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Pen, U.L.; Yang, I.S.

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Strong lensing interferometry for compact binariesUe-Li Pen^{1,*} and I-Sheng Yang^{2,†}¹*Canadian Institute for Theoretical Astrophysics, University of Toronto, M5S 3H8 Ontario, Canada and Canadian Institute for Advanced Research CIFAR program in Gravitation and Cosmology, Toronto, M5G 1Z8 Ontario, Canada*²*IOP and GRAPPA, Universiteit van Amsterdam, Science Park 904, 1090 GL Amsterdam, Netherlands*
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We propose a possibility to improve the current precision measurements on compact binaries. When the orbital axis is almost perpendicular to our line of sight, a pulsar behind its companion can form two strong lensing images. These images cannot be resolved, but we can use multiwavelength interferometry to accurately determine the passage through superior conjunction. This method does not depend strongly on the stability of the pulse profile and applies equally well to both slow and fast pulsars. We discuss the possible improvement this can bring to the bound on stochastic gravitational wave background and to determine black hole spin. We also discuss the possibility of discovering a suitable binary system by the Square Kilometer Array to which our method can apply.

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I. INTRODUCTION

High-precision timing of compact binaries has been one of the best tools to study general relativity. The post-Keplerian orbital changes provided excellent confirmation of gravitational wave emission [1–3]. More recently, it was pointed out that the same data can be used to bound the stochastic gravitational wave background at an orbital frequency $\sim 10^{-4}$ Hz, and such a bound can be lowered to the interesting range if the timing precision can be improved by a few orders of magnitude [4].

Current techniques to achieve high timing precision rely on a stable millisecond pulsar in the binary system. After establishing an average pulse profile, one can look for Doppler shifts due to the orbital motion. Mapping out such periodic behavior of the pulse arrival times can identify the occurrence of a particular orbital phase, for example, the periastron passage. For the Hulse–Taylor system, PSR B1913+16, the current technique of millisecond pulsar timing can reach an orbital phase resolution ($\delta T_{\text{periastron}}/P_{\text{orbit}}$) $\sim 10^{-6}$.

In this paper, we will explore an alternative method that may provide a better precision. Our method also requires a pulsar, but unlike traditional timing techniques, it does not depend directly on the stability nor the short period of the pulses. For us, a pulsar serves as a good source of radio waves, $f \sim \text{GHz}$, $\lambda \sim 10^{-1}$ m. We exploit such a short wavelength in an interferometer setup, in which the two paths of light interfering with each other come from a strong lensing effect [5]. After decomposing the signal into multiple frequency channels, we get a two-dimensional interference pattern [6] on which the passage through

superior conjunction (when the pulsar is right behind its companion) is a vertical line. If the pulsar is bright enough, recognizing such a line is quite robust against the fluctuations of single pulse behavior such as microstructure [7,8] or jitters [9]. Given a solar mass binary with a period of a few hours, the Einstein radius R_{lens} is about 10^6 m. Observing such an interference pattern then provides a native resolution of the orbital phase that is about $(\lambda/R_{\text{lens}}) \sim 10^{-7}$.

The lensing effect requires the binary to have a small inclination. Along our line of sight, the pulsar needs to go behind its companion within a few Einstein radii. Note that the radius of a white dwarf is about 10^7 m $> R_{\text{lens}}$, so it blocks the strong lensing signals. The pulsar’s companion has to be a neutron star or a black hole. There is one known example, the double pulsars PSR J0737-3039A/B, that seems to have a small enough inclination. Unfortunately, the magnetospheres of these two pulsars are rather large, $\sim 10^7$ m $> R_{\text{lens}}$, so the observation is about eclipsing instead of strong lensing [10,11]. There is not yet a clear theoretical reason why the magnetosphere has to be this large, so in the near future, we might hope to discover pulsar-neutron star binaries, or simply pulsar-black hole binaries that meet our requirements [12].

