Making real options credible: Incomplete markets, dynamics and model ambiguity
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Citation for published version (APA):
Zhao, L. (2016). Making real options credible: Incomplete markets, dynamics and model ambiguity

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Investment decisions in the energy industry are often undertaken sequentially and are sensitive to information regarding energy markets and geographic conditions. Information may arrive gradually over time and as a consequence of early stage decisions. Therefore, NPV-based frameworks are unsuitable for decision making because they do not allow for the fact that new information may change later stage decisions. Real option valuation (ROV) serves as an effective evaluation tool for projects with flexibilities. However, it has remained a fringe field; practitioners believe it is not practically applicable in complex real world environments. The reason is that the real-life problems violate basic assumptions required by standard option pricing techniques. This thesis therefore aims to provide feasible solutions by applying real option theory to several highly complex energy problems with unhedgeable risks, complicated underlying dynamics, and model ambiguity taken into consideration.

Lin Zhao (1986) holds a BSc in Mathematics from Shandong University, an MSc in Quantitative Economics from Shanghai University of Finance and Economics, and an MPhil in Finance from Tinbergen Institute. Under the supervision of Prof. Sweder van Wijnbergen, Lin Zhao conducted her PhD thesis at the University of Amsterdam between 2012 and 2015. The financial supports from EBN B.V. and Duisenberg School of Finance are greatly acknowledged.
MAKING REAL OPTIONS CREDIBLE:

INCOMPLETE MARKETS, DYNAMICS, AND MODEL AMBIGUITY
This book is no. 628 of the Tinbergen Institute Research Series, established through cooperation between Rozenberg Publishers and the Tinbergen Institute. A list of books which already appeared in the series can be found in the back.
MAKING REAL OPTIONS CREDIBLE: INCOMPLETE MARKETS, DYNAMICS, AND MODEL AMBIGUITY

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Universiteit van Amsterdam
op gezag van de Rector Magnificus
prof. dr. D. C. van den Boom
ten overstaan van een door het College voor Promoties ingestelde
commissie, in het openbaar te verdedigen in de Agnietenkapel
op woensdag 13 januari 2016 om 10.00 uur

door

Lin Zhao

geboren te Shandong, China
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                                 Erasmus Universiteit Rotterdam

Faculteit: Faculteit Economie en Bedrijfskunde
Acknowledgement

It has been a long long journey to finally complete my thesis, both physically and mentally. The day has eventually come that I am a student no longer. But if there is one thing I have learnt from the 20+ years of studying, it has to be the humbleness one has to possess through their lifetime. On one hand, I know that I know nothing: the more one learns, the less one knows. On the other hand, whatever I have today could not have been achieved without the many remarkable people, who have so generously helped and supported me in the past years. I would like to take this opportunity to thank and to acknowledge them all.

First and foremost I would like to express my special appreciation and thanks to Prof. Sweder van Wijnbergen, who has always been a great inspiration since the first day we met. Especially during the very difficult times of my PhD, he gave me enormous advice, patience, and trust. Without his guidance, support, and encouragement, my three-year PhD would have not been so enjoyable. I am truly grateful for being able to have such a tremendous mentor.

I would also like to show my gratitude to Prof. Ton Vorst. To start with, thanks to him I have had the opportunity to join Tinbergen Institute and started an amazing life in Amsterdam, and secondly he has always been most generous to offer me helps in both academia and industry.

My thanks also go out to the support I have received from the collaborative work with EBN. I would like to thank Jan Boekelman, for his excellent supports and advice; Jacqueline Platenburg, for always being so kind and thoughtful; Peter de Vries, for his patience and all the helps with my thesis; Ruben Swart, for providing constructive feedback and being supportive; Bas Brouwer, Danial Smith, for providing me with assistance throughout my thesis.

I am extremely grateful to be able to spend my three-year PhD at MInt, an inspiring group with amazing intellectuals. I would like to thank all the faculty members, Franc Klaassen, Ward Romp, Christian Stoltenberg, John Lorié, Massimo Giuliodori, Naomi Leefmans, Kostas Mavromatis, and Dirk Veestraeten. Special thanks go to Stephanie Chan, for not only being my friend but also my spiritual sister, and to Christiaan van der Kwaak, for always being a supportive and sincere friend. I am lucky to have met the brilliant PhD students. Lucy Gornicka, Egle Jakucionyte, and Liu Yang: It is truly pleasant to work in one office with you, as colleagues, and as friends; Damiaan Chen, Rutger Teulings, Julien Pinter, and Swapnil Singh: The delightful conversations with you have always been so enjoyable. I thank Moutaz Alta, Nicole Ciurila, Oana Furtuna, Jesper Hanson, Zina Lekniute, Ron van Maurik, Jante Parlevliet, for providing inspiring discussions and the stimulating environment at MInt.

Many thanks go to my dearest friends. Teresa Yang, I am so lucky to have met you and
have you as my friend. Yang Yang, Liting Zhou, Zhenzhen Fan, Xiao Xiao, Zhiling Wang, thank you for creating such wonderful memories during my years at TI. Yusi Wang, Sisi Zhang, Zhaochun Ren, Tony Zhang, thank you for always being there for me and for all the laughter we had and will have. Thank you, Andrew Pua, Zhida Xu, Marios Spyropoulos, Shawn Xiang, Hao Fang, for all the helps you generously offered me and for being my friends.

A special mention goes to a special person, Erkki. Thank you for helping me improve my English in the last years – The end justifies the means.

Finally, I thank my beloved parents, who have been unconditionally loving and supporting me since the moment I was born. Words are too limited to describe my gratitude to you. Mama, Baba, xie xie, wo ai ni men!
# Contents

1 Introduction 1

2 A Real Option Perspective on Valuing Gas Fields 5

   2.1 Introduction .......................................................... 5

   2.2 Literature Review ................................................... 8

      2.2.1 Real Option Analysis vs. NPV ............................... 8

      2.2.2 Incomplete Market Setting ................................. 8

      2.2.3 GARCH Option Pricing Model ............................ 11

      2.2.4 Least Square Monte Carlo Method ...................... 12

   2.3 Problem Description .............................................. 13

      2.3.1 Reservoir Size Distribution .............................. 14

      2.3.2 Option Model Setup ........................................ 17

   2.4 Methodology ........................................................ 18

      2.4.1 Predicting Future Gas Prices ............................. 18

         2.4.1.1 Gas Price Data ....................................... 18

         2.4.1.2 GARCH Model Estimation ........................... 19

         2.4.1.3 GARCH Option Pricing Model ..................... 19

      2.4.2 Integrated Valuation Method ............................. 21

   2.5 Application .......................................................... 22

      2.5.1 Cost-of-capital Method ..................................... 23

         2.5.1.1 Results without reservoir information update ...... 23

         2.5.1.2 GARCH Model vs. Constant Volatility ............ 25

         2.5.1.3 Spot Prices vs. Option Values ........................ 26

         2.5.1.4 Results with reservoir information update ........ 26

      2.5.2 Integrated Valuation Method ............................. 28

         2.5.2.1 Results with and without reservoir information updates . 28

         2.5.2.2 GARCH vs. Constant volatility ..................... 29

   2.6 Conclusion .......................................................... 30

   2.7 Appendix ............................................................ 32

      2.7.1 LSMC for Cost-of-capital Method ....................... 32

      2.7.2 LSMC for Integrated Valuation Approach ................. 33

      2.7.3 Estimation Results and Diagnostic Tests for GARCH models ...... 35

         2.7.3.1 Unit Root Test ....................................... 35

         2.7.3.2 Model Estimation and Diagnostic Tests ............ 36
## CONTENTS

2.7.3.3 Seasonal Effects ........................................... 38
2.7.3.4 EGARCH .................................................. 41

3 Decision Making in Incomplete Markets with Ambiguity ........................................ 45
  3.1 Introduction ................................................... 45
  3.2 Literature Review ............................................. 48
  3.3 Problem Description ........................................... 50
    3.3.1 Reservoir Distribution .................................. 50
    3.3.2 Option Characteristics of the Valuation Problem ........ 52
  3.4 Methodology .................................................. 52
    3.4.1 Generalized Autoregressive Score (GAS) Models ........... 52
    3.4.2 Estimation Results and Diagnostic Tests .................. 53
    3.4.3 Utility Indifference Pricing .............................. 56
  3.5 Results ...................................................... 57
    3.5.1 Cost-of-capital Method .................................. 57
      3.5.1.1 NPVs vs. Option Values ............................ 57
      3.5.1.2 Good State vs. Bad State .......................... 59
    3.5.2 Utility Indifference Pricing (UIP) ....................... 59
    3.5.3 Model Ambiguity ........................................ 61
    3.5.4 Reservoir Correlation .................................. 64
      3.5.4.1 Cost-of-capital method ............................ 65
      3.5.4.2 Utility Indifference Pricing ........................ 67
  3.6 Conclusion .................................................. 68
  3.7 Appendix ..................................................... 70
    3.7.1 GAS Models ............................................. 70
      3.7.1.1 Gaussian GARCH model ............................ 70
      3.7.1.2 Gaussian GAS model ............................... 70
      3.7.1.3 Student’s t GARCH model .......................... 70
      3.7.1.4 Student’s t GAS model ............................ 70
    3.7.2 Results under a specification of the gas price volatility process as a Student’s t-GARCH model .................. 71
    3.7.3 Option Results in Details ................................ 76

4 Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity ..................... 83
  4.1 Introduction ................................................... 83
  4.2 Literature Review ............................................. 85
    4.2.1 Pricing Storage Capacity ............................... 85
    4.2.2 GAS Models ............................................. 87
Introduction

Investment decisions in the energy industry are often undertaken sequentially and are sensitive to information regarding energy markets and geographic conditions. Information may arrive gradually over time and as a consequence of early stage decisions. Therefore, NPV-based frameworks are unsuitable for decision making because they do not allow for the fact that new information may change later stage decisions. Real option valuation (ROV), which quantifies the value of embedded flexibilities through option pricing techniques, serves as an effective evaluation tool for projects with flexibilities. McDonald and Siegel [1986] initiated the application of option pricing technology to decisions involving irreversible “real” projects. Following their work, Pindyck [1991] provides a simple and applicable methodology for practitioners. He emphasizes two major characteristics of investment opportunities: irreversible expenditure and postponement of execution. These features both have a profound effect on investment decisions, and share similarities with financial options.

ROV builds on NPV but allows for managerial response to new information coming on stream in the course of the project. The NPV scenarios are embedded in the analysis because a manager can always choose to adhere to NPV scenarios instead of switching between them. Dixit and Pindyck [1994] also stress that the differences between these two methods lie in the ability of ROV to incorporate and value flexibility and path-dependency among projects. As a consequence, an investment is always valued equally or higher by real option valuation than by NPV. In other words, NPV undervalues investments as long as there exists any possible flexibility.

However, real option theory has remained a fringe field; practitioners believe it is not practically applicable in complex real world environments. The reason is that the real-life problems violate basic assumptions required by standard option pricing techniques. This thesis therefore aims to provide feasible solutions to real-life problem by applying real option theory to several highly complex energy problems with unhedgeable risk, time-varying volatilities and endogenous exercise dates.
1. Introduction

**Incomplete Markets**

The application of standard option pricing formula (e.g. Black and Scholes [1973]) requires the assumption of market completeness. However, this may not be satisfied by the complicated structure of investment problems in reality, which results in problematic ROV applications. Markets are often times incomplete which makes preference free pricing impossible and thus standard option pricing inapplicable. For example, one of the underlying assets we consider is the risk embedded in reservoir size, which cannot be hedged through the activities in existing markets. Another factor that may cause market incompleteness is the time-varying volatility structure existing in the gas price dynamics. Therefore, the no-arbitrage assumption fails and the individual risk aversion needs to be parametrized into the models.

To deal with this matter, this thesis adopts mainly three methods, namely the cost-of-capital method (Chapter 2 and 3), integrated valuation procedure (Chapter 2), and utility indifference pricing (Chapter 3 and 4). The first method simply calculates the value of project under a carefully selected range of cost-of-capital rates. It is very easy to implement and is most relevant and attractive for practitioners. However, the choice of the most appropriate cost-of-capital is unclear and has to be decided upon subjective experiences. The integrated valuation procedure follows Smith and Nau [1995], Smith [1996], Smith and McCardle [1998], and Smith and McCardle [1999], where the effective certainty equivalent is employed to deal with market incompleteness caused by the reservoir distribution. However, the integrated valuation procedure again becomes unsatisfying as we adopt alternative time-varying volatility models other than GARCH ones. Hence we then employ utility indifference pricing models to study the relationship between option prices and individual risk appetite.

**Dynamics of Underlying Processes**

Black-Scholes formula (Black and Scholes [1973]) provides a closed-form solution with the assumption of the underlying asset following a geometric Brownian motion process. However, the gas price series under concern cannot be fully described with such a simple assumption. Time-varying volatility exists in our data, and later the diagnostic tests also prove the existence of fat-tails. To characterize the heteroskedasticity, we first employ a GARCH model in Chapter 2 and then extend it by introducing fat-tails in Chapter 3. In addition, to further study certain unique features of the latent volatilities, we introduce Generalized Autoregressive Score (GAS) models which are developed by Creal et al. [2013], Harvey [2013]. Besides from the univariate models, Chapter 4 investigates two correlated gas prices (day-ahead and month-ahead) with heavy-tailed multivariate GAS models.
Model Ambiguity

Model ambiguity or Knightian uncertainty (Knight [1921]) refers to the situation when one has to make a decision without knowing the true probability distribution. This has to be distinguished from a decision problem under risk, i.e. one has to make a decision given the true probability distribution known. Model ambiguity is particularly an important issue to be considered in the energy development stage, since the geophysicists have to estimate the reservoir distribution based not only on limited fact sheets, but also on their own experiences. Therefore it is of large interest to assume the presence of model ambiguity in case of gas field valuations (Chapter 3).

Furthermore, market incompleteness and model ambiguity may mutually reinforce each other. Mukerji and Tallon [2001] argue that the markets are less complete due to the effect of ambiguity aversion.

Least Square Monte Carlo Method

As mentioned before, closed-form solutions do not exist given the complicated structure of the problem presented. We thus adopt and modify Least Square Monte Carlo (LSMC) method proposed by Longstaff and Schwartz [2001], which easily reduces the dimensionality problem and provides accurate results. In addition, with this method, we are able to incorporate complicated dynamics of the underlying assets, as well as the complex option structure. Details of LSMC will be discussed in each chapter separately.

Thesis Outline

The structure of this thesis is presented as follows. In Chapter 2, we apply two real option approaches to exploitation decisions for a cluster of Dutch gas fields, where the two main sources of uncertainty are gas prices and field reservoir size. Gas price returns are modeled with a GARCH specification due to the volatility clustering; while the cost-of-capital or risk tolerance has to be parametrized in face of the incomplete market issue raised by the uncertainty of reservoir size. Moreover investment decisions can be postponed or delayed, which implies a non-European option setting. Least Square Monte Carlo method is therefore applied for improving the computational efficiency.

Correctly modeling the structure of volatility has a major impact: Option values shrink by 70% if the time varying nature of volatility is ignored. We also show that a high correlation between reservoir sizes at different locations creates extra option values. As for the two approaches, option values decrease with the cost-of-capital in use while increase with the investor’s risk tolerance. The non-standard features of our approaches result in a major impact:
1. Introduction

option values are large so real options based valuations substantially exceed corresponding NPV calculations.

The third chapter improvises the methodology demonstrated in Chapter 2. We apply utility indifference pricing to solve a contingent claim problem, valuing a connected pair of gas fields where the underlying process is not standard geometric Brownian motion and the assumption of complete markets is not fulfilled. First, empirical data are often characterized by time-varying volatility and fat tails; therefore we use Gaussian GAS (Generalized AutoRegressive Score) and GARCH models, extending them to Student’s t-GARCH and t-GAS. Second, an important risk (reservoir size) is not hedgeable. Therefore we parametrize the investor’s risk preference and use utility indifference pricing techniques. We use again Least Square Monte Carlo simulations as a dimension reduction technique. Moreover, an investor often only has an approximate idea of the true probabilistic model underlying variables, making model ambiguity a relevant problem. We show empirically how model ambiguity affects project values, and importantly, how option values change as model ambiguity gets resolved in later phases of the projects considered. We show that traditional valuation approaches will consistently underestimate the value of project flexibility and in general lead to overly conservative investment decisions in the presence of time dependent stochastic structures.

