SUPPLEMENTAL MATERIAL

DETAILS ON LUX AND COUPP LIKELIHOOD FUNCTIONS

Here we provide a detailed description of the likelihood functions and the nuisance parameters proper to the LUX and COUPP experiments. Before that, let us notice that details on the treatment of the other experiments included in this letter can be found in [1, 2], with the following exceptions. We disregard scattering off carbon and chlorine in PICASSO, SIMPLE and COUPP due to the lack of nuclear form factors; however, the most relevant WIMP interaction for these experiments occurs with fluorine, that we consider. For KIMS, WIMP scatterings off cesium and iodine are expected to occur in equal number due to the similar nuclear properties (mass, proton and neutron spin content) of these two elements, equal number due to the similar nuclear properties (mass, proton and neutron spin content) of these two elements, see e.g. [3]. Due to the lack of a form factor for Cs, we substitute it therefore with the I form factor.

LUX The Large Underground Xenon (LUX) experiment consists of a dual-phase xenon detector located at the Sanford Underground Research Facility in the USA. The detector has a fiducial volume of 118 kg and the first science run took place from April to August 2013 for a total of 85.3 live days [4].

Signals of DM scatterings on nuclei are searched by combining the scintillation light signal (S1) with the secondary ionization signal (S2). In the S1 channel, the detector threshold is set to 2 photoelectrons, which roughly correspond to 3 keVnr (nuclear recoil keV), by using the indicative $L_{\text{eff}}$ function in [5]. The signal is conservatively set to zero below 3 keVnr, hence the Poisson fluctuations below threshold do not contribute to the estimated signal. The analysis pipeline of LUX is different from the one of XENON100: instead of keeping separated the S1 and S2 signals and use $L_{\text{eff}}$, these two quantities are related and modeled with the NEST software [6]. However we will not use this procedure but a simplified approach to specify a likelihood function for LUX.

After cuts, 160 events were observed by the collaboration in a non-blind analysis. Only one event is placed (slightly) below the mean nuclear recoil line extracted from calibration events (see Fig. 4 of [4]), where a background of $\bar{B} \pm \sigma_B = 0.64 \pm 0.16$ events is expected. The likelihood of observing $N = 1$ event at fixed signal $S$ and background $B$ is given by the Poisson distribution as

$$\ln L_{\text{LUX}}(N|S + B) = -S + \frac{1}{2} S B + \ln \left( e^{-z} \frac{\sigma_B}{\sqrt{2\pi}} + \frac{1}{2} \left( S - \sigma_B^2 \right) \left( 1 + \text{erf}(z) \right) \right),$$

where we have marginalized analytically over the background (as described in [1]) with $z = (\bar{B} - \sigma_B)/2\sigma_B$ and erf the error function. In computing the signal rate we have considered the acceptance as given in the bottom panel of Fig. 1 of [4] and an additional factor of 1/2 to account for the 50% nuclear recoil acceptance. With this approximation for the likelihood function there are no nuisance parameters proper to the LUX experiment.

COUPP The Chicagoland Observatory for Underground Particle Physics (COUPP) has been operated at the SNOLAB underground laboratory in the USA between September 2010 and August 2011 [7]. It consisted of a 4 kg CF$_3$I bubble chamber, with fluorine and iodine being sensitive to spin-dependent interactions with protons.

If the energy density injected in the bubble chamber exceeds a certain critical value, a recoiling nucleus traversing the liquid might generate a phase transition i.e. a bubble. The detector then operates as a threshold device, controlled by setting the temperature $T$. The relation between the energy threshold $E_{\text{th}}(T)$ and the temperature is obtained at a fixed pressure during the calibration process. The observed rate per day per kg of target material is

$$S = \int_{E_{\text{th}}(T)}^{\infty} dE_R P(E_R, E_{\text{th}}(T)) \frac{dR}{dE_R},$$

where $P(E_R, E_{\text{th}}(T))$ is a temperature-dependent nucleation efficiency. This is $P(E_R, E_{\text{th}}(T)) = \Theta(E_R - E_{\text{th}}(T))$ for iodine, while for fluorine it can be parametrized either by

