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Missing top of the AdS resonance structure

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We study a massless scalar field in AdS_{d+1} with a nonlinear coupling ϕ^N and not limited to spherical symmetry. The free-field-eigenstate spectrum is strongly resonant, and it is commonly believed that the nonlinear coupling leads to energy transfer between eigenstates. We prove that when Nd is even, the most efficient resonant channels to transfer energy are always absent. In particular, for $N = 3$ this means no energy transfer at all. For $N = 4$, this effectively kills half of the channels, leading to the same set of extra conservation laws recently derived for gravitational interactions within spherical symmetry.

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I. INTRODUCTION AND SUMMARY

Anti-de Sitter space has many intriguing properties. One of those is a strongly resonant spectrum. For any massless field, for example gravitational waves, the eigenstates all have integer frequencies. When there is a nonlinear (self-) coupling, for example, through gravitational backreactions, such resonance allows energy to be transferred between different eigenstates. Thus, no matter how small the coupling is, after a correspondingly long time, the initial energy can end up being anywhere, leading to rich and unpredictable dynamics.

Some authors have argued that these many channels to transfer energy generically lead to energy cascade: starting from a few low frequency modes, energy continuously spreads out into higher ones. This naturally accumulates to a significant effect at the time scale set by the inverse coupling strength. This “resonant-cascade” theory has been widely quoted as the explanation for a nonlinear gravitational instability: black hole formation often observed in numerical simulations at this time scale [1–5].

However, as emphasized in [6,7], the existence of these channels is only a necessary condition for energy cascade, and energy cascade is again only a necessary condition for black hole formation.¹ Since it is doubly insufficient, the resonant cascade theory cannot be a good dynamical explanation of why black hole forms in those cases. In other words, the resonant-cascade theory only derives a lower bound of the time scale in which a black hole can form. It does not really explain how a black hole forms and why, in some cases, it happens so fast, almost saturating this lower bound. Such dynamical understanding likely requires one to go beyond any theory in the eigenstate spectrum and directly study the position space behavior [7].

On the other hand, since it is a necessary condition, analyzing the energy flow between eigenstates can provide excellent arguments against black hole formation. This has been a fruitful line of thought. For example, it was argued that a small modification of the AdS space makes the spectrum only asymptotically resonant, which will not be enough for energy cascade [8]. Later numerical simulations indeed supported such an argument [9]. It was also observed that half of the naïvely expected channels to transfer energy are actually absent, which leads to extra conserved quantities. This may explain the stable, quasi-periodic solutions which are also often observed during numerical simulations [6,10–12].

In this paper, we follow this fruitful line of thought. The missing channels and the consequent conservation laws were established within spherical symmetry, but an explicit evaluation of several low energy eigenstates demonstrated the same behavior even without spherical symmetry [5].² Thus, a natural next step is to establish the missing channels beyond spherical symmetry. We study a massless scalar field with a general nonlinear coupling, ϕ^N with $N \geq 3$, and in AdS_{d+1} with $d \geq 2$. We show that the missing channels are generic for AdS eigenstates, and those missing are exactly the most efficient ones for transferring energy into higher eigenstates.

More technically, the energy spectrum of a massless scalar field is given by

$$w_{nl} = 2n + l + d, \quad (1)$$

where the radial wave number n and the total angular momentum l are both non-negative integers. The resonant condition is

$$w_{n_1 l_1} = \sum_{i=2}^N \pm w_{n_i l_i}, \quad (2)$$

¹The AdS (in)stability problem is indeed one important motivation of this work, but many related articles are not directly relevant for this paper. For a more complete list of references about the (in)stability problem, please see [7].

²Although the former is done for a scalar field but the later is for pure gravity, the general structure of the eigenstates are the same.

with $(N - 1)$ arbitrary choices of $\pm$ while maintaining the positivity of frequencies. Basically, the $(N - 1)$ eigenstates on the rhs can conspire to transfer energy to the eigenstate on the lhs if this condition is met.³ If we choose all $+$ signs in this resonant condition, we get

$$w_{n_1 l_1} = \sum_{i=2}^N w_{n_i l_i}. \quad (3)$$

These channels represent the most efficient ways to transfer energy into high-energy states if we start with only low-energy ones. We call it the “top” of the AdS resonance structure.

