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DOI
10.1103/PhysRevLett.109.100401

Publication date
2012

Document Version
Other version

Published in
Physical Review Letters

Citation for published version (APA):
https://doi.org/10.1103/PhysRevLett.109.100401

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Download date: 13 Jun 2021
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– Supplementary material –

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(Dated: May 23, 2012)

In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation
(equation (1) of the main text), or in other terms, \(P(a = b|\mathbf{x}, \mathbf{y}) = \frac{1}{2}\).

Correlation from Protocol 1

We first note that the definitions of \(a = \text{sign}(\sin \phi_A)\) and of \(\phi'_A = (\phi_A - jA \frac{\pi}{4} \mod \pi) \in [0, \frac{\pi}{4}]\) on Alice’s side, and
the definitions of \(b = \text{sign}(\sin(\phi_B - jA \frac{\pi}{4}))\) (and then of \(b = \pm \beta\)) and of \(\phi'_B = (\phi_B - jA \frac{\pi}{4} \mod \pi) \in [0, \pi]\) on Bob’s
side, ensure that the following relations hold, as required:

\[
P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi)
\]

\[
= P(a = b|\phi_A + jA \frac{\pi}{4}, \phi_B + jA \frac{\pi}{4}) \text{ for any } j \in \mathbb{Z}.
\]

It is therefore sufficient to check that the correct correlation is obtained for \(\phi_A \in [0, \frac{\pi}{4}]\) and \(\phi_B \in [0, \pi]\).

For such values of \(\phi_A, \phi_B\) (for which \(\phi'_A = \phi_A\) and \(\phi'_B = \phi_B\)), the probability \(P(a = b|\phi_A, \phi_B)\) obtained from
Protocol 1 can be calculated as follows:

\[
P(a = b|\phi_A < \phi_B - jB \frac{\pi}{4}) = \frac{16}{\pi} \left( \int_{0}^{\phi_A} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi_{001}^{\lambda_{AR}}(\phi_B - \lambda_{RB}) \right.
\]

\[
+ \int_{0}^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_A - jA \frac{\pi}{4}} d\lambda_{RB} \psi_{001}^{\lambda_{RB}}(\phi_B - \lambda_{RB}) \right.
\]

\[
+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{0}^{\lambda_{AR}} d\lambda_{RB} \psi_{101}^{\lambda_{RB}}(\phi_B - \lambda_{RB}) \right.
\]

\[
+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{RB} \psi_{101}^{\lambda_{RB}}(\phi_B - \lambda_{RB}) \right.
\]

\[
+ \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{RB} \psi_{101}^{\lambda_{RB}}(\phi_B - \lambda_{RB}) \].


and

\[ P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2}, \]

as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \( x = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A) \) and \( y = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B) \), the probability that Alice and Bob's outputs are the same is

\[ P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) P(a = b|\theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2} \]

\[ = P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2} \]

\[ = \frac{2 - \cos(\theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos(\phi_a - \phi_B))}{2} = \frac{1 - x \cdot y}{2}. \]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).