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Unsupervised, Efficient and Semantic Expertise Retrieval

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ABSTRACT

We introduce an unsupervised discriminative model for the task of retrieving experts in online document collections. We exclusively employ textual evidence and avoid explicit feature engineering by learning distributed word representations in an unsupervised way. We compare our model to state-of-the-art unsupervised statistical vector space and probabilistic generative approaches. Our proposed log-linear model achieves the retrieval performance levels of state-of-the-art document-centric methods with the low inference cost of so-called profile-centric approaches. It yields a statistically significant improved ranking over vector space and generative models in most cases, matching the performance of supervised methods on various benchmarks. That is, by using solely text we can do as well as methods that work with external evidence and/or relevance feedback. A contrastive analysis of rankings produced by discriminative and generative approaches shows that they have complementary strengths due to the ability of the unsupervised discriminative model to perform semantic matching.

Categories and Subject Descriptors

H.3.1 [Information Storage and Retrieval]: Content Analysis and Indexing

Keywords

Expertise retrieval; Language models; Semantic matching

1. INTRODUCTION

The transition to the knowledge and information economy [1] introduces a great reliance on cognitive capabilities [50]. It is crucial for employers to facilitate information exchange and to stimulate collaboration [17]. In the past, organizations would set-up special-purpose database systems for their members to maintain a profile [7]. However, these systems required employees to be proactive. In addition, self-assessments are known to diverge from reality [10, 32] and document collections can quickly become practically infeasible to manage manually. Therefore, there has been an active interest in automated approaches for constructing expertise profiles [7, 61] and retrieving experts from an organization’s heterogeneous document repository [15]. Expert finding (also known as expertise retrieval or expert search) addresses the task of finding the right person with the appropriate skills and knowledge [6]. It attempts to provide an answer to the question:

Given a topic X, who are the candidates with the most expertise w.r.t. X?

The expertise retrieval task gained popularity in the research community during the TREC Enterprise Track [61] and has remained relevant ever since, while broadening to social media and to tracking the dynamics of expertise [5, 6, 10, 19, 21–23, 47, 49, 62]. Existing methods fail to address key challenges: (1) Queries and expert documents use different representations to describe the same concepts [26, 34]. Term mismatches between queries and experts [34] occur due to the inability of widely used maximum-likelihood language models to make use of semantic similarities between words [53]. (2) As the amount of available data increases, the need for more powerful approaches with greater learning capabilities than smoothed maximum-likelihood language models is obvious [63]. (3) Supervised methods for expertise retrieval [23, 47] were introduced at the turn of the last decade. However, the acceleration of data availability has the major disadvantage that, in the case of supervised methods, manual annotation efforts need to sustain a similar order of growth. This calls for the further development of unsupervised methods. (4) In some expertise retrieval methods, a language model is constructed for every document in the collection. These methods lack efficient query capabilities for large document collections, as each query term needs to be matched against every document [6]. Our proposed solution has a strong emphasis on unsupervised model construction, efficient query capabilities and semantic matching between query terms and candidate experts.

Specifically, we propose an unsupervised log-linear model with efficient inference capabilities for the expertise retrieval task. We show that our approach improves retrieval performance compared to vector space-based and generative language models, mainly due to its ability to perform semantic matching [34]. Our method does not require supervised relevance judgments and is able to learn from raw textual evidence and document-candidate associations alone. The purpose of this work is to provide insight in how discriminative language models can improve performance of core retrieval tasks compared to maximum-likelihood language models. Therefore, we avoid explicit feature engineering and the incorporation of external evidence in this paper. In terms of performance, the current best-performing formal language model [4] exhibits a worst-case time complexity linear in the size of the document collection. In contrast, the inference time complexity of our approach is asymptotically bounded by the number of candidate experts.
Our research questions are as follows: (1) How does our discriminative log-linear model compare to vector space-based methods and generative language models for the expert retrieval task in terms of retrieval performance? (2) What can we learn regarding the different types of errors made by generative and discriminative language models? (3) How does the complexity of inference in our log-linear model compare to vector-space based and generative models? (4) How does the log-linear model handle incremental indexing and what are its limitations?

We contribute: (i) An unsupervised log-linear model with efficient inference capabilities for the expertise retrieval task, together with an open-source implementation.\(^1\) (ii) Comparison of the retrieval performance of the log-linear model with traditional vector space-based models and language model methods on well-known benchmarks. (iii) Insights in how certainty in predictions by the log-linear model correlates with performance. (iv) In-depth analysis of the inference complexity of the log-linear model. (v) Comparative error analysis between the semantic log-linear model and traditional generative language models that perform exact matching. (vi) Insight in the relative strengths of semantic matching and exact matching for the expert retrieval task.

The remainder of this paper is organized as follows. In Section 2, we briefly discuss related work. Section 3 introduces the log-linear model for the expert retrieval problem. In Section 4 we state our research questions and detail our experimental set-up and implementations. We provide an overview of our experimental results, followed by a discussion and analysis in Section 5. Section 6 concludes the paper and discusses ideas for future work.

