L + C \Leftrightarrow C < R - 2L \xrightarrow{\text{by (a)}} R - L > \frac{2}{3}$ (candidate C receives the most votes)

²⁴ Note, that the center candidate can never win an absolute majority.

(c) $C - L = R - C \xLeftrightarrow{\text{by (a)}} 2 - R - 3L = 3R + L - 2 \Leftrightarrow R + L = 1 \Rightarrow C = \frac{1}{2}$ (the center-left and a center-right coalitions' policies are equidistant from candidate C's position)

Given that in this situation all candidates receive office rents (in expectation), they all want to enter. Now we need to ensure that no other player wants to enter.

Let us consider an entrant located at position Y between candidates L and C. First, we can note that the entrant cannot become the formateur since her vote share is smaller than the center candidate's vote share. Therefore either candidate R or candidate C have a plurality of the votes. Candidate R will either form a coalition with C or L but never with the entrant (R and the entrant never have a majority) which always leads to a more right-wing policy than before entry. Therefore the entrant only wants to enter if the center candidate stays the formateur which happens if:

$$2 - R - C < R - Y \xLeftrightarrow{C=\frac{1}{2}; R=1-L} Y < 0.5 - 2L$$

The center party will form a coalition with the entrant if:²⁵

$$P_C + P_Y > 0.5 \Leftrightarrow R - Y + C - L > 1 \xLeftrightarrow{C=\frac{1}{2}; R=1-L} Y < 0.5 - 2L \text{ (the coalition is viable)}$$

$$C - L \leq 2 - R - C \xLeftrightarrow{C=\frac{1}{2}; R=1-L} L \geq 0 \text{ (C wants to form a coalition with the entrant)}$$

From this follows that to support the equilibrium we need that $\nexists Y \in (L; 0.5 - 2L) \Rightarrow L > \frac{1}{6}$. But $R - L > 2/3$ and $R = 1 - L$ imply that $L < \frac{1}{6}$. Therefore this is not an equilibrium.

II: The two extreme candidates are formateur

This is the case if the following conditions hold:

$$(a) L + C = 2 - R - C \Leftrightarrow C = \frac{2-R-L}{2} \xLeftrightarrow{L < C < R} 2 - 3R < L < \frac{2-R}{3} \text{ (They are tied)}$$

$$(b) L + C > R - L \Leftrightarrow C > R - 2L \xLeftrightarrow{\text{using (a)}} R - L < \frac{2}{3} \text{ (C has fewer votes)}$$

Since all candidates receive office rents they all want to enter. Now we have to make sure that no additional player wants to enter.

An entrant at position Z that is located between L and C

Candidate R will then become the sole formateur. To support this equilibrium it cannot be the case that the entrant will be part of the coalition (since then she would want to enter to get part of the office rents). The conditions for different coalitions being viable are:

²⁵ L+C only have 50% of the votes; C+R always has a majority

- R and Z: $P_Z + P_R = \frac{C-L}{2} + \frac{2-R-C}{2} > \frac{1}{2} \Leftrightarrow R + L < 1$
- R and L: $P_L + P_R = \frac{L+Z}{2} + \frac{2-R-C}{2} > \frac{1}{2} \Rightarrow Z > \frac{R-3L}{2}$
- R and C: $P_C + P_R = \frac{R-Z}{2} + \frac{2-R-C}{2} > \frac{1}{2} \Rightarrow Z < 1 - C = \frac{R+L}{2}$
- R, L and C: always larger than 50%

If a coalition between the entrant and candidate is viable it will be formed if it is the minimal-winning coalition, because this maximizes the partners' shares in the office rents. Since $\frac{R-3L}{2} < \frac{R+L}{2}$ if $Z < \frac{R-3L}{2}$ (i.e. a coalition with L is not viable) only the two-party coalitions with Z or C need to be considered.²⁶ In this case the entrant Z will be part of the coalition if $\frac{R-Z}{2} > \frac{C-L}{2} \Leftrightarrow Z < 1.5R + 1.5L - 1$. Therefore, if there exists a $Z \in \left[L; \min\left\{\frac{R-3L}{2}; \frac{3R+3L-2}{2}\right\} \right]$ and $R + L < 1$ this configuration cannot be an equilibrium. Given that such a Z always exists it is a necessary condition for the existence of the grand coalition equilibrium configuration that $R + L \geq 1$.