In Chapter 4, we investigate the relationship between the gas spot market and the price of gas storage capacity. Contrary to the common belief, the auction prices for gas storage are mostly affected by the volatility of current market prices rather than by the winter-summer price differences. This chapter provides a numerical solution for pricing storage capacity, by taking investor’s activities through the spot market and storage service into account. A bivariate Generalized Autoregressive Score (GAS) model is employed for modeling the dynamics of the day-ahead and month-ahead spot market prices, as well as the time-varying volatilities and correlations. Under an incomplete market setting, our model is able to approximate the realized auction prices. Moreover, one interesting implication is that the implied average risk aversion of investor for a storage contract increases with the volatility of the spot market. This is an intuitive result because storage capacity can serve as an effective hedging product for the spot market, and the demand of this product is high when the market becomes risky: more risk averse investors are participating in the auctions. Moreover, a sensitivity analysis on different injection/withdrawal rates is also included, and particularly, contracts with higher capacity rates are priced at a higher level.

A short summary is then given in Chapter 5.
2 A Real Option Perspective on Valuing Gas Fields

2.1 Introduction

Net Present Value (NPV) and Discounted Cash Flow (DCF) methods are widely used in capital budgeting processes. Their popularity results from their straightforwardness and convenience, but both methods ignore path-dependency\(^2\) embedded in projects and fail to incorporate the value of managerial flexibility to change or to revise decisions as new information becomes available. Real option analysis can be looked at as an extension of NPV, providing more accurate estimates of project values by taking both flexibility and path-dependence into consideration. This is of particular importance in the energy sector: investment decisions in the energy industry are often undertaken sequentially and are substantially influenced by new information on market conditions as well as on geographic conditions. During a year, a gas company routinely has to take all kinds of day-to-day management decisions to which our analytical tools would apply:

- When to start exploration projects, e.g. seismic programs or drilling exploration wells;
- When to start development projects, e.g. field development programs including development drilling, installing platforms and pipelines, etc;
- When/Whether to apply enhanced production techniques such as compression;

This chapter is coauthored with Prof. Sweder van Wijnbergen. We thank EBN B.V. for financial support, and Ton Vorst (VU University Amsterdam), Bob Pindyck (MIT), and seminar participants at EBN, the Tinbergen Institute and the TopQuant and E&Y Best Quant Finance Thesis event for helpful discussions and comments.

\(^2\)Two projects are said to be path-dependent when one project can only be initiated upon completion of the other.
Abandonment decisions, such as plugging wells, abandoning and cleaning platforms and pipelines.

Each decision is usually assessed by its stand-alone parameters and value outcomes. However, in sequential investment projects where later investment opportunities are dependent on the outcome of earlier projects, starting early investments often brings in additional opportunities for further investments, which should be seen as an extra value of those early investments. Real option analysis is more appropriate in such circumstances than NPV as a decision tool in the capital budgeting process. But in practice, applications of real option theory have been limited to highly simplified investment decisions, modeled as simple option type problems. Anything approaching real world complexity is typically considered too difficult to solve using this approach. As a result, real option theory has remained something of a niche product, nice in theory but not useful for real world problems. In this chapter, we show that such a view is mistaken: we provide a complex but trackable solution to much more complicated option style and strategic investment problems.

Real option analysis is often criticized for its misuse of standard option pricing models (i.e. the Black-Scholes formula) since the assumption of market completeness does not hold in most environments where it is or could be applied, which makes the preference free risk-neutral valuation methods and traditional option pricing formulas inapplicable. These methods are derived from no arbitrage conditions with respect to a replicating portfolio of tradable underlying assets, which do not (fully) exist in an incomplete market setting. With regard to the application considered in this chapter, there are two sources of risks associated with the value of underlying assets, market gas prices and reservoir volume. Gas contracts are traded on public markets so we assume gas price uncertainty can be hedged; but investment projects also carry idiosyncratic risks, such as reservoir size, which cannot be hedged by appropriately structured replicating portfolios due to a lack of correlated traded instruments. This lack of market completeness makes standard option pricing methods inapplicable; we instead demonstrate two alternative approaches to solving the resulting valuation problems.

The value of embedded options is strongly influenced by the stochastic process of gas prices and the assumption of reservoir distribution. Reservoir size uncertainty is modeled by given geographic analysis. As for the spot price returns, we use a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Engle [1982]; Bollerslev [1986]) to incorporate the clustering and heteroskedasticity in volatility. This is due to recent research findings in finance that the second moment evolution of a price process should not be neglected. Based on econometric analysis we show that a GARCH model is the appropriate model for fitting and predicting the stochastic process of natural gas price returns and volatility. That structure turns out to have a major impact on option values. Although a few attempts have been made to predict commodity prices and volatility with GARCH models...
2.1. Introduction

(e.g. Pindyck [2003]), this chapter is to our knowledge the first application within a real option pricing framework. It also provides an empirical evidence of the impact of assuming a GARCH process rather than the standard assumption of constant volatility on option pricing problems.

The case study at hand, investment decisions in offshore gas fields, presents more non-standard features. First, because decisions can be brought forward or backward in time, the embedded options have endogenously determined exercise dates. We model the problem as a set of compound Bermuda options. Solving this complex Bermuda style option pricing problem with multiple state variables is a challenge because of spiraling dimensionality problems, which we bring down to manageable proportions through the use of a simulation-based technique, Least Square Monte Carlo simulation (Longstaff and Schwartz [2001]). Second, we initially make a simple assumption of no reservoir size correlation among gas fields, but relax it later on for more general and realistic analysis. A key characteristic of our problem setup is that early investments provide information that is of use in making investment decisions at a later stage. Of course information as a byproduct of investment decisions is not a new idea, see for example the very early application of option theory to the valuation of oil leases in Paddock et al. [1988]. See also Dixit and Pindyck [1994] for similar discussions. But by and large this literature focuses on the option to delay or abandon continuous investment programs as new information becomes available, but does not take into account the fact that information release itself may be influenced by the early investment decisions. That interdependence is at the core of the issues we consider in this chapter.

To deal with the market incompleteness problem, our first approach is based on ideas similar to the ambiguous method for incomplete markets (see Floroiu and Pelsser [2013]), by setting a reasonable range of cost-of-capital rates. The cost-of-capital is the rate of return that capital could be expected to earn in an alternative investment with equivalent risks. The results yield a consistently higher project value than one obtained with NPV, i.e. NPV substantially underestimates the project value. Sensitivity analysis of the option price relative to changes of the spot gas price (i.e. Delta) shows that, as expected, option values are increasing in spot gas prices. So a higher spot price affects the project value through two channels, directly and indirectly, through its traditional NPV and through option value respectively. Furthermore, as more information on reservoir size is obtained, higher option value is accordingly observed from the results.

Our second approach is called integrated valuation method or utility indifference pricing for derivatives. If the assets on which options are written are neither traded in public markets nor are the associated risks otherwise hedgeable, it is impossible to derive valuations that are preference free. Therefore this approach makes the valuation an explicit function of the investor’s degree of risk tolerance. In line with the intuition, we find that risk attitude has a
significant influence on project valuation: option values are higher for investors with higher
risk tolerance (lower risk aversion). With this method we again confirm that option values
increase substantially when we introduce correlation between reservoir sizes of different
projects.

Finally, the compound option model analyzed and priced in this study can be easily
reduced to a simple model by leaving out the early or the late project. With appropriate
modifications, it can be applied to valuing a wide variety of options that are different from
the ones considered in this chapter, for example options to wait, options to abandon projects,
etc.

The remainder of this chapter is organized as follows. Section 2.2 discusses a selected
bibliography on real options including commodity prices modeling and option pricing method-
ology. Section 2.3 describes the general compound option problem that is refined from a real
life puzzle. Section 2.4 includes the methodology of gas pricing modeling and the GARCH
option pricing model, as well as the introduction of a utility function necessary for the utility
indifference pricing approach. Section 2.5 provides the results of applying the framework
to exploration decisions concerning a combination of offshore Dutch gas fields. Section 2.6
concludes.

2.2 Literature Review

2.2.1 Real Option Analysis vs. NPV

Real Options Analysis (ROA) is best seen as a complement rather than a substitute for tra-
ditional NPV calculations. For instance, when evaluating mining plans under uncertainty,
Dimitrakopoulos and Sabour [2007] shows the project value as valued by real option anal-
ysis is 11-18% higher than that by NPV. Quigg [1993] first presents empirical predictions
of a real option-pricing model using a large sample of market real estate prices. She finds
the wait to invest option accounts for a premium of 6% of the theoretical asset value; her
analysis shows how option models help in understanding and predicting transactions prices
over and above the intrinsic value.

2.2.2 Incomplete Market Setting

In the gas field investment problem we focus on in this study, underlying asset value depends
on two state variables, gas price and reservoir size. We assume that these variables are not
correlated (Borison [2005]). Gas prices in the public market are obviously independent of the
reservoir size of one small field; while with fixed extraction costs, the market developments
2.2. Literature Review

also do not impact on the distribution of reservoir size\(^3\). Both reservoir uncertainty and our assumption of stochastic volatility introduce risk factors that are not directly related to any financial instrument traded in financial market. This precludes the use traditional option valuation formulas; the replicating portfolios on which they are based do not exist. The market incompleteness problem resulting from stochastic volatility is easier to deal with than the reservoir size related risk factor. Using a GARCH model to capture the stochastic volatility embedded in gas prices adds the second moment evolution of a price process. Duan [1995] provides a solution that approximates risk-neutral pricing to deal with the additional risk source brought by stochastic volatility. For this locally risk-neutral valuation relationship (LRNVR) approach to be valid one needs to assume a CARA utility function. Note that by assuming such a utility function, we allow the investor to consider only the change of wealth brought by the corresponding investment instead of considering their total wealth. Although the CARA assumption is empirically not likely to hold for individual investors because of its corollary (independence between initial wealth and portfolio allocation over risky and safe assets), we consider the independence of initial wealth an advantage for the corporate environment in which this analysis is likely to be most often used. And it is the only assumption under which the LRNVR approach of Duan [1995] can be used.

The other risk factor resulting in market incompleteness is the unhedgeable risk caused by recoverable reservoir size uncertainty. Hubalek and Schachermayer [2001] also state that no-arbitrage arguments yield no information on the option value in case of non-existence of a matching trading asset. They propose that an adjustment to the cost-of-capital should be made accordingly, which corresponds to our first cost-of-capital approach. Floroiu and Pelsser [2013] make a similar argument in a situation with model ambiguity. A more structured approach recognizes that in the absence of a replication portfolio it is not possible to achieve preference free valuation. Explicitly specifying the investor’s risk preference leads to a utility-based valuation method, Utility Indifference Pricing. The indifference price of a financial security is the price at which an investor would have the same expected utility level by buying (selling) a financial security as by not doing so, with optimal trading in both cases otherwise. Typically UIP results in a range, with an indifference price for a buyer and a seller. We take a buyer’s approach. An example of this approach can be found in by De Jong [2008] and a very comprehensive treatment in Carmona [2009]. Note by the way that we can use UIP for any regular utility function, we do not need to restrict ourselves to CARA utility functions, as we do to use Duan’s LRNVR approach.

This dual approach is advocated in a series of papers by Smith and Nau [1995], Smith [1996], Smith and McCardle [1998], and Smith and McCardle [1999]. This implies the

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\(^3\)Recoverable reservoir size may be dependent on market gas prices if marginal extraction costs are increasing as more gas is extracted. We do not incorporate this dependence in this chapter here, but it is an interesting extension and adds more managerial flexibility.
2. A Real Option Perspective on Valuing Gas Fields

following procedure. The underlying assets under concern here are separated into two parts. First, with the assumption of complete trading markets, unique risk-neutral probabilities for the market states can be determined. In our case of GARCH models, we approximate risk-neutral pricing for gas derivatives using Duan [1995] local risk-neutral valuation approach. Second, subjective risk assessment of reservoir size is processed for project evaluation; and utility functions are introduced to capture an investor’s preferences towards risks that cannot be eliminated because of market incompleteness.

There is substantial literature on weather derivatives that is of obvious relevance for our analysis, since the underlying asset (the weather index) is not traded either. That literature has emerged since the derivatives themselves started to be traded, initially as OTC contracts, as far as we know the first contract was structured in 1996, and later as exchange traded contracts: the first CME traded futures contract based on a weather index was introduced in 1999. The literature largely focuses on imperfections in the use of weather derivatives in hedging weather related risks, mostly by agricultural producers and energy companies. The imperfections arise because weather derivatives are either OTC, which leads to an exposure to counter-party risk, or exchange traded based on a fixed location weather index. The latter leads to what is called basis risk, risk arising from the fact that in most cases local weather conditions will be imperfectly correlated with the weather conditions at the basis where the weather related variables are measured. See Brockett et al. [2007] for an extensive description and an analysis of hedging performance in a one period mean variance framework. Li et al. [1998] are a typical example, also using a mean variance framework to evaluate similar hedging problems arising in the use of aggregate yield based derivative contracts. This comes down to evaluation of hedging performance by measuring the success in reducing variance. Most authors resort to CARA Utility and normally distributed one or two dimensional processes, which implies variance reduction as an objective. Generally authors analyze fixed maturity futures contracts or single period European options (referred to as non-linear hedging strategies). The issue of limited or non-tradability of the underlying assets is not addressed in this part of the literature, authors typically use an actuarial approach ignoring financial markets, and use the safe rate of interest to calculate expected NPVs in simulation runs under the physical measure, which raises the problem of weather prediction. An exception is Davis [2001], who does not sidestep the issue of the absence of a replicating portfolio but derives the proper discount rate and drift factor based on the assumptions of logarithmic utility and constant variance log normal processes, also in a simplified fixed maturity framework. Brockett et al. [2009] use a similar framework and also derive analytical solutions for the similarly simplified case of a quadratic utility function, fixed maturity derivatives and constant variance, paying specific attention to the very relevant case of dual stochastic processes for which partial hedging instruments are available,
also the setup we are interested in in this chapter. Such simplifying assumptions have their advantages of course, by allowing for analytical solutions they lead to substantial insights in the impact of underlying process and preference parameters on derivative prices. However they preclude application of the incomplete markets real option concept to a wide range of real world problems where these simplifying assumptions are very clearly violated. In this chapter we show that the assumption of constant volatility is decisively rejected by the data on energy prices; furthermore the basic structure of the decision problem involves endogenous exercise dates (American options rather than European ones) and a time pattern of endogenous information release: early decisions have an impact on later information, introducing additional option like characteristics. The contribution of this chapter is to show how stochastic dynamic programming can be used to numerically solve the implied asset pricing problems while circumventing the curse of dimensionality plaguing most real world applications of this technique by using the Longstaff Schwartz Least Squares Monte Carlo approach to dimension reduction.

2.2.3 GARCH Option Pricing Model

Most of the real option literature assumes that the price of underlying asset follows a geometric Brownian motion (GBM) process (McDonald and Siegel [1986]; Paddock et al. [1988]), where commodity prices at any future time are lognormally distributed. But historical price return data shows strong volatility clustering, which violates the constant instantaneous volatility assumption used in traditional option analysis (Bates [2003]). Sadorsky [2006] finds that a single-equation GARCH model outperforms more sophisticated models in forecasting volatility of commodity returns. Hansen and Lunde [2005] obtain similar results, in terms of the ability to describe the conditional variance and forecasting ability, in their comparison of no less than 330 ARCH-type models. But a GARCH option pricing model is a function of the volatility risk premium embedded in the underlying asset, which invalidates the risk-neutral valuation relationship. An extension of the risk neutral valuation (i.e. LRNVR) is explored by Duan [1995] to deal with the complex nature of GARCH(1,1) processes in option pricing. This property guarantees that the GARCH option pricing model is a well-specified model that at least locally does not depend on preferences. Two further conditions are required to satisfy LRNVR according to Duan [1995]. With regard to our problem, these two conditions are

1. The investor is an expected utility maximizer and the utility function is time separable and additive; The utility function displays constant absolute risk aversion;

2. Relative changes in the aggregate cash flow (NPV) are distributed normally with constant mean and variance under real measure $\mathbb{P}$. 


We assume the investor maximizes a utility function satisfying the above conditions. Therefore LRNVR holds and our GARCH option pricing model is well specified. The LRNVR guarantees the invariance of the one-period ahead conditional variance with respect to a change in risk-neutralized pricing measure.

The time-varying volatility under (G)ARCH processes results in solutions which are not analytically tractable, since the distribution of underlying asset prices cannot be derived in closed form even for European type options. So more advanced numerical techniques are needed, to which we turn in the following subsection.

### 2.2.4 Least Square Monte Carlo Method

The applied real option literature usually simplifies the kinds of projects encountered in practice by setting them up as simple European call/put options with fixed exercise dates (Smith and McCardle [1999], McDonald and Siegel [1986]). This has the advantage that a closed form solution can be easily derived from the classical option pricing model such as the Black-Scholes formula (Black and Scholes [1973]). But firms run into much more complicated option problems in reality, involving not only path-dependence (Copeland and Tufano [2004]) and intercorrelation among investment projects, but also various risk sources. And managerial flexibility implies in most cases the possibility to postpone or accelerate investment decisions; this makes the real option problem more the equivalent of an American (or, in discrete time, a Bermuda) option with endogenous exercise dates rather than a European option with one fixed exercise date.