$$P(E_R, E_{\text{th}}(T)) = 1 - \exp \left[ a \left( 1 - \frac{E_R}{E_{\text{th}}(T)} \right) \right]$$

or by a step function

$$P(E_R, E_{\text{th}}(T)) = \eta \Theta(E_R - E_{\text{th}}(T)).$$

We explore both possibilities. The parameter $a$ defines the steepness of the energy threshold, while $\eta$ has the role of a nucleation efficiency. The values of $a$ and $\eta$ are uncertain and therefore we treat them as nuisance parameters with Gaussian priors centered at $a = 0.15 \pm 0.02$ and $\eta = 0.49 \pm 0.02$.

The total exposure after cuts is 553 kg-days, subdivided into three run periods, which have a different threshold for the bubble nucleation. The first period is characterized by $N_1 = 2$ events with an expected background $\bar{B}_1 = 0.8$ events and has a total exposure of 55.8 kg-days. The second run has $N_2 = 3$ events, $\bar{B}_2 = 0.7$ events for 70 kg-days, while the third one has $N_3 = 8$ events, $\bar{B}_3 = 3$ events for 311.7 kg-days. The background comes mainly from neutrons and alpha particles. The exposures take into account the efficiency for single bubble production.

The likelihood is therefore given by the Poisson probability of observing $N$ events in each of the three runs

$$\ln L_{\text{COUPP}}(N|S) = \sum_{j=1}^{3} \ln P(N_j|S + \bar{B}_j).$$
By considering the uncertainties on the nucleation parameter (either \(a\) or \(\eta\)) and on the energy thresholds of the three runs, we have four nuisance parameters. For the energy thresholds, we use Gaussian priors with mean values and standard deviations provided in [7]: \(E_1^{th} = 7.8\) keVnr, \(\sigma_{E_1} = 1.1\) keVnr, \(E_2^{th} = 11.0\) keVnr, \(\sigma_{E_2} = 1.6\) KeVnr, \(E_3^{th} = 15.5\) keV and \(\sigma_{E_3} = 2.3\) keVnr.

Finally the likelihood for fluorine in COUPP is given by

\[
\ln L_{\text{COUPP}} = \ln L_{\text{COUPP}}(N|S + B) - \frac{(a - \bar{a})^2}{2\sigma_a^2} - \sum_{i=1}^{3} \frac{(E_i^{th} - E_i)^2}{2\sigma_{E_i}^2},
\]

(6)

for the nucleation efficiency in Eq. (3), and analogous expression for the one in Eq. (4) with the substitution \(a \rightarrow \eta\).

We have computed the bounds with both nucleation efficiencies (3) and (4). In the region of interest no significant deviation between the two is found since scatterings occur dominantly off iodine, hence we only show the result corresponding to the choice (3).

**DETAILS ON THE DM ANNIHILATION CROSS-SECTION**

The s-channel DM annihilation cross section into SM fermions is

\[
\sigma(\bar{\chi}\chi \rightarrow \bar{f}f) = N_c \frac{g_f^2 g_{\chi}^2}{64\pi} \frac{s}{(s-m_a^2)^2} \sqrt{s-4m_f^2},
\]

(7)

where \(N_c\) is the number of colors for the final fermions, and we neglected the pole resonance in the propagator because the mediator is always off-shell for the values of \(m_a\) and \(m_{DM}\) needed to explain the DAMA regions.

The annihilation cross section into two pseudo-scalars is

\[
\sigma(\bar{\chi}\chi \rightarrow aa) = \frac{g_{\Delta M}^4}{256\pi} \frac{h(t_0) - h(t_1)}{s(s-4m_{DM}^2)},
\]

(8)

with

\[
t_i = -\frac{1}{4} \left( \sqrt{s-4m_{DM}^2} + \sqrt{s-4m_a^2} \right)^2,
\]

(9)

the integration extrema, and the undefined integral

\[
h(t) \equiv 4(m_{DM}^2 - t) + \frac{m_a^4(u-t)}{(m_{DM}^2 - t)(m_{DM}^2 - u)} - \frac{2m_a^4 + (s-2m_a^2)^2}{s-2m_a^2} \log\left( \frac{m_{DM}^2 - t}{m_{DM}^2 - u} \right),
\]

(10)

with \(u = 2m_{DM}^2 + 2m_a^2 - s - t\).