By the same reasoning, for an entrant at a position located between C and R we need $R + L \leq 1$. Therefore, from now on we will use $R = 1 - L$ which implies $C = \frac{1}{2}$ and $L > \frac{1}{6}$. Also, since we are now in a symmetric case without loss of generality we can restrict attention to entrants to the left of the median.

An entrant at $X < L$

Now the right-wing candidate will be a formateur and can certainly form a coalition with candidate L or C.²⁷ This also implies that a three-party coalition will never be formed since such a coalition is not minimal-winning. Therefore the entrant will be part of the coalition if:

$$P_X + P_R = L + 0.5X + 0.25 > 0.5 \text{ (it is viable)}$$

$$P_X < \min\{P_L; P_C\} \Rightarrow L + X < \min\{C - X; R - L\} \text{ (it is the preferred option)}$$

$$\Rightarrow \max\{0.5 - 2L; 0\} < X < \min\{0.25 - 0.5L; 1 - 3L; L\}$$

So to deter entry we need that such an X cannot exist. This is the case if $L < \frac{1}{6}$ or $L > \frac{1}{3}$. Since $L < \frac{1}{6}$ violates the assumption that the extreme candidates receive more votes than candidate C to support the equilibrium we need $L > \frac{1}{3}$. We also have to consider entry for policy reasons. Since a center-right coalition after entry leads to a worse (i.e. more right-

²⁶ A coalition of R and Z, if possible, is always preferred to the three-party coalition since $P_Z = \frac{0.5-L}{2} < P_L + P_C = \frac{R+L}{2}$.

²⁷ The vote shares of the different coalitions are: $P_{RX} = L + 0.5X + 0.25$, $P_{RL} = 0.5(1 + L - X) > 0.5$, $P_{RC} = 0.75 - 0.5L > 0.5$

wing) policy we only need to analyze the case where a left-right coalition will be formed (which is the case if $P_L < P_C \Leftrightarrow C - X < R - L \Rightarrow X > 2L - 0.5$).

The left-right policy is $\frac{(C-X)L+(2-R-C)R}{2-R-X} = \frac{0.5+L-LX-L^2}{1+L-X}$ which is increasing in X . We therefore evaluate it at most left-wing entry position possible since this leads to the most favorable outcome for the entrant. The policy for an entrant at $X = 2L - 0.5$ is $\frac{0.5+1.5L-3L^2}{1.5-L}$ which for $L > \frac{1}{3}$ is larger than $\frac{1}{2}$ (the expected policy without entry). Therefore no extreme candidate will enter to influence the policy.

Reconsidering an entrant at position Y between L and C .

This entrant will never be part of a two-party coalition since $P_Y + P_R = \frac{C-L+2-R-C}{2} = \frac{1}{2}$ and both a coalition with L ($P_L + P_R = \frac{C+L+2-R-C}{2} = \frac{1+L}{2}$) and C ($P_C + P_R = \frac{R-Y+2-R-C}{2} = \frac{1.5-Y}{2}$) are viable. This implies that the entrant will never receive any office rents. So the only reason for entry would be to influence the policy. If the center-right coalition forms this unambiguously moves the policy to the right. Therefore, entry might only occur if the left-right coalition will be formed, i.e. if $P_L < P_C \Leftrightarrow C + L < R - Y \Rightarrow Y < 0.5 - 2L$. But for $L > \frac{1}{3}$ it is the case that $0.5 - 2L < L$ and therefore a left-right coalition will never arise. Therefore $C = \frac{1}{2}$ and $R = 1 - L$ and $L > \frac{1}{3}$ constitute a set of equilibria.

III: All parties are tied

This happens if

$$(a) R - L = L + C \Leftrightarrow C = R - 2L \text{ (C and L are tied)}$$

$$(b) R - L = 2 - R - C \Leftrightarrow C = 2 - 2R + L \stackrel{\text{by (a)}}{\iff} R = \frac{2}{3} + L \Rightarrow C = \frac{2}{3} - L \text{ (C and R are tied)}$$

Since all candidates receive office rents they all want to enter. We again consider all possible types of entrants.

An entrant located at $X < L$.