By rephrasing the investment problem as a dynamic programming problem, the decision maker at each exercise date considers a two-step optimal strategy. The first step is to compare immediate exercise payoff with the expected payoff of continuing (waiting) and possibly exercising later; and next, the decision maker chooses not to exercise the option now if waiting is more valuable than exercising now. At the next exercise date, the same structure of choice presents itself again. The choice of exercise dates and the calculation of continuation payoffs both become even more complicated in the case of compound options, because values of unlocked options in the future should also be taken into consideration in calculating continuation values now.

Without a closed form solution, advanced numerical techniques are required to solve the resulting dynamic programming problem. Lattice methods (Cox et al. [1979], Trigeorgis [1996]) are simple from a computational point of view and do not require closed-form solutions. However, despite its flexibility and ease of application in valuing projects with many embedded options, this technique suffers from dimensionality difficulties and is therefore in practice unable to handle multidimensional problems of a significant size.

Longstaff and Schwartz [2001] propose a simulation-based technique instead. Similar
to Carr and Yang [1998], this Least Square Monte Carlo (LSMC) method is able to value various styles of options including American options or more exotic options and to manage multiple uncertainties described by complex stochastic processes without sacrificing option pricing tractability. It approximates conditional continuation values as a function of the state variables based on linear regressions run on backward simulation results. The backward simulations form the basis of the regressions linking continuation values to state variables; although the backward simulations cannot be used in the valuation exercise, the regression functions can be used in a subsequent forward simulation study to approximate continuation values. It essentially solves the dimensionality problem, the complexity now increases linearly in dimension size instead of exponentially.

Moreover, this algorithm is not restricted to Markovian processes; similar algorithms can easily be applied to non-Markovian processes, like the (G)ARCH process we use. See for example Stentoft [2005], who applies the LSMC method to price options with early exercise features within a GARCH context. Of relevance to our study is the conclusion reached by Stentoft [2005] based on his empirical analysis of out-of-sample performance, that GARCH effects are of substantial importance and lead to significant improvements over constant volatility model results.

2.3 Problem Description

Drilling for near-field prospective resources is appealing because it both reduces unit operating cost and extends infrastructure life as many viable prospects are located outside platform owner’s acreages. More accurate estimates of reservoir sizes of near fields can be obtained through accessing the information of first drilling. Thereafter an option problem rises. Here we simplify an investment problem as follows.

Figure 2.3.1 illustrates a compound real option problem concerning both Prospect A and Prospect B⁴, where the Platform, main pipeline, and Pipeline C for transportation are previously constructed infrastructure. Prospect A and Prospect B are both prospective gas fields sharing similar geologic and geographic properties, and the dashed lines represent pipelines to be built for developing A or B.

The reservoir uncertainty of Prospect B can be resolved after one-year production, which, due to similar geological structure, will provide additional information on the reservoir distribution of Prospect A. Based on new information, the firm continues to decide whether and when to explore Prospect A. Moreover, higher gas prices also make new investment projects more attractive. For instance, if the gas price turns out to drop dramatically after one year,

⁴Here a prospect is defined as a gas field where recoverable reserves have been identified by initial exploration.
the decision maker can simply choose to delay or abandon further investment if reservoir size of A is proved to be poor.

To summarize, under the schematic layout, the investor’s problem can be written as a combination of two options. The first option is when/whether to start developing B or when/whether to exercise the waiting option on B. The second option is subject to the exercise of the first one: Once the waiting option of B is exercised, the firm holds the right to decide whether and when to exercise the option to undertake project A. So the two combined form a compound option problem. The underlying assets of the option problem depend on market gas price and reservoir distribution. Our aim is to evaluate prospect B by taking both fixed cash flows and future opportunities into consideration, i.e. an extra option value is added to the project evaluation on top of the traditional NPV method, where this extra option value also depends on the profitability of Prospect A.

2.3.1 Reservoir Size Distribution

In conformity with industrial standard terminologies, the reservoir distribution of a gas field is decomposed into probability of success (POS) and a distribution for reservoir size given success, $R$. We assume POS is related to porosity, the fraction of a given volume area made up by empty space: by definition, porosity is a variable ranging from zero to one. Higher porosity areas can possibly contain more hydrocarbons, and are more likely to lead

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5This section relies extensively on Hanna and Willot [2011], which provides extensive further references to the relevant Geophysics literature.
to a successful exploratory drilling resulting in the discovery of recoverable reserves. The actual reservoir distribution depends on a great many more factors influencing the three-dimensional geometry and geophysical characteristics of a well area. Prior estimates of reservoir distribution are typically arrived at by combining well specific information (well logs, pressure tests, cores, geological depositional knowledge and other information from exploration and appraisal wells), with data obtained by applying geophysical methods, principally 2-D or 3-D seismic surveys (the geophysics literature also refers to 4-D seismic data, these are 3-D data from surveys that are repeated at different points in time). Both porosity and $R$ distribution parameters are more likely to be correlated at different locations the closer these locations are, although there still is a high degree of stochastics involved even in closely located areas.

Figure 2.3.2a shows the simplified setup we use based on the geophysical literature (Hanna and Willot [2011]). A reservoir amount $R > 0$ is found with a probability equal to POS; so the probability of no discovery (i.e. zero recoverable amount) is 1-POS. Since the actual value of $R$ is the non-negative product of a great many other non-negative variables, we can apply a variant of the Central Limit Theorem and assume that $R$ is lognormally distributed Rempala and J.Wesolowsky [2002]: conditional on a positive finding, the reservoir size $R$ is a random variable with a truncated lognormal distribution as illustrated in Figure 2.3.2b. Reservoir size is estimated as a product of a set of variables and lognormal is the most widely used distributional assumption for fields with simple geographic structures. However, a domain extending out to positive infinity is evidently inappropriate, so we use a simple weight truncation at the 99th percentiles of a lognormal distribution.

In practical project evaluation processes, companies typically use three representative cases, often labeled as case P10, P50, and P90. Here P90 stands for the most pessimistic reservoir estimate. This is described as triangular distribution. We incorporate this practice by scaling the truncated lognormal such that the probability that recoverable reserves exceed the P90 case is 90%. Analogously, the P10 case is the most optimistic reservoir estimate that is likely to be met or exceeded with a probability of only 10%. P50 case is a moderate outcome, best interpreted as the median of the distribution. We choose the various parameters of the truncated lognormal so as to most closely match these definitions. Obviously, the different cases imply different production profiles, both as to output levels and production lifetime. For instance, in our case, the production lifetime of P90 and P50 is four years, while for the high case P10, the active period is five years. In our example of a lognormal distribution, we use appropriate linear combinations of the three production profiles corresponding to the three cases for values in the distribution domain other than the three case points.
We first investigate a simplified case where the reservoir distributions of A and B are uncorrelated. However, the Geophysics literature documents extensively how, as development wells are drilled and put in production, seismic data are revised and recalibrated to take advantage of the new borehole information and production histories. "Aspects of the seismic interpretation that initially were considered ambiguous become more reliable and detailed as uncertainties in the relationships between seismic parameters and field properties are reduced" (Hanna and Willot [2011]). Drilling one well thus leads to improved information, not just about the well location, but also about potential wells in nearby locations (cf Naevdal and Vefring [2002] for a sophisticated example of using modern econometric techniques in updating estimates based on improved near reservoir information availability). This obviously has major implications for our analysis of option values. Therefore after presenting the analysis of our simplified set up without correlation between A and B, we subsequently extend that analysis by assuming that a positive result for Prospect B has additional informa-
2.3. Problem Description

2.3.2 Option Model Setup

Figure 2.3.3 demonstrates the timeline of the investment problem. $T$ is the minimum license duration of Prospect A, B, and their facilities; $T_A$ and $T_B$ are production periods of Prospect A and B respectively; $t_A$ and $t_B$ are the production starting dates of A and B. Interval I contains all possible starting dates of Prospect B; and Interval II contains all possible starting dates of Prospect A, whose lower boundary is subject to the starting date of Prospect B, i.e. $t_B$.

We construct two sequential Bermuda-style options, which can be exercised at a set of predetermined dates before maturity. The first option is a waiting option on Prospect B starting from time 0 with maturity $T - 1 - \max(T_A, T_B)$. Investor has the option to wait until the market gas price increases so that higher profits are realized. Once the waiting option is exercised\(^6\) at time $t_B$ and further information on POS (or $R$, or both) of A is gained at time $t_B + 1$, the firm decides whether and when to explore Prospect A by taking both reservoir size and future gas prices into consideration. The option for Prospect A arises after one-year production of B (i.e. $t_B + 1$), when the reservoir quantity of B is revealed. It has a maturity $T_B - 1$ with the assumption that the second option disappears once the development of Prospect B is finished; so we have $t_A \in II \equiv [t_B + 1, t_B + T_B]$. Since B unlocks the option on A, the project value of B should include the value of managerial flexibility embedded in Prospect A.

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\(^6\)i.e. the firm starts drilling at Prospect B.
2. A Real Option Perspective on Valuing Gas Fields

2.4 Methodology

2.4.1 Predicting Future Gas Prices

2.4.1.1 Gas Price Data

The Dutch gas market has been considered as a mature and stable trading market on the continent especially since the Title Transfer Facility (TTF) was set up in 2003. Operated by (TSO) GTS, a wholly owned subsidiary of Gasunie, the TTF virtual hub is the largest continental OTC gas trading market in terms of trading volumes and number of trades. It is also the virtual gas hub covering all high calorific entry and exit points in the Netherlands.

TTF records volume-weighted average natural gas price (Euro/Megawatt Hour) of all orders that are executed and delivered on the same day. A weekly spot day-ahead data set with 365 observations was obtained from Datastream covering the period of Mar 7, 2005 through May 18, 2012, where the starting date is constrained by the beginning of data available on Datastream. Statistical descriptions are given in Table 2.1.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot Prices</td>
<td>365</td>
<td>18.7072</td>
<td>5.6142</td>
<td>31.5197</td>
<td>-0.0495</td>
<td>2.3026</td>
</tr>
</tbody>
</table>

Phillips-Perron unit root test and Dickey-Fuller unit root test were used to detect the existence of unit roots in historical gas data. The test results (Appendix) show that time series $P_t$ or $\ln P_t$ has a unit root while first difference of gas prices $\Delta P_t$, or first difference of log prices $\Delta \ln P_t$ are both stationary time series with p-value 0.0000. Only stochastic trend exists in this time series. By observing Figure 2.4.1b, the logarithm price returns series moves with gradual upward and downward fluctuations around a long-term mean. We continue investigating the properties of logarithm return time series since its first difference series is stationary and contains no trend. Figure 2.4.1b suggests changes in variance; there is evidence of volatility clustering. For instance, larger fluctuations during the periods 2005-2006 and 2009-2010 are followed by less volatile periods.
2.4. Methodology

Figure 2.4.1: Spot Price and Log Price Returns

(a) Weekly Spot Prices
(b) Log Price Returns

2.4.1.2 GARCH Model Estimation

Due to techniques such as hydraulic fracturing, or “fracking”, gas development costs in US
have been dramatically driven down since shale gas extraction became economically viable.
However this technique is still controversial in Europe and therefore has not been applied
on a large scale yet or will not be expanded in the near future. So we expect no structural
change for the foreseeable future.

Let $P_t$ be the spot gas price at time $t$. Suppose under probability measure $\mathbb{P}$, its one-
period rate of return has conditionally lognormal distribution. Following Duan [1995], we have

\[
\ln \frac{P_t}{P_{t-1}} = \mu + \lambda \sqrt{h_t} - \frac{1}{2}h_t + \tilde{\varepsilon}_t,
\]

\[
h_t = \alpha_0 + \alpha_1 \tilde{\varepsilon}_{t-1}^2 + \alpha_2 h_{t-1},
\]

where $\tilde{\varepsilon}_t | \mathcal{F}_{t-1} \sim N(0, h_t)$

where $\mu$ is constant one-period risk-free rate of return and $\lambda$ is the constant unit risk premium.

Here, $\mathcal{F}_{t-1}$ is the information set, up to and including time $t - 1$; $\alpha_1, \alpha_2 \geq 0$ so that
non-negative variance is guaranteed. To ensure its stationarity, $\alpha_1$ and $\alpha_2$ need to satisfy
$\alpha_1 + \alpha_2 < 1$. The process $\tilde{\varepsilon}_t^2$ follows an ARMA(1,1) process.

2.4.1.3 GARCH Option Pricing Model

Duan [1995] shows that under risk-neutral pricing measure $\mathbb{Q}$, $\ln \frac{P_t}{P_{t-1}}$ follows a normal dis-
2. A Real Option Perspective on Valuing Gas Fields

distribution conditional on $\mathcal{F}_{t-1}$ under assumptions mentioned in 2.2.3 and that

$$Var^p \left[ \ln \frac{P_t}{P_{t-1}} \mid \mathcal{F}_{t-1} \right] = Var^Q \left[ \ln \frac{P_t}{P_{t-1}} \mid \mathcal{F}_{t-1} \right]$$

Hence under the risk neutral measure $Q$, the LRNVR is sufficient to reduce all preference
considerations and it implies the logarithm return follows a stochastic process as

$$\ln \frac{P_t}{P_{t-1}} = \mu - \frac{1}{2} h_t + \varepsilon_t,$$

$$h_t = \alpha_0 + \alpha_1 \left( \varepsilon_{t-1} - \lambda \sqrt{h_t} \right)^2 + \alpha_2 h_{t-1},$$

where $\varepsilon_t \mid \mathcal{F}_{t-1} \sim N(0, h_t)$.

where annual risk-free return is 3%.

An MA(2)-GARCH(1,1) model is selected for predicting returns and volatilities of future
gas prices. The estimation results is shown as follows, where standard errors are presented
in the parenthesis.

$$\ln \frac{P_t}{P_{t-1}} = 0.00058 - \frac{1}{2} h_t + \varepsilon_t + 0.114 \varepsilon_{t-1} - 0.178 \varepsilon_{t-2},$$

$$h_t = 0.002 + 0.735 \left( \varepsilon_{t-1} - 0.0001 \sqrt{h_t} \right)^2 + 0.232 h_{t-1}$$

where $\varepsilon_t \mid \mathcal{F}_{t-1} \sim N(0, h_t)$

Model comparisons and diagnostic tests are presented in Appendix 2.7.3.

Furthermore, Figure 2.4.2 also confirms that the MA(2)-GARCH(1,1) model also
performs well at out-of-sample prediction, by comparing the mean square errors (MSE) of the
stationary GARCH models. We take a set of out-of-sample data spreading from May 21st,
2012 to Feb 18th, 2015. Here we standardize the MSE of MA(2)-GARCH(1,1) to 1 and it
can be observed that all the other models considered here have a relative MSE larger than 1.
Thus we conclude MA(2)-GARCH(1,1) also has the best out-of-sample forecasting performance comparing to other GARCH models and therefore it is chosen by our study for further studies.
2.4. Methodology

Figure 2.4.2: Comparison of Mean Squared Errors of Out-of-sample Predictions

2.4.2 Integrated Valuation Method

Given a complete market, we value a claim $OP$ by replicating it with the unique self-financing portfolio, which yields a price $X_T$ at the final date $T$. Therefore $X_0$ gives the price of $OP$ at time 0. However, a claim in an incomplete market cannot be perfectly replicated and we run into a problem of finding a unique price for this claim. More specifically, selling such a claim entails exposing oneself to an idiosyncratic/unhedgeable risk, which can be represented by $X_T - OP$ (or $OP - X_T$) at time $T$. This difference can be solved by specifying the investor’s preference towards the risk. Therefore the price of the claim should be

$$OP_0 = X_0 + \text{utility value of } (OP_T - X_T)$$

which results in the failure of preference free pricing. This leads to the necessity of introducing utility function to capture individual’s preference.

Assume the investor has a utility function which is additive among individual periods such that the investor’s utility at time 0 equals the sum of the utility of discounted cash flow over all periods. We assume the investor’s preferences exhibit constant absolute risk aversion (CARA):

$$u_t(x_t) = -\exp \left( -\frac{x_t}{\rho_t} \right)$$

where $\rho_t > 0$ represents the decision maker’s period-t risk tolerance. This assumption of utility function implies that the investor’s behavior does not depend on his initial wealth, and he only cares about the change of his wealth caused by an investment. These two assumptions

21
2. A Real Option Perspective on Valuing Gas Fields

(additivity and CARA) imply a certainty equivalence expression:

\[
\exp\left(-\tilde{x}_t^{CE}/\rho_t\right) = \mathbb{E}_t\left[\exp\left(-\tilde{x}_t/\rho_t\right)\right],
\]

with

\[
\tilde{x}_t^{CE} = -\rho_t \ln\left(\mathbb{E}_t\left[\exp\left(-\tilde{x}_t/\rho_t\right)\right]\right)
\]

with \(\tilde{x}_t\) as an uncertain cash flow at period-\(t\).