In Sec. II, we explicitly prove that these channels are absent whenever Nd is even. In particular, when $N = 3$, all resonant channels take this form, so this implies no energy transfer at all between eigenstates. The ϕ^3 coupling only introduces subleading corrections to the free eigenstates, and any solution can still be expressed as a superposition of these approximate eigenstates. It is rather intriguing that such behavior is limited to even spatial dimensions.

In Sec. III we demonstrate that when $N = 4$, this “missing channels” property is a direct generalization of what was previously proven within spherical symmetry for weak gravitational self-interaction [6]. Based on the frequently observed similarity between ϕ^4 theory and weak gravity, and the explicit evaluation of several coupling terms of the gravity theory without spherical symmetry [5], it is very likely that general gravitational interactions also have these channels missing. As a first step toward such generalization, we provide a slightly simpler proof of the conservation laws recently pointed out in [11–13], such that their nonspherically symmetric versions are obviously also valid.

Finally, in Sec. IV, we emphasize two particular properties which suggest that there should be a more elegant group theory method to prove the “missing top.” For gravitational interactions without spherical symmetry, it is probably wiser to formulate such a method instead of trying to explicitly evaluate the couplings between eigenstates.

II. THE MISSING TOP

A. The general proof

Consider the global AdS_{d+1} spacetime,

$$ds^2 = \frac{1}{\cos^2 x} (-dt^2 + dx^2 + \sin^2 x d\Omega_{d-1}^2), \quad (4)$$

and a massless scalar field with a nonlinear self-coupling,

³Of course, certain angular momentum summation rules must also be satisfied between these eigenstates, but there is a large degeneracy involved so it can always be done, and we will ignore such complications in this paper.

$$S = \int \left(\frac{g_{\mu\nu}}{2} \partial^\mu \phi \partial^\nu \phi + \frac{\phi^N}{N} \right) \sqrt{g} dV_{d+1}. \quad (5)$$

This leads to the equation of motion,

$$-\ddot{\phi} + \frac{1}{\cos^2 x} \hat{L}_d \phi = \frac{\phi^{N-1}}{\cos^2 x}, \quad (6)$$

where $\hat{L}_d$ is an effective Laplacian operator for the spatial dimensions.⁴ Ignoring the coupling term on the rhs, the free field solutions with Dirichlet boundary condition can be decomposed into separable eigenstates [14,15]:

$$\phi_0(x, t, \Omega_{d-1}) = \sum_{n,l,\vec{m}} (A_{nl\vec{m}} e^{-i w_{nl} t} + \bar{A}_{nl\vec{m}} e^{i w_{nl} t}) e_{nl\vec{m}}(x, \Omega_{d-1}), \quad (7)$$

$$w_{nl} = 2n + l + d, \quad (8)$$

$$e_{nl\vec{m}}(x, \Omega_{d-1}) = \cos^d x \sin^l x Y_{l\vec{m}}(\Omega_{d-1}) P_n^{(d/2+l-1, d/2)}(\cos 2x). \quad (9)$$

$P_n^{\alpha,\beta}$ is the Jacobi polynomial, l is the magnitude of the total angular momentum, $\vec{m}$ describes its components, and $Y_{l\vec{m}}$ is the generalized spherical harmonics. These eigenstates form an orthogonal basis,

$$\int e_{n_i l_i \vec{m}_i} e_{n_j l_j \vec{m}_j} \tan^{d-1} x dx d\Omega_{d-1} \propto \delta_{n_i n_j} \delta_{l_i l_j} \delta_{\vec{m}_i \vec{m}_j}. \quad (10)$$