She only wants to enter if she is a part of the coalition since otherwise the policy will move to the right, decreasing her payoff. Given that $P_C = P_R = \frac{1}{3}$, X is part of the coalition if $P_X = \frac{X+L}{2} > \frac{1}{6} \Rightarrow X > \frac{1}{3} - L$.²⁸ Since $X < L$ an extreme entrant that is able to join the coalition exists if $L > \frac{1}{6}$. This implies that only for $L \leq \frac{1}{6}$ can this configuration be supported as an equilibrium.

²⁸ Note, that in this case L is not a viable coalition partner since her vote share is less than $1/6$.

An entrant at $L < Y < C$

This leads to candidate R being the formateur. The different coalitions are viable if:

- R and Y: $P_Y = \frac{C-L}{2} > \frac{1}{6} \Rightarrow \frac{2}{3} - 2L > \frac{1}{3} \Rightarrow L < \frac{1}{6}$.
- R and L: $P_L = \frac{L+Y}{2} > \frac{1}{6} \Rightarrow Y > \frac{1}{3} - L$
- R and C if $P_C = \frac{R-Y}{2} > \frac{1}{6} \Rightarrow Y < \frac{1}{3} + L$.

Since for $L < \frac{1}{6}$ it is the case that $\frac{1}{3} + L < C = \frac{2}{3} - L$ there exist entrants such that for R only a coalition with L or Y is feasible. In this case to support the equilibrium we need that candidate L receives fewer votes than the entrant. This is the case if $C - L > L + Y \Rightarrow Y < \frac{2}{3} - 3L$. But $\exists Y \in \left[\max \left\{ \frac{1}{3} + L; \frac{2}{3} - 3L \right\}; \frac{2}{3} - L \right]$ (i.e. entrants that make only a coalition with the entrant or candidate L feasible and that are preferred to candidate L) and therefore we need $L \geq \frac{1}{6}$ (i.e. the moderate entrant is not a viable coalition partner) to support this equilibrium configuration. This only leaves the case where $L = \frac{1}{6}$ (which results in the perfectly symmetric situation) as an equilibrium candidate.

Again consider an entrant located at $X < L$. We established above that she will not enter to be part of a coalition, but we still need to confirm that she will not enter for policy reasons. Her entrance would move the policy to the right, however decreasing her payoff. The reason is that both candidate C and R will form a coalition with candidate L independently of the entrant's decision but if she enters this reduces candidate L's bargaining power and therefore the policy will move to the right.

Finally, consider an entrant at $\frac{1}{6} < Z < \frac{1}{2}$ which earns a vote share of $\frac{1}{6}$. Given that both L and C earn a vote share larger than $\frac{1}{6}$ also this type of entrant does not want to enter since she will not become a part of the coalition.

Therefore the perfectly symmetric situation constitutes an equilibrium.

IV: One extreme candidate (wlog candidate L) and the center candidate are formateur

This happens if:

- $L + C = R - L \Leftrightarrow C = R - 2L$ (L and C are tied)
- $R - L > 2 - R - C \Leftrightarrow C > 2 - 2R + L \stackrel{\text{by (a)}}{\Leftrightarrow} R - L > \frac{2}{3}$ (R has least votes)

Since all candidates receive office rents they all want to enter. We now analyze all potential entrants.

An entrant at $X < L$

A necessary condition for the entrant becoming candidate C's coalition partner is $\frac{R-L}{2} + \frac{L+X}{2} > \frac{1}{2} \Leftrightarrow X > 1 - R$ (which makes sure that she is a viable partner). Furthermore, it has

to be the case that $P_X < P_R \Leftrightarrow L + X < 2 - R - C \xLeftrightarrow{C=R-2L} X < 2 - 2R + L$ (X is preferred over R).²⁹

Additionally, candidate L can form a coalition with candidate C if

$$P_L + P_C > \frac{1}{2} \Leftrightarrow R - L + C - X > 1 \xLeftrightarrow{C=R-2L} 2R - 3L - X > 1 \Leftrightarrow X < 2R - 3L - 1$$

Should candidate L and the entrant both be a viable partners, the entrant becomes part of the coalition if:

$$P_X < P_L \Leftrightarrow L + X < C - X \Leftrightarrow X < \frac{C - L}{2} \xLeftrightarrow{C=R-2L} X < \frac{R - 3L}{2}$$

This implies that for this situation to be an equilibrium we need that neither of the following conditions is satisfied:

- $1 - R < X < \min\left\{\frac{R-3L}{2}; 2 - 2R + L; L\right\}$ (L is a viable partner)
- $\max\{1 - R; 2R - 3L - 1\} < X < \min\{2 - 2R + L; L\}$ (L not viable)

It turns out these conditions can both be satisfied and therefore this situation is not an equilibrium.