Suppose a project has a series of future cash flows \(\{CF_0, CF_1, ..., CF_T\}\) where \(CF_t = P_t \times G_t - C_t\), with gas price \(P_t\), production \(G_t\), and cost \(C_t\) at time \(t\). More generally, we have

\[
v_t = \text{NPV}_t(P_t, R_t, u_t) = \begin{cases} 
0 & \text{if } u_t = 0, \text{i.e. no exercise} \\
\mathbb{E}_t\left(\text{POS} \times \sum_{i=t}^{T+T} e^{-(i-t)r_f} (P_i \times G_i - C_i) | R_t, R_i) & \text{if } u_t = 1, \text{i.e. exercise}\end{cases}
\]

where \(R_t\) is the realized reservoir volume; and \(u_t\) is a dummy variable, representing the decision to exercise.

Effective certainty equivalent is represented as \(ECE_{t+1}(\cdot)\), defined by taking expectations over period-\(t\)’s private uncertainties (reservoir) conditional on the outcome of period-\(t\)’s market uncertainties (gas market). The calculation of \(ECE_{t+1}(\cdot)\) depends on assumptions of the utility function. For instance, an exponential utility function with an effective risk tolerance equal to the sum of the decision maker’s discounted future risk tolerance leads to

\[
ECE_{t+1}\left[v_{t+1} | \mathcal{F}^m_{t+1}, \mathcal{F}_t \right] = -\gamma_{t+1} \ln\left(\mathbb{E}_t\left[e^{-\gamma_{t+1}v_{t+1} | \mathcal{F}^m_{t+1}, \mathcal{F}_t} \right]\right)
\]

Here \(v_t\) denotes the project value at time \(t\) and \(\gamma_t = \sum_{t=1}^T \frac{\rho_t}{(1+r_f)}\) is the NPV of the future period risk tolerances, where \(\rho_t\), same as before, denotes the decision maker’s period-\(t\) risk tolerance. Therefore the integrated valuation approach uses effective certainty equivalent instead of NPV as a proxy of project value. Note that if \(\gamma \to +\infty\), this decision make becomes risk neutral and the option pricing problem becomes identical as if one in complete market.

### 2.5 Application

In this section, we apply the proposed model to a Dutch offshore case in Netherlands, where two prospects are distributed as shown in Figure 2.3.1. Evaluated separately from Prospect B, Prospect A has a possibly large reserve quantity but conditional on a low probability of successful drilling; furthermore, initial explorations have proved a poor reservoir potential of Prospect B. Knowing that the cost of initial exploration can be considered as sunk costs which can never be recovered, the investor needs to decide whether to abandon Prospect B or to continue developing it with its POS equal to 80%. Here the maturity of the option on starting B is defined as the shortest license duration of Prospect B, Prospect A and other
facilities, and its exercise time is at the end of every following year. Unlike B, which has a high POS after thorough exploration procedures, Prospect A has a much smaller POS of 30%. The tradeoff embedded in the problem is that Prospect B in itself is not economically attractive enough for the investor, but giving up B effectively blocks future investment opportunity of A. However if B would turn out to be a failure, A would lose investor’s interests too. The decision tree is shown in Figure 2.5.1.

As mentioned before, for both methods of Cost-of-capital and integrated valuation, we first consider a simple case where the revealed reservoir quantity of B adds no further information on A, and then proceed on to a complicated case when further updates based on the outcome of B are possible. Moreover, results and empirical tests on the properties of option values are also conducted under both methods.

Figure 2.5.1: Decision Tree

2.5.1 Cost-of-capital Method

We choose a reasonable range for cost-of-capital\(^7\) that reveals the underlying risk of a project. The dotted line in Figure 2.5.2 exhibits the simulated NPVs of Prospect B with respect to a range of cost-of-capital (from 3% to 15%), where the red horizontal line separates projects with positive and negative NPVs. It is clear that due to its low NPV, Prospect B is not economically attractive enough to be developed in itself. Even if the manager chooses projects only based on the sign of NPV, Prospect B is still rejected if the cost-of-capital is higher than 9%.

2.5.1.1 Results without reservoir information update

Option values are calculated for cost-of-capital rates varying from 3% to 15%, with risk-free rate equal to 3%. Figure 2.5.2 shows that the integrated value of Prospect B is greatly

\(^7\)The range includes all cost-of-capital being used for the investor’s budgeting process.
increased when option values are taken into consideration. For instance, with a cost-of-capital equal to the risk-free rate (3%), we assume the risk of the underlying asset can be perfectly hedged in the market. The simulated NPV of Prospect B is then €1.93mln. But its corresponding option value is €19.47mln, resulting in an integrated value of €23.05mln. However, one has completely ignored the risk embedded in reservoir by using the risk-free rate as the cost-of-capital. To take the reservoir risk into consideration, we effectively assume the reservoir risk is captured by subtracting the risk-free rate from the cost-of-capital; therefore the cost-of-capital presumably reflects risk preferences and serves as a solution to deal with the incomplete market problem.

Figure 2.5.2: Option Values vs. NPV

Integrated Value on B = NPV of B + Option Value on B.

Take another example when the cost-of-capital is 15%. The negative NPV of Prospect B (€−1.82mln) leads to a rejection of this project based on the traditional selection criterion. While real option analysis gives a positive integrated project value of B (€10.36mln) implying its commercial profitability; as a result it leads to an opposite investment decision to one made under traditional NPV.

As shown above, real option valuation plays a crucial role in investment decisions by accepting projects that could have been rejected under traditional evaluation rules. Under real option valuation, the prospect B is valuable for development under all cost-of-capital rates considered. As a result, the firm should not simply abandon Prospect B; in fact, with the further exploration opportunity on A, the project yields a positive expected value and is worth investing. Furthermore, given the more precise evaluation of the projects, the firm
can compare them with other investment opportunities and then choose (one of) those with highest values.

Two observations from Figure 2.5.2 are also worth commenting. First, as a natural result, Prospect A is less valued with higher cost-of-capital rates. Therefore as expected, the option value decreases in discount rates as well due to the shrinking value of Prospect A. Second, note that the declining option value does not imply a negative Greek $\rho$, which is defined as a sensitivity measure evaluating the sensitivity of option value to the risk-free rate: it is the derivative of the option value with respect to the risk-free interest rate. Risk-free rate here remains unchanged. What is shown in Figure 2.5.2 is the interaction between option value and cost-of-capital rates, where cost-of-capital is used to adjust payoffs.

2.5.1.2 GARCH Model vs. Constant Volatility

Due to the limited downside of options, an option becomes more valuable as the volatility of underlying assets increases. Thus precise structure of volatility process is important in valuing options. This subsection discusses how option value changes along with alternative variance modeling.

Suppose the logarithm price return time series follows a log-normal distribution with mean and constant variance calculated from the same TTF data set as used for the above GARCH model. Following the same analogy, we show option values of the offshore case in Figure 2.5.3.

Figure 2.5.3: Option Values vs. NPV with constant variance
2. A Real Option Perspective on Valuing Gas Fields

In comparison to Section 2.5.1.1, the real option value is still positive but is only half size as the result under GARCH model (Figure 2.5.2). Thus, neglecting clustered volatility dramatically undervalues the corresponding options. Now the simulated NPV of B is negative under all cost-of-capital rates considered. Both option values and NPVs are significantly underestimated and the project is more likely to be rejected under constant volatility assumption. Failure to capture the main characteristic of volatility leads to severely biased results.

This finding is also consistent with standard finance option pricing theories where higher uncertainty results in higher option values.

2.5.1.3 Spot Prices vs. Option Values

Figure 2.5.4 shows how the integrated value and option value of Prospect B change with spot prices, i.e. Delta.

In addition to the 2-D plot, the 3-D plot presents the dynamics of option values against spot prices and cost-of-capital rates. It can be observed from both 2-D and 3-D plots in Figure 2.5.4 that option values are increasing in spot prices. This means that by observing a high spot price on the market, the investor is more likely to accept a project if it takes option values into account. On the other hand, similar as what we have found before, the option value decreases as the cost-of-capital increases. In other words, Delta is decreasing in cost-of-capital.

2.5.1.4 Results with reservoir information update

Until now we considered the case without reservoir correlation and that the option problem is an issue caused only by the dynamics of future gas prices. In this section, we introduce one extra dimension to our model by taking reservoir correlation into account. For practical reasons, given the structure of uncertainty we work with, we can distinguish three cases of correlations. The first one considers that only POS of A and POS of B are correlated. Assume that POS of A rises to 50% given a success drilling of B. The second case considers when POS remains unchanged but focuses on correlation between the probability distribution functions of reservoir (i.e. $R$) in particular. We assume that if the reservoir size of B turns out to be equal to that of P10 case, then the reservoir size of A equals the outcome of its P10 case as well. Similarly, a P50 (P90) outcome of B also implies a P50 (P90) outcome of A. Lastly, Case III combines the first two cases, where both information updates in POS and $R$ are considered. Note that the three cases can be reduced to only one if an alternative reservoir distribution is assumed with POS and $R$ mingled together.

---

8The reason why the results of different gas price modeling differ so much is that, we are modeling log price return instead of gas price itself. Therefore a slight change in log price return may result in a large change in gas price prediction, which has a significant influence on option values.
Figure 2.5.4: Option Value vs Spot Price

(a) 2-D

(b) 3-D
Figure 2.5.5 presents all the results from the three cases as well as the one with no correlation. For all three cases with correlation, option values are (much) larger than what is obtained without reservoir correlation. This is an obvious result because the more information can be gained in the future, the more valuable the option is. In addition, the option value of Case III is larger than either option value in Case I or in Case II. This is to be expected since information has been updated to the largest extent in Case III. The positive correlation between option price and reservoir size is consistent with the intuition that the option is more valuable when more information of the underlying assets can be acknowledged in the future.

2.5.2 Integrated Valuation Method

2.5.2.1 Results with and without reservoir information updates

Next we explore the preference-dependent valuation given a particular utility structure of investors. The investor maximizes her utility function with idiosyncratic risk aversion (risk tolerance) $\gamma$. Instead of exploring a reasonable set of cost-of-capital, we use effective certainty equivalent to calculate option values based on different $\gamma$ within the context of incomplete market.

Figure 2.5.6 shows how option values change along with increasing risk tolerance, with or without reservoir correlation between A and B. From left to right, as $\gamma$ becomes larger, the investor becomes more risk tolerant (i.e. less risk averse). It is clear that more risk averse investors value options less than investors with higher risk tolerance. Moreover, option value
is a concave function of risk tolerance, meaning that the instantaneous acceleration of the option value is decreasing along with the risk tolerance. Although option values increase as the investor becomes more risk seeking (while still risk averse), preference dependence is moderate, resulting from the observation that the difference of option values between risk neutral and extremely risk averse investors is less than 8mln for a given reservoir correlation.

Figure 2.5.6: Results of Integrated Valuation Approach

2.5.2.2 GARCH vs. Constant volatility

Similar to Section 2.5.2.1, we compare the results with gas prices specified under a GARCH process and one under a constant volatility assumption. Figure 2.5.7 shows that with a constant volatility setting, not only the option values largely shrink, but also the effect of future information update becomes smaller. This provides empirical evidence that option value is largely undervalued if underlying asset process is modeled with constant volatility instead of a more appropriate GARCH model.

2.5.3 Cost-of-capital vs. Integrated Valuation Approach

We apply two different methods for capturing individual’s preference in incomplete market asset pricing. The choice of cost-of-capital reflects an individual’s risk preference; while the integrated valuation approach models investor’s risk aversion explicitly by introducing a parameter of risk tolerance.
Despite their different theoretical backgrounds, these two approaches yield similar results to a large extent. First, project value increases substantially comparing to one obtained by traditional NPV method, which leads to different investment decisions. Second, the real option approach allows incorporation of future information as it becomes available, which again raises project values when reservoir distributions are correlated. Third, the GARCH specification is preferred over a model with constant volatility, since the latter undervalues the investment opportunity due to its oversimplification. In short, the value of embedded options is strongly influenced by the correlation among reservoirs and the stochastic process of gas prices.

However, both approaches have their own pros and cons, which is why we present both. The cost-of-capital method is straightforward and closest to the traditional capital budgeting processes in practice. However, it is not clear what the most appropriate value of cost-of-capital for use is. The integrated valuation approach provides the best results if one knows the investor’s risk preference, for which a survey method could be used to pin down the investor’s risk tolerance $\gamma$ (Cohen et al. [1987], Holt and Laury [2002]).

2.6 Conclusion

We have presented evidence regarding the effectiveness of investment valuation between real option analysis and NPV. In this chapter, we have found the option value is crucial in decision
2.6. Conclusion

making process with both cost-of-capital approach and integrated valuation approach. We have also found that in the presence of clustered volatility, incorrectly assuming constant variance leads to an underestimation of project values.

An integrated approach is applied by taking investor’s risk aversion into account. Results show that when the investor is not risk-neutral, option value increases as her risk aversion becomes smaller. Moreover, the correlation between reservoirs has also a positive effect on option values.

The results obtained from both methods are coherent in the sense that a choice of low cost-of-capital rate corresponds to a high risk tolerance.

Further analysis can be extended for research, among which one interesting direction is to investigate the dependence of productivity on market gas prices. A high market price might drive the firm to accelerate the producing rate. As the marginal cost of extraction is decreasing in the remaining amount of reservoir, the production plan may be adjusted with unexpected gas price realization accordingly. Thus the $ex$ $post$ production profile and costs become endogenous variables for project valuation.

Note that the algorithm and valuation approach in this chapter can also be applied to general option pricing other than real options. For instance, it is very common that traders in reality hedge with proxy assets when a liquid market for the asset of interest does not exist. However the proxy asset is hardly perfectly correlated with the underlying asset, therefore the residual risk should be taken into consideration by specifying the investor’s risk preference.
2. A Real Option Perspective on Valuing Gas Fields

2.7 Appendix

2.7.1 LSMC for Cost-of-capital Method

Inputs include risk-free return $r_f$, cost-of-capital $r$, production profile: \{${G^B_1, G^B_2, ..., G^B_{T_B}}$\} and \{${G^A_1, G^A_2, ..., G^A_{T_A}}$\} and corresponding costs: \{${C^B_1, C^B_2, ..., C^B_{T_B}}$\} and \{${C^A_1, C^A_2, ..., C^A_{T_A}}$\}.

1. Generate $N$ paths, with gas prices simulated from MA(2)-GARCH(1,1) and reservoir size simulated from the distribution illustrated in Section 2.3.

2. Compound Option on B

(a) Exercise Strategy Matrix: $E$ is $N \times (M + 1)$ matrix, with 1 as exercise and 0 as no exercise, where $M + 1$ is the number of predetermined exercise dates.

(b) $NPV^B_{t_B} = POS^B \times \sum_{t_i = t_B}^{t_B + T_B} \exp \left( -r \left( t_i - t_B \right) \right) \left( P^B_{t_i} G^B_{t_i} - C^B_{t_i} \right)$, with $G_t = 0$, where $t_B \in I$ as shown in Figure 2.3.3.

(c) The analysis starts at the final moment when the firm can make a decision, i.e. $t_M = T - 1 - T_A - T_B$.

i. At $t = t_M$, if exercise at path $j$, the exercise value includes not only the payoff from B, i.e. $X_{t_M,j} = NPV^B_{t_M,j}$, but also an option on A. The option value of A at time $t_M$ for path $j$ is

$$A_{t_M,j} = \max \mathbb{E}_{t_M} \left( \exp \left( -r_f \left( t - t_M \right) \right) \max \left( NPV^A_{t_M,j} | B, 0 \right) \right)$$

where interval $II$ is shown in Figure 2.3.3 and

$$NPV^A_{t_M,j} = POS^A \sum_{t_i = t_A}^{t_A + T_A} \exp \left( -r \left( t_i - t_A \right) \right) \left( P^A_{t_i} G^A_{t_i} - C^A_{t_i} \right).$$

Suppose $\tau$ is the set of stopping time \{${t : t_0^A, t_1^A, ..., t^A_L}$\}, while $\tau \setminus \{t < t_M\}$ represents the possible exercising date set of starting drilling A, i.e. $II = \{t : t_0^A, t_1^A, ..., t_L^A\} \cap \{t \geq t_M\}$, with regard to $\sigma$-algebra \{${\mathcal{F}_t}$\}. Therefore, the compound option value on B becomes

$$\max_{t \leq t_B} \mathbb{E}_0 \left( \exp \left( -r_f t \right) \max \left( X_{t,M,j} + A_{t,M,j}, 0 \right) \right)$$

ii. Define $X'_{t_M,j} = X_{t_M,j} + A_{t_M,j}$ and $Y'_{t_{M-1},j} = e^{-r_f \left( t_{M-1} - t_{M-1} \right)} X'_{t_M,j}$, for all $j \in \{j : X'_{t_M,j} > 0\}$. 