We compute the thermally averaged annihilation cross section for a non-relativistic DM gas by expanding the cross section in powers of the DM relative velocity \(v\), \(s \approx m_{DM}^2(4-v^2)\), weighting with the appropriate Maxwell-Boltzmann distribution, and then summing over all possible annihilation channels. We obtain

\[
\langle\sigma v\rangle(x) = \sum_f A_f \frac{3B}{2x} + O(x^{-2}),
\]

(11)

where \(x \equiv m_{DM}/T\) with \(T\) the temperature of the gas. The first coefficient,

\[
A_f = \frac{N_c g_f^2 g_{\chi}^2 m_{DM}^2}{8\pi (4m_{DM}^2 - m_a^2)^2} \sqrt{1 - \frac{m_f^2}{m_{DM}^2}},
\]

(12)

is the contribution of the s-wave annihilation into SM fermion pairs, while the second coefficient,

\[
B = \frac{g_{\Delta M}^4 m_{DM}^2}{96\pi} \frac{(m_{DM}^2 - m_a^2)^2}{(2m_{DM}^2 - m_a^2)^4} \sqrt{1 - \frac{m_a^2}{m_{DM}^2}},
\]

(13)

is given by the annihilation into pseudo-scalar pairs, which occurs in p-wave. The p-wave contribution of the \(\bar{\chi}\chi \rightarrow \bar{f}f\) process is much smaller than the \(\bar{\chi}\chi \rightarrow aa\) cross section and therefore we neglect it.

To obtain the value of the thermally averaged cross section at present time, which accounts for the GC \(\gamma\)-ray excess, we use:

\[
2\langle\sigma v\rangle_{\text{best}} = \langle\sigma v\rangle(x_0) \simeq \sum_f A_f,
\]

(14)

with \(x_0 \gg 1\) the present value of \(x\), and \(\langle\sigma v\rangle_{\text{best}}\) and the adopted value of the DM mass \(m_{DM}^{\text{best}}\) taken from Fig. 15 of Ref. [8], as explained in the letter. The factor of 2 in front of \(\langle\sigma v\rangle_{\text{best}}\) takes into account the fact that \(\chi\) is here not self-conjugated, unlike in Ref. [8].

**BOUNDS FROM ELECTRON AND MUON’S ANOMALOUS MAGNETIC MOMENT**

The presence of a pseudo-scalar mediator coupled to SM fermions may produce detectable effects in various precision measurements, e.g. in the electroweak sector. These observables however usually probe new physics coupled to the electroweak gauge bosons, while the pseudo-scalar state only couples to the SM fermions at tree level and therefore contributes only through two-loop or higher order processes. This, plus the smallness of the couplings favored by DAMA data and the GC excess, makes it easy to exclude any sizable contribution to these observables.

An observable that is able to directly probe new physics coupled to the SM fermions is the anomalous magnetic moment (AMM) of charged leptons. The
AMMs of electron and muon are in fact known to a high precision. The experimental values are

\[ a_e^{\text{exp}} = (11596521807.6 \pm 2.7) \times 10^{-13}, \]
\[ a_\mu^{\text{exp}} = (11659209.1 \pm 6.3) \times 10^{-10}, \]

(as per CODATA recommendations [9], also endorsed by the Particle Data Group [10]), while the value predicted by the SM is [11, 12]

\[ a_e^{\text{SM}} = (11596521817.8 \pm 7.7) \times 10^{-13}, \]
\[ a_\mu^{\text{SM}} = (11659180.2 \pm 4.9) \times 10^{-10}. \]

The difference between experimental and theoretical values, \( \Delta a \equiv a^{\text{exp}} - a^{\text{SM}} \), is

\[ \Delta a_e = (-10.2 \pm 8.2) \times 10^{-13}, \]
\[ \Delta a_\mu = (+28.9 \pm 8.0) \times 10^{-10}. \]

Notice from this last result that there is a 3.6\( \sigma \) tension between the measured and theoretical value of the muon’s AMM, while for the electron the two are in very good agreement.