V: One extreme candidate (wlog candidate L) is formateur and forms coalition with R

This happens if:

- (a) $1 \geq L + C > R - L \Leftrightarrow 1 - L \geq C > R - 2L$ (L has plurality of votes)
- (b) $R - L > 2 - R - C \Leftrightarrow C > 2 - 2R + L \xrightarrow{\text{by (a)}} R \geq \max\left\{0.5 + L; \frac{L+2}{3}\right\}$ (R is preferred)

Again we consider all possible entrants.

An entrant at $X > R$

She receive a vote share $P_X = \frac{2-X-R}{2}$ and therefore a two-party coalition with X is feasible if $P_X + P_L \geq \frac{1}{2} \Leftrightarrow X < 1 + C + L - R$. If furthermore $P_R + P_L \leq \frac{1}{2} \Leftrightarrow X \leq 1 - L$ (i.e. the left-right coalition is not viable) this coalition will be formed for certain.

Since $1 - L < 1 + C + L - R$ is implied by $C > R - 2L$ there are three cases to considered.

- $X < 1 - L$: The entrant is a viable coalition partner but candidate R is not. Since the entrant is preferred over a coalition with the (larger) center partner the entrant will enter.

²⁹ Since neither candidate C nor R loses votes, the center-right coalition is always viable.

- $1 - L < X < 1 + C + L - R$: Both the entrant and the right-wing candidate are viable coalition partners. If $P_X < P_R \Leftrightarrow 2 - X - R < X - C \Rightarrow X > \frac{2-R+C}{2}$ the entrant will be in the coalition. Since $C > R - 2L$ we have that $1 + C + L - R > \frac{2-R+C}{2}$ and therefore there always exists an entrant that wants to enter.
- $X > 1 + C + L - R$: Neither a two-party coalition with the entrant nor with candidate R is viable. But since the entrant and the right-wing candidate have the same combined vote share as candidate R had before the entry a three-party coalition with the entrant and candidate R will be formed. Therefore the entrant wants to enter.

Since there always exists an extreme entrant that will be part of the coalition this situation cannot be an equilibrium.

Conclusion

Entry at positions L , C and R is an equilibrium if:

(a) $L = \frac{1}{6}$, $C = \frac{1}{2}$ and $R = \frac{5}{6}$ OR

(b) $L > \frac{1}{3}$, $C = \frac{1}{2}$ and $R = 1 - L$

3.A.8 Proof of proposition 8

Table 3A.3 shows the equilibria under the different electoral rules for the case of purely office-motivated candidates.³⁰

Table 3A.3: Equilibria for purely office-motivated candidates

	PL	PR	PR+C
One-candidate equilibrium		$X = \frac{1}{2}$	
Two-candidate equilibrium	$R = 1 - L$ and $\frac{1}{6} < L < \frac{1}{2}$	$1 - L < R < \frac{3L+2}{3}$ and $\frac{1}{6} \leq L \leq \frac{1}{3}$ and $\frac{L^2}{2} < c < \frac{2-R-L}{2}(R-L)$ OR $1 - L < R < 2 - 3L$ and $\frac{1}{3} < L \leq \sqrt{2} - 1$ and $\frac{L^2}{2} < c < \frac{2-R-L}{2}(R-L)$ OR $1 - \sqrt{1-2L} < R < 2 - 3L$ and $\sqrt{2} - 1 < L < \frac{4}{9}$ and $\frac{L^2}{2} < c < \frac{2-R-L}{2}(R-L)$ OR $R = 1 - L$ and $\frac{1}{6} < L < \frac{1}{2}$ and $c > \frac{1}{2}L^2$ OR $\frac{2-L}{3} < R < \frac{3L+2}{3}$ and $0 \leq L \leq \frac{1}{6}$ and $(1-R)^2 < c < \frac{R^2-L^2}{2}$ OR $\frac{2-L}{3} < R < 1 - L$ and $\frac{1}{6} < L \leq \frac{1}{3}$ and $(1-R)^2 < c < \frac{R^2-L^2}{2}$ OR $\frac{1+L^2}{2} < R < 1 - L$ and $\frac{1}{3} < L < \sqrt{2} - 1$ and $(1-R)^2 < c < \frac{R^2-L^2}{2}$	Does not exist

³⁰ The equilibria for proportional representation without coalitions and plurality voting are derived in appendix B.

