Classical simulation of entanglement swapping with bounded communication

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Correlation from Protocol 1

We first note that the definitions of $a = \text{sign}(\sin(\phi_A))$ and of $\phi'_A = (\phi_A - jA \pi \text{ mod } \pi) \in [0, \pi]$ on Alice’s side, and the definitions of $\beta = \text{sign}(\sin(\phi_B - jA \pi))$ (and then of $b = \pm \beta$) and of $\phi'_B = (\phi_B - jA \pi \text{ mod } \pi) \in [0, \pi]$ on Bob’s side, ensure that the following relations hold, as required:

$$P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi)$$

$$= P(a = b|\phi_A + j\frac{\pi}{4}, \phi_B + j\frac{\pi}{4}) \text{ for any } j \in \mathbb{Z}.$$ 

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0, \pi]$ and $\phi_B \in [0, \pi]$.

For such values of $\phi_A, \phi_B$ (for which $\phi'_A = \phi_A$ and $\phi'_B = \phi_B$), the probability $P(a = b|\phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$P(a = b|\phi_A < \phi_B - jB \frac{\pi}{4}) = \frac{16}{\pi} \left( \int_{\phi_A}^{\phi_B} d\phi_A \int_{0}^{\lambda_{AR}} d\lambda_{AR} \int_{\phi_B_{010}}^{\lambda_{010}} (\phi_B - \lambda_{RB}) d\phi_{010} \right)$$

$$+ \int_{0}^{\phi_A} d\phi_A \int_{\lambda_{AR}}^{\phi_A - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B_{011}}^{\lambda_{011}} (\phi_B - \lambda_{RB}) d\phi_{011}$$

$$+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\phi_A \int_{\lambda_{AR}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B_{110}}^{\lambda_{110}} (\phi_B - \lambda_{RB}) d\phi_{110}$$

$$+ \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B} d\phi_A \int_{\lambda_{AR}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B_{111}}^{\lambda_{111}} (\phi_B - \lambda_{RB}) d\phi_{111}.$$
Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).

and

\[
P(a = b|\phi_A \geq \phi_B - jB|\pi_4) = \frac{16}{\pi^2} \left( \int_0^{\phi_n-j\theta} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{\lambda_{AR}}^{i_{10}}(\phi_B - \lambda_{RB}) + \int_0^{\phi_n-j\theta} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{\lambda_{AR}}^{i_{01}}(\phi_B - \lambda_{RB}) + \int_0^{\phi_n-j\theta} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{\lambda_{AR}}^{i_{00}}(\phi_B - \lambda_{RB}) \right).
\]

One can then check that with the choice of functions $\psi_{\lambda_{AR}}^{i_{00}} \in [0, 1]$ indicated in Table I (see main text), this leads (for all values of $j_B$) to

\[
P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2},
\]

as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs $x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)$ and $y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)$, the probability that Alice and Bob’s outputs are the same is

\[
P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2}
\]

\[
= P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2}
\]

\[
= \frac{2}{2} - \frac{2}{2} = \frac{1}{2}. + \frac{1 + \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A - \theta_B)}{2}
\]

\[
= \frac{1}{2} - \frac{1}{2} = \frac{1}{2}.
\]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).