2. RELATED WORK

We first discuss prior work on expert retrieval and its relation to document retrieval. Then, we review semantic search methods. The ideas we present in this paper are inspired by neural language models used in automated speech recognition (ASR) and natural language processing (NLP). Consequently, we also review work from those fields.

2.1 Expert retrieval

Early expert retrieval systems were often referred to as expert locator and expertise management systems [38]. These database systems often relied on people to self-assess their expertise against a predefined set of topics [39], which is known to generate unreliable results [7].

With the introduction of the P@NOPTIC system [15], and later the TREC Enterprise track [61], there has been an active research interest in automated expertise profiling methods. It is useful to distinguish between profile-based methods, which create a textual representation of a candidate’s knowledge, and document-based methods, which represent candidates as a weighted combination of documents. The latter generally performs better at ranking, while the former is more efficient as it avoids retrieving all documents relevant to a query [6, p. 221].

There has been much research on generative probabilistic models for expert retrieval [21, 49]. Such models have been categorized in candidate generation models [13], topic generation models [4, 5], and proximity-based variants [5, 54]. Of special relevance to us are the unsupervised profile-centric (Model 1) and document-centric (Model 2) models of Balog et al. [4], which focus on raw textual evidence without incorporating collection-specific information (e.g., query modeling, document importance or document structure). Supervised discriminative models [23, 47, 60] are preferred when query-candidate relevance pairs are available for training. Unlike their generative counterparts these models have no issue combining complex and heterogeneous features (e.g., link-based features, document importance features, etc.); they resemble Learning to Rank (L2R) methods for document retrieval [6, 35]. However, a lack of training data may greatly hinder their applicability [6, p. 179]. Beyond unsupervised generative and supervised discriminative approaches, there are graph-based approaches based on random walks [55] and voting-based approaches based on data fusion [36]. Demartini et al. [19] propose a vector space-based method for the entity ranking task; their framework extends vector spaces operating on documents to entities. See [6] for a survey on the topic.

2.2 Latent semantic models for document retrieval

Li and Xu [34] note that the query document mismatch poses the most critical challenge in search. Semantic matching is an important attempt to remedy this problem. There has been much work on bridging the semantic gap for various different tasks [19, 26, 28, 30, 31, 41, 48, 53]. Expertise and document retrieval [6, p. 224] are closely related as performance of the latter can greatly impact that of the former [37].

Latent Semantic Models (LSM) first became popular through the introduction of Latent Semantic Indexing (LSI) [18], followed by probabilistic LSI (pLSI) [27]. LSMs based on neural networks [28, 53, 58] emerged in the last decade. Salakhutdinov and Hinton [53] employ unsupervised deep auto-encoders to map documents to bit patterns using semantic addressing. Huang et al. [28] perform semantic matching of documents and queries by leveraging click-through data optimizing for Web document ranking. Further, deep models [12, 20] have been proposed to learn to rank [35].

2.3 Neural language modeling

Large-vocabulary neural probabilistic language models for modeling word sequence distributions have become very popular recently [8, 42, 43, 44]. These models learn continuous-valued distributed representations for words, also known as embeddings [42, 45, 48], in order to fight the curse of dimensionality and increase generalization by introducing the expectation that similar word vectors signify semantically or syntactically similar words. Recurrent neural language models have shown to perform well in ASR [40]. Collobert et al. [14] propose a unified neural network architecture for various NLP tasks. Even more recently, there has been a surge in multimodal neural language models [31], which lend themselves to the task of automated image captioning.

What we add on top of the related work described above is the following. In this work we model the conditional probability of the expertise of a candidate given a single query term (contrary to binary relevance given a character-based n-gram [28]). In the process we learn a distributed vector representation (similar to LSI, pLSI and semantic hashing) for both words and candidates such that nearby representations indicate semantically similar concepts. We propose a log-linear model that is similar to neural language models. The important difference is that we predict a candidate expert instead of the next word. To the best of our knowledge, we are the first to propose such a solution. We employ an embedding layer in our shallow model for the same reasons as mentioned above: we learn continuous word representations that incorporate semantic and syntactic similarity tailored to an expert’s domain.

\(^1\)https://github.com/cvangysel/SERT
3. A LOG-LINEAR MODEL FOR EXPERT SEARCH

In the setting of this paper we have a document collection $D$ and a predefined set of candidate experts $C$ (entities to be retrieved). Documents $d \in D$ are represented as a sequence of words $w_1, \ldots, w_{|d|}$ originating from a vocabulary $V$, where $w_i \in V$ and the operator \(|\cdot|\) denotes the document length in tokens. For every document $d \in D$ we write $C_d$ to denote the set of candidates $c \in C$ associated with it (i.e., $C = \bigcup_{d \in D} C_d$). These document-candidate associations can be obtained explicitly from document meta-data (e.g., the author of an e-mail) or implicitly by mining references to candidates from the document text. Notice that some documents might not be associated with any candidate. When presented with a query $q$ of constituent terms $t_1, \ldots, t_q$, the expert retrieval task is to return a list of candidates $p(C)$ ordered according to topical expertise. We generate this ranking using a relatively shallow neural network which directly models $P(e \mid q)$.

