Classical simulation of entanglement swapping with bounded communication

Branciard, C.; Brunner, N.; Buhrman, H.M.; Cleve, R.; Gisin, N.; Portmann, S.; Rosset, D.; Szegedy, M.

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Classical simulation of entanglement swapping with bounded communication
– Supplementary material –

Cyril Branciard,1 Nicolas Brunner,2 Harry Buhrman,3 Richard Cleve,4,5
Nicolas Gisin,6 Samuel Portmann,6 Denis Rosset,6 and Mario Szegedy7

1School of Mathematics and Physics, The University of Queensland, St Lucia, QLD 4072, Australia
2H.H. Wills Physics Laboratory, University of Bristol, Bristol, BS8 1TL, United Kingdom
3Centrum Wiskunde & Informatica, and University of Amsterdam, Science Park 123, 1098 XG Amsterdam, The Netherlands
4Institute for Quantum Computing and School of Computer Science, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, Canada N2L 3G1
5Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario, Canada N2L 2Y5
6Group of Applied Physics, University of Geneva, Chemin de Pinchat 22, CH-1211 Geneva 4, Switzerland
7Department of Computer Science, Rutgers, the State University of NJ, 110 Frelinghuysen Road, Piscataway, NJ 08854-8019 USA

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In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation
(equation (1) of the main text), or in other terms, \( P(a = b|\mathbf{x}, \mathbf{y}) = \frac{1 - \pi}{2} \).

Correlation from Protocol 1

We first note that the definitions of \( a = \text{sign} (\sin \phi_A) \) and of \( \phi_A' = (\phi_A - jA\frac{\pi}{4}) \mod \pi \in [0, \frac{\pi}{4}] \) on Alice’s side, and the
definitions of \( \beta = \text{sign} (\sin (\phi_B - jA\frac{\pi}{4})) \) (and then of \( b = \pm \beta \)) and of \( \phi_B' = (\phi_B - jA\frac{\pi}{4}) \mod \pi \in [0, \pi] \) on Bob’s
side, ensure that the following relations hold, as required:

\[
P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi) = P(a = b|\phi_A + j\frac{\pi}{4}, \phi_B + j\frac{\pi}{4}) \quad \text{for any } j \in \mathbb{Z}.
\]

It is therefore sufficient to check that the correct correlation is obtained for \( \phi_A \in [0, \frac{\pi}{4}] \) and \( \phi_B \in [0, \pi] \).

For such values of \( \phi_A, \phi_B \) (for which \( \phi_A' = \phi_A \) and \( \phi_B' = \phi_B \)), the probability \( P(a = b|\phi_A, \phi_B) \) obtained from
Protocol 1 can be calculated as follows:

\[
P(a = b|\phi_A < \phi_B - jB\frac{\pi}{4}) = \frac{16}{\pi} \left( \int_{0}^{\phi_A} d\lambda_A \int_{0}^{\lambda_A} d\lambda_B \psi_{01}^{A}(\phi_B - \lambda_B) + \int_{0}^{\phi_A} d\lambda_A \int_{\lambda_A}^{\phi_A} d\lambda_B \psi_{10}^{B}(\phi_A - \lambda_A) \right)
\]

\[
+ \int_{0}^{\phi_A - jB\frac{\pi}{4}} d\lambda_A \int_{0}^{\lambda_A} d\lambda_B \psi_{01}^{A}(\phi_B - \lambda_B) + \int_{0}^{\phi_A - jB\frac{\pi}{4}} d\lambda_A \int_{\lambda_A}^{\phi_A} d\lambda_B \psi_{10}^{B}(\phi_A - \lambda_A)
\]

\[
+ \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_A \int_{0}^{\lambda_A} d\lambda_B \psi_{01}^{A}(\phi_B - \lambda_B) + \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_A \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_B \psi_{10}^{B}(\phi_A - \lambda_A)
\]

\[
+ \int_{0}^{\phi_A} d\lambda_A \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_B \psi_{01}^{A}(\phi_B - \lambda_B) + \int_{0}^{\phi_A} d\lambda_A \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_B \psi_{10}^{B}(\phi_A - \lambda_A)
\]

\[
+ \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_A \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_B \psi_{01}^{A}(\phi_B - \lambda_B) + \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_A \int_{\phi_A - jB\frac{\pi}{4}}^{\phi_A} d\lambda_B \psi_{10}^{B}(\phi_A - \lambda_A).
\]
and

\[
P(a = b | \phi_A \geq \phi_B - j_B \frac{\pi}{4}) = \frac{16}{\pi^2} \left( \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{001}^{j_B}(\phi_B - \lambda_{RB}) \right. \\
+ \left. \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{010}^{j_B}(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{000}^{j_B}(\phi_B - \lambda_{RB}) \right) \\
+ \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{010}^{j_B}(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{001}^{j_B}(\phi_B - \lambda_{RB}) \\
+ \frac{\pi}{4} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{010}^{j_B}(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{001}^{j_B}(\phi_B - \lambda_{RB}) \right).
\]

One can then check that with the choice of functions $\psi_{010}^{j_B} \in [0, 1]$ indicated in Table I (see main text), this leads (for all values of $j_B$) to

\[
P(a = b | \phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2},
\]
as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs $x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)$ and $y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)$, the probability that Alice and Bob’s outputs are the same is

\[
P(a = b | x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0 | \phi_A, \phi_B) P(a = b | \theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0 | \phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2}
\]

\[
= P(a_0 = b_0 | \phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0 | \phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2}
\]

\[
= \frac{1 - \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A + \theta_B)}{2} + \frac{1 + \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A - \theta_B)}{2}
\]

\[
= \frac{1 - \cos \theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos(\phi_A - \phi_B)}{2} = \frac{1 - x \cdot y}{2}.
\]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).