$L + 1$ is the number of predetermined exercise dates for A.
iii. At $t = t_M$, if $X_{t_M,j}' > 0$, exercise the option and let $E(j, M + 1) = 1$; if $X_{t_M,j}' \leq 0$, abandon the project and let $E(j, M + 1) = 0$.

iv. Move one step backward. At $t = t_{M-1}$, regress $Y_{t_{M-1},j}'$ on a set of basis functions of $X_{t_{M-1},j}'$, for instance, a set of a constant, $X_{t_{M-1},j}'$, and $X_{t_{M-1},j}^2$.

\[
Y_{t_{M-1},j}' = \beta_0 + \beta_1 X_{t_{M-1},j}' + \beta_2 X_{t_{M-1},j}^2 + u_j'
\]

Longstaff and Schwartz [2001] show that the results from the Least Square Monte Carlo algorithm are significantly robust to the choice of basis functions. Adding extra high degree of basis functions does not help improving accuracy noticeably.

v. The investment opportunity is take-it-or-leave-it choice that the investor will exercise the option to produce if $X_{t_M,j}' > 0$ and not otherwise. Compare the continuation value $\hat{Y}_{t_{M-1},j}'$ with immediate payoff $X_{t_M,j}'$. If $\hat{Y}_{t_{M-1},j}' > X_{t_M,j}'$, which means continuation value is higher than exercise at $t_{M-1}$, the corresponding exercise strategy is not to exercise, i.e. $E(j, M) = 0$; while if $\hat{Y}_{t_{M-1},j}' < X_{t_M,j}'$, the investor finds it more profitable to exercise right now rather than to wait for next exercise date, resulting in $E(j, M) = 1$, and $E(j, M + 1)$ is updated to 0 whatever it was given last step at time $t_M$. Table 2.2-a and Table 2.2-b explain the idea of LSMC. It compares the net present value at each exercise moment with the optimal net present value of next exercise date discounted into the current exercise date.

vi. By proceeding recursively, we again obtain an updated exercise strategy matrix $E$ with only zero or one as its elements. Similarly, there is at most one exercise date denoted by one for each simulation path, i.e. each row of $E$, which is followed by zeros till the expiration date of the option.

(d) The option value is calculated by averaging discounted current cash flow of $X_{t,j}'$ on the exercising date across paths.

### 2.7.2 LSMC for Integrated Valuation Approach

The main procedure for LSMC applied to Integrated valuation approach is very much similar to the one for Cost-of-capital method, except for a few adjustments. Here we modify the above 2-c-(i) as below:

At $t = t_M$, if exercise at path $j$, the exercise value includes not only the payoff from $B$, i.e. $X_{t_M,j}' = -\gamma_t \ln \left( E_{t_M} \left( e^{-\frac{NPV_{B,t_{M-1},j}}{\gamma_{t_{M-1}}}} | \mathcal{F}_{t_{M-1}} \right) \right)$, but also an option on $A$. The
Table 2.2: Examples of Exercising Decisions

<table>
<thead>
<tr>
<th>Simulations</th>
<th>( X )</th>
<th>( A )</th>
<th>Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = t_{M-1} ) at Exercise</td>
<td>( X )</td>
<td>( A )</td>
<td>Simulations</td>
</tr>
<tr>
<td>( 0 \geq \nu_{t_{M-1}}X )</td>
<td>( 0 \geq \nu_{t_{M-1}}X )</td>
<td>( 0 \geq \nu_{t_{M-1}}X )</td>
<td>( \left( 0, \nu_{t_{M-1}}X \right) ) max</td>
</tr>
<tr>
<td>( 0 \geq \nu_{t_{M-1}}X )</td>
<td>( 0 &lt; \nu_{t_{M-1}}X )</td>
<td>( 0 &lt; \nu_{t_{M-1}}X )</td>
<td>( \left( \nu_{t_{M-1}}X f_{A-\nu}, \nu_{t_{M-1}}X \right) ) max</td>
</tr>
<tr>
<td>( 0 &lt; \nu_{t_{M-1}}X )</td>
<td>( \nu_{t_{M-1}}X f_{A-\nu} &lt; \nu_{t_{M-1}}X )</td>
<td>( \nu_{t_{M-1}}X f_{A-\nu} &lt; \nu_{t_{M-1}}X )</td>
<td>( \left( \nu_{t_{M-1}}X f_{A-\nu}, \nu_{t_{M-1}}X \right) ) max</td>
</tr>
</tbody>
</table>

\( E \) represents exercising options and \( NE \) represents not exercising options.

Here \( t \) represents exercising options and \( t \) represents not exercising options.

(a) Exercising Decisions

(b) Regression at time \( t_{M-1} \):

\[
\hat{Y} = E(Y | X) 
\]

If exercise at \( t_{M-1} \):

\[
\begin{align*}
\hat{Y}_1 &= X'_{t_{M-1}}, 1 \\
\hat{Y}_2 &= X'_{t_{M-1}}, 2 \\
\hat{Y}_n &= X'_{t_{M-1}}, N \\
\end{align*}
\]
option value of A at time $t_M$ for path $j$ is

$$A_{t_M,j} = -\gamma t_M \ln \left( \mathbb{E}_{t_M} \left[ e^{-\max_{t \in \mathcal{I}} \mathbb{E}_{t_M} \left( \exp \left( -r_f(t-t_M) \right) \max_{\mathcal{I}} \left( NPV_{t,j} \mid B_{t,j}, 0 \right) \right) \mid \mathcal{F}_{t_M}, \mathcal{F}_{t_M-1} \right] \right).$$

All the following procedures stay the same with $X_{t_M,j}$ and $A_{t_M,j}$ now replaced with the effective certainty equivalent values.

### 2.7.3 Estimation Results and Diagnostic Tests for GARCH models

#### 2.7.3.1 Unit Root Test

Table 2.3 exhibits the results of Phillips-Perron Unit Root Test and Dickey-Fuller Unit Root Test.

The Phillips-Perron unit root test on $P_t$ indicates rejecting the stationarity hypothesis of $P_t$ under 95% confidence with a p-value 0.0639. While the corresponding Dickey-Fuller test yields different result and suggests no presence of unit root in the times series $P_t$. The problem with Dickey-Fuller test is that if time series varies much across time, it is more likely to accept the stationarity hypothesis. And unit root test on $\ln P_t$ also rejects the hypothesis that the price time series is stationary under 95% confidence level. So in this case, we are more inclined to accepting the results suggested by Phillips-Perron test that $P_t$ has a deterministic trend.

Furthermore, observed from the unit root test, first difference of gas prices, or first difference of log prices are stationary time series with p-value 0.0000. Only stochastic trend exists in this time series. By observing Figure 2.4.1b, the log price returns series moves with gradual upward and downward fluctuations around a long-term mean. Since the first difference series is stationary and contains no trend, we proceed to investigate the logarithm return time series.
2. A Real Option Perspective on Valuing Gas Fields

Figure 2.7.1: AC and PAC of $\Delta \ln P_t$

(a) Autocorrelation Plot of $\Delta \ln P_t$  
(b) Partial Autocorrelation Plot of $\Delta \ln P_t$

2.7.3.2 Model Estimation and Diagnostic Tests

The sample autocorrelations and partial autocorrelations are plotted as in Figure 2.7.1 to give a first indication on how to choose the lag structure of a possibly adequate ARMA model.

The second order of ACF and PACF of log weekly returns are significant. Both ACF and PACF are small but slowly decaying, which implies an ARMA model might be suitable for $\Delta \ln P_t$. Note that the 20th order of ACF and PACF is also significant, but this has no intuitive meaning and might be due to random effect. Now we determine the choice of lag terms by comparing several estimated models, i.e. autoregressive models, moving average models, or mixed ARMA models as shown in Table 2.4.

Table 2.4 illustrates the results of AR, MA and ARMA models with lag terms up to 3. Ljung-Box $Q$ tests examine the existence of autocorrelation in the standardized residuals, while Ljung-Box $Q^2$ tests examine the existence of autocorrelation in the squared standardized residuals. Of these ten models, MA(2) model is preferred according to the AIC and SIC selection criteria. However, the squared residuals of the MA(2) model have some significant autocorrelations (for instance at lag 40, corresponding to a lag of three quarters). Thus it is worthwhile comparing this model with other models that allow for a richer correlation pattern in the time series. Elimination of this correlation cannot be achieved only by adding extra lags to the model, since the correlations still exist in MA(3). Evaluated by diagnostic tests, current ARMA models are not able to capture the main correlations in the time series $\Delta \ln P_t$ under a confidential level 95%, since the volatility clustering tests reject the non-existence of GARCH effects. Although MA(2) minimizes the information criteria AIC and BIC, we cannot simply apply it since the diagnostic checking of volatility clustering reveals that the variances of residuals are correlated. Models with higher lag structures correspond
Table 2.4: Estimated ARMA(p,q) models

<table>
<thead>
<tr>
<th></th>
<th>AR(1)</th>
<th>AR(2)</th>
<th>AR(3)</th>
<th>MA(1)</th>
<th>MA(2)</th>
<th>MA(3)</th>
<th>ARMA (1,1)</th>
<th>ARMA (2,2)</th>
<th>ARMA (1,2)</th>
<th>ARMA (2,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
</tr>
<tr>
<td>DlnP cons</td>
<td>0.001 (0.005)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.005)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.004)</td>
<td>0.001 (0.004)</td>
</tr>
<tr>
<td>ARMA L.ar</td>
<td>0.031 (0.040)</td>
<td>0.036 (0.041)</td>
<td>-0.20*** (0.045)</td>
<td>0.026 (0.041)</td>
<td>-0.198*** (0.046)</td>
<td>-0.055 (0.038)</td>
<td>-0.535* (0.264)</td>
<td>0.299 (0.202)</td>
<td>-0.045 (0.204)</td>
<td>0.267 (0.198)</td>
</tr>
<tr>
<td>L2.ar</td>
<td>0.026 (0.041)</td>
<td>-0.198*** (0.046)</td>
<td>-0.055 (0.038)</td>
<td>0.008 (0.037)</td>
<td>-0.216*** (0.042)</td>
<td>-0.060 (0.044)</td>
<td>0.632** (0.243)</td>
<td>-0.278 (0.197)</td>
<td>-0.174 (0.199)</td>
<td>-0.247 (0.193)</td>
</tr>
<tr>
<td>L3.ar</td>
<td>0.050 (0.041)</td>
<td>0.008 (0.037)</td>
<td>-0.216*** (0.042)</td>
<td>0.020 (0.040)</td>
<td>-0.217*** (0.043)</td>
<td>-0.060 (0.044)</td>
<td>0.632** (0.243)</td>
<td>-0.278 (0.197)</td>
<td>-0.174 (0.199)</td>
<td>-0.247 (0.193)</td>
</tr>
<tr>
<td>L.ma</td>
<td>0.096*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.096*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.095*** (0.002)</td>
<td>0.093*** (0.002)</td>
<td>0.093*** (0.002)</td>
<td>0.093*** (0.002)</td>
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<tr>
<td>L2.ma</td>
<td>0.096*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.096*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.095*** (0.002)</td>
<td>0.093*** (0.002)</td>
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<tr>
<td>L3.ma</td>
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<td>0.094*** (0.002)</td>
<td>0.094*** (0.002)</td>
<td>0.096*** (0.002)</td>
<td>0.094*** (0.002)</td>
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<td>0.093*** (0.002)</td>
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<td>31.2863</td>
<td>47.4597</td>
<td>31.8765</td>
<td>30.0042</td>
<td>42.6613</td>
<td>29.9293</td>
<td>30.0709</td>
<td>30.0297</td>
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<tr>
<td>Ljung-Box test Q(40)</td>
<td>0.1800</td>
<td>0.7871</td>
<td>0.8363</td>
<td>0.1947</td>
<td>0.8165</td>
<td>0.8751</td>
<td>0.3574</td>
<td>0.8772</td>
<td>0.8732</td>
<td>0.8744</td>
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<tr>
<td>Ljung-Box test Q(2) (40)</td>
<td>61.8430</td>
<td>76.8285</td>
<td>74.5196</td>
<td>61.3505</td>
<td>78.4971</td>
<td>76.1496</td>
<td>64.2830</td>
<td>75.2732</td>
<td>75.4161</td>
<td>72.9037</td>
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<tr>
<td>Ljung-Box test Q(2) (40)</td>
<td>0.0149</td>
<td>0.0004</td>
<td>0.0008</td>
<td>0.0165</td>
<td>0.0003</td>
<td>0.0008</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

* 99.9% confidence level; ** 99% confidence level; *** 95% confidence level. Standard errors are reported in parentheses.
to over-fitting, which also results in even worse predictions than MA(2). In order to produce reliable forecasts, an ARCH or GARCH factor is introduced to AR, MA or mixed ARMA model.

The evidence from Table 2.5 and Table 2.6 shows that the variance in the log returns is correlated and changes over time. The rejection of non-correlation of squared residuals indicates the assumption that the innovations $\varepsilon_t$ have the same variance $\sigma_t^2$ is not realistic and the conditional variance of the time series may have lagged effects. According to Ljung-Box test results, an MA(2)-GARCH(1,1) model is preferred over an MA(2)-ARCH(1) model, since the latter still yields variance clustering of innovations. This is also consistent with Poon and Granger [2003] that empirical findings suggest GARCH model is a more parsimonious model than ARCH model. Stentoft [2005] argues that allowing different mean specifications generally does not change the dynamics under the equivalent martingale measure.

Let $P_t$ be spot gas price at time $t$. Suppose under risk-neutral probability measure $Q$, its one-period rate of return has conditionally lognormal distribution, i.e.

$$\ln \frac{P_t}{P_{t-1}} = \mu - \frac{1}{2} h_t + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2},$$

where

$$h_t = \alpha_0 + \alpha_1 \left( \varepsilon_{t-1} - \lambda \sqrt{h_{t-1}} \right)^2 + \alpha_2 h_{t-1},$$

Here, $\mathcal{F}_{t-1}$ is the information set up to and including time $t - 1$; $\alpha_0 > 0$, $\alpha_1$, $\alpha_2 \geq 0$ so that non-negative variance is guaranteed. Similarly, to ensure its stationarity, $\alpha_1$ and $\alpha_2$ need to satisfy $\alpha_1 + \alpha_2 < 1$.

Note that both AR(2)-GARCH(1,1) model and MA(2)-GARCH(1,1) have the smallest AIC or BIC values. However, the coefficients of ARCH and GARCH effects of the AR(2)-GARCH(1,1) model are not significant. The coefficients of ARCH and GARCH effects of the AR(1)-GARCH(1,1) model add up to bigger than one, violating the stationary condition of GARCH model. An IGARCH model might be employed to solve the non-stationary problem. Yet IGARCH is not covariance stationary and is not attractive from empirical point of view. So an MA(2)-GARCH(1,1) model is chosen over AR(2)-GARCH(1,1) and AR(1)-GARCH(1,1).

### 2.7.3.3 Seasonal Effects

The time series of gas price has a significant seasonal component due to changes in demand. This seasonal effect can be detected from Figure 2.4.1 by observing that the prices are much higher in winter (Q1 and Q4) than in summer (Q2 and Q3). However, the existence of a
2.7. Appendix

Table 2.5: AR(p)/MA(q)-ARCH/GARCH

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99.9% confidence level; * 99% confidence level; ** 95% confidence level. Standard errors are reported in parentheses.
## Table 2.6: ARMA(p,q)-ARCH/GARCH

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</table>

- **Table 2.6: ARMA(p,q)-ARCH/GARCH**
- Standard errors are reported in parentheses.
- 99.9% confidence level; 99% confidence level; 95% confidence level.
Table 2.7: Seasonality Effect Tests

<table>
<thead>
<tr>
<th></th>
<th>MA(2)-GARCH(1,1)</th>
<th>MA(2)-GARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Seasonality b/se</td>
<td>Seasonality b/se</td>
</tr>
<tr>
<td>DlnP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>s1</td>
<td>-0.017</td>
<td>(0.013)</td>
</tr>
<tr>
<td>s2</td>
<td>-0.007</td>
<td>(0.012)</td>
</tr>
<tr>
<td>s3</td>
<td>0.006</td>
<td>(0.013)</td>
</tr>
<tr>
<td>cons</td>
<td>-0.020</td>
<td>(0.015)</td>
</tr>
<tr>
<td>ARCHM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sigma2ex</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>ARMA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.ma</td>
<td>0.118</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.095)</td>
</tr>
<tr>
<td>L2.ma</td>
<td>-0.175*</td>
<td>-0.178*</td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>ARCH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.arch</td>
<td>0.785*</td>
<td>0.735*</td>
</tr>
<tr>
<td></td>
<td>(0.308)</td>
<td>(0.301)</td>
</tr>
<tr>
<td>L.garch</td>
<td>0.232</td>
<td>0.232</td>
</tr>
<tr>
<td></td>
<td>(0.126)</td>
<td>(0.152)</td>
</tr>
<tr>
<td>cons</td>
<td>0.002*</td>
<td>0.002*</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>ll(model)</td>
<td>389.1714</td>
<td>390.3984</td>
</tr>
<tr>
<td>AIC</td>
<td>-764.3428</td>
<td>-760.7968</td>
</tr>
<tr>
<td>BIC</td>
<td>-737.0627</td>
<td>-721.8528</td>
</tr>
</tbody>
</table>

* 99.9% confidence level; ** 99% confidence level; *** 95% confidence level. Standard errors are reported in parentheses.