We employ vector-based distributed representations [26], for both words (i.e., word embeddings) and candidate experts, in a way that motivates the unsupervised construction of features that express regularities of the expertise finding domain. These representations can capture the similarity between concepts (e.g., words and candidate experts) by the closeness of their representations in vector space. That is, concepts with similar feature activations are interpreted by the model as being similar, or even interchangeable.

3.1 The model

To address the expert search task, we model $P(c_j \mid q)$ and rank candidates $c_j$ accordingly for a given $q$. We propose an unsupervised, discriminative approach to obtain these probabilities. We construct our model solely from textual evidence: we do not require query-candidate relevance assessments for training and do not consider external evidence about the corpus (e.g., different weightings for different sub-collections), the document (e.g., considering certain parts of the document more useful) nor link-based features.

Let $e$ denote the size of the vector-based distributed representations of both words in $V$ and candidate experts in $C$. These representations will be learned by the model using gradient descent [42] (Section 3.2). For notational convenience, we write $P(e \mid \cdot)$ for the (conditional) probability distribution over candidates, which is the result of vector arithmetic. We define the probability of a candidate $c_j$ given a single word $w_i \in V$ as the log-linear model

$$P(e \mid w_i) = \frac{1}{Z_d} \exp (W_e \cdot (W_p \cdot v_i) + b_e),$$

where $W_p$ is the $e \times |V|$ projection matrix that maps the one-hot representation (i.e., 1-of-$|V|$) of word $w_i$, $v_i$ to its $e$-dimensional distributed representation, $b_e$ is a $|C|$-dimensional bias vector and $W_e$ is the $|C| \times e$ matrix that maps the word embedding to an unnormalized distribution over candidates $C$, which is then normalized by $Z_d = \sum_{c \in C} \exp (W_e \cdot (W_p \cdot v_i) + b_c)$. If we consider Bayes’ theorem, the transformation matrix $W_e$ and bias vector $b_e$ can be interpreted as the term log-likelihood $\log P(w_i \mid e)$ and candidate log-prior $P(c \mid e)$, respectively. The projection matrix $W_p$ attempts to soften the curse of dimensionality introduced by large vocabularies $V$ and maps words to word feature vectors [8]. Support for large vocabularies is crucial for retrieval tasks [28, 53].

We then assume conditional independence of a candidate’s expertise given an observation of data (i.e., a word). Given a sequence of words $w_1, \ldots, w_k$ we have:

$$P(e \mid w_1, \ldots, w_k) = \frac{1}{Z_{2d}} \prod_{i=1}^{k} P(e \mid w_i) = \frac{1}{Z_{2d}} \exp \left( \sum_{i=1}^{k} \log \left( P(e \mid w_i) \right) \right)$$

where $\tilde{P}(e \mid w_1, \ldots, w_k)$ denotes the unnormalized score and $Z_{2d} = \sum_{e \in C} \exp \left( \sum_{c \in C} \log \left( P(c \mid w_i) \right) \right)$ is a normalizing term. The transformation to log-space in (2) is a well-known trick to prevent floating point underflow [46, p. 445]. Given (2), inference is straightforward. That is, given query $q = t_1, \ldots, t_k$, we compute $P(e \mid t_1, \ldots, t_k)$ and rank the candidate experts in descending order of probability.

Eq. 1 defines a neural network with a single hidden layer. We can add additional layers. Preliminary experiments, however, show that the shallow log-linear model (1) performs well enough in most cases. Only for larger data sets did we notice a marginal gain from adding an additional layer between projection matrix $W_p$ and the softmax layer over $C (W_e$ and bias $b_e)$, at the expense of longer training times and loss of transparency.

3.2 Parameter estimation

The matrices $W_p$, $W_e$ and the vector $b_e$ in (1) constitute the parameters of our model. We estimate them using error back propagation [51] as follows. For every document $d_j \in D$ we construct an ideal distribution over candidates $\hat{p} = P(e \mid d_j)$ based on the document-candidate associations $C_{d_j}$ such that

$$P(e \mid d_j) = \begin{cases} \frac{1}{|C|} & c \in C_{d_j} \\ 0 & c \not\in C_{d_j} \end{cases}$$