Before such a binary is found, our method is futuristic. The exact precision depends on the signal-to-noise analysis that cannot be predicted beforehand. In this paper, we will first go over the basic idea of this strong lensing interferometer. We then calculate the projected precision and discuss some practical issues to show that a pulsar binary can indeed reach it. Finally, we discuss the improvement this might bring to the upper bound of a stochastic gravitational wave provided by the binary-resonance detector [4] and the possibility to determine the spin of the companion neutron star or black hole.

*pen@cita.utoronto.ca
†isheng.yang@gmail.com

II. BASIC IDEA

Consider a binary system with a circular orbit of radius R_{orbit} . On the plane perpendicular to the line of sight, we can parametrize the projected orbit as

$$D_{\text{proj}} = R_{\text{orbit}} \sqrt{\sin^2 \phi_{\text{orbit}} + \cos^2 \phi_{\text{orbit}} \sin^2 \theta_{\text{tilt}}}. \quad (1)$$

In this toy model, the binary includes a point source of an electromagnetic (EM) wave, and its partner is purely a gravitational lens. ϕ_{orbit} is the orbital phase, and we will focus on the time during which it is small, which is defined to be the time when the signal source is near the superior conjunction (the farthest point behind the lens). θ_{tilt} is the tilt of the orbit, and we will also focus on the cases of a small tilt such that the orbital axis is almost perpendicular to the line of sight.

In this case, it is possible to have a strong lensing effect near the superior conjunction. We will use the unit that $G = 1$, so the mass of the lens M equals (half) of its Schwarzschild radius. A far-away observer can define the Einstein radius as

$$R_{\text{lens}} = \sqrt{MR_{\text{orbit}}}. \quad (2)$$

When $D_{\text{proj}} \lesssim R_{\text{lens}}$, there will be two strongly lensed images with magnifications [5,13]

$$\mu_{\pm} = \frac{u^2 + 2}{2u\sqrt{u^2 + 4}} \pm \frac{1}{2}, \quad (3)$$

where $u = (D_{\text{proj}}/R_{\text{lens}})$. This can only happen when $\theta_{\text{tilt}} \lesssim (R_{\text{orbit}}/R_{\text{lens}})$. The orbit and the lensing images can be visualized in Fig. 1.

In practice, the binary is so far away, and these two images cannot be resolved. What we really observe is an overall magnification¹

$$\mu_{\text{total}} = \mu_+ + \mu_- = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}. \quad (4)$$

This leads to a feature on the signal amplitude within a range of orbital phases:

$$|\phi_{\text{orbit}}| \lesssim \Delta\phi_{\text{amp}} = \frac{R_{\text{lens}}}{R_{\text{orbit}}}. \quad (5)$$

Thus, observing this feature already implies a native resolution up to $\Delta\phi_{\text{amp}}$, but that is not the main effect we would like to use here.

¹This diverges at $u = 0$ when the source is right behind the lens because that produces a full Einstein ring instead of two images. This will only happen for θ_{tilt} extremely close to zero, and we will not address such a rare situation.

FIG. 1 (color online). The binary orbit in the rest frame of the lens and viewed from a small inclination angle. When the source goes behind the lens and approaches the Einstein radius (the dotted circle), we can see both a far image (blue) and a close image (red). When far from the Einstein radius, the interior image becomes very faint.

When the EM wave is monochromatic, these two images will interfere with each other. The length difference between their paths is given by [5,13]

$$\Delta l = M \left(\frac{u\sqrt{u^2 + 4}}{2} + \ln \frac{\sqrt{u^2 + 4} + u}{\sqrt{u^2 + 4} - u} \right). \quad (6)$$

The intensity as a function ϕ_{orbit} should really be an interference pattern,

$$I(\phi_{\text{orbit}}) = \mu_+ + \mu_- + 2\sqrt{\mu_+\mu_-} \cos 2\pi \frac{\Delta l}{\lambda}. \quad (7)$$

