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– Supplementary material –

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In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation (equation (1) of the main text), or in other terms, \( P(a = b|x, y) = \frac{1 - x y}{2} \).

Correlation from Protocol 1

We first note that the definitions of \( a = \text{sign} (\sin \phi_A) \) and of \( \phi'_A = (\phi_A - j \pi \frac{A}{4} \mod \pi) \in [0, \pi] \) on Alice’s side, and the definitions of \( \beta = \text{sign} (\sin (\phi_B - j \pi \frac{B}{4})) \) (and then of \( b = \pm \beta \)) and of \( \phi'_B = (\phi_B - j \pi \frac{B}{4} \mod \pi) \in [0, \pi] \) on Bob’s side, ensure that the following relations hold, as required:

\[
P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi) = P(a = b|\phi_A + j \frac{\pi}{4}, \phi_B + j \frac{\pi}{4}) \text{ for any } j \in \mathbb{Z}.
\]

It is therefore sufficient to check that the correct correlation is obtained for \( \phi_A \in [0, \pi] \) and \( \phi_B \in [0, \pi] \).

For such values of \( \phi_A, \phi_B \) (for which \( \phi'_A = \phi_A \) and \( \phi'_B = \phi_B \)), the probability \( P(a = b|\phi_A, \phi_B) \) obtained from Protocol 1 can be calculated as follows:

\[
P(a = b|\phi_A < \phi_B - jB \frac{\pi}{4}) = \frac{16}{\pi} \left( \int_{0}^{\phi_A} d\lambda_A \int_{0}^{\lambda_A} d\lambda_R \psi_{010}^{j} (\phi_B - \lambda_B) \right.
\]

\[
+ \int_{0}^{\phi_A} d\lambda_A \int_{\lambda_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_R \psi_{011}^{j} (\phi_B - \lambda_B) + \int_{0}^{\phi_A} d\lambda_A \int_{\phi_B - jB \frac{\pi}{4}}^{\pi} d\lambda_R \psi_{010}^{j} (\phi_B - \lambda_B) \Bigg)
\]

\[
+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_A \int_{0}^{\lambda_A} d\lambda_R \psi_{101}^{j} (\phi_B - \lambda_B) + \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_A \int_{\phi_B - jB \frac{\pi}{4}}^{\pi} d\lambda_R \psi_{100}^{j} (\phi_B - \lambda_B) \Bigg)
\]

\[
+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_A \int_{\phi_B - jB \frac{\pi}{4}}^{\lambda_A} d\lambda_R \psi_{100}^{j} (\phi_B - \lambda_B) + \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_A \int_{\phi_B - jB \frac{\pi}{4}}^{\pi} d\lambda_R \psi_{111}^{j} (\phi_B - \lambda_B) \Bigg)
\]

\[
+ \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_A \int_{\phi_B - jB \frac{\pi}{4}}^{\lambda_A} d\lambda_R \psi_{100}^{j} (\phi_B - \lambda_B) + \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_A \int_{\phi_B - jB \frac{\pi}{4}}^{\pi} d\lambda_R \psi_{111}^{j} (\phi_B - \lambda_B) \Bigg).
\]
and
\[
P(a = b|\phi_A \geq \phi_B - jB/4) = \frac{16}{\pi^2} \left( \int_0^{\phi_A - jB/4} d\lambda_A \int_0^{\lambda_A} d\lambda_R B \psi_{001}^{jn} (\phi_B - \lambda_{RB}) d\lambda_{RB} \right)
+ \int_0^{\phi_A} d\lambda_A \int_0^{\lambda_A} d\lambda_R B \psi_{001}^{jn} (\phi_B - \lambda_{RB}) d\lambda_{RB} \psi_{001}^{jn} (\phi_B - \lambda_{RB})
+ \int_0^{\phi_A} d\lambda_A \int_0^{\lambda_A} d\lambda_R B \psi_{001}^{jn} (\phi_B - \lambda_{RB}) d\lambda_{RB} \psi_{001}^{jn} (\phi_B - \lambda_{RB})
+ \int_0^{\phi_A} d\lambda_A \int_0^{\lambda_A} d\lambda_R B \psi_{001}^{jn} (\phi_B - \lambda_{RB}) d\lambda_{RB} \psi_{001}^{jn} (\phi_B - \lambda_{RB})
+ \int_0^{\phi_A} d\lambda_A \int_0^{\lambda_A} d\lambda_R B \psi_{001}^{jn} (\phi_B - \lambda_{RB}) d\lambda_{RB} \psi_{001}^{jn} (\phi_B - \lambda_{RB})
\]

One can then check that with the choice of functions \(\psi_{001}^{jn} \in [0, 1]\) indicated in Table 1 (see main text), this leads (for all values of \(j_B\)) to
\[
P(a = b|\phi_A, \phi_B) = \frac{1 - \cos (\phi_A - \phi_B)}{2},
\]
as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \(x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)\) and \(y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)\), the probability that Alice and Bob’s outputs are the same is
\[
P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos (a_0 \theta_A + b_0 \theta_B)}{2}
= P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos (\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos (\theta_A - \theta_B)}{2}
= \frac{2}{2} - \cos \theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos (\phi_A - \phi_B)
\]
Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).