The pseudo-scalar contribution to the AMM of charged leptons has been computed up to two loops. Phenomenological consequences have been already studied e.g. in [13], and in [14] in the framework of Coy DM, for pseudo-scalar masses above 1 GeV. However we are here interested in masses of the order of tens of MeV. In this regime, the result is dominated by the one-loop contribution which is always negative. Therefore, it is sound to compare the pseudo-scalar contribution to the electron’s AMM, \( a_e \), with \( \Delta a_e \), and requiring that \( a_e \leq \Delta a_e \). However, the same can not be done with \( a_\mu \), since this has opposite sign respect to \( \Delta a_\mu \) and therefore the presence of the new particle will only make the deviation of the theoretical result from the experimental measure worse (unless higher loop orders change the sign of \( a_\mu \)). Therefore we derive a bound from the muon’s AMM imposing \( a_\mu \leq \Delta a_\mu \), where \( \delta a_\mu = 8.0 \times 10^{-10} \) is the error on \( \Delta a_\mu \) in Eq. (20). By means of the formulas in [13], these bounds can be converted into upper limits on the pseudo-scalar coupling to SM fermions \( g \) and lower limits on \( \Lambda_a = m_a/\sqrt{g_D M} \) (the parameter of interest for direct DM detection searches), for any given value of the pseudo-scalar mass \( m_a \). These limits are shown as dashed curves in Figs. 1 and 2 (blue for electron, purple for muon), for Universal (democratic) and Universal (heavy-flavors) couplings respectively. The best-fitting points to the GC excess, denoted with black dots, as well as the fit to DAMA data, are excluded in both cases.

Notice that, as said above, these bounds are dominated by the one-loop contribution which only depends on the pseudo-scalar couplings to leptons, but not to quarks. Therefore, the dashed lines in Figs. 1 and 2 can be intended as bounds on the lepton coupling alone. To avoid these bounds we can assume that the pseudo-scalar state has leptophobic couplings, i.e. it doesn’t couple to charged leptons. As noted in the main text, the lepton couplings are free parameters that play very little role in fitting the GC excess, and no role whatsoever in direct detection of WIMPs. By setting the couplings of the pseudo-scalar to charged leptons to zero, the one- and two-loop contribution to electron and muon’s AMM vanishes, and the first non-zero contribution is expected to arise at three loops. To assess the bound coming from the three-loop-generated AMM, we assume that the most important contribution at this perturbative order is obtained by adding an internal photon line to the two-loop diagrams. Accordingly, we estimate the three-loop \( a_\ell \) (\( \ell = e, \mu \)) to be \( \alpha/\pi \) times the two-loop result (when all leptons have been removed from the internal loops), with \( \alpha \approx 1/137 \) the electromagnetic fine structure constant. Since we do not know the sign of \( a_\ell \), we make the conservative assumption that it has opposite sign respect to \( \Delta a_\ell \) for both electron and muon; therefore, we produce bounds on the model parameters by requiring that \( a_\ell \leq \delta \Delta a_\ell \). The new limits for the leptophobic case are shown as solid lines in Figs. 1 and 2. The best-fitting points to the GC excess (as well as the DAMA regions) are now perfectly compatible with the bounds.
FIG. 2. Same as in Fig. 1 but for Universal (heavy-flavors) couplings, i.e. when the pseudo-scalar couples universally only to the heavier SM fermions. Dashed lines are for the two-loop limit, while solid lines apply in the leptophobic limit where the first non-zero contribution to the leptons’ AMM arises at three loops. The black dot denotes the Universal (heavy-flavors) best-fit in parameter space, as explained in the main text.