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DOI
10.1103/PhysRevLett.109.100401

Publication date
2012

Document Version
Other version

Published in
Physical Review Letters

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– Supplementary material –

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(Dated: May 23, 2012)

In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation (equation (1) of the main text), or in other terms, $P(a=b|\boldsymbol{x},\boldsymbol{y}) = \frac{1-\boldsymbol{x}\cdot\boldsymbol{y}}{2}$.

Correlation from Protocol 1

We first note that the definitions of $a = \mathrm{sign}(\sin\phi_A)$ and of $\phi'_A = (\phi_A - j_A\frac{\pi}{4} \mod \pi) \in [0, \frac{\pi}{4}[$ on Alice's side, and the definitions of $\beta = \mathrm{sign}[\sin(\phi_B - j_A\frac{\pi}{4})]$ (and then of $b = \pm\beta$) and of $\phi'_B = (\phi_B - j_A\frac{\pi}{4} \mod \pi) \in [0,\pi[$ on Bob's side, ensure that the following relations hold, as required:

$$\begin{aligned}
P(a=b|\phi_A,\phi_B) &= P(a\neq b|\phi_A,\phi_B+\pi) = P(a\neq b|\phi_A+\pi,\phi_B) = P(a=b|\phi_A+\pi,\phi_B+\pi) \\
&= P(a=b|\phi_A+j\frac{\pi}{4},\phi_B+j\frac{\pi}{4}) \quad \text{for any } j\in\mathbb{Z}.
\end{aligned}$$

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0,\frac{\pi}{4}[$ and $\phi_B \in [0,\pi[$.

For such values of ϕ_A, ϕ_B (for which $\phi'_A = \phi_A$ and $\phi'_B = \phi_B$), the probability $P(a=b|\phi_A,\phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$\begin{aligned}
P(a=b|\phi_A<\phi_B - j_B\frac{\pi}{4}) \;=\; \frac{16}{\pi^2}\Bigg(& \int_0^{\phi_A} d\lambda_{AR}\int_0^{\lambda_{AR}} d\lambda_{RB}\;\wp_{001}^{j_B}(\phi_B-\lambda_{RB}) \\
&+ \int_0^{\phi_A} d\lambda_{AR}\int_{\lambda_{AR}}^{\phi_B-j_B\frac{\pi}{4}} d\lambda_{RB}\;\wp_{011}^{j_B}(\phi_B-\lambda_{RB}) + \int_0^{\phi_A} d\lambda_{AR}\int_{\phi_B-j_B\frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{RB}\;\wp_{010}^{j_B}(\phi_B-\lambda_{RB}) \\
&+ \int_{\phi_A}^{\phi_B-j_B\frac{\pi}{4}} d\lambda_{AR}\int_0^{\lambda_{AR}} d\lambda_{RB}\;\wp_{101}^{j_B}(\phi_B-\lambda_{RB}) + \int_{\phi_A}^{\phi_B-j_B\frac{\pi}{4}} d\lambda_{AR}\int_{\lambda_{AR}}^{\phi_B-j_B\frac{\pi}{4}} d\lambda_{RB}\;\wp_{111}^{j_B}(\phi_B-\lambda_{RB}) \\
&+ \int_{\phi_A}^{\phi_B-j_B\frac{\pi}{4}} d\lambda_{AR}\int_{\phi_B-j_B\frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{RB}\;\wp_{110}^{j_B}(\phi_B-\lambda_{RB}) + \int_{\phi_B-j_B\frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{AR}\int_0^{\phi_B-j_B\frac{\pi}{4}} d\lambda_{RB}\;\wp_{101}^{j_B}(\phi_B-\lambda_{RB}) \\
&+ \int_{\phi_B-j_B\frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{AR}\int_{\phi_B-j_B\frac{\pi}{4}}^{\lambda_{AR}} d\lambda_{RB}\;\wp_{100}^{j_B}(\phi_B-\lambda_{RB}) + \int_{\phi_B-j_B\frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{AR}\int_{\lambda_{AR}}^{\frac{\pi}{4}} d\lambda_{RB}\;\wp_{110}^{j_B}(\phi_B-\lambda_{RB})\Bigg),
\end{aligned}$$