We plot this pattern in Fig. 2. Qualitatively, this is a double slit with a separation $\sim R_{\text{lens}}$. Therefore, the interference pattern provides a native resolution up to

$$\Delta\phi_{\text{int}} \sim \frac{\lambda}{R_{\text{lens}}}. \quad (8)$$

FIG. 2 (color online). The amplitude profile as a function of orbital phase ϕ_{orbit} . The orbital parameters are $R_{\text{orb}} = 10^9$ m, $R_{\text{lens}} = 10^6$ m, and $\theta_{\text{tilt}} = 0.5(R_{\text{lens}}/R_{\text{orb}})$. We chose $\lambda = 50$ m, which is about $f = 6$ MHz, such that both the magnification envelope and the interference pattern are visible. The GHz observation will have a denser interference pattern (by a factor of hundreds).

III. APPLICATION

Let us put in some real numbers to estimate the expected precision of our method. For convenience, we use orbital parameters comparable to that of the Hulse–Taylor binary [1]:

$$M = 10^3 \text{ m}, \quad R_{\text{orbit}} = 10^9 \text{ m}, \quad R_{\text{lens}} = 10^6 \text{ m}. \quad (9)$$

This leads to an orbital period of a few hours, and we will use $P_{\text{orbit}} = 10^4 \text{ s}$ for simplicity. The lensing magnification profile leads to a resolution $\Delta\phi_{\text{amp}} \sim 10^{-3}$. In other words, in every orbital period, the signal will be magnified for

$$\Delta T_{\text{amp}} = P_{\text{orbit}} \frac{\Delta\phi_{\text{amp}}}{2\pi} \sim \text{a few seconds}. \quad (10)$$

The interference pattern for a radio wave $f = 1 \text{ GHz}$ leads to $\Delta\phi_{\text{int}} = 3 \times 10^{-7}$. Near the center, the duration of each interference peak is about

$$T_{\text{int}} = P_{\text{orbit}} \frac{\Delta\phi_{\text{int}}}{2\pi} \approx 5 \times 10^{-4} \text{ s}. \quad (11)$$

Recognizing these peaks in the data leads to the desired improvement of timing precision. The mass is a little bit of an underestimation for black holes, so the real situation might be slightly better.

Up to this point, we have pretended that the radio wave comes from a continuous, isotropic, and monochromatic

point source. A real pulsar is significantly different from this idealization. We will proceed to address these differences and show that our method is applicable.

A. Multiple channels

A real pulsar emission is broadband, and different wavelengths lead to different interference patterns. The typical phase difference between two light paths during the strong lensing effect is

$$\left\langle \frac{\Delta l}{\lambda} \right\rangle = \frac{\Delta\phi_{\text{amp}}}{\Delta\phi_{\text{int}}} = \frac{M}{\lambda} \approx 3 \times 10^3. \quad (12)$$

This means that a narrow-band filter with $\Delta f \sim 10^{-4} f = 10^5 \text{ Hz}$ is required to get a clear pattern.

The typical observation at GHz is broadband, $\Delta f \sim \text{GHz}$. We can decompose it into $\sim 10^4$ narrow-band channels, and their interference patterns will be correlated. This technique is similar to the secondary spectrum of timing data that reconstructs multipath scattering events [6,14]. As shown in Fig. 3, the interference peaks form a family of parabolas on a time-wavelength dynamic spectrum, and their common center corresponds to the superior conjunction.

Note that the narrow-band observation is essentially the original broadband observation with a longer Fourier integration time. The $\Delta f \sim \text{GHz}$ observation can have a time resolution up to nanoseconds, but every narrow-band channel requires a Fourier integral over $\sim 10 \mu\text{s}$. Fortunately, this

FIG. 3 (color online). The top panel is the standard two-dimensional interference pattern. White regions are peaks. The vertical axis is the wavelength in meters, which scans over frequency channels $(1 \pm 10^{-3}) \text{ GHz}$. The horizontal axis is the orbital phase ϕ_{orb} . Its range is 10% of the full magnification envelope $\Delta\phi_{\text{amp}}$, which is about 0.1 sec, given the orbital parameters of the Hulse–Taylor binary. The middle panel shows how a long pulse illuminates a segment near the center. The bottom panel shows how multiple short pulses illuminate different vertical lines, along which we will get several one-dimensional interference patterns. We assume a uniform intensity across channels.

