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# Holographic de Sitter Geometry from CFT Entanglement

## SUPPLEMENTAL MATERIAL

*Explicit Examples of States.*— Local propagation of  $\delta S$  in auxiliary de Sitter geometry holds for any states which are small perturbations of the vacuum state everywhere in space. In particular, we require that for the excitations under consideration, the first law applies not just for a particular sphere but for all spheres (and their complements) on a given time slice. Below we discuss two examples of such states.

Let us first consider a pure state in a  $d$ -dimensional CFT on a plane, which is created by an infinitesimal insertion of the energy density operator  $T_{tt}$  at time  $t_0 + i\tau$  and position  $\vec{x}_0$

$$|\phi\rangle = (1 + \epsilon T_{tt}) |0\rangle, \quad (\text{S1})$$

see also Ref. [19]. The parameter  $\epsilon$  is taken to be small in the sense of  $\epsilon/\tau^d \ll 1$ . The evolution in imaginary time  $\tau$  is included to regulate potential UV divergences and ensures that the state is a small perturbation of the vacuum. The energy density of the state (S1) is determined by the two-point function of the stress tensor [S1]

$$\begin{aligned} \langle\phi|T_{tt}(t, x)|\phi\rangle &= \epsilon C_T \left[ \frac{1}{(\Delta x^2 - (\Delta t + i\tau)^2)^d} \right. \\ &\quad \left. \times \left( \frac{(\Delta x^2 + (\Delta t + i\tau)^2)^2}{(\Delta x^2 - (\Delta t + i\tau)^2)^2} - \frac{1}{d} \right) + \text{c.c.} \right] + \mathcal{O}(\epsilon^2), \end{aligned} \quad (\text{S2})$$

where  $C_T$  is the central charge and we have defined  $\Delta x^2 = |\vec{x} - \vec{x}_0|^2$  and  $\Delta t^2 = |t - t_0|^2$ . An explicit illustration of the energy density for this state and corresponding dS propagation of the perturbation in the EE is shown in Fig. S1. Note that the energy density (S2) is a spherical shell expanding out from  $(t_0, \vec{x}_0)$  at the speed of light. As expected from our general argument, the energy density profile (S2) obeys the constraints (15) and, hence, the holographic propagation respects the antipodal symmetry on the auxiliary dS background.

Our second example is the following mixed state

$$\rho = |0\rangle\langle 0| + \eta |E\rangle\langle E|, \quad (\text{S3})$$

where  $|E\rangle$  is an energy eigenstate (with constant energy density), and  $\eta$  is a small parameter. In this case, we assume that the constant time slice has topology  $\mathbb{S}^{d-1}$  with radius  $r$ . Let us now look at  $(d-2)$ -dimensional spherical entangling surfaces surrounding a cap of the  $\mathbb{S}^{d-1}$  specified by the angle  $\theta_0$ . The first law reads now

$$\begin{aligned} \delta S &= 2\pi \int_0^{\theta_0} r^{d-1} \Omega_{d-2} \sin^{d-2} \theta \, d\theta \times \\ &\quad \times r \frac{\cos \theta - \cos \theta_0}{\sin \theta_0} \times \frac{\eta E}{r^{d-1} \Omega_{d-1}}, \end{aligned} \quad (\text{S4})$$

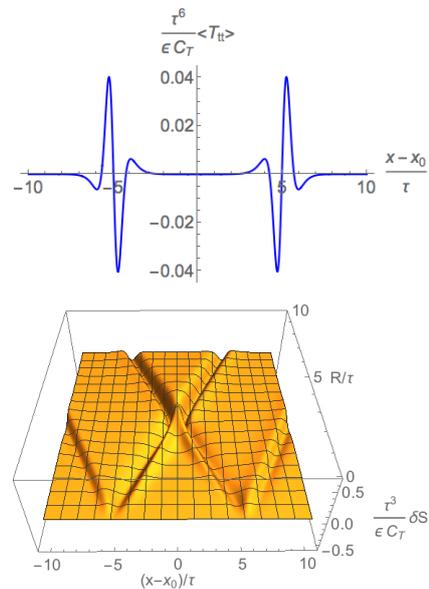


FIG. S1. Top: Rescaled energy density  $\langle T_{tt} \rangle$  for the state (S1) with  $d = 3$  plotted along the  $x$ -axis at  $t = t_0 + 5\tau$ , see Eq. (S2). Bottom: Corresponding change in the entanglement entropy  $\delta S$  as a function of dS time  $R$  and position along the  $x$ -axis. The causal nature of propagation is clearly visible.