and

$$
\begin{aligned}
P(a=b|\phi_A \geq \phi_B - j_B\frac{\pi}{4}) \; = \; & \frac{16}{\pi^2}\Bigg(\int_0^{\phi_B - j_B\frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \; \wp^{j_B}_{001}(\phi_B - \lambda_{RB}) \\
& + \int_0^{\phi_B - j_B\frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - j_B\frac{\pi}{4}} d\lambda_{RB} \; \wp^{j_B}_{011}(\phi_B - \lambda_{RB}) \; + \int_0^{\phi_B - j_B\frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j_B\frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{RB} \; \wp^{j_B}_{010}(\phi_B - \lambda_{RB}) \\
& + \int_{\phi_B - j_B\frac{\pi}{4}}^{\phi_A} d\lambda_{AR} \int_0^{\phi_B - j_B\frac{\pi}{4}} d\lambda_{RB} \; \wp^{j_B}_{001}(\phi_B - \lambda_{RB}) \; + \int_{\phi_B - j_B\frac{\pi}{4}}^{\phi_A} d\lambda_{AR} \int_{\phi_B - j_B\frac{\pi}{4}}^{\lambda_{AR}} d\lambda_{RB} \; \wp^{j_B}_{000}(\phi_B - \lambda_{RB}) \\
& + \int_{\phi_B - j_B\frac{\pi}{4}}^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\frac{\pi}{4}} d\lambda_{RB} \; \wp^{j_B}_{010}(\phi_B - \lambda_{RB}) \; + \int_{\phi_A}^{\frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B\frac{\pi}{4}} d\lambda_{RB} \; \wp^{j_B}_{101}(\phi_B - \lambda_{RB}) \\
& + \int_{\phi_A}^{\frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j_B\frac{\pi}{4}}^{\lambda_{AR}} d\lambda_{RB} \; \wp^{j_B}_{100}(\phi_B - \lambda_{RB}) \; + \int_{\phi_A}^{\frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\frac{\pi}{4}} d\lambda_{RB} \; \wp^{j_B}_{110}(\phi_B - \lambda_{RB}) \Bigg).
\end{aligned}
$$

One can then check that with the choice of functions $\wp^{j_B}_{c_A c_R c_B} \in [0,1]$ indicated in Table I (see main text), this leads (for all values of j_B) to

$$
P(a=b|\phi_A, \phi_B) \; = \; \frac{1-\cos(\phi_A - \phi_B)}{2},
$$

as desired.

Correlation from Protocol 2

After running Protocol 2 for inputs $\boldsymbol{x} = (\sin\theta_A\cos\phi_A, \sin\theta_A\sin\phi_A, \cos\theta_A)$ and $\boldsymbol{y} = (\sin\theta_B\cos\phi_B, \sin\theta_B\sin\phi_B, \cos\theta_B)$, the probability that Alice and Bob's outputs are the same is

$$
\begin{aligned}
P(a=b|\boldsymbol{x},\boldsymbol{y}) \; &= \sum_{a_0,b_0=\pm1} P(a_0,b_0|\phi_A,\phi_B)\, P(a=b|\theta_A,\theta_B,a_0,b_0) \; = \sum_{a_0,b_0=\pm1} P(a_0,b_0|\phi_A,\phi_B)\, \frac{1-\cos(a_0\theta_A + b_0\theta_B)}{2} \\
&= P(a_0=b_0|\phi_A,\phi_B)\, \frac{1-\cos(\theta_A+\theta_B)}{2} + \; P(a_0\neq b_0|\phi_A,\phi_B)\, \frac{1-\cos(\theta_A-\theta_B)}{2} \\
&= \frac{1-\cos(\phi_A-\phi_B)}{2}\,\frac{1-\cos(\theta_A+\theta_B)}{2} + \frac{1+\cos(\phi_A-\phi_B)}{2}\,\frac{1-\cos(\theta_A-\theta_B)}{2} \\
&= \frac{1-\cos\theta_A\cos\theta_B - \sin\theta_A\sin\theta_B\cos(\phi_A-\phi_B)}{2} \; = \; \frac{1-\boldsymbol{x}\cdot\boldsymbol{y}}{2}\,.
\end{aligned}
$$

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).