time resolution is enough to see the interference pattern, which is about a millisecond wide. Since information is conserved, the loss of time resolution is the only consequence of decomposing into narrow-band channels, and we do not otherwise lose S/N . If individual pulses have enough S/N to be seen at the original broadband observation, we will have enough narrow-band S/N in multiple channels. We are simply rearranging the same information in the conjugate Fourier domain where they are more localized.

B. Finite pulse duration

Typically, the pulse duration is about $(1/10) \sim (1/100)$ of the pulse period. Since that comes from the neutron star's spinning motion, it implies that the emission has an opening angle of a few degrees [15]. To have two lensing images, the emission needs to cover the entire gravitational lens at the same time. That means $(R_{\text{lens}}/R_{\text{orbit}})$ has to be less than a few degrees, which is satisfied (within a factor of 10) by Hulse–Taylor-like orbits.

Although the emission is wide enough to cover the entire lens, it does not last long enough to illuminate the entire interference pattern. To see this effect, we should first qualitatively put the pulsars into two categories: slow pulsars with $P_{\text{pulse}} \sim \text{seconds}$ and pulse duration $10^{-1} \sim 10^{-2}$ s and fast pulsars with $P_{\text{pulse}} \sim \text{milliseconds}$ and pulse duration $10^{-1} \sim 10^{-2}$ ms.

In the first case, we are looking for pulses that happen to be emitted when the pulsar is near the superior conjunction. Such a pulse will illuminate a large segment of the interference pattern. As shown in Fig. 3, if such a segment includes the center or is sufficiently close to the center, we can locate the center up to the projected precision $\Delta\phi_{\text{int}} \sim 10^{-7}$. Although slow pulsars are known to have microstructure [7,8], those have no reason to behave in a similarly coherent way across multiple channels. Thus, it should not be difficult to tell them apart using multichannel techniques [6,16]. It is typical for slow pulsars to have a single-pulse $S/N \gtrsim 1$, so they are ideal for our purpose.

For fast pulsars, a pulse duration is comparable to (or shorter than) the interference peak, but we will get many pulses. This means that the full pattern is illuminated in several places but for shorter durations. As shown in Fig. 3, every single pulse illuminates a vertical line, along which we have a secondary interference pattern in the Fourier space—as we scan through different frequency channels. The separation of the peaks in this secondary pattern is largest for pulses near the superior conjunction, and it decreases as we move away. A simple interpolation can locate the maximum with a native precision similar to $\Delta\phi_{\text{int}}$. Pulsar jitters [9] might affect the brightness and S/N for individual pulses, but the structure of the secondary interference pattern is immune to such noise. Although single-pulse S/N is typically small for fast pulsars, we are using many of them together to determine one superior

conjunction. Thus, it is still possible to get a precise measurement.

C. Coherence

To have an interference pattern, the waves coming through two paths must be coherent. Since the emission mechanism is still under debate [17,18], we cannot directly know whether it will be true. Here, we will provide three scenarios consistent with observed properties of pulsars so far.

First of all, the pulse can be intrinsically a single-mode emission, which guarantees coherence. In this case, the electric field excitations in the pulsar magnetosphere are perfectly correlated, meaning the correlation function $\langle E(x, t)E(x', t) \rangle$ is rank 1 for any two points separated by less than the inverse bandwidth over which the electric field is averaged. This implies 100% polarization. Evidence for such properties comes from pulsar giant pulses, which are often consistent with being 100% polarized.

