Classical simulation of entanglement swapping with bounded communication

Branciard, C.; Brunner, N.; Buhrman, H.; Cleve, R.; Gisin, N.; Portmann, S.; Rosset, D.; Szegedy, M.

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Correlation from Protocol 1

We first note that the definitions of $a = \text{sign}(\sin(\phi_A))$ and of $\phi'_A = (\phi_A - j \pi A^2 \mod \pi) \in [0, \pi]$, and the definitions of $b = \text{sign}[\sin(\phi_B - j \pi A^2)]$ (and then of $b = \pm b'$) and of $\phi'_B = (\phi_B - j \pi A^2 \mod \pi) \in [0, \pi]$ on Bob’s side, ensure that the following relations hold, as required:

$$P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi) = P(a = b|\phi_A + j \frac{\pi}{4}, \phi_B + j \frac{\pi}{4}) \text{ for any } j \in \mathbb{Z}.$$

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0, \pi]$ and $\phi_B \in [0, \pi]$.

For such values of $\phi_A, \phi_B$ (for which $\phi'_A = \phi_A$ and $\phi'_B = \phi_B$), the probability $P(a = b|\phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$P(a = b|\phi_A < \phi_B - j \pi B^2 \frac{\pi}{4}) = \frac{16}{\pi} \left( \int_{\phi_A}^{\phi_B} d\lambda_{AB} \int_{0}^{\lambda_{AB}} d\lambda_{RB} \psi_{010}^{j_0}(\phi_B - \lambda_{RB}) \right.$$
and

\[ P(a = b|\phi_A \geq \phi_B - j_B) = \frac{16}{\pi^2} \left( \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AB} \int_0^{\lambda_{AB}} d\lambda_{RB} \psi^{j_B}_{001}(\phi_B - \lambda_{RB}) \right. \]
\[ + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi^{j_B}_{001}(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_\phi_{B - j_B \frac{\pi}{4}}^{\phi_A} d\lambda_{RB} \psi^{j_B}_{001}(\phi_B - \lambda_{RB}) \]
\[ + \int_\phi_{B - j_B \frac{\pi}{4}}^{\phi_B} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi^{j_B}_{001}(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_{B - j_B \frac{\pi}{4}}}^{\phi_B} d\lambda_{RB} \psi^{j_B}_{001}(\phi_B - \lambda_{RB}) \]
\[ + \int_{\phi_{B - j_B \frac{\pi}{4}}}^{\phi_B} d\lambda_{AR} \int_{\phi_{B - j_B \frac{\pi}{4}}}^{\phi_B} d\lambda_{RB} \psi^{j_B}_{001}(\phi_B - \lambda_{RB}) \right). \]

One can then check that with the choice of functions \( \psi^{j_B}_{001,0} \in [0,1] \) indicated in Table I (see main text), this leads (for all values of \( j_B \)) to

\[ P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2}, \]

as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \( x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A) \) and \( y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B) \), the probability that Alice and Bob’s outputs are the same is

\[ P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2} \]
\[ = P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2} \]
\[ = \frac{1 - \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A + \theta_B)}{2} + \frac{1 + \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A - \theta_B)}{2} \]
\[ = \frac{1 - \cos(\theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos(\phi_A - \phi_B))}{2} = \frac{1 - x \cdot y}{2}. \]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).