Classical simulation of entanglement swapping with bounded communication

Branciard, C.; Brunner, N.; Buhrman, H.; Cleve, R.; Gisin, N.; Portmann, S.; Rosset, D.; Szegedy, M.

DOI
10.1103/PhysRevLett.109.100401

Publication date
2012

Document Version
Other version

Published in
Physical Review Letters

Citation for published version (APA):
Correlation from Protocol 1

We first note that the definitions of $a = \text{sign} (\sin \phi_A)$ and of $\phi'_A = (\phi_A - j A_T \frac{\pi}{4} \mod \pi)$ (for which $\phi' = \phi_A = \phi_B = \phi_B'$), the probability $P(a = b| \phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$P(a = b| \phi_A, \phi_B) = \frac{16}{\pi} \left( \int_{0}^{\phi_A} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi_{101}^{10} (\phi_B - \lambda_{RB}) \right)$$

$$+ \int_{0}^{\phi_A} d\lambda_{AR} \int_{0}^{\phi_{\phi' - j A_{T}}} d\lambda_{RB} \psi_{101}^{10} (\phi_B - \lambda_{RB}) + \int_{0}^{\phi_A} d\lambda_{AR} \int_{0}^{\phi_{\phi' - j A_{T}}} d\lambda_{RB} \psi_{101}^{10} (\phi_B - \lambda_{RB})$$

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0, \frac{\pi}{4}]$ on Alice’s side, and $\phi_B \in [0, \pi]$. 

For such values of $\phi_A, \phi_B$ (for which $\phi'_A = \phi_A$ and $\phi'_B = \phi_B$), the probability $P(a = b| \phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:
and
\[ P(a = b|\phi_A \geq \phi_B - j_B \frac{\pi}{4}) = \frac{16}{\pi^2} \left( \int_0^{\phi_A - j_B \frac{\pi}{4}} d\lambda_A \int_0^{\lambda_{\lambda A}} d\lambda_R B \varphi_{001}^j(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_A \int_0^{\lambda_{\lambda A}} d\lambda_R B \varphi_{001}^j(\phi_B - \lambda_{RB}) \right) \]

One can then check that with the choice of functions $\varphi_{001}^j \in [0, 1]$ indicated in Table I (see main text), this leads (for all values of $j_B$) to
\[ P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2}, \]
as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs $x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)$ and $y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)$, the probability that Alice and Bob’s outputs are the same is
\[ P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(a_0\theta_A + b_0\theta_B)}{2} \]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).