A second, more generic, scenario is a multimode excitation mechanism, in which the outgoing radiation converts from alvenic plasma modes into freely propagating radiation. The magnetic field could act as a multimode wave guide, which directs radiation into a narrow direction, analogous to a microwave feed horn. The observed narrow beam widths of pulsars, often even at low frequency, require electromagnetic coherence of a hundred wave lengths. There could be multiple such emission regions. If each individual region sends out a beam that is wide enough to cover the lens, $\Delta\phi_{\text{beam}} > \Delta\phi_{\text{amp}} \sim 10^{-3}$, then we can see a clear interference pattern if the total emission area is not resolved by the double-slit lens pattern. This means that the length dimension of the emission area needs to be smaller than $(R_{\text{orbit}})\Delta\phi_{\text{int}} \sim 10^2$ m. Currently, the area upper bound is a few kilometers [19,20], and there are no lower bounds.

Even if the emission is highly beamed, such that individual emission region does not cover the lens, fast pulsars may still produce interference patterns. That is because one emission region will point to two paths within a time separation $\sim 10^{-5}$ s, and we are doing a Fourier integral of that duration for the multichannel observation anyway. Therefore, an interference is forced to happen as long as the emission mechanism stays coherent for time duration $> 10^{-5}$ s, although, theoretically, such an emission mechanism may be more contrived. Observationally, there is no sharp difference between this temporal coherence and the spatial coherence due to a wide beaming angle.

Because of the uncertainties in the emission mechanism, there might be other possibilities for the pulse to be coherent for our purpose. It is also possible that there is indeed no coherence. In fact, this suggests that we can also invert the logic. Whether an interference pattern appears or not, such a multichannel analysis in a lensing system can

improve our understanding about the pulsar itself and further distinguish different emission mechanisms.

D. Interstellar scintillation

In addition to multipath propagation due to gravitational lensing, pulsars are known to undergo multipath scattering due to structures in the interstellar medium. The latter leads to prominent scintillation, which can be undone using interstellar holography to reconstruct the original signal before scattering [20,21]. Since this scattering happens when two strongly lensed light paths are almost parallel, such reconstruction is unlikely to lead to confusion with gravitational lensing, which happens near the binary, has the orbital periodicity, and exhibits a very specific pattern as we demonstrated.

IV. DISCUSSION

The binary-resonance effect can provide an upper bound on the stochastic gravitational wave background at the orbital frequency of a binary [4]. Such an upper bound is roughly given by

$$h_c \sim 5 \left(\frac{\delta T}{P_{\text{orbit}}} \right) n_{\text{data}}^{1/2} \left(\frac{T_{\text{tot}}}{P_{\text{orbit}}} \right)^{-2}. \quad (13)$$

The first factor is the orbital phase precision that our method can improve to $\Delta\phi_{\text{int}} \lesssim 10^{-7}$. To be more thorough, we should also consider whether our method affects the other two factors. In the last factor, T_{tot} is the time difference between the first and the last data points. We can leave it as a constant while comparing to our method. The middle factor, n_{data} , is the inverse data density: the number of orbital periods required to produce a precision measurement of the orbital phase. Our method may limit how small this number can be.

As described earlier, for slow pulsars, our method requires extrapolating to the center of the interference pattern while only seeing a segment of it. When such a segment is further away, a larger error is involved in this extrapolation. Let us be very conservative and demand to use only those pulses that cover the superior conjunction passage. Since the pulse duration is roughly $(1/100)$ of the pulse period, this only happens once every 100 periods. Thus, our method requires $n_{\text{data}} > 100$. For an 8 h orbit, this means around ten data points per year. The upper bound derived from the Hulse–Taylor binary has one data point per year and can only be improved up to 26 since each data point requires two weeks of observations [4]. This is the only possible side effect our method may have, and it does not make a big difference. Therefore, it is fair to just compare the orbital phase precisions. Our projected native precision is already 1 order of magnitude better than the actual precision from the Hulse–Taylor data after a thorough pattern fitting and signal-to-noise analysis. If we really get such a pulsar of

which the individual pulses are directly observable, a multichannel analysis is very likely to reduce the upper bound on h_c by a few orders of magnitude.