In general, the nonlinear coupling allows energy to be transferred between these eigenstates. One can model that by a perturbative expansion also in the eigenstate basis,

$$\phi = \phi_0 + \phi_1 + \cdots, \quad \phi_1 = \sum_{n,l,\vec{m}} c_{nl\vec{m}}(t) e_{nl\vec{m}}, \quad (11)$$

$$\begin{aligned} & \ddot{c}_{n_1 l_1 \vec{m}_1} + w_{n_1 l_1}^2 c_{n_1 l_1 \vec{m}_1} \\ &= \sum_{n_j, l_j}^{2 \leq j \leq N} S_{\{nl\vec{m}\}} \prod_{j=2}^N (A_{n_j l_j \vec{m}_j} e^{-i w_{n_j l_j} t} + \bar{A}_{n_j l_j \vec{m}_j} e^{i w_{n_j l_j} t}), \end{aligned} \quad (12)$$

$$\begin{aligned} S_{\{nl\vec{m}\}} &\equiv S_{n_1 l_1 \vec{m}_1 n_2 l_2 \vec{m}_2 \dots n_N l_N \vec{m}_N} \\ &= \int \prod_{i=1}^N e_{n_i l_i \vec{m}_i} \tan^{d-1} x \frac{dx d\Omega_{d-1}}{\cos^2 x}. \end{aligned} \quad (13)$$

Through the nonzero coupling coefficients $S_{\{nl\vec{m}\}}$, combinations of $(N - 1)$ zeroth-order modes source the first-order correction in one mode.

⁴It is not exactly the Laplacian for the spatial metric, but the exact form does not matter too much here.

Of course, many of these coefficients will be zero from the integral of eigenstates. We will worry about those later. First we should note that we do not care too much about some of them even if they are not zero. If the resonant condition, Eq. (2), is not satisfied, then these combinations of $(N-1)$ modes drive the $c_{n_1 l_1 \bar{m}_1}$ harmonic oscillator not at resonance. If those are the only nonzero coefficients, then for a small zeroth-order magnitude, we can have $\phi_1 \sim \phi_0^{N-1} \ll \phi_0$, and the nonlinear coupling simply leads to small corrections of the free eigenstates. Most of the energy stays within the original eigenstates.

When Eq. (2) is satisfied, those combinations drive the oscillator at resonance, thus leading to a secular growth of $c_{n_1 l_1 \bar{m}_1}$. In this case, ϕ_1 will soon become comparable to ϕ_0 , and the naïve perturbation theory breaks down. In other words, a significant amount of energy is transferred from the original eigenstates into others. Now, the interesting question is how many of these coefficients satisfying the resonant condition are actually nonzero? Here we will prove that when the sign choices in the resonant condition are all +, namely in the form of Eq. (3), then those coefficients are all zero.

We first rewrite Eq. (3) more explicitly as

$$2n_1 + l_1 = (N-2)d + \sum_{i=2}^N (2n_i + l_i), \quad (14)$$

and also the explicit integral of the coupling coefficients,

$$\begin{aligned} S_{\{nl\bar{m}\}} &= \int \left(\prod_{i=1}^N Y_{l_i \bar{m}_i} \right) d\Omega_{d-1} \\ &\times \int_0^{\pi/2} \left(\prod_{i=1}^N \sin^{l_i} x P_{n_i}^{d/2+l_i-1, d/2} \right) \\ &\times \cos^{Nd-2} x \tan^{D-1} x dx. \end{aligned} \quad (15)$$

The Ω_{d-1} integral leads to $(d-2)$ conditions for matching the angular momentum components and the generalized triangular inequality for total angular momentum, $l_i \leq \sum_{j \neq i} l_j$. Neither will be very important for our main purpose. Our proof only requires the x integral.