where  $\Omega_n = 2\pi^{(n+1)/2}/\Gamma(\frac{n+1}{2})$  denotes the volume of a unit  $\mathbb{S}^n$ . The factors in the integrand then correspond to, in order, the volume element of  $\mathbb{S}^{d-1}$ , the boundary-to-bulk propagator for dS in global coordinates, and the (constant) expectation value of the energy density. A special case of this expression appears in Ref. [S2], which discusses universal thermal corrections to the vacuum entanglement entropy. There, the energy is given by  $E = \frac{\Delta}{r}$ , where  $\Delta$  is the smallest scaling dimension in the spectrum (apart from the identity), and  $\eta$  is the product of the degeneracy of the energy eigenstate and the corresponding Boltzmann factor, *i.e.*,  $\eta = g e^{-\beta\Delta/r}$ . Clearly, this and other mixed states of the form (S3) violate the first constraint in Eq. (17). Hence, the corresponding  $\delta S$  propagates on dS without antipodal symmetry.

Note that  $\delta S$  in Eq. (S4) diverges as  $\theta_0 \rightarrow \pi$ , *i.e.*, as the dS propagation reaches the past boundary  $\mathcal{I}^-$ . This divergence is related to a breakdown of the first law and corresponding free propagation in dS space when  $\sin \theta_0 \sim \eta E r$  (with  $\theta_0 > \pi/2$ ).

*Alternative derivation of wave equation on dS geometry.*— Let us now present another perspective on the wave equation (3). The conformal group relevant for a  $d$ -dimensional CFT is  $SO(2, d)$ . However, only the subgroup  $SO(1, d)$  leaves a constant time slice invariant. Hence the corresponding spherical entangling surfaces are

mapped onto one another under the action of  $SO(1, d)$ . Now considering the perturbations  $\delta S$  for these ball-shaped regions, the  $SO(1, d)$  generators  $K_i$  act as

$$\partial_{K_i} \delta S [\langle T_{tt} \rangle] = -\delta S [\langle \partial_{K_i} T_{tt} \rangle] . \quad (\text{S5})$$

Here, the  $\partial_{K_i}$  on the RHS can be viewed as generating an “active” conformal transformation that changes the CFT state, while the  $\partial_{K_i}$  on the LHS generates a “passive” transformation that instead changes the spherical entangling surface. Comparing now the “active” and “passive” action of the quadratic Casimir of  $SO(1, d)$ ,  $\nabla^2 \equiv c_{ij} \partial_{K_i} \partial_{K_j}$ , we get

$$\nabla^2 \delta S [\langle T_{tt} \rangle] = \delta S [\langle \nabla^2 T_{tt} \rangle] = -d \delta S [\langle T_{tt} \rangle] , \quad (\text{S6})$$

where the second expression above uses the linearity in  $T_{tt}$  of the modular Hamiltonian (8). Further, the last expression appears after using the fact that the energy density transforms as a scalar of weight  $d$  with

respect to the  $SO(1, d)$  subgroup. Now, a particular spherical entangling surface is left invariant by the stabilizer group  $SO(1, d - 1)$ . Hence, on the LHS of Eq. (S6), the nontrivial action of  $\nabla^2$  is on the coset space  $SO(1, d)/SO(1, d - 1)$ . The latter coset is precisely the anticipated  $d$ -dimensional dS geometry, and  $\nabla^2$  becomes the d’Alembertian on this space. Hence this group theoretic approach produces precisely the Klein-Gordon equation (3) on the auxiliary dS space. Note that this analysis implicitly normalizes the dS radius  $L$  to unity.

Finally, let us mention that the group theoretic argument above can be also generalized to the higher-spin case and, as expected, yields the mass given by Eq. (17).

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- [S1] H. Osborn and A. C. Petkou, *Annals Phys.* **231**, 311 (1994), arXiv:hep-th/9307010 [hep-th].  
[S2] C. P. Herzog, *JHEP* **10**, 28 (2014), arXiv:1407.1358 [hep-th].