In reality, only a few known millisecond pulsars (less than 10 out of the known ~ 130) are bright enough to have observable single pulses. For those yet to be discovered, we should conservatively expect their signal to be barely above the noise, which requires a superposition of a few pulses to get a significant S/N. As a quick estimation, if a superposition of ten pulses is required to get a S/N = 10, then we pretend that we get one clear pulse out of every ten. If we look at the bottom panel of Fig. 3, effectively one out of ten passages through the superior conjunction has a pulse close enough to see it in the precision we derived. Of course, this is only a very rough estimation, since many technical issues and tricks cannot be predicted beforehand.

A lot of slower pulsars can be seen in individual pulses, but the intensity during one pulse can fluctuate significantly. Such fluctuation will not be confused with the interference pattern, but the situation will not be as ideal as the middle panel of Fig. 3. In addition to the interference parabola, the brightness will also vary in other patterns. The actual technique to achieve the best precision in this case is again beyond the scope of this paper.

In addition to improving the bound on the gravitational wave background, an interferometer is a generally powerful tool to probe the geometry of space-time. For example, if the companion has a large spin, there will be a frame dragging effect. Without interference, this has been considered unmeasurable from timing information alone [22]. We can estimate the sensitivity of our interferometer for frame dragging by considering a black hole with spin a where $|a| = 1$ means extremal. When the source is within the Einstein radius, its frame dragging effect can lead to an additional length difference between two strongly lensed paths [22]:

$$\Delta l_{FD} \sim \frac{aM^2}{R_{\text{lens}}}. \quad (14)$$

For an order 1 value of a and the particular parameters we used in this paper, this is slightly less than 1 m. In other words, it is about one wavelength at GHz. A shift of one wavelength causes exactly a maximum-to-minimum change in the interference pattern, so this is in principle observable. However, one should be slightly more cautious here.² This shift exists in a comparison between a spinless lens and one with a nonzero spin. In reality, we only get to observe one interference pattern, and thus an overall shift is not observable by itself. On top of this unobservable overall shift, the entire pattern will also develop additional asymmetry in the shape. For a nonspinning black hole, the

²We thank the Physical Review D referee who brought this to our attention.

interference pattern is exactly symmetric about the superior conjunction. The spin phase shift is an asymmetry in the reflection symmetry relative to superior conjunction. Thus, it might still be observable without a comparison system, but further analysis is required.

Finally, let us comment on the possibility to find a suitable binary for our method. The Square Kilometer Array projected the discovery of hundreds of compact pulsar binaries [12]. Population synthesis suggests that in about one-fifth of those the companion shall be a black hole [23]. If we are optimistic and say that an order-1 fraction of neutron stars has magnetospheres smaller than 10^6 m, then there will be hundreds of likely candidates. The most limiting requirement seems to be the inclination angle. We require the binary rotation axis to be almost orthogonal to the line of sight. For a Hulse–Taylor-like orbit, such probability for that is roughly

$$\mathcal{P}\left(|\theta_{\text{tilt}}| < \frac{R_{\text{lens}}}{R_{\text{orbit}}}\right) \sim \frac{2\pi \times (2R_{\text{lens}}/R_{\text{orbit}})}{4\pi} = 10^{-3}. \quad (15)$$

One might want to scan through different orbital sizes. For example, at $R_{\text{orbit}} = 10^7$ m, the above probability

improves to 10^{-2} . However, at this distance or smaller, the pulse may not have a wide enough opening angle to cover the entire lens. Also, the remaining inspiral will not last longer than the pulsar lifetime, so we cannot expect to find any pulsar binary with a smaller orbit. Thus, scanning over orbital sizes still gives about 10^{-3} probability in finding a suitable pulsar binary.

Given hundreds of binaries, a 10^{-3} chance cannot guarantee that we will find one, but we can still keep our hopes high. On top of improving the bound on the stochastic gravitational wave background, such a high-precision measurement can also improve our knowledge about the pulsar and its companion in various ways [13,19,20].

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