<p>Three-candidate equilibrium</p>	$L = \frac{2}{3} - C \text{ and}$ $\frac{1}{3} < C < \frac{2}{3} \text{ and}$ $R = \frac{4}{3} - C$	$C = \frac{2}{3} - L \text{ and}$ $R = \frac{2}{3} + L; L < \frac{1}{3} \text{ and}$ $c > \max\left\{\frac{1}{2}L^2; \frac{1}{2}(1-R)^2\right\}$ <p style="text-align: center;">OR</p> $C = R - 2L \text{ and}$ $R > \frac{2}{3} + L \text{ and}$ $L < \frac{1}{3} \text{ and}$ $\max\left\{\frac{1}{2}L^2; \frac{1}{2}(1-R)^2\right\}$ $< c < 2[L^2 - RL + L]$ <p style="text-align: center;">OR</p> $C = 2 - 2R + L \text{ and}$ $1 - \frac{(\sqrt{2}-1)L}{2} > R > \frac{2}{3} + L \text{ and}$ $L < \frac{2\sqrt{2}-2}{3} \text{ and}$ $\max\left\{\frac{1}{2}L^2; \frac{1}{2}(1-R)^2\right\} < c$ $< 2[1 - 2R + L - RL + R^2]$ <p style="text-align: center;">OR</p> $2 - 2R + L < C < R - 2L \text{ and}$ $R > \frac{2}{3} + L \text{ and}$ $L < \frac{1}{3} \text{ and}$ $\max\left\{\frac{1}{2}L^2; \frac{1}{2}(1-R)^2\right\} < c$ $< \min\left\{\frac{C^2 - 2C + 2R - R^2}{2}; \frac{C^2 - L^2}{2}\right\}$	$L = \frac{1}{6} \text{ and}$ $C = \frac{1}{2}$ <p style="text-align: center;">and $R = \frac{5}{6}$</p> <p style="text-align: center;">OR</p> $L > \frac{1}{3} \text{ and}$ $C = \frac{1}{2} \text{ and}$ $R = 1 - L$
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Notes. The table shows the equilibria for the three electoral rules. PL (PR; PR+C) denotes plurality voting (proportional representation without coalitions; proportional representation with coalitions). L, R, C and X denotes the candidates' positions in the policy space and c denotes the costs of entry.

Part (a) of proposition 8 states that under proportional representation with coalitions equilibria are more likely to involve multiple candidates. This can be seen from table 3A.3 since equilibria with two candidates exist for plurality voting and proportional representation without coalitions but not for proportional representation with coalitions. This implies that if the equilibrium involves more than one candidate, (note that such equilibria are the same for all electoral rules), the equilibrium with coalitions has at least three candidates while this is not the case for the other two rules.

Parts (b) of the proposition is concerned with the polarization of candidate positions. I again define the polarization of the equilibria as the distance between the left- and right-wing candidate's position, i.e. $R - L$. Table 3A.4 shows the possible range of polarization for the different electoral rules and types of equilibria.

From the table we see that proportional representation with coalitions leads to least polarized three candidate equilibria since polarization is at most $\frac{2}{3}$ while under plurality voting it is exactly $\frac{2}{3}$ and for proportional representation without coalitions it is at least $\frac{2}{3}$. This proves part (b) of the proposition.

Table 3A.4: Polarization of equilibria

	PL	PR	PR+C
Two-candidate equilibrium	$R - L \in \left(0; \frac{2}{3}\right)$	$R - L \in \left(0; \frac{2}{3}\right)$	Does not exist
Three-candidate equilibrium	$R - L = \frac{2}{3}$	$R - L \in \left(\frac{2}{3}; 1\right)$	$R - L \in \left(0; \frac{1}{3}\right) \cup \left\{\frac{2}{3}\right\}$

Notes. The table shows the possible range of polarization of two-candidate equilibria for the three electoral rules. PL (PR; PR+C) denotes plurality voting (proportional representation without coalitions; proportional representation with coalitions).