Seasonal effect is not observable when it comes to logarithm returns instead of price levels. Table 2.7 shows the results of adding seasonal components to the MA(2)-GARCH(1,1) model. All the four season dummies are insignificant and adding season dummies does not yield a smaller information criteria AIC or BIC. These results lead to rejecting the existence of deterministic seasonal effects and confirming our choice of an MA(2)-GARCH(1,1) model.

2.7.3.4 EGARCH

In addition, we continue to examine the existence of asymmetric shocks, i.e. whether the gas trading market as a whole responds differently to unanticipated increases in spot prices than it does to unanticipated decreases. One of these models is Exponential GARCH (EGARCH) model, first proposed by Nelson [1991]. Stentoft [2005] finds the specifications of EGARCH generally have the smallest pricing errors concerning options on individual stocks. We mod-
ify the conditional variance of the GARCH model proposed above as follows:

\[
\log (h_t) = \alpha_0 + \alpha_1 z_{t-1} + \alpha_2 \log (h_{t-1}) + \alpha_3 (|z_{t-1}| - E[|z_{t-1}|])
\]

where \( z_t = \varepsilon_t / h_t \) is the standardized residual, which is distributed as \( N(0, 1) \). \( \alpha_3 \) is the asymmetric component and \(|\alpha_2| < 1\) ensures stationarity and ergodicity for the EGARCH(1,1) model.

Table 2.8 shows the results of ARMA(p,q)-EGARCH(1,1) models, where only ARMA(p,q) models without over fitting are considered. Judging from the information criteria, MA(2)-EGARCH(1,1) outperforms the other models, with the smallest AIC and BIC, which confirms the model selection above. MA(2)-EGARCH(1,1) fits the data better in terms of the model loglikelihood. We observe that EGARCH models yield larger loglikelihood than the GARCH model, implying a slightly better fit. Furthermore, MA(2)-EGARCH(1,1) results in the following estimations, where standard errors are reported in the parenthesis:

\[
\begin{align*}
\log h_t &= -0.588 + 0.009z_{t-1} + 0.873 \log h_{t-1} + 0.672 (|z_{t-1}| - E[|z_{t-1}|]) \\
&\text{(.)} (0.068) (0.005) (0.094)
\end{align*}
\]

Table 2.8: ARMA(p,q)-EGARCH

<table>
<thead>
<tr>
<th>Estimated Coefficients</th>
<th>AR(2) ARCHM (1,1) EGARCH(1,1)</th>
<th>MA(2) ARCHM (1,1) EGARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ARCHM</strong></td>
<td><strong>b/se</strong></td>
<td><strong>b/se</strong></td>
</tr>
<tr>
<td>sigma2ex</td>
<td>0.000 (0.001)</td>
<td>-0.000*** 0.000</td>
</tr>
<tr>
<td><strong>ARMA</strong></td>
<td><strong>L.ar</strong></td>
<td><strong>L2.ar</strong></td>
</tr>
<tr>
<td>L.ar</td>
<td>0.118 (0.073)</td>
<td>-0.147* -0.069</td>
</tr>
<tr>
<td>L2.ar</td>
<td>0.110*** -0.001 -0.131</td>
<td></td>
</tr>
<tr>
<td><strong>ARCH</strong></td>
<td><strong>L.earch</strong></td>
<td><strong>L.earcha</strong></td>
</tr>
<tr>
<td>L.earch</td>
<td>0.010 (0.076)</td>
<td>0.009 (0.068)</td>
</tr>
<tr>
<td>L.earcha</td>
<td>0.671*** (0.153)</td>
<td>0.672*** (0.094)</td>
</tr>
<tr>
<td><strong>L.egarch</strong></td>
<td>0.876*** (0.066)</td>
<td>0.873*** (0.005)</td>
</tr>
<tr>
<td>cons</td>
<td>-0.571 (0.322)</td>
<td>-0.588 (.)</td>
</tr>
<tr>
<td>Il(model)</td>
<td>397.7379</td>
<td>397.5245</td>
</tr>
<tr>
<td>AIC</td>
<td>-779.4758</td>
<td>-783.0490</td>
</tr>
<tr>
<td>BIC</td>
<td>-748.2985</td>
<td>-759.6660</td>
</tr>
</tbody>
</table>

* 99.9% confidence level; ** 99% confidence level; *** 95% confidence level. Standard errors are reported in parentheses.
A positive search coefficient implies that positive innovations (unanticipated price increases) are more destabilizing than negative innovations and vice versa. The effect appears insignificant (0.009) and is substantially smaller than the symmetric effect (0.672) which is significantly positive. Results indicate that there is no strong indication for a leverage effect. Therefore the data do not support this asymmetric supposition by specifying a nonlinear GARCH model that allows for asymmetric shocks to volatility. Thus despite a slightly larger loglikelihood obtained by EGARCH model, GARCH is comparably a more appropriate model for our gas price return data.
3 Decision Making in Incomplete Markets with Ambiguity

3.1 Introduction

Firms need project evaluation techniques for many purposes: capital budgeting assessment, risk management, mergers and acquisitions (M&A) activities and so forth. The most popular and well-adopted evaluation method over the past decades is the net present value (NPV) approach, for whose calculations only one time discount rate and a series of future cash flows are required. The NPV approach is simple and straightforward, but to achieve that needs strong assumptions and therefore it suffers from rigidity and inflexibility. Problems arise when investors believe that they may benefit from the flexibilities embedded in the projects: within a NPV framework, there is no way of quantifying the benefits of such flexibilities. As a consequence, NPV structurally underestimates the value of projects with flexible investment opportunities.

Heteroskedasticity The volatility of underlying processes obviously matters for option pricing problems. Chapter 2 shows in an application of ROV to an energy related project that a Gaussian GARCH specification outperforms one which assumes constant variance in modeling the dynamics of the underlying asset returns, in this case gas prices. We have also shown that switching from constant variance to Gaussian GARCH has a dramatic impact on option values, so modeling the structure of volatility matters. In this chapter, we again consider GARCH models as well as a more general volatility modeling approach, Generalized

This chapter is coauthored with Prof. Sweders van Wijnbergen. We are thankful to EBN B.V. for financial support.
Autoregressive Score (GAS) models which are capable of capturing some unique characteristics of the latent volatilities of the time series. This GAS model family, first proposed and developed by Harvey [2013] and Creal et al. [2013], is a more general set-up compared to GARCH models. By adding the first derivatives of its likelihood function as the latent factor of dynamics, this model takes full advantage of the changing directions of likelihood. As shown in Creal et al. [2013], the GAS framework demonstrates superior features and better empirical fit over Gaussian GARCH models, which is also consistent with our findings in this chapter in case of Dutch gas prices. Note that GARCH(1,1) processes can be seen as a special case of a more general GAS(1,1) structure, which allows a direct Likelihood ratio test of one specification against the other.

Furthermore, the diagnostic test on residuals from both Gaussian GARCH and Gaussian GAS models rejects the normality assumption. This feature is commonly found in financial data, often referred to as “black swans”: extreme outcomes happen more often than people expect, which results in the failure of normal distribution assumptions. Therefore, we proceed on with Student’s t-GARCH and t-GAS models, aiming to capture the fat-tail characteristics of the data. The estimated degrees of freedom for both models are smaller than 4 and statistically significant, which again confirms our fat-tail hypothesis. Hence a simple Gaussian assumption would undermine the high occurrence of extreme events and lead to incorrect descriptions of the data.

To our knowledge, this is the first study to derive option pricing results under a process of t-GAS and compare the results with option valuations based on the constant variance and results based on Gaussian GARCH assumptions. Even though we solve the problem within a real-option set-up, the results are doubtlessly relevant to the valuation of finance options traded in the market.

Incomplete Markets One difficulty of applying standard option techniques to real life problems is that often, the decision maker is facing an asset pricing problem in an incomplete market, where not all underlying risks are hedgeable through the market. For instance, in our gas field valuation problem, reservoir uncertainty cannot be hedged away in any existing market. In fact even price risk cannot be fully hedged since the derivative market built on the Dutch gas contracts is still young and immature2. Yet another cause of market incompleteness is the stochastic volatility characterizing the underlying process driving asset returns (gas prices), because the dynamics of the second moment of the process cannot be hedged through the market either. Therefore, classical option pricing models such as the Black-Scholes formula (Black and Scholes [1973]), which lay their foundations on the assumption of complete markets, are not applicable in an incomplete markets environment;

2 As already mentioned in Chapter 2, the Dutch gas spot market is called Title Transfer Facility (TTF). The first TTF Natural Gas Options were launched by ICE in December 2011.
in fact preference free (risk neutral) pricing becomes impossible: an individual’s risk preference has to be parametrized and be taken into account. Accordingly, utility indifference pricing is an appropriate valuation method to be applied for our real option problem.

Model Ambiguity  Another important issue in decision-making problem involves model ambiguity. This occurs when the decision maker is uncertain about the true probabilistic model, which is often referred to as Knightian uncertainty (Knight [1921]) or model ambiguity. Note that a decision problem with ambiguity is different from one under risk: the latter refers to a decision problem with the true probability distribution known and the former is one without the true probability known. Model ambiguity is a realistic and robust assumption because an investor often does not have access to the true probabilistic model underlying relevant variables and may only have an approximation for the true one at the best.

In applications like the one analyzed in this chapter concerning gas field evaluation, model ambiguity occurs often due to the relatively unsophisticated existing technology for reservoir size estimation. Geophysicists estimate parameters for reservoir distribution based both on imperfect exploration data, often supplemented by insights derived from their own past experiences, which makes model ambiguity a particularly important issue for valuation problems.

Choice of Discount Rate  In an incomplete market setting, a proper discount rate should not only reflects the decision maker’s risk aversion and her time discount value, but also the structure of the uncertainty embedded in the project itself. The appropriate choice of discount rates is not always clear to the decision maker. In fact, managers are struggling in determining the time discount factor, especially for individual projects. Often, as Borison [2005] point out, the WACC (weighted average cost-of-capital) is used without clearly identifying its risk coverage, i.e., whether it reflects private risk only or the overall investment risk. Therefore, we decompose the discount rate and discuss how each aspect inherent in discount rate determination affects the decision making process.

Guide to The Reader  This chapter is arranged as follows. Section 3.2 reviews related literature on real options, model ambiguity, GARCH and GAS models, etc. Our representative gas field case study problem is described in Section 3.3. The econometric models and option pricing models are explained in the following Section 3.4. Section 3.5 demonstrates the results and Section 3.6 concludes.
3. Decision Making in Incomplete Markets with Ambiguity

3.2 Literature Review

**Real Option Valuation**  Borison [2005] criticizes existing applications of real option theory for requiring assumptions that are not satisfied in practice, thereby invalidating the pricing methodologies chosen. He surveys the applicability and assumptions of all existing approaches, including the classic approach (Brennan and Schwartz [1985], Amram and Kulatilaka [1999]), the subjective approach (Howell [2001]), the MAD approach (Copeland et al. [1994]), the revised classic approach (Dixit and Pindyck [1994]), and the integrated approach (Smith and Nau [1995], Smith and McCardle [1998]). All of them except the last one assume market completeness (hedgeable risks), which is however rarely the case in real-life problems where real options are under consideration. Furthermore, the first two explicitly assume the underlying asset follows a constant variance geometric Brownian motion process (GBM), which is also not always a good approximation of real life situations.

**Discount Rate**  According to a survey conducted by Mukerji and Tallon [2001], the most popular valuation method chosen by CFOs is discounted cash flow, i.e. Net Present Value (NPV) method. However, when applying this approach, the CFOs are (understandably...) not clear on the choice of discount rate. Apparently, they typically use one of the following four discount rates: the acquiring firm’s weighted average cost of capital, the acquiring firm’s cost of equity, the target’s weighted average cost of capital, or other rates such as the target’s cost of equity. Each discount rate has its pros and cons, and the choice may also depend on the (size of the) M&A project itself. This may confuse the CFOs and lead to biased (too optimistic or pessimistic) results. And in fact project structures may be such that the use of any constant discount rate is wrong because the risk structure changes over time.

**Dynamic Processes of Underlying Assets**  As is explained in Chapter 2, the dynamic of gas prices follows a complicated structure with time varying volatilities, which cannot be captured by a GBM process. Therefore the classic approach, the subjective approach, and the revised classic approaches mentioned before become inapplicable, since they rely on the GBM assumption for their pricing formulas.

In this chapter, we consider GAS/GARCH models accounting for volatility, where both are able to reproduce the volatile volatility. Creal et al. [2013] explain that GAS models can be specialized into GARCH models by selecting appropriate factors. They also compare different dynamic copula models and conclude that the likelihood information is extensively exploited under a GAS framework. As shown in Andres [2014], the model with dynamic scores outperforms autoregressive conditional duration (ACD) models in terms of the rate of convergence and reliability. Note that an ACD model, as proposed in Engle and Russell [1998], is analogous to a Gaussian GARCH model.
Furthermore, the financial data often contain fat-tails: extreme outcomes happen too often that a normal distribution is not capable of accounting for the outliers. By applying GARCH and GAS models to global equity returns, Creal et al. [2011] find that t-GAS produces highly persistent estimated factors and improves loglikelihood substantially.

**Real Options and Incomplete Markets** As mentioned above and in Borison [2005], most real option approaches assume market completeness, which results in problematic applications when there are unhedgeable risks. For example, the subjective approach (Howell [2001]) uses subjective probability; therefore, it is incapable of shading lights on the market trading price. The MAD approach (Copeland et al. [2001]) argues that traditional NPV serves as an unbiased replicated portfolio; however, still, the no-arbitrage assumption cannot be satisfied with this argument only, because arbitrage opportunities may exist due to the use of subjective data. Several attempts have been made for resolving the incomplete market problem. For example, Smith and Nau [1995], Smith and McCardle [1998] remedy the issue by assuming a partial complete market and solve the partial market incompleteness by utility indifference pricing, as well as we have presented in Chapter 2. Carmona [2009] states the effectiveness of utility indifference pricing mechanism for option pricing problems in an incomplete market, where risk preferences are built into the model to acknowledge risks.

**Model Ambiguity** The concern for modeling ambiguity can be traced back to Knight [1921], where ambiguity is also described as uncertainty. The essential difference between risk and (Knightian) uncertainty (or ambiguity, as we refer to it here) is whether the true probability is known or unknown. The breakthrough made by Gilboa and Schmeidler [1989] solves the ambiguity problem numerically through a maxmin utility with multiple priors, by assuming the agent is ambiguity averse and therefore considers the worst case scenario.

Camerer and Weber [1992] give an extensive survey on ambiguity aversion, including both theoretical and empirical analysis. In earlier experimental studies, e.g. Heath and Tversky [1991], subjects were shown to exhibit strong ambiguity aversion in many circumstances. However, the results for the effect of ambiguity on asset prices are not always coherent. For example, Camerer and Kunreuther [1989] show that even though ambiguity has changed the market structure, it did not affect the asset prices systematically; whereas, Sarin and Weber [1993] draw a different conclusion, and claim that ambiguity drives prices down slightly but significantly.

Chen and Epstein [2002] investigate the effect of ambiguity by considering multiple-priors utility. Their model is able to decompose excess returns into a risk premium and an ambiguity premium. Maccheroni et al. [2006] consider models of decision-making under ambiguity of variational preferences, which focus both on multiplier preferences and on
3. Decision Making in Incomplete Markets with Ambiguity

multiple priors as in Chen and Epstein [2002].

**Real Options and LSMC** There is no closed form solution to the option valuation problems we analyze since we are adopting the GARCH and GAS frameworks, in addition to the presence of unhedgeable reservoir size risks. Another complication in our problem is the endogenous exercise moments, but no analytical solution exists for such options either (in fact it is appropriate to characterize the options as Bermuda type due to the discrete exercise dates). We solve the valuation problem using Stochastic Dynamic Programming, and reduce the dimensionality problem by again using the Least Squares Monte Carlo approach proposed by Longstaff and Schwartz [2001]. The continuation value of the claim is then approximated as a function of the state variables by repeated application of regression techniques. A flowchart (Figure 3.7.8) in the Appendix explains how this method works.