We continue by extracting n-grams where $n$ remains fixed during training. For every n-gram $w_1, \ldots, w_n$ generated from document $d$ we compute $\tilde{p} = P(e \mid w_1, \ldots, w_n)$ using (2). During model constructing we then optimize the cross-entropy $H(p, \tilde{p})$ (i.e., the joint probability of the training data if $|C_{d_j}| = 1$ for all $j$) using batched gradient descent. The loss function for a single batch of $m$ instances with associated targets $(\hat{p}^{(i)}, \tilde{p}^{(i)})$ is as follows:

$$L(W_p, W_e, b_e) = \frac{1}{m} \sum_{i=1}^{m} \frac{d_{\text{max}}^{(i)}}{|d^{(i)}|} \sum_{i,j} W_{p_{i,j}} \tilde{p}^{(i)} + \frac{\lambda}{2m} \sum_{i,j} W_{e_{i,j}}^2 \hat{p}^{(i)}$$

where $d^{(i)}$ refers to the document from which n-gram $w_1^{(i)}, \ldots, w_n^{(i)}$ was extracted, $d_{\text{max}} = \arg \max_{d \in D} |d|$ indicates the longest document in the collection, and $\lambda$ is a weight regularization parameter.

The update rule for a particular parameter $\theta$ ($W_p$, $W_e$ or $b_e$) given a single batch of size $m$ is:

$$\theta^{(t+1)} = \theta^{(t)} - \alpha^{(t)} \frac{\partial L(W_p^{(i)}, W_e^{(i)}, b_e^{(i)})}{\partial \theta}$$

where $\alpha^{(t)}$ is the learning rate.
where $\alpha^{(t)}$ and $\theta^{(t)}$ denote the per-parameter learning rate and parameter $\theta$ at time $t$, respectively. The learning rate $\alpha$ consists of the same number of elements as there are parameters; in the case of a global learning rate, all elements of $\alpha$ are equal to each other. The derivatives of the loss function (3) are given in the Appendix.

In the next section we will discuss our experimental setup, followed by an overview of our experimental results and further analysis in Section 5.

4. EXPERIMENTAL SETUP

4.1 Research questions

As indicated in the introduction, we seek to answer the following research questions:

RQ1 How does our discriminative log-linear model compare to vector-space-based methods and generative language models for the expert retrieval task in terms of retrieval performance? In particular, how does the model perform when compared against vector-space-based (LSI and TF-IDF) and generative approaches (profile-centric Model 1 and document-centric Model 2)?

RQ2 What can we learn regarding the different types of errors made by generative and discriminative language models? Does the best-performing generative model simply perform slightly better on the topics for which the other models perform decent as well, or do they make very different errors? If the latter holds, an ensemble of the rankings produced by both model types might exceed performance of the individual rankings.

RQ3 How does the complexity of inference in our log-linear model compare to vector-space based and generative models? The worst-case inference cost of document-centric models makes them unattractive in online settings where the set of topics is not defined beforehand and the document collection is large. Profile-centric methods are preferred in such settings as they infer from one language model per candidate expert for every topic (i.e., a pseudo-document consisting of a concatenation of all documents associated with an expert) [6]. Vector-space-based methods [19] have similar problems due to the curse of dimensionality [29] and consequently their inferential time complexity is likewise asymptotically bounded by the number of experts.

RQ4 How does the log-linear model handle incremental indexing and what are its limitations?

4.2 Benchmarks

The proposed method is applicable in the setting of the Expert Search task of the TREC Enterprise track from 2005 to 2008 [61]. We therefore evaluate on the W3C and CERC benchmarks released by the track. The W3C dataset [16] is a crawl of the W3C’s sites in June 2004 (mailing lists, web pages, etc.). The CSIRO Enterprise Research Collection (CERC) [2] is a dump of the intranet by the track. The W3C dataset [16] is a crawl of the W3C’s sites in June 2004 (mailing lists, web pages, etc.). The CSIRO Enterprise Research Collection (CERC) [2] is a dump of the intranet by the CSIRO. The W3C dataset [16] is a crawl of the W3C’s sites in June 2004 (mailing lists, web pages, etc.).

We propose two models for expert finding based on generative language models. The first takes a profile-centric approach comparing the language model of every expert to the query, while the second is document-centric. We consider both models with different smoothing configurations: Jelinek-Mercer (jm) smoothing with $\lambda = 0.5$ [4] and Dirichlet (d) smoothing with $\beta$ equal to the average document length [5] (see Table 1). Significance of results produced by the baselines (compared to our method) is determined using a two-tailed paired randomization test [59].

4.3 Baselines

We compare our approach to existing unsupervised methods for expert retrieval that solely rely on textual evidence and static document-candidate associations. (1) Demartini et al. [19] propose a generic framework to adapt vector spaces operating on documents to entities. We compare our method to TF-IDF (raw frequency and inverse document frequency) and LSI (300 latent topics) variants of their vector space model for entity ranking (using cosine similarity). (2) In terms of language modeling, Balog et al. [4] propose two models for expert finding based on generative language models. The first takes a profile-centric approach comparing the language model of every expert to the query, while the second is document-centric. We consider both models with different smoothing configurations: Jelinek-Mercer (jm) smoothing with $\lambda = 0.5$ [4] and Dirichlet (d) smoothing with $\beta$ equal to the average document length [5] (see Table 1). Significance of results produced by the baselines (compared to our method) is determined using a two-tailed paired randomization test [59].