Changing the variable to $y = \cos 2x$, the x integral becomes

$$\begin{aligned} &\int_{-1}^1 \left(\prod_{i=1}^N P_{n_i}^{d/2+l_i-1, d/2}(y) \right) \\ &\times (1-y)^{L/2+d/2-1} (1+y)^{(N-1)d/2-1} dy, \end{aligned} \quad (16)$$

where $L = \sum_i l_i$. We then use the definition of the Jacobi polynomial,

$$P_n^{(\alpha, \beta)}(y) \propto (1-y)^{-\alpha} (1+y)^{-\beta} \frac{d^n}{dy^n} [(1-y)^{\alpha+n} (1+y)^{\beta+n}], \quad (17)$$

to write down P_{n_i} explicitly in the integral,

$$\begin{aligned} &\int_{-1}^1 \left(\prod_{i=2}^N P_{n_i}^{d/2+l_i-1, d/2}(y) \right) (1-y)^{L/2-l_1} (1+y)^{(N-2)d/2-1} \\ &\times \frac{d^{n_1}}{dy^{n_1}} [(1-y)^{d/2+l_1+n_1-1} (1+y)^{d/2+n_1}] dy. \end{aligned} \quad (18)$$

We note that when Eq. (14) is satisfied, L and Nd must be together even or odd. Thus, when either N or d is even, the first line in the above integrand is a polynomial of y . Since integration by part produces no boundary terms, the condition for the integral to not vanish is

$$n_1 \leq \left(\sum_{i=2}^N n_i + \frac{l_i}{2} \right) - \frac{l_1}{2} + \frac{(N-2)d}{2} - 1. \quad (19)$$

Since this contradicts Eq. (14), we have proven that $S_{\{nl\bar{m}\}} = 0$ for the resonant channels satisfying Eq. (3).

B. $N = 3$ and other odd numbers

For $N = 3$, every resonant coefficient is associated with a channel of the form $w_{n_1 l_1} = w_{n_2 l_2} + w_{n_3 l_3}$. By proving that such coupling coefficients all vanish, we showed that no energy transfer actually occurs despite a strongly resonant spectrum.⁵ The original eigenstates only receive small corrections through the coupling, and nothing dramatic will happen given any initial condition. Note that our proof only works when d is even. When d is odd, we explicitly evaluated some coefficients, and they are indeed nonzero.

For larger odd number N and in even d dimensions, these “top” resonant channels remain missing, but lower resonant channels, those with more “−” signs in Eq. (2), do exist. We also explicitly evaluated some of those to confirm that. The radial integral does not seem to give rise to other constraints, which agrees with our physical intuitions. Other constraints will only come from the angular Ω_{d-1} integral. This “missing top” behavior is again only true in even d . We evaluated a few coefficients in odd d and saw that these top channels do exist.

On the other hand, when d is odd, something interesting already happens when limited to spherical symmetry. All the frequencies, with $l = 0$, are odd numbers. When N is odd, the resonant condition, Eq. (2), involves an odd number of frequencies and, thus, cannot be satisfied

⁵The coupling coefficient is invariant under any permutation of (1), (2), and (3). The channels of the form $w_{n_1 l_1} = w_{n_2 l_2} - w_{n_3 l_3}$ use the same coefficient as $w_{n_2 l_2} = w_{n_1 l_1} + w_{n_3 l_3}$. Thus, those are absent, too.

independently of the those “ $\pm$ ” sign choices. As a result, when both N and d are odd, if we start with spherically symmetric initial data, there is again no energy transfer at all. These interesting behaviors related to the parity of spatial dimensions might provide further insight to express and prove the “missing top” and other constraints in a more elegant formalism.⁶

III. $N = 4$ AND CONSERVED QUANTITIES

A. Extra conserved quantities

For $N = 4$, the resonant condition has two qualitatively different forms, either “3-1” or “pairwise,”

$$\text{either } w_{n_1 l_1} = w_{n_2 l_2} + w_{n_3 l_3} + w_{n_4 l_4}, \quad (20)$$

$$\text{or } w_{n_1 l_1} + w_{n_2 l_2} = w_{n_3 l_3} + w_{n_4 l_4}. \quad (21)$$

Our result shows that the 3-1 channels do not exist; only the pairwise ones do. The same conclusion was reached in [6] when they studied gravitational self-interactions within spherical symmetry. More recently, combining the absence of these channels and the symmetry properties of the coupling coefficients, extra conserved quantities were found [11,12]. Here we will revisit the proof of those conservation laws and show that they are also valid in the ϕ^4 theory even without spherical symmetry.

