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Classical simulation of entanglement swapping with bounded communication
– Supplementary material –

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In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation (equation (1) of the main text), or in other terms, $P(a = b|\mathbf{x}, \mathbf{y}) = \frac{1-\pi}{2}$.

Correlation from Protocol 1

We first note that the definitions of $a = \text{sign} (\sin \phi_A)$ and of $\phi'_A = (\phi_A - j A \frac{\pi}{4} \mod \pi) \in [0, \frac{\pi}{4}]$ on Alice’s side, and the definitions of $\beta = \text{sign} (\sin (\phi_B - j B \frac{\pi}{4}))$ (and then of $b = \pm \beta$) and of $\phi'_B = (\phi_B - j B \frac{\pi}{4} \mod \pi) \in [0, \pi]$ on Bob’s side, ensure that the following relations hold, as required:

$$P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi)$$

$$= P(a = b|\phi_A + j \frac{\pi}{4}, \phi_B + j \frac{\pi}{4}) \quad \text{for any } j \in \mathbb{Z}.$$

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0, \frac{\pi}{4}]$ and $\phi_B \in [0, \pi]$.

For such values of $\phi_A, \phi_B$ (for which $\phi'_A = \phi_A$ and $\phi'_B = \phi_B$), the probability $P(a = b|\phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$P(a = b|\phi_A < \phi_B - j B \frac{\pi}{4}) = \frac{16}{\pi} \left( \int_{0}^{\phi_A} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi^{01}_{001}(\phi_B - \lambda_{RB}) \right.$$ 

$$+ \int_{0}^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - j A \frac{\pi}{4}} d\lambda_{RB} \psi^{01}_{011}(\phi_B - \lambda_{RB}) + \int_{0}^{\phi_A} d\lambda_{AR} \int_{\phi_B - j A \frac{\pi}{4}}^{\pi} d\lambda_{RB} \psi^{01}_{010}(\phi_B - \lambda_{RB})$$

$$+ \int_{\phi_A}^{\phi_B - j A \frac{\pi}{4}} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi^{11}_{111}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\phi_B - j A \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j A \frac{\pi}{4}}^{\phi_B - j B \frac{\pi}{4}} d\lambda_{RB} \psi^{11}_{110}(\phi_B - \lambda_{RB})$$

$$+ \int_{\phi_A}^{\phi_B - j B \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j B \frac{\pi}{4}}^{\phi_B \frac{\pi}{4}} d\lambda_{RB} \psi^{11}_{101}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\phi_B - j B \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j B \frac{\pi}{4}}^{\pi} d\lambda_{RB} \psi^{11}_{100}(\phi_B - \lambda_{RB})$$

$$+ \int_{\phi_B - j B \frac{\pi}{4}}^{\pi} d\lambda_{AR} \int_{\phi_B - j B \frac{\pi}{4}}^{\phi_B \frac{\pi}{4}} d\lambda_{RB} \psi^{11}_{011}(\phi_B - \lambda_{RB}) + \int_{\phi_B - j B \frac{\pi}{4}}^{\pi} d\lambda_{AR} \int_{\phi_B - j B \frac{\pi}{4}}^{\phi_B \frac{\pi}{4}} d\lambda_{RB} \psi^{11}_{010}(\phi_B - \lambda_{RB})$$

$$\left. + \int_{\phi_B - j B \frac{\pi}{4}}^{\phi_B \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j B \frac{\pi}{4}}^{\pi} d\lambda_{RB} \psi^{11}_{001}(\phi_B - \lambda_{RB}) \right).$$
and

\[
P(a = b|\phi_A \geq \phi_B - j_B) = \frac{16}{\pi^2} \left( \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{001}^j(\phi_B - \lambda_{RB}) \right. \\
+ \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{010}^j(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{000}^j(\phi_B - \lambda_{RB}) \\
+ \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{010}^j(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{000}^j(\phi_B - \lambda_{RB}) \left. \right) \\
+ \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{001}^j(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{010}^j(\phi_B - \lambda_{RB})
\]

One can then check that with the choice of functions \(\psi_{\alpha\beta\gamma\delta} \in [0, 1]\) indicated in Table I (see main text), this leads (for all values of \(j_B\)) to

\[
P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2},
\]

as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \(x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)\) and \(y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)\), the probability that Alice and Bob’s outputs are the same is

\[
P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2}
\]

\[
= P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2}
\]

\[
= \frac{1 - \cos \theta_A \cos \theta_B}{2} + \frac{1 - \cos \theta_A \sin \theta_B}{2} \frac{1 - \cos \theta_B}{2} = \frac{1 - x \cdot y}{2}.
\]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).