3.3 Problem Description

We explain the investment problem under concern in this section, regarding neighboring gas fields. The investor needs precise valuations for the project for acquisition reasons.

A and B are two gas fields geographically close to each other. The recoverable size of Field A is currently estimated to be low and not economically attractive by itself. However, if the development of Field B turns out to be successful, the reservoir estimate of Field A can be revisited and a more precise estimate might then be expected. So given the possible information updates, the investor designs a strategic developing plan as displayed in Figure 3.3.1. As is shown, if the drilling on B is successful and the reservoir of Field B turns out to be high, the producer may decide to build a new platform on Field A, which allows the productions of both A and B at the same time due to a platform’s larger capacity compared to a single pipeline. Thus Field A can be considered as an extension option on Field B, which should be exercised only when the reservoir of B reveals a good state (P10).

Our aim is to value Field B accurately, so that the valuation can serve as a reserve price for the acquisition of the area combining both A and B.

3.3.1 Reservoir Distribution

Possibility of success (POS) stands for the probability of a successful drilling. So the probability of a dry well is \(1 - \text{POS}\). Based on a drilling success, the investor expects a recovery size \(R\). Three commonly used assumptions for the distribution of \(R\) are a triangular distribution, the lognormal distribution, and a variant on the latter, the truncated lognormal distribution.
3.3. Problem Description

The triangular distribution is the industry standard for capital budgeting problems due to its simplicity. It considers only three outcomes of $R$, namely cases $P_{10}$, $P_{50}$, and $P_{90}$. Naturally in line with the concept of a CDF (cumulative density function), reservoir amount $R_{P_{10}}$ of $P_{10}$ case means the probability that the realization of the gas reserve is lower than $R_{P_{10}}$ is 90%. This simple representation is popular among decision makers, because it provides reasonable and easy proxies for low/medium/high reserve size cases, based on which a scenario analysis can be set up for capital budgeting procedures.

Despite the simplicity of triangular distribution, geophysicists prefer a lognormal distribution which provides more insights into the reservoir distribution as explained in Chapter 2. However, a standard lognormal distribution ranges from 0 to infinity, which is unrealistic in case of gas reservoir. Therefore, we apply a truncated lognormal distribution for the simulation of the reservoir volume size. To fully approximate the estimations provided by the geophysicists, the reservoir distribution sometimes cannot solely be characterized by a single truncated lognormal one. For example, the reserve size of Field B here is well described by a sum of two weighted lognormal probability distribution functions. The two lognormal distributions are truncated at 99% quantile, one with parameters (-0.1772, 0.5336)$^3$ and a weight of 0.8661, the other with parameters (-26.6623, 0.0002)$^4$ and a weight of 0.1339.

The POS of A may be updated from the exploration of Field B and the ambiguity of A, if any, may disappear too. Note that even if the reservoir size of A in our case study is given as one number instead of a whole distribution, our methodology can still be easily adjusted to one with complicated distributions, where the input of the reservoir size would

---

3. (Mean, standard deviation).
4. (Mean, standard deviation).
be replaced by a simulated number from a distribution, such as a lognormal distribution, or a weighted lognormal distribution, etc. Furthermore, the information update could also be extended to that of a more precise distribution estimation of A after B’s development. We give an example of such an extension in Section 3.5.4.

3.3.2 Option Characteristics of the Valuation Problem

The strategic plan followed by the firm is divided into two steps. First, the firm might wait and meanwhile observe the market price of gas to decide whether to start developing B or not. This decision has to be made within three years, due to the remaining life of the relevant exploration licenses. This setup means that the firm has a wait-and-see Bermuda-type\(^5\) option on B with a maturity of three years. Once this development option is exercised, the firm may build a platform and further develop A based on a not-worse-than-P10 realized reservoir amount of B (Figure 3.3.1), which can thus be seen as the unlocking of a European option. Thus, Project B has multiple and compound option characteristics with a sequential structure, whose values are calculated in Section 3.5.

3.4 Methodology

Below, we present the econometric model used for fitting and predicting gas prices and the utility indifference pricing setting for the option pricing problem. We apply both GARCH and GAS models to analyze daily returns obtained from Title Transfer Facility (TTF), the Dutch gas market.

3.4.1 Generalized Autoregressive Score (GAS) Models

The gas daily return series ranges from Mar 5, 2012 to Sep 27, 2013, shown in Figure 3.4.1. The time series is stationary by both Dickey-Fuller test and Phillips–Perron test.

The Generalized Autoregressive Score model follows Creal et al. [2013]. \(y_t\) is the demeaned daily log return of gas on the TTF market and has a probability distribution function \(p(y_t|f_t; \theta_t)\), where \(f_t\) stands for unobserved time-varying factors and \(\theta_t\) contains unknown parameters.

\(^5\)A Bermuda option is an American option with a set of predetermined exercise timing possibilities.
3.4. Methodology

Figure 3.4.1: TTF Daily Logarithmic Return Data

\[ y_t = \sigma_t \varepsilon_t \]
\[ f_{t+1} = \omega + A s_t + B f_t \]
\[ s_t = S_t \nabla_t \]
\[ S_t = -E_{t-1} [\nabla_t \nabla_t']^{-1} \]
\[ \nabla_t = \frac{\partial \log p (y_t | f_t; \theta_t)}{\partial f_t} \]

The scaling matrix \( S_t \) equals the Fisher information matrix and \( \nabla_t \) stands for the “score” as in “Generalized Autoregressive Score”. Hence, \( s_t \) is also called the scaled score function. We assume that \( \varepsilon_t \) follows a standard normal (Gaussian) distribution but also investigate the possibility of fatter tails by basing the GAS model on a student’s t distribution with estimated degrees of freedom, thereby testing for normality. The model collapses into a Gaussian GARCH or a t-GARCH one with the appropriate assumptions on the factor \( f_t \) as we show later in Appendix 3.7.1.1; as GARCH is a special case of GAS, this allows for a simple loglikelihood ratio test.

3.4.2 Estimation Results and Diagnostic Tests

Our econometric analysis shows that the Gaussian GAS model yields a lower log-likelihood value of -672.36, comparing to -614.76 for the Gaussian GARCH model. Thus a Gaussian
3. Decision Making in Incomplete Markets with Ambiguity

Table 3.1: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Gaussian</th>
<th>Student’s t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GARCH</td>
<td>GAS</td>
</tr>
<tr>
<td>omega</td>
<td>4.6086</td>
<td>0.1823</td>
</tr>
<tr>
<td></td>
<td>(1.4039)</td>
<td>(0.6241)</td>
</tr>
<tr>
<td>A</td>
<td>0.3822***</td>
<td>0.2245***</td>
</tr>
<tr>
<td></td>
<td>(9.4001)</td>
<td>(8.7826)</td>
</tr>
<tr>
<td>B</td>
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<td>0.9329***</td>
</tr>
<tr>
<td></td>
<td>(57.7331)</td>
<td>(56.8993)</td>
</tr>
<tr>
<td>nu</td>
<td>3.9511***</td>
<td>3.9096***</td>
</tr>
<tr>
<td>(Degree of Freedom)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LogLikelihood</td>
<td>-614.759</td>
<td>-672.361</td>
</tr>
</tbody>
</table>

\[ t \text{ statistics are reported in parentheses. *** represents 1\% significance level; ** represents 5\% significance level; * represents 10\% significance level.} \]

GARCH model outperforms a Gaussian GAS model by 9\% in terms of log-likelihood. And the kernel density plots and the QQ-plots in Figure 3.4.2 imply that the residuals from both Gaussian GARCH and Gaussian GAS models present fat tail leading to a rejection of the normality hypothesis. Therefore we proceed with a Student’s t-based GAS model, to capture the impact of the fat tails embedded in the data.

Table 3.1 lists the estimation results from all four models considered. The estimated degree of freedom for the Student’s t distribution is 3.95 and 3.91 for Student’s t-GARCH and Student’s t-GAS model respectively, and is significant for both models. In addition, the loglikelihood from the models with Student’s t distribution is significantly larger than the one from those with Gaussian distributions, which also confirms our findings from the residual plots. The result of a loglikelihood-ratio test between Gaussian-GAS and Student’s t-GAS is significant, implying the existence of fat-tails in the data series.

Figure 3.4.3 demonstrates the variances \( \sigma^2_t \) estimated from all four models. As can be seen, a model with constant variance assumption is not able to capture all the relevant features of this time series adequately. All Gaussian/ Student’s-t GARCH/ GAS models are able to characterize the high volatility periods, compared to a constant volatility model. As illustrated, two highly volatile periods in early 2007 and mid 2012 can be easily observed correspondingly to Figure 3.4.1.

The Student’s t-GAS model produces a slightly more smooth volatility series than the Student’s t-GARCH model: during the high volatility periods, e.g. early 2007, the estimated...

---

6 Arguably more important, the degrees of freedom parameter is very significantly lower than the number where the difference between t-distribution and the normal becomes negligible (higher than 30).

7 LR = 2 (−593.894 + 672.361) = 156.934 > χ^2 (1).
3.4. Methodology

Figure 3.4.2: Residual Tests

- Probability density estimate of residuals from Gaussian model (top left).
- QQ-plot of residuals versus standard normal (top right).
- Probability density estimate of residuals from Gaussian model (bottom left).
- QQ-plot of residuals versus standard normal (bottom right).

Data analysis: Distribution and normality tests.
3. Decision Making in Incomplete Markets with Ambiguity

volatility given by a Student’s t-GAS model is smaller than one by a Student’s t-GARCH model. This observation is attributed to the feature of Student’s t-GAS model, which adjusts quickly to new observations. Also it implies that extreme outcomes of returns does not necessarily stem from high volatility, the occurrence of tail events can result in the rare outcomes as well.

Figure 3.4.3: Comparison of Estimated Volatility with Gaussian/ Student’s-t GARCH/ GAS Models

Volatility data is rescaled by 10 times the original data for illustration purposes.

3.4.3 Utility Indifference Pricing

Chapter 2 applied an integrated approach adjusted from Smith and Nau [1995], Smith and McCardle [1998] based on the assumption that (A) gas price risk can be hedged so risk neutral valuation can be used in that dimension, but (B) reservoir risk is not hedgeable so we used a preference based valuation method in that dimension. Although a Gaussian GARCH(1,1) model for gas prices in the previous chapter violates the constant variance property necessary for the applicability of risk neutral pricing methods, we used a local variant on risk neutral pricing as proposed by Duan [1995]. But that mixed approach cannot be used here because the general GAS models do not satisfy Duan’s conditions necessary for the applicability of his local variant on risk neutral pricing. Since now the gas price volatility risks are unhedgeable too, we thus simply assume that neither risk can be hedged and adopt multidimensional utility indifference pricing for both the gas price and reservoir size risks. Also, for
comparison with methods more used in practice, we present results adopting the so-called
cost-of-capital method where a range of discount rates is used instead of explicit preference
parameters.

3.5 Results

The number of Monte Carlo simulations is 100,000. Here POS for drilling at A is 90% and
POS for drilling at B is 30%.

3.5.1 Cost-of-capital Method

3.5.1.1 NPVs vs. Option Values

The results over a range of values for the cost-of-capital (3% - 15%) rates are shown in
Figure 3.5.1. Figure 3.5.1a gives the results based on assuming a t-GARCH(1,1) process for
gas prices, and Figure 3.5.1b the same set of results but now based on assuming a t-GAS(1,1)
structure for the volatility process of gas prices.

In both sub-figures, option values and NPVs of both fields are declining as the assumed
cost of capital increases, as one should expect given the time structure of cash flows. The
horizontal solid (red) line stands for a break-even project, which sets a standard for accepting
and rejecting investment projects. The gray-circled line illustrates the net present value of
B at time zero without any option values counted. The dashed gray line stands for the net
present value of the strategic plan without the wait-and-see Bermuda option on B, i.e. a fixed
starting time at t=0.

The first interesting result stems from comparing the two graphs: the option values as-
suming t-GAS(1,1) are about 0.5-1 million euros higher than the project values based on
assuming a t-GARCH(1,1) specification. Note that the t-GARCH and t-GAS models ex-
plains the data with similar power in terms of loglikelihood, therefore this difference of
option values result from the different volatility structure predicted by two models.

As is shown in Figure 3.5.1b (and Table 3.3 in the Appendix), with a t-GAS specifica-
tion, the gray circled line intersects the break-even line at a cost-of-capital of 10%, so on
a NPV should be positive criterium, the firm would reject the entire project B for any cost-
of-capital higher than (or equal to) 10%. But taking into account the various waiting option
values changes that outcome: The discounted net project value with all options incorporated
values more than doubles for a WACC of 3%, declines with higher WACC but the overall
project value with options included stays significantly positive for all values of the WACC
considered. It does decline with higher discount rates, obviously, because the high CAPEX
come upfront but the revenues come later in time. Note also that a platform may have further
uses that we do not incorporate: for example, it can be used for gas storage at a later stage. Nevertheless, it is evident that the strategic development plan including waiting option is worthwhile because the project value including option values stays positive given cost-of-capitals varying from 3% to 15%. A second conclusion one can draw is that incorporating the option values is a meaningful exercise: otherwise the wrong investment decision would be taken under a wide range of cost-of-capital estimates.

Figure 3.5.1: Option pricing results
3.5. Results

Similar patterns can be found under the t-GARCH specification with slightly lower option values, as demonstrated in Figure 3.5.1a (and Table 3.3 in the Appendix). For example, the break-even point of NPV of B is at a cost-of-capital of 9.5%, compared to 10% in case of a t-GAS model.

The strategic plan is valued less than the NPV of sole project B. Later we show that the strategic plan becomes valuable if more information will be brought in in the future.

From here on we will not report the GARCH results anymore. They are obviously qualitatively similar to the GAS based results, but the GAS specification has a stronger basis in the econometric results of our data analysis.

3.5.1.2 Good State vs. Bad State

As in regular option pricing theory, the option value in our analysis depends on the current market state, in this case the gas price, since the econometric analysis suggests that the best prediction for the future return is mainly influenced by the current state. Figure 3.5.2 shows that the value of the project increases with the spot market price. For example, when the spot price is lower than 15 euros per megawatt hour, the option value is close to zero, so for extremely low spot prices, the project has not only negative NPV but also almost worthless options, resulting in a definite rejection at all discount rates.

3.5.2 Utility Indifference Pricing (UIP)

We again assume the investor has an exponential utility function: \( u_t(x_t) = -\exp(-x_t/\rho_t) \), where \( \rho_t \) represents the decision maker’s risk tolerance. A high \( \rho_t \) implies a high tolerance for risk (low risk aversion). Our basic criterion then relies on the discounted value of certainty equivalence cash flows where the certainty equivalence is calculated using a specific value for \( \rho \). Given the degree of risk tolerance, the certainty equivalent therefore represents the project value, in comparison to the NPVs used above.

The option values calculated based on utility indifference pricing for a range of values for the risk tolerance parameter \( \rho \) are given in Figure 3.5.3 and Table 3.2 in the Appendix. For both models, the option values ranges between 10 to 15 million euros. As one should expect, option values are increasing in the investor’s risk tolerance. Or, to put it differently, the more risk averse an investor is, the less value she attaches to a risky project. It is evident that taking into account the option values once again leads to higher project values, and to a different outcome in terms of the decision to proceed or not. Note that the valuation increases

8 Corresponding GARCH based results can be found in the Appendix.

9 This may sound plausible, but it is not trivial: note that such a result is typical for real options (unhedgeable risk) set ups only. For standard risk neutral option pricing methodology to be applicable all risks need to be hedgeable and option values do not depend on risk aversion in such a hedgeable risk environment.
3. Decision Making in Incomplete Markets with Ambiguity

Figure 3.5.2: Spot prices vs. option value under a Student’s t-GAS specification
initially steadily as the risk tolerance of the decision maker goes up from 5 to 40; however, from a risk tolerance of 40 onwards, the valuation flattens out. These findings are similar to the results obtained by Chapter 2.

One interesting observation can be made by comparing Figure 3.5.3 and Figure 3.5.1b. In Figure 3.5.3, when the risk tolerance of the investor increases and she/he becomes risk neutral, the valuations are similar to the ones from Cost-of-capital method with a cost-of-capital of 3%. Since we assume a risk-free rate of 3%, these two methods coincide provided complete market assumption holds.

As one should expect, the UIP approach leads to a comparison between the outcomes based on the different stochastic specifications of the volatility processes that is similar to what we saw comparing the outcomes under different cost of capital values. For all levels of risk tolerance, the t-GAS based analysis leads to slightly higher valuations than the t-GARCH based approach due to higher estimated volatilities.