4.4 Implementation details

The vocabulary $V$ is constructed from each corpus by ignoring punctuation, stop words and case; numbers are replaced by a numerical placeholder token. During our experiments we prune $V$ by only retaining the 2$^{16}$ most-frequent words so that each word can be encoded by a 16-bit unsigned integer. Incomplete n-gram instances are padded by a special-purpose token.

In terms of parameter initialization, we sample the initial parameters $W_c$ and $W_p$ (1) uniformly in the range

$$[-\sqrt{\frac{6.0}{m+n}}, \sqrt{\frac{6.0}{m+n}}]$$

for an $m \times n$ matrix, as this initialization scheme improves model training convergence [25], and take the bias vector $b_c$ to be null. The projection layer $W_p$ is initialized with pre-trained word representations trained on Google News data [41]; the number of word features is set to $v_c = 300$, similar to pre-trained representations.

We used adadelta ($\rho = 0.95$, $\epsilon = 10^{-6}$) [64] with batched gradient descent ($\alpha = 1024$) and weight decay $\lambda = 0.01$ during training on NVidia GTX480 and NVidia Tesla K20 GPUs. We only iterate once over the entire training set for each experiment.

5. RESULTS AND DISCUSSION

We start by giving a high-level overview of our experimental results and then address issues of scalability, provide an error analysis and discuss the issue of incremental indexing.

5.1 Overview of experimental results

We evaluate the log-linear model on the W3C, CERC and TU benchmarks (Section 4.2). During training we extract non-overlapping n-grams for the W3C and CERC benchmarks and overlapping n-grams for the TU benchmark. As the TU benchmark is considerably smaller, we opted to use overlapping n-grams to counter data sparsity. The architecture of each benchmark model (e.g., number of candidate experts) is inherently specified by the benchmarks themselves (see Table 1). However, the choice of n-gram size during training remains open. Errors for input $w_1, \ldots, w_n$ are propagated back through $W_c$ until the projection matrix $W_p$ is reached; if a single word $w_i$ causes a large prediction error, then this will influence its neighboring words $w_{i-1}, w_{i+1}, \ldots, w_n$ as well.
This allows the model to learn continuous word representations tailored to the expert retrieval task and the benchmark domain.

A larger window size has a negative impact on batch throughput during training. We are thus presented with the classic trade-off between model performance and construction efficiency. Notice, however, that the number of n-grams decreases as the window size increases if we extract non-overlapping instances. Therefore, larger values of n lead to faster wall-clock time model construction for the W3C and CERC benchmarks in our experiments.

We sweep over the window width \( n = 2^i \) (0 ≤ i < 6) for all three benchmarks and their corresponding relevance assessments. We report MAP and MRR for every configuration (see Figure 1). We observe a significant performance increase between \( n = 1 \) and \( n = 2 \) on all benchmarks, which underlines the importance of the window size parameter. The increase in MAP implies that the performance achieved is not solely due to initialization with pre-trained representations (Section 4.4), but that the model efficiently learns word representations tailored to the problem domain. The highest MAP scores are attained for relatively low \( n \). As \( n \) increases beyond \( n = 8 \) a gradual decrease in MAP is observed on all benchmarks. In our remaining experiments we choose \( n = 8 \) regardless of the benchmark.

Words \( w_i \) that mainly occur in documents associated with a particular expert are learned to produce distributions \( P(c \mid w_i) \) with less uncertainty than words associated with many experts in (1). The product of \( P(c \mid w_i) \) in (2) aggregates this expertise evidence generated by query terms. Hence, queries with strong evidence for a particular expert should be more predictable than very generic queries. To quantify uncertainty we measure the normalized entropy [56] of \( P(c \mid q) \):

\[
\eta(c \mid q) = -\frac{1}{\log(|C|)} \sum_{j=1}^{n} p(c_j \mid q) \log(p(c_j \mid q)).
\]

Equation 5 can be interpreted as a similarity measure between the given distribution and the uniform distribution. Importantly, Figure 2 shows that there is a statistically significant negative correlation between query-wise normalized entropy and average precision for all benchmarks.

Table 2 presents a comparison between the log-linear model and the various baselines (Section 4.3). Our unsupervised method significantly (\( p < 0.01 \)) outperforms the LSI-based method consistently. In the case of the TF-IDF method and the profile-centric generative language models (Model 1), we always perform better and statistical significance is achieved in the majority of cases. The document-centric language models (Model 2) perform slightly better than our method on two (out of six) benchmarks in terms of MAP and NDCG@100: (1) For the CERC 2007 assessment we match performance of the document-centric generative model with Jelinek-Mercer smoothing. (2) For TU GT1 the generative counterpart seems to outperform our method.

Notice that over all assessments, the log-linear model consistently outperforms all profile-centric approaches and is only challenged by the smoothed document-centric approach. In addition, for the precision-based measures (P@k and MRR), the log-linear model consistently outperforms all other methods we compare to.

