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Classical simulation of entanglement swapping with bounded communication
– Supplementary material –

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In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation (equation (1) of the main text), or in other terms, \(P(a = b| x, y) = \frac{1}{2} \text{sign}(x - y)\).

Correlation from Protocol 1

We first note that the definitions of \(a = \text{sign} (\sin (\phi_A))\) and of \(\phi_A' = (\phi_A - j \frac{\pi}{4} \mod \pi) \in [0, \frac{\pi}{4}]\) on Alice’s side, and the definitions of \(\beta = \text{sign} (\sin (\phi_B - j \frac{\pi}{4} \mod \pi))\) and of \(\phi_B' = (\phi_B - j \frac{\pi}{4} \mod \pi) \in [0, \pi]\) on Bob’s side, ensure that the following relations hold, as required:

\[
P(a = b| \phi_A, \phi_B) = P(a \neq b| \phi_A, \phi_B + \pi) = P(a \neq b| \phi_A + \pi, \phi_B) = P(a = b| \phi_A + \pi, \phi_B + \pi) = P(a = b| \phi_A + j \frac{\pi}{4}, \phi_B + j \frac{\pi}{4}) \quad \text{for any } j \in \mathbb{Z}.
\]

It is therefore sufficient to check that the correct correlation is obtained for \(\phi_A \in [0, \frac{\pi}{4}]\) and \(\phi_B \in [0, \pi]\).

For such values of \(\phi_A, \phi_B\) (for which \(\phi_A' = \phi_A\) and \(\phi_B' = \phi_B\)), the probability \(P(a = b| \phi_A, \phi_B)\) obtained from Protocol 1 can be calculated as follows:

\[
P(a = b| \phi_A, \phi_B) = \frac{16}{\pi} \left( \int_{0}^{\phi_A} d\lambda_{\text{AR}} \int_{0}^{\lambda_{\text{AR}}} d\lambda_{\text{RB}} \, \psi_{001}^{(\phi_B - \lambda_{\text{RB}})} \right)
\]

\[
+ \int_{0}^{\phi_A} d\lambda_{\text{AR}} \int_{\lambda_{\text{AR}}}^{\phi_A - j \frac{\pi}{4}} d\lambda_{\text{RB}} \, \psi_{011}^{(\phi_B - \lambda_{\text{RB}})} + \int_{0}^{\phi_A} d\lambda_{\text{AR}} \int_{\phi_A - j \frac{\pi}{4}}^{\lambda_{\text{AR}}} d\lambda_{\text{RB}} \, \psi_{100}^{(\phi_B - \lambda_{\text{RB}})}
\]

\[
+ \int_{\phi_A}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{AR}} \int_{\phi_A}^{\lambda_{\text{AR}}} d\lambda_{\text{RB}} \, \psi_{011}^{(\phi_B - \lambda_{\text{RB}})} + \int_{\phi_A}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{AR}} \int_{\lambda_{\text{AR}}}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{RB}} \, \psi_{001}^{(\phi_B - \lambda_{\text{RB}})}
\]

\[
+ \int_{\phi_A}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{AR}} \int_{\phi_A}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{RB}} \, \psi_{110}^{(\phi_B - \lambda_{\text{RB}})} + \int_{\phi_A}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{AR}} \int_{\phi_B - j \frac{\pi}{4}}^{\lambda_{\text{AR}}} d\lambda_{\text{RB}} \, \psi_{100}^{(\phi_B - \lambda_{\text{RB}})}
\]

\[
+ \int_{\phi_B - j \frac{\pi}{4}}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{AR}} \int_{\phi_B - j \frac{\pi}{4}}^{\lambda_{\text{AR}}} d\lambda_{\text{RB}} \, \psi_{110}^{(\phi_B - \lambda_{\text{RB}})} + \int_{\phi_B - j \frac{\pi}{4}}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{AR}} \int_{\lambda_{\text{AR}}}^{\phi_B - j \frac{\pi}{4}} d\lambda_{\text{RB}} \, \psi_{100}^{(\phi_B - \lambda_{\text{RB}})}.
\]
and

\[
P(a = b|\phi_A \geq \phi_B - j_B \pi/4) = \frac{16}{\pi^2} \left( \int_0^{\phi_a - j_B \pi/4} d\lambda_A \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{001}^{010}(\phi_B - \lambda_{RB}) \right. \\
+ \left. \int_0^{\phi_a - j_B \pi/4} d\lambda_A \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{010}^{011}(\phi_B - \lambda_{RB}) \right)
\]

One can then check that with the choice of functions \(\psi_{ACERB}^{j_B} \in [0, 1]\) indicated in Table I (see main text), this leads (for all values of \(j_B\) to

\[
P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2},
\]

as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \(x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)\) and \(y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)\), the probability that Alice and Bob's outputs are the same is

\[
P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2} \\
= P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2} \\
= \frac{2}{2} \frac{1 - \cos(\phi_A - \phi_B)}{2} + \frac{2}{2} \frac{1 - \cos(\phi_A + \phi_B)}{2} \\
= \frac{2 - \cos \theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos(\phi_A - \phi_B)}{2} = \frac{1 - \mathbf{x} \cdot \mathbf{y}}{2}.
\]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).