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Classical simulation of entanglement swapping with bounded communication – Supplementary material –

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In this Supplementary material we prove that Protocols 1 and 2 generate the desired correlation (equation (1) of the main text), or in other terms, $P(a = b|\mathbf{x}, \mathbf{y}) = \frac{1-\mathbf{x}\cdot\mathbf{y}}{2}$.

Correlation from Protocol 1

We first note that the definitions of $a = \text{sign}(\sin \phi_A)$ and of $\phi'_A = (\phi_A - j_A \frac{\pi}{4} \bmod \pi) \in [0, \frac{\pi}{4}[$ on Alice's side, and the definitions of $\beta = \text{sign}[\sin(\phi_B - j_A \frac{\pi}{4})]$ (and then of $b = \pm\beta$) and of $\phi'_B = (\phi_B - j_A \frac{\pi}{4} \bmod \pi) \in [0, \pi[$ on Bob's side, ensure that the following relations hold, as required:

$$\begin{aligned} P(a = b|\phi_A, \phi_B) &= P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi) \\ &= P(a = b|\phi_A + j \frac{\pi}{4}, \phi_B + j \frac{\pi}{4}) \quad \text{for any } j \in \mathbb{Z}. \end{aligned}$$

It is therefore sufficient to check that the correct correlation is obtained for $\phi_A \in [0, \frac{\pi}{4}[$ and $\phi_B \in [0, \pi[$.

For such values of ϕ_A, ϕ_B (for which $\phi'_A = \phi_A$ and $\phi'_B = \phi_B$), the probability $P(a = b|\phi_A, \phi_B)$ obtained from Protocol 1 can be calculated as follows:

$$\begin{aligned} P(a = b|\phi_A < \phi_B - j_B \frac{\pi}{4}) &= \frac{16}{\pi^2} \left(\int_0^{\phi_A} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \wp_{001}^{j_B}(\phi_B - \lambda_{RB}) \right. \\ &+ \int_0^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{011}^{j_B}(\phi_B - \lambda_{RB}) + \int_0^{\phi_A} d\lambda_{AR} \int_{\phi_B - j_B \frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{RB} \wp_{010}^{j_B}(\phi_B - \lambda_{RB}) \\ &+ \int_{\phi_A}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \wp_{101}^{j_B}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{111}^{j_B}(\phi_B - \lambda_{RB}) \\ &+ \int_{\phi_A}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_{\frac{\pi}{4}}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{110}^{j_B}(\phi_B - \lambda_{RB}) + \int_{\phi_B - j_B \frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{101}^{j_B}(\phi_B - \lambda_{RB}) \\ &\left. + \int_{\phi_B - j_B \frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j_B \frac{\pi}{4}}^{\lambda_{AR}} d\lambda_{RB} \wp_{100}^{j_B}(\phi_B - \lambda_{RB}) + \int_{\phi_B - j_B \frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\frac{\pi}{4}} d\lambda_{RB} \wp_{110}^{j_B}(\phi_B - \lambda_{RB}) \right), \end{aligned}$$

and

$$\begin{aligned}
P(a = b | \phi_A \geq \phi_B - j_B \frac{\pi}{4}) &= \frac{16}{\pi^2} \left(\int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \wp_{001}^{j_B}(\phi_B - \lambda_{RB}) \right. \\
&+ \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{011}^{j_B}(\phi_B - \lambda_{RB}) + \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j_B \frac{\pi}{4}}^{\frac{\pi}{4}} d\lambda_{RB} \wp_{010}^{j_B}(\phi_B - \lambda_{RB}) \\
&+ \int_{\phi_B - j_B \frac{\pi}{4}}^{\phi_A} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{001}^{j_B}(\phi_B - \lambda_{RB}) + \int_{\phi_B - j_B \frac{\pi}{4}}^{\phi_A} d\lambda_{AR} \int_{\phi_B - j_B \frac{\pi}{4}}^{\lambda_{AR}} d\lambda_{RB} \wp_{000}^{j_B}(\phi_B - \lambda_{RB}) \\
&+ \int_{\phi_B - j_B \frac{\pi}{4}}^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\frac{\pi}{4}} d\lambda_{RB} \wp_{010}^{j_B}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\frac{\pi}{4}} d\lambda_{AR} \int_0^{\phi_B - j_B \frac{\pi}{4}} d\lambda_{RB} \wp_{101}^{j_B}(\phi_B - \lambda_{RB}) \\
&\left. + \int_{\phi_A}^{\frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - j_B \frac{\pi}{4}}^{\lambda_{AR}} d\lambda_{RB} \wp_{100}^{j_B}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\frac{\pi}{4}} d\lambda_{AR} \int_{\lambda_{AR}}^{\frac{\pi}{4}} d\lambda_{RB} \wp_{110}^{j_B}(\phi_B - \lambda_{RB}) \right).
\end{aligned}$$

One can then check that with the choice of functions $\wp_{C_A C_R C_B}^{j_B} \in [0, 1]$ indicated in Table I (see main text), this leads (for all values of j_B) to

$$P(a = b | \phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2},$$

as desired.

Correlation from Protocol 2

After running Protocol 2 for inputs $\mathbf{x} = (\sin \theta_A \cos \phi_A, \sin \theta_A \sin \phi_A, \cos \theta_A)$ and $\mathbf{y} = (\sin \theta_B \cos \phi_B, \sin \theta_B \sin \phi_B, \cos \theta_B)$, the probability that Alice and Bob's outputs are the same is

$$\begin{aligned}
P(a = b | \mathbf{x}, \mathbf{y}) &= \sum_{a_0, b_0 = \pm 1} P(a_0, b_0 | \phi_A, \phi_B) P(a = b | \theta_A, \theta_B, a_0, b_0) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0 | \phi_A, \phi_B) \frac{1 - \cos(a_0 \theta_A + b_0 \theta_B)}{2} \\
&= P(a_0 = b_0 | \phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0 | \phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2} \\
&= \frac{1 - \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A + \theta_B)}{2} + \frac{1 + \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A - \theta_B)}{2} \\
&= \frac{1 - \cos \theta_A \cos \theta_B - \sin \theta_A \sin \theta_B \cos(\phi_A - \phi_B)}{2} = \frac{1 - \mathbf{x} \cdot \mathbf{y}}{2}.
\end{aligned}$$

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).