Next, we turn to a topic-wise comparative analysis of discriminative and generative models. After that, we analyze the scalability and efficiency of the log-linear model and compare it to that of the generative counterparts, and address incremental indexing.

### 5.2 Error analysis

How does our log-linear model achieve its superior performance over established generative models? Figure 3 depicts the per-topic differences in average precision between the log-linear model and Model 2 (with Jelinek-Mercer smoothing) on all benchmarks. For each plot, the vertical bars with a positive AP difference correspond to test topics for which the log-linear model outperforms Model 2 and vice versa for bars with a negative AP difference.

The benefit gained from the projection matrix \( W_p \) is two-fold. First, it avoids the curse of dimensionality introduced by large vocabularies. Second, term similarity with respect to the expertise domain is encoded in latent word features. When examining words nearby query terms in the embedding space, we found words to be related to the query term. For example, word vector representations of \( \text{xml} \) and \( \text{nonterminal} \) are very similar for the W3C benchmark (\( l_2 \) norm). This can be further observed in Figure 1: log-linear models trained on single words perform significantly worse compared to those that are able to learn from neighboring words.

We now take a closer look at the topics for which the log-linear model outperforms Model 2 and vice versa. More specifically, we investigate textual evidence related to a topic and whether it is considered relevant by the benchmark. For the log-linear model, we examine terms nearby topic terms in \( W_p \) (\( l_2 \)-norm), as these terms are considered semantically similar by the model and provide a means for semantic matching. For every benchmark, we first consider top-

<table>
<thead>
<tr>
<th>W3C</th>
<th>CERC</th>
<th>TU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of documents</td>
<td>331,037</td>
<td>370,715</td>
</tr>
<tr>
<td>Average document length( ^{a} )</td>
<td>1,237.23</td>
<td>460.48</td>
</tr>
<tr>
<td>Number of candidates( ^{b} )</td>
<td>715</td>
<td>3,479</td>
</tr>
<tr>
<td>Number of document-candidate associations</td>
<td>200,939</td>
<td>236,958</td>
</tr>
<tr>
<td>Number of documents (with ( C_d &gt; 0 ))</td>
<td>93,826</td>
<td>123,934</td>
</tr>
<tr>
<td>Number of associations per document( ^{c} )</td>
<td>2.14 ± 3.29</td>
<td>1.91 ± 3.70</td>
</tr>
<tr>
<td>Number of associations per candidate</td>
<td>281.03 ± 666.63</td>
<td>68.11 ± 1,120.74</td>
</tr>
<tr>
<td>Queries</td>
<td>49 (2005)</td>
<td>50 (2007)</td>
</tr>
<tr>
<td></td>
<td>50 (2006)</td>
<td>77 (2008)</td>
</tr>
</tbody>
</table>

\(^{a} \) Measured in number of tokens.
\(^{b} \) Only candidates with at least a single document association are considered.
\(^{c} \) Only documents with at least one association are considered.

**Table 1:** An overview of the three datasets (W3C, CERC and TU) used for evaluation and analysis.
Significance of results is determined using a two-tailed paired randomization test [59] (**p < 0.01; ***p < 0.05; ****p < 0.1) with respect to the log-linear model (adjusted using the Benjamini-Hochberg procedure for multiple testing [9]).

Table 2: Evaluation results for models trained on the W3C, CERC and TU benchmarks. Suffixes (d) and (jm) denote Dirichlet and Jelinek-Mercer smoothing, respectively (Section 4.3). Significance of results is determined using a two-tailed paired randomization test [59] (**p < 0.01; ***p < 0.05; ****p < 0.1) with respect to the log-linear model (adjusted using the Benjamini-Hochberg procedure for multiple testing [9]).

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<td><strong>0.656</strong>*</td>
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<td><strong>0.092</strong>*</td>
</tr>
</tbody>
</table>

CERC For CSIRO, topic nanohouse (CE-035) is mentioned in many irrelevant contexts (i.e., spam) and therefore semantic matching fails. The term fish oil (CE-126) is quickly associated with different kinds of fish, oils and organizations related to marines and fisheries. On the other hand, we observe sensor networks (CE-018) to be associated with sensor/networking jargon and sensor platforms. Topic forensic science workshop (CE-103) expands to syntactically-similar terms (e.g., plural), the names of science laboratories and references to support/law-protection organizations.