We first follow the two-time formalism introduced in [10]. Instead of the naïve perturbative expansion in Eq. (11), we assume that the amplitudes of the eigenstates are also time dependent, but evolve much more slowly,

$$\begin{aligned} \phi(x, t, \Omega_{d-1}) &= \sum_{n, l, \bar{m}} (A_{nl\bar{m}}(t) e^{-i w_{nl} t} + A_{nl\bar{m}}(t) e^{i w_{nl} t}) \\ &\times e_{nl\bar{m}}(x, \Omega_{d-1}), \end{aligned} \quad (22)$$

$$|\dot{A}_{nl\bar{m}}|^2 \ll w_{nl}^2 |A_{nl\bar{m}}|^2. \quad (23)$$

This assumption will not easily break down as easily the naïve perturbation theory. The leading-order effect of the coupling only requires us to solve

$$\begin{aligned} -2i w_{n_1 l_1} \frac{dA_{n_1 l_1 \bar{m}_1}}{dt} &= \sum_{n_2 l_2 \bar{m}_2} \sum_{n_3 l_3 \bar{m}_3} \sum_{n_4 l_4 \bar{m}_4} \\ &\times S_{\{1234\}} \bar{A}_{n_2 l_2 \bar{m}_2} A_{n_3 l_3 \bar{m}_3} A_{n_4 l_4 \bar{m}_4}, \end{aligned} \quad (24)$$

$$S_{\{1234\}} \equiv S_{\{n_1 l_1 \bar{m}_1\} \{n_2 l_2 \bar{m}_2\} \{n_3 l_3 \bar{m}_3\} \{n_4 l_4 \bar{m}_4\}}. \quad (25)$$

The solutions $A_{nl\bar{m}}(t)$ then model how energy is being slowly transferred between eigenstates. As the subscript becomes too long, we will resort to the above abbreviation.

⁶We thank Luis Lehner for pointing out the possible connection to the Huygen-Fresnel principle, although we have not been able to make use of that further.

While limited to $l = \bar{m} = 0$, in [11,12] it was shown that the symmetry properties of $S_{1234} = S_{2134} = S_{3412}$ lead to a conserved “particle number,”

$$\begin{aligned} \frac{d}{dt} \sum_n w_n |A_n|^2 \\ = \frac{-1}{2i} \sum_{\{1234\}} S_{\{1234\}} (\bar{A}_{n_1} \bar{A}_{n_2} A_{n_3} A_{n_4} - A_{n_1} A_{n_2} \bar{A}_{n_3} \bar{A}_{n_4}) = 0. \end{aligned} \quad (26)$$

This is simply because under the exchange of $\{12\} \leftrightarrow \{34\}$, $S_{\{1234\}}$ is symmetric but the next factor is antisymmetric. These symmetries are still there including eigenstates of nonzero angular momenta, so it is straightforward to verify that

$$\begin{aligned} \frac{d}{dt} \sum_{nl\bar{m}} w_{nl\bar{m}} |A_{nl\bar{m}}|^2 \\ = \frac{-1}{2i} \sum_{\{1234\}} S_{\{1234\}} (\bar{A}_1 \bar{A}_2 A_3 A_4 - A_1 A_2 \bar{A}_3 \bar{A}_4) = 0. \end{aligned} \quad (27)$$

