Classical simulation of entanglement swapping with bounded communication

Branciard, C.; Brunner, N.; Buhrman, H.; Cleve, R.; Gisin, N.; Portmann, S.; Rosset, D.; Szegedy, M.
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Correlation from Protocol 1

We first note that the definitions of \(a = \text{sign}(\sin \phi_A)\) and of \(\phi'_A = (\phi_A - jA_4 \frac{\pi}{4} \mod \pi) \in [0, \frac{\pi}{4}]\) on Alice’s side, and the definitions of \(\beta = \text{sign}(\sin(\phi_B - jA_4 \frac{\pi}{4}))\) (and then of \(b = \pm \beta\)) and of \(\phi'_B = (\phi_B - jA_4 \frac{\pi}{4} \mod \pi) \in [0, \pi]\) on Bob’s side, ensure that the following relations hold, as required:

\[
P(a = b|\phi_A, \phi_B) = P(a \neq b|\phi_A, \phi_B + \pi) = P(a \neq b|\phi_A + \pi, \phi_B) = P(a = b|\phi_A + \pi, \phi_B + \pi)
\]

\[
= P(a = b|\phi_A + j \frac{\pi}{4}, \phi_B + j \frac{\pi}{4}) \text{ for any } j \in \mathbb{Z}.
\]

It is therefore sufficient to check that the correct correlation is obtained for \(\phi_A \in [0, \frac{\pi}{4}]\) and \(\phi_B \in [0, \pi]\).

For such values of \(\phi_A, \phi_B\) (for which \(\phi'_A = \phi_A\) and \(\phi'_B = \phi_B\)), the probability \(P(a = b|\phi_A, \phi_B)\) obtained from Protocol 1 can be calculated as follows:

\[
P(a = b|\phi_A < \phi_B - jB \frac{\pi}{4}) = \frac{16}{\pi} \left( \int_0^{\phi_A} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{001}(\phi_B - \lambda_{RB}) \right.
\]

\[
+ \int_0^{\phi_A} d\lambda_{AR} \int_{\lambda_{AR}}^{\phi_A - jA_4 \frac{\pi}{4}} d\lambda_{RB} \psi_{011}(\phi_B - \lambda_{RB}) + \int_0^{\phi_A} d\lambda_{AR} \int_{\phi_B - jA_4 \frac{\pi}{4}}^{\phi_B} d\lambda_{RB} \psi_{010}(\phi_B - \lambda_{RB})
\]

\[
+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{110}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B} d\lambda_{RB} \psi_{111}(\phi_B - \lambda_{RB})
\]

\[
+ \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{100}(\phi_B - \lambda_{RB}) + \int_{\phi_A}^{\phi_B - jB \frac{\pi}{4}} d\lambda_{AR} \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B} d\lambda_{RB} \psi_{101}(\phi_B - \lambda_{RB})
\]

\[
+ \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B} d\lambda_{AR} \int_0^{\lambda_{AR}} d\lambda_{RB} \psi_{100}(\phi_B - \lambda_{RB}) + \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B} d\lambda_{AR} \int_{\phi_B - jB \frac{\pi}{4}}^{\phi_B} d\lambda_{RB} \psi_{101}(\phi_B - \lambda_{RB}) \right).
\]
and

\[ \begin{align*}
P(a = b|\phi_A, \phi_B) &= \frac{16}{\pi^2} \left( \int_0^{\phi_n-\frac{\pi}{4}} d\lambda_A \int_0^{\lambda_A} d\lambda_B \, \psi_{001}^n(\phi_B - \lambda_B) \right. \\
&+ \left. \int_0^{\phi_n-\frac{\pi}{4}} d\lambda_A \int_{\lambda_A}^{\frac{\pi}{2}} d\lambda_B \, \psi_{010}^n(\phi_B - \lambda_B) \right) + \int_0^{\phi_n-\frac{\pi}{4}} d\lambda_A \int_0^{\frac{\pi}{4}} d\lambda_B \, \psi_{100}^n(\phi_B - \lambda_B) \right) \\
&+ \int_0^{\phi_n-\frac{\pi}{4}} d\lambda_A \int_{\lambda_A}^{\frac{\pi}{2}} d\lambda_B \, \psi_{110}^n(\phi_B - \lambda_B) \right) .
\end{align*} \]

One can then check that with the choice of functions \( \psi_{A(\phi_B)}^n \in [0, 1] \) indicated in Table I (see main text), this leads (for all values of \( j_B \)) to

\[ P(a = b|\phi_A, \phi_B) = \frac{1 - \cos(\phi_A - \phi_B)}{2} , \]

as desired.

**Correlation from Protocol 2**

After running Protocol 2 for inputs \( x = (\sin\theta_A \cos\phi_A, \sin\theta_A \sin\phi_A, \cos\theta_A) \) and \( y = (\sin\theta_B \cos\phi_B, \sin\theta_B \sin\phi_B, \cos\theta_B) \), the probability that Alice and Bob’s outputs are the same is

\[ P(a = b|x, y) = \sum_{a_0, b_0 = \pm 1} P(a_0, b_0|\phi_A, \phi_B) \cdot P(a = b|\theta_A, \theta_B, a_0, b_0) \]

\[ = P(a_0 = b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A + \theta_B)}{2} + P(a_0 \neq b_0|\phi_A, \phi_B) \frac{1 - \cos(\theta_A - \theta_B)}{2} \]

\[ = \frac{1 - \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A + \theta_B)}{2} + \frac{1 + \cos(\phi_A - \phi_B)}{2} \frac{1 - \cos(\theta_A - \theta_B)}{2} \]

\[ = \frac{1 - \cos(\theta_A \cos\theta_B - \sin\theta_A \sin\theta_B \cos(\phi_A - \phi_B))}{2} = \frac{1 - x \cdot y}{2} . \]

Protocol 2 thus reproduces the desired entanglement swapping correlation (equation (1) of the main text).