In order to further quantify the effect of the embedding matrix $W_p$, we artificially expand benchmark topic terms by $k$ nearby terms. We then examine how the performance of a profile-centric generative language model [4, Model 1] evolves for different values of $k$ (Figure 4). The purpose of this analysis is to provide further insight in the differences between maximum-likelihood language models and the log-linear model. Figure 4 shows that, for most benchmarks, MAP increases as $k$ goes up. Interestingly enough, the two benchmarks that exhibit a decrease in MAP for larger $k$ (CERC 2007 and TU GT1) are likewise those for which generative language models outperform the log-linear model in Table 2. This suggests that the CERC 2007 and TU GT1 benchmarks require exp-
5.3 Scalability and efficiency

Inference in the log-linear model is expressed in linear algebra operations (Section 3). These operations can be efficiently performed by highly optimized software libraries and special-purpose hardware (i.e., GPUs). But the baseline methods against which we compare do not benefit from these speed-ups. Furthermore, many implementation-specific details and choice of parameter values can influence runtime considerably (e.g., size of the latent representations). Therefore, we opt for a theoretical comparison of the inference complexity of the log-linear model and compare these to the baselines (Section 4.3).

The log-linear model generates a ranking of candidate experts by straight-forward matrix operations. The look-up operation in the projection matrix $W_p$, occurs in constant time complexity, as the multiplication with the one-hot vector $v_i$ comes down to selecting the $i$-th column from $W_p$. Multiplication of the $|C| \times e$ matrix $W_e$ with the $e$-dimensional word feature vector exhibits $O(|C| \cdot e)$ runtime complexity. If we consider addition of the bias term and division by the normalizing function $Z_1$, the time complexity of (1) becomes

$$O(|C| \cdot (e + (e - 1)) + |C| + 2 \cdot |C| - 1).$$

Notice, however, that the above analysis considers sequential execution. Modern computing hardware has the ability to parallelize common matrix operations [24, 33]. The number of candidate experts $|C|$ is the term that impacts performance most in the log-linear model (under the assumption that $|C| \gg e$).

If we consider $n$ terms, where $n$ is the query length during inference or the window size during training, then the complexity of (2) becomes

$$O(n \cdot |C| \cdot (2 \cdot e - 1) + n \cdot (3 \cdot |C| - 1))$$

$$+ (n - 1) \cdot |C| + 2 \cdot |C| - 1).$$

Notice that $Z_2$ does not need to be computed during inference as it does not affect the candidate expert ranking.
Figure 2: Scatter plot of the normalized entropy of distribution $P(c | q)$ (2) returned by the log-linear model and per-query average precision for W3C, CERC and TU benchmarks. Pearson’s $R$ and associated $p$-value (two-tailed paired permutation test: *** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$) are between parentheses. The depicted linear fit was obtained using an ordinary least squares regression.

<table>
<thead>
<tr>
<th></th>
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Table 3: Comparison of Model 2, the log-linear model and an ensemble of the former on W3C, CERC and TU benchmarks. Significance of results is determined using a two-tailed paired randomization test [59] (*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$) with respect to the ensemble ranking (adjusted using the Benjamini-Hochberg procedure for multiple testing [9]).

In terms of space complexity, parameters $W_p$, $W_e$ and $b_e$, in addition to the intermediate results, all require memory space proportional to their size. Considering (2) for a sequence of $k$ words and batches of $m$ instances, we require $O(mnk | C|)$ floating point numbers for every forward-pass to fit in-memory. While such an upper bound seems reasonable by modern computing standards, it is a severely limiting factor when considering large-scale communities and while utilizing limited-memory GPUs for fast computation.

The inferential complexity of the vector space-based models for entity retrieval [19] depends mainly on the dimensionality of the vectors and the number of candidate experts. The dimensionality of the latent entity representations is too high for efficient nearest neighbor retrieval [29] due to the curse of dimensionality. Therefore, the time complexity for the LSI- and TF-IDF-based vector space models are respectively $O(\gamma \cdot |C|)$ and $O(|V| \cdot |C|)$, where $\gamma$ denotes the number of latent topics in the LSI-based method. As hyperparameters $\epsilon$ and $\gamma$ both indicate the dimensionality of latent entity representations, the time complexity of the LSI-based method is comparable to that of the log-linear model. We note that $|V| \gg |C|$ for all benchmarks ($|V|$ is between 18 to 91 times larger than $|C|$) we consider in this paper and therefore conclude that the TF-IDF method loses to the log-linear model in terms of efficiency.

Compared to the unsupervised generative models of Balog et al., we have the profile-centric Model 1 and the document-centric Model 2 with inference time complexity $O(n \cdot |C|)$ and $O(n \cdot |D|)$, respectively, with $|D| \gg |C|$. In the previous section we showed that the log-linear model always performs better than Model 1 and nearly always outperforms Model 2. Hence, our log-linear model generally achieves the expertise retrieval performance of Model 2 (or higher) at the complexity cost of Model 1 during inference.

5.4 Incremental indexing

Existing unsupervised methods use well-understood maximum-likelihood language models that support incremental indexing. We now briefly discuss the incremental indexing capabilities of our proposed method. Extending the set of candidate experts $C$ requires the log-linear model to be re-trained from scratch as it changes the topology of the network. Moreover, every document associated with a candidate expert is considered as a negative example for all other candidates. While it is possible to reiterate over all past documents and only learn an additional row in matrix $W_e$, the final outcome is unpredictable.

