Classical simulation of entanglement swapping with bounded communication

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Published in:
Physical Review Letters

DOI:
10.1103/PhysRevLett.109.100401

Citation for published version (APA):
In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation (equation (1) of the main text), or in other terms, $P(a = b|x, y) = \frac{1 - \pi^2}{4}$.

Correlation from Protocol 1

We first note that the definitions of $a = \text{sign} (\sin (\phi_A))$ and of $\phi_A' = (\phi_A - jA \frac{\pi}{4} \mod \pi) \in [0, \frac{\pi}{4}]$ on Alice’s side, and the definitions of $b = \text{sign} (\sin (\phi_B - jB \frac{\pi}{4}))$ (and then of $b = \pm \beta$) and of $\phi_B' = (\phi_B - jB \frac{\pi}{4} \mod \pi) \in [0, \pi]$ on Bob’s side, ensure that the following relations hold, as required:

$$P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi) = P(a = b|\phi_A + j\frac{\pi}{4}, \phi_B + j\frac{\pi}{4}) \text{ for any } j \in \mathbb{Z}.$$ 

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0, \frac{\pi}{4}]$ and $\phi_B \in [0, \pi]$.

For such values of $\phi_A, \phi_B$ (for which $\phi_A' = \phi_A$ and $\phi_B' = \phi_B$), the probability $P(a = b|\phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$P(a = b|\phi_A < \phi_B - jB \frac{\pi}{4}) = \frac{16}{\pi^2} \left( \int_{0}^{\phi_A} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi_{010}^{AB}(\phi_{B} - \lambda_{RB}) \right. 
\left. + \int_{0}^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_{B} - jB \frac{\pi}{4}} d\lambda_{RB} \psi_{011}^{AB}(\phi_{B} - \lambda_{RB}) \right. 
\left. + \int_{\phi_{A}}^{\phi_{B} - jB \frac{\pi}{4}} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi_{101}^{AB}(\phi_{B} - \lambda_{RB}) \right. 
\left. + \int_{\phi_{A}}^{\phi_{B} - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_{B} - jB \frac{\pi}{4}} d\lambda_{RB} \psi_{110}^{AB}(\phi_{B} - \lambda_{RB}) \right. 
\left. + \int_{\phi_{B} - jB \frac{\pi}{4}}^{\phi_{B}} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi_{111}^{AB}(\phi_{B} - \lambda_{RB}) \right)$$

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– Supplementary material –

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(Dated: May 23, 2012)
and
\[ P(a = b|\phi_A \geq \phi_B - j_B \frac{\pi}{4}) = \frac{16}{\pi^2} \left( \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_A \int_0^{\lambda_{\lambda A}} d\lambda_{RB} \psi_{001}^{j_B}(\phi_B - \lambda_{RB}) \right. \]
\[ + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_A \int_{\lambda_{\lambda A}}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{001}^{j_B}(\phi_B - \lambda_{RB}) \]
\[ + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_A \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{100}^{j_B}(\phi_B - \lambda_{RB}) \]
\[ + \int_{\phi_A}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_A \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{100}^{j_B}(\phi_B - \lambda_{RB}) \]
\[ \left. + \int_{\phi_A}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_A \int_{\phi_A}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \psi_{100}^{j_B}(\phi_B - \lambda_{RB}) \right) \cdot \int_{\lambda_{\lambda A}}^{\lambda_{\lambda A}} d\lambda_{RB} \psi_{100}^{j_B}(\phi_B - \lambda_{RB}). \]

One can then check that with the choice of functions \( \psi_{CA,\phi B}^{j_B} \in [0, 1] \) indicated in Table I (see main text), this leads (for all values of \( j_B \)) to
\[ P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2}, \]
as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \( x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A) \) and \( y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B) \), the probability that Alice and Bob’s outputs are the same is
\[ P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2} \]
\[ = P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2} \]
\[ = \frac{1}{2} \left( 1 - \cos(\theta_A - \phi_B) \right) \frac{1 - \cos(\theta_A + \theta_B)}{2} + \frac{1}{2} \left( 1 + \cos(\theta_A - \phi_B) \right) \frac{1 - \cos(\theta_A - \theta_B)}{2} \]
\[ = \frac{2}{2} \left( 1 - \cos \theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos(\phi_A - \phi_B) \right) \frac{1}{2} \]
\[ \frac{1 - x \cdot y}{2}. \]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).