Combining these symmetry properties and the fact that only pairwise channels exist, there is also a conserved “leading-order energy,”

$$\begin{aligned} \frac{d}{dt} \sum_n w_n^2 |A_n|^2 \\ = \frac{-1}{2i} \sum_{\{1234\}} w_{n_1} S_{\{1234\}} (\bar{A}_1 \bar{A}_2 A_3 A_4 - A_1 A_2 \bar{A}_3 \bar{A}_4) \\ = \frac{-1}{4i} \sum_{\{1234\}} (w_{n_1} - w_{n_3}) S_{\{1234\}} (\bar{A}_1 \bar{A}_2 A_3 A_4 - A_1 A_2 \bar{A}_3 \bar{A}_4) \\ = \frac{-1}{8i} \sum_{\{1234\}} (w_{n_1} + w_{n_2} - w_{n_3} - w_{n_4}) \\ \times S_{\{1234\}} (\bar{A}_1 \bar{A}_2 A_3 A_4 - A_1 A_2 \bar{A}_3 \bar{A}_4) = 0. \end{aligned} \quad (28)$$

In the third line we used the antisymmetry when $\{12\} \leftrightarrow \{34\}$, and in the fourth line we used the symmetry when $1 \leftrightarrow 2$ and $3 \rightarrow 4$. Now since the coefficient $S_{\{1234\}}$ for a resonant channel is only nonzero when the pairwise condition is met, the last line is zero. Even after including eigenstates of nonzero angular momenta, exactly the same proof goes through without change. Thus, the ϕ^4 theory, beyond spherical symmetry, also has a conserved “leading-order energy,”

$$\frac{d}{dt} \sum_{nl\bar{m}} w_{nl\bar{m}}^2 |A_{nl\bar{m}}|^2 = 0. \quad (29)$$

B. Physical implications

Note that the above two quantities are only approximately conserved, namely, up to the leading-order effect

of the ϕ^4 coupling. Among these two approximately conserved quantities, we think that the conserved particle number, Eq. (28), is not very surprising. If we have a complex scalar field with $|\phi|^4$ coupling, this becomes an exactly conserved $U(1)$ current. Thus, the above observation means that after being limited to the real axis, the same quantity is approximately conserved. This is very similar to the fact that in flat space, if a complex scalar field theory allows exactly stable Q-balls [16,17], then the same theory limited to the real axis allows very long-lived oscillons [18–21].

One might think that the conservation of leading-order energy, Eq. (29), is not surprising either. It seems to follow from the exactly conserved total energy and the fact that energy in the coupling is further suppressed by the small field amplitude. We would like to disagree with such intuition. Remember that if there are n eigenstates with nonzero amplitudes, there will only be n terms in the leading-order energy. However, there will generically be n^2 terms in the coupling energy. Thus, a conserved leading-order energy means either one of the following:

- (i) Energy does not spread out too much, $n \ll |\phi^2|$, so it does not compete with the amplitude suppression.
- (ii) Energy spreads out but there is a conspiracy in the relative phases between eigenstates such that many cross terms vanish.

We recommend [11,12] for further discussions about the implications to the AdS (in)stability problem.

IV. DISCUSSION

We should note that the physical origin of the “missing top” is a selection rule in the form of an inequality, Eq. (19), which is equivalent to

$$w_1 < \sum_{i=2}^N w_i. \quad (30)$$

This immediately reminds us that a nonlinear coupling between spherical harmonics is subjected to a similar rule

about total angular momentum: the generalized triangular inequality:

$$l_1 \leq \sum_{i=2}^N l_i. \quad (31)$$

This suggests that there should be a group theory representation of the eigenstates such that the frequency plays the role of total angular momentum. This, together with the intriguing dependence on the parity of spatial dimensions, might be useful in proving the “missing top” property, or even in discovering further restrictions in the resonance structure, when the ϕ^N coupling is replaced by something more complicated.

For example, the explicit evaluation of the coupling coefficients for gravitational self-interaction, despite the simplification of spherical symmetry [6], is already much more involved than the ϕ^4 theory. A direct generalization beyond spherical symmetry needs to include gravitational waves [5], and it appears to be a daunting task. A group theory method would be a much more preferable tool to figure out the AdS resonance structure in general.

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