If we consider a stream of documents instead of a predefined set $D$, the log-linear model can be learned in an online fashion. However, stochastic gradient descent requires that training examples are
picked at random such that the batched update rule (4) behaves like the empirical expectation over the full training set [11]. While we might be able to justify the assumption that documents arrive randomly, the $n$-grams extracted from those documents clearly violate this requirement.

Considering a stream of documents leads to the model forgetting expertise evidence as an (artificial) shift in the underlying distribution of the training data occurs. While such behavior is undesirable for the task considered in this paper, it might be well-suited for temporal expert finding [22, 52], where expertise drift over time is considered. However, temporal expertise finding is beyond the scope for this paper and left for future work.

6. CONCLUSIONS

We have introduced an unsupervised discriminative, log-linear model for the expert retrieval task. Our approach exclusively employs raw textual evidence. Future work can focus on improving performance by feature engineering and incorporation of external evidence. Furthermore, no relevance feedback is required during training. This renders the model suitable for a broad range of applications and domains.

We evaluated our model on the W3C, CERC and TU benchmarks and compared it to state-of-the-art vector space-based entity ranking (based on LSI and TF-IDF) and language modeling (profile-centric and document-centric) approaches. The log-linear model combines the ranking performance of the best maximum-likelihood language modeling approach (document-centric) with inference time complexity linear in the number of candidate experts. We observed a notable increase in precision over existing methods. Analysis of our model’s output reveals a negative correlation between the per-query performance and ranking uncertainty: higher confidence (i.e., lower entropy) in the rankings produced by our approach often occurs together with higher rank quality.

An error analysis of the log-linear model and traditional language models shows that the two make very different errors. These errors are mainly due to the semantic gap between query intent and the raw textual evidence. Some benchmarks expect exact query matches, others are helped by our semantic matching. An ensemble of methods employing exact and semantic matching generally out-performs the individual methods. This observation calls for further research in the area of combining exact and semantic matching.

One current limitation of our work is its scalability with respect to the number of candidate experts. We have started investigating trade-offs between model performance and time/space complexity. In the future we hope to apply scalable variants of this method on large-scale social media communities, for the purpose of determining topic ownership. While in this work we focus on expertise retrieval, the ideas we proposed can easily be transferred to the more general entity retrieval task. Moreover, our approach is likely to be applicable to authorship attribution and various other entity retrieval and prediction tasks.

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Computing resources were provided by the Netherlands Organisation for Scientific Research (NWO) through allocation SH-322-15 of the Cartesius system and by the Advanced School for Computing and Imaging (ASCI) by allocation of the Distributed ASCI Supercomputer 4 (DAS-4) system.
The derivative of (3) w.r.t. bias term $b_c$ equals

$$\frac{\partial L(W_p, W_c, b_c)}{\partial b_c} = \frac{1}{m} \left( \sum_{i=1}^{m} \left[ \frac{1}{d_{iw}} \frac{\partial P(c_j \mid w_i, \ldots, w_n)}{\partial P(c_j \mid w_i, \ldots, w_n)} \right] \right)$$

and w.r.t. an arbitrary matrix parameter $\theta$ ($W_p$ or $W_c$):

$$\frac{\partial L(W_p, W_c, b_c)}{\partial \theta} = \frac{1}{m} \left( \sum_{i=1}^{m} \left[ \frac{1}{d_{iw}} \frac{\partial P(c_j \mid w_i, \ldots, w_n)}{\partial P(c_j \mid w_i, \ldots, w_n)} \right] \right)$$

Further differentiation for parameter $\theta$ ($W_p$, $W_c$ or $b_c$):

$$\frac{\partial \log \left( P(c_j \mid w_1, \ldots, w_n) \right)}{\partial \theta} = \frac{1}{P(c_j \mid w_1, \ldots, w_n)} \frac{\partial P(c_j \mid w_1, \ldots, w_n)}{\partial \theta}$$

Further differentiation for parameter $\theta$ ($W_p$, $W_c$ or $b_c$):

$$\frac{\partial L(W_p, W_c, b_c)}{\partial \theta} = \frac{1}{m} \left( \sum_{i=1}^{m} \frac{\partial P(c_j \mid w_i, \ldots, w_n)}{\partial \theta} \right)$$

For a given candidate $c_j$ and word $w_i$, following (1) we have

$$P(c_j \mid w_i) = \frac{\hat{P}(c_j \mid w_i)}{Z_i}$$

and consequently, with $W_p^T$ denoting the $i$-th column of matrix $W_p$,

$$\frac{\partial P(c_j \mid w_i)}{\partial W_p} = \frac{Z_i \hat{P}(c_j \mid w_i)}{Z_i} \hat{P}(c_j \mid w_i) \frac{Z_i}{Z_i}$$

As can be seen in (7), the distributed representations of candidates $c_j$ at time $t + 1$ are updated using the representation of words $w_i$ at time $t$ and vice versa.