### 3.5.3 Model Ambiguity

In the discussion so far we have proceeded on the assumption that specific values for the reserve levels were unknown, but that their probability distribution was known with full certainty. Of course that is overly optimistic: there is ambiguity about the distribution itself, model ambiguity in short. Therefore, in this subsection, we take this ambiguity into con-
consideration and show how it affects the project values. We also assume that the investor is ambiguity averse, which means she considers the worst-case scenario when facing ambiguity. In technical terms, the investor follows a maxmin strategy: take the minimum value of the maximized outcomes/valuations over the different distributional possibilities (see Gilboa and Schmeidler [1989]). In this example, we assume ambiguity exists in the mean of the reservoir distribution only, and we take the variance of the reservoir distribution as known from the geological structure of the locations. Initially we assume the same ambiguity on the reservoir sizes of both A and B. Of course we can apply similar methods based on the assumption of different ambiguity levels for A and B. Of special interest is the case where ambiguity levels get reduced when information becomes available half way the project. We consider that possibility explicitly in the next section, 3.5.3.

Figure 3.5.4: Option Values with Persistent Model Ambiguity

Figure 3.5.4 shows that the project value decreases with higher ambiguity levels. The levels chosen here are only for demonstration purpose. A higher ambiguity level means the decision maker is less certain about the mean of the reservoir distribution, which leads to a lower level of valuation due to the Minimax strategy followed. As can be seen, given a high level of ambiguity, the valuation differences between decision makers with different risk tolerance shrink accordingly.

These results provide interesting implications for insurance. The mirror image (upside down) of these graphs can be interpreted as how much the agent would be willing to pay for insurance against a certain risk the agent faces. It implies that for a given ambiguity level,
risk averse agents are likely to buy insurance product because they attach a high value to the insurance. On the contrary, risk tolerant agents would be less likely to buy such insurance. On the other hand, for those people with the same risk tolerance, the decision of purchasing the insurance contract depends on their ambiguity level on the underlying processes. For example, people with high ambiguity levels tend to buy insurance comparing to those with low ambiguity levels.

**Project Values When Ambiguity is Resolved Halfway the Process**

In the preceding section, we introduced persistent ambiguity, i.e. uncertainty about the probability structure that remains constant over time. However it is more reasonable to assume that once production in B has started, more information about A, and more specifically, about the probability distribution of possible outcomes of A, will become available, since the geological structures of B and A are related. And declining ambiguity again brings in rewards for waiting, in a sense once again real option value. We explore the additional value project B gets if its exploration reduces ambiguity over well A once B is brought in production. In particular, we focus on reservoir A ambiguity only, and assume it gets resolved after starting on reservoir B. In other words, starting on B leads not only to more specific information but also narrows down the range of distributional possibilities.

**Figure 3.5.5: Ambiguity in Field A Only (Student’s t-GAS)**

For simplicity and focus we demonstrate the effect for the case where there is just ambiguity about A, which gets resolved once B is brought into operation. The no-ambiguity case
is obviously the same as shown in Figure 3.5.4. But the interesting results come once we assume that starting on B leads to reduced ambiguity on A because the fields are contiguous. If the ambiguity level of A is at a particular level at the beginning and we know that ambiguity disappears after the development of B, then the difference between no-ambiguity and the project value at that particular Ambiguity-level should be added to the project value of B. 3.5.5 makes the point for the two moderate ambiguity level (Level 2 and Level 3): it shows the option values that resolution of ambiguity leads to as a percentage of the original project value of B with ambiguity persistent, and for different levels of risk tolerance.

It is clear from Figure 3.5.5 that option values go up with risk tolerance and also increase as the initial ambiguity level that gets resolved is higher. And the option value numbers are substantial: in this example the increase in project value due to the reduction in ambiguity ranges between approximately 5% and 15% of the original project value depending on risk tolerance and level of pre-existing ambiguity.

3.5.4 Reservoir Correlation

Finally we consider another plausible example of correlated information, for when the reservoir size of A follows a truncated lognormal distribution, with mean equal to the one mass point before, and with variance equal to 0.5, shown in Figure 3.5.6a.

Assume now that if B turns out to be a successful development, the information about the reservoir size distribution of A will be updated correspondingly. This can also be interpreted as one example of ambiguity reduction. Figure 3.5.6 shows some possible distribution updates for the distribution of A, with the original distribution given in Figure 3.5.6a. The following diagrams 3.5.6b-d show three different ways the distributional information could change: In Figure 3.5.6b we show how the distribution changes when the truncation point shifts inwards, the range of possible outcomes narrows down, as in the shadow area displayed in Figure 3.5.6b. Alternatively, the mean could shift; Figure 3.5.6c shows an example where the mean shifts up.

Finally, mean and truncation points could be left unchanged but the variance could be reduced as information from B becomes available (Figure 3.5.6d). In what follows we focus on the case where the truncation point shifts inwards once B has started up, the case shown in Figure 3.5.6b, to demonstrate how our option technique works. We again present the results both for the Cost-of-capital approach and for UIP.
3.5. Results

Figure 3.5.6: Reservoir Correlation Examples

(a) A has a truncated lognormal distribution
(b) Truncation Update
(c) Mean Update
(d) Variance Update

3.5.4.1 Cost-of-capital method

By comparing Figure 3.5.7 with Figure 3.5.1, one can easily find out that this reservoir correlation has increased the option value by about 1 to 5 million over the range of cost-of-capital rates considered (as also shown in Figure 3.5.8). The reservoir correlation of course does not change the NPV of B, therefore the red dashed line and the gray circled line stay the same as in Figure 3.5.1.

Moreover, the strategic plan now outperforms the stand-alone project B for low cost-of-capital estimates; this happens for rates below 10% under a t-GAS specification.
3. Decision Making in Incomplete Markets with Ambiguity

Figure 3.5.7: Option Pricing Results for the General Case with Reservoir Correlation (Student’s t-GAS)

![Option Values v.s. NPV (under a t-GAS specification)](image)

Figure 3.5.8: Comparison: Option Values With and Without Reservoir Correlation (Student’s t-GAS)

![Comparisons of option values and strategic plans, without and with reservoir correlation (t-GAS)](image)

**Comparison with the case without reservoir correlation**  Furthermore, the shadow areas in Figure 3.5.8 represent the differences of values between the projects with and without
reservoir correlation. It is evident that both the option and strategic plan are valued higher when reservoir correlation exists. In other words, the halfway resolution of reservoir distribution ambiguity/correlation increases the project value significantly by adding option value.

### 3.5.4.2 Utility Indifference Pricing

When the evaluation is based on UIP instead of on fixed cost-of-capital estimates, similar to the comparison in Section 3.5.4.1, the strategic plan presented brings in more revenues (in NPV terms) than project B on its own, shown in Figure 3.5.9 and Figure 3.5.10. But the more important point is that reduction of uncertainty, this time a narrowing down of the range of possible outcomes, once again leads to substantial option values and correspondingly higher project value. Again, ignoring option values and information acquisition would lead to overly conservative project valuation and excessively conservative project decisions.

**Comparison with the case without reservoir correlation**  Similar to Figure 3.5.8, the shadow areas in Figure 3.5.10 represent the differences of values when comparing the projects with and without reservoir correlation.

Once again, the fact that ambiguity on A is reduced once B has been brought into operation causes the option values to increase: the more future information gets updated as the project moves ahead, the higher the initial project value is.

Figure 3.5.9: Utility Indifference Pricing Results for the General Case with Reservoir Correlation (Student’s t-GAS)
3. Decision Making in Incomplete Markets with Ambiguity

Figure 3.5.10: Comparison: Option Values With and Without Reservoir Correlation (Student’s t-GAS)

3.6 Conclusion

This chapter has focused on the real option approach to solving a contingent claim problem as an alternative method for decision making under uncertainty. We incorporate many aspects that complicate asset pricing problems, such as incomplete markets and unhedgeable risks, dynamic release of distributional information and non-normal volatility assumptions, all of which invalidate traditional risk neutral approaches to asset pricing. Utility indifference pricing is applied in face of market incompleteness and t-GARCH/t-GAS models are used for volatility modeling of gas prices. We show in a real world example that the Student’s t-GARCH/ -GAS model, with its fatter tails, fits the observed data better than the Gaussian GARCH/GAS model in terms of loglikelihood ratio.

We also take the analysis one step further by introducing deep uncertainty, of the type that cannot be summarized by formulating a probability density function, because it concerns uncertainty about that very density function. In the literature this sort of uncertainty is referred to as Knightian uncertainty or, the word we prefer, model ambiguity. In our case study we show that the existence of model ambiguity reduces asset values in a risk averse world and will ceteris paribus lead to more conservative project continuation decisions.

In addition, we also introduce a new angle to this debate by pointing out that for time structured projects with correlated distributions, a new source of option value can emerge. If executing one part of the project leads to reduced model ambiguity concerning the later com-
ponents of the project, the initial blocks acquire additional option values, which in our case study are shown to be substantial. As the ambiguity level decreases with project progress, the initial project becomes more valuable due to the information that will be brought in along with development. The value of projects that allow for that sort of flexibility will be underestimated consistently by more traditional NPV-based valuation approaches. In our real world case study, the biases are shown to be substantial.
3.7 Appendix

3.7.1 GAS Models

3.7.1.1 Gaussian GARCH model

The model above can be reduced to a Gaussian GARCH model if \( f_t = \sigma_t^2 \) and \( \epsilon_t \sim N (0, 1) \), i.e.

\[
y_t = \sigma_t \epsilon_t
\]

\[
\sigma_{t+1}^2 = \omega + A \left( \frac{y_t^2}{\sigma_t^2} - \sigma_t^2 \right) + B \sigma_t^2
\]

where \( \omega, A, \) and \( B - A \) are parameters in a classical Gaussian GARCH model.

3.7.1.2 Gaussian GAS model

Alternatively, if take \( f_t = \log \sigma_t^2 \), we obtain a Gaussian GAS(1,1) model, i.e.

\[
y_t = \sigma_t \epsilon_t
\]

\[
\log \sigma_{t+1}^2 = \omega + A \left( \frac{y_t^2}{\sigma_t^2} - 1 \right) + B \log \sigma_t^2
\]

In this model, next period’s variance depends in a linear manner on a constant, the current period’s variance and the square of the standardized observations, \( \frac{y_t^2}{\sigma_t^2} \).

3.7.1.3 Student’s t GARCH model

If the error term \( \epsilon_t \) follows a Student’s t distribution with degree of freedom \( \nu \), then it again becomes a t-GARCH model. Similarly, if we still fit it into a GAS framework, the model can be written as follows:

\[
y_t = \sigma_t \epsilon_t
\]

\[
\sigma_{t+1}^2 = \omega + A \nu + \frac{3}{\nu} \left( \left( 1 + \frac{y_t^2}{(\nu - 2) \sigma_t^2} \right)^{\nu - 1} \frac{1}{(\nu - 2)} y_t^2 - \sigma_t^2 \right) + B \sigma_t^2
\]

3.7.1.4 Student’s t GAS model

A Student’s t GAS(1,1) model is obtained by choosing \( f_t = \log \sigma_t^2 \), and \( \epsilon_t \sim t (\nu) \).
\[ y_t = \sigma_t \varepsilon_t \]
\[ \log \sigma_{t+1}^2 = \omega + A \frac{\nu + 3}{\nu} \left( 1 + \frac{y_t^2}{(\nu - 2) \sigma_t^2} \right)^{-1} \left( \nu + 1 \right) \frac{y_t^2}{(\nu - 2) \sigma_t^2} - 1 \] + B \log \sigma_t^2.

### 3.7.2 Results under a specification of the gas price volatility process as a Student’s t-GARCH model
Figure 3.7.1: Spot prices vs. option value under a Student’s t-GARCH specification
Figure 3.7.2: Option Values with Persistent Model Ambiguity (t-GARCH)

![Graph showing option values with different ambiguity levels](image1)

Figure 3.7.3: Ambiguity in Field A Only (t-GARCH)

![Graph showing ambiguity in Field A only](image2)
3. Decision Making in Incomplete Markets with Ambiguity

Figure 3.7.4: Option Pricing Results for the General Case with Reservoir Correlation (t-GARCH)

![Option Values v.s. NPV (under a GARCH specification)](image)

Figure 3.7.5: Comparison: Option Values With and Without Reservoir Correlation (Gaussian GARCH)

![Comparisons of option values and strategic plans, without and with reservoir correlation (t-GARCH)](image)
Figure 3.7.6: Utility Indifference Pricing Results for the General Case with Reservoir Correlation (t-GARCH)

Figure 3.7.7: Comparison: Option Values With and Without Reservoir Correlation (t-GARCH)
### 3.7.3 Option Results in Details

Table 3.2: Utility Indifference Pricing Comparison (million euros)

<table>
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<th>Option Value (t-GAS)</th>
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Table 3.3: Option values versus NPV of B at time 0 (million euros)

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### 3. Decision Making in Incomplete Markets with Ambiguity

#### Table 3.4: Model Ambiguity

(a) Student’s t-GARCH

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(b) Student’s t-GAS

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Table 3.5: Model Ambiguity of A Only

(a) Student’s t-GARCH

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(b) Student’s t-GAS

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### Table 3.6: Option values vs. NPV of B at time 0 (million euros) with reservoir correlation

<table>
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<th>Cost-of-capital</th>
<th>Option Values</th>
<th>NPV of strategic plan starting at ( t = 0 )</th>
<th>Difference</th>
<th>( % ) Difference</th>
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<tr>
<td>9%</td>
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<tr>
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<tr>
<td>15%</td>
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<td>10.37</td>
<td>6.26</td>
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</tr>
</tbody>
</table>

Note: Table 3.6: Option values vs. NPV of B at time 0 (million euros) with reservoir correlation.
Table 3.7: Utility Indifference Pricing Comparison (million euros) with Reservoir Correlation

<table>
<thead>
<tr>
<th>Risk Tolerance</th>
<th>Option Value under a Student’s t-GARCH specification</th>
<th>Option Value under a Student’s t-GAS specification</th>
<th>Difference</th>
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</thead>
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<td>11.33</td>
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<td>12.92</td>
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<tr>
<td>55</td>
<td>15.40</td>
<td>15.94</td>
<td>-0.54</td>
</tr>
</tbody>
</table>
A put option with exercise price $K$; risk free rate $r$

Simulate paths $P^n_t$: $N$ paths $\times T$ periods.

Value of the option at expiry date $T$ is $OP^n_T = \max(K - P^n_T, 0)$.

$t = T$

Define $\Gamma = \{n : OP^n_T > 0\}$.

$Y^{(n)} = e^{-rt}OP^n_T$

$X^{(n)} = P^n_{t-\Delta t}$. Regress $Y$ on a constant, $X$, and $X^2$.

$\hat{Y} = E(Y|X) = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 X^2$

The option value at time $t-\Delta t$ is $OP^{n}_{t-\Delta t} = \max(Y^{(n)}, \max(K - P^n_{t-\Delta t}, 0))$

The value of the put option is $\frac{1}{N} \sum_{n=1}^{N} e^{-rt}OP^n_t$

Stop
4.1 Introduction

Gas storage allows the investors to inject a certain amount of gas when gas demand and spot prices are low (e.g. in summer) and to withdraw when market gas demand and spot prices are high (e.g. in winter). By purchasing the storage capacity, the investors gain the right to choose both the timing and the quantity of gas injections/withdrawals in accordance with the anticipated or realized demand changes. Therefore, theoretically, the value of the storage capacity consists of both intrinsic and extrinsic values: the former is intuitive, the price difference between winter and summer, and the latter values the flexibility of the storage opportunity. Many studies have been conducted in order to correctly model the two components. However, there has been a mismatch between the theory and practice of the capacity pricing. This chapter tries to bridge that gap by on the one hand taking the theoretical problems seriously into account, but on the other hand modeling the problem with sufficient complexity to do justice to the real world structure of the contracts traded. In addition to constructing a satisfactory statistical model to solve the capacity pricing problem, we will in the process also shed light on investors’ behavior by comparing the modeling results with actual auction outcomes. The aim of this chapter is to value storage capacity and to investigate its interaction with the Dutch gas spot market, namely the Title Transfer Facility (TTF) market, by taking the complex option structure into consideration.

In the Netherlands, storage capacity is traditionally traded through semi-annual auctions. All eligible capacity buyers submit their bids including the price for one Standard Bundled

This chapter draws on joint research with Prof. Sweder van Wijnbergen. We thank EBN B.V. for their financial support and helpful discussions.
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

Unit (SBU) and the amount of SBUs they require. After rounds of auctions, successful bidders will be offered the demanded storage correspondingly at their bidding prices. During the contract period, natural gas is then delivered at TTF market. Therefore, in essence, storage capacities serve as a bundle of options on the gas spot market, because it offers the capacity buyer the right to ask for gas delivery or storage at a predetermined cost.

Only European-style fixed maturity options on the TTF market are publicly traded. The lack of flexibility and low liquidity of that market are the main reason for the popularity of storage capacity contracts. Since the storage capacity contract is essentially an option-like contract on gas, its pricing can be determined through option pricing techniques. However, the classical pricing methods can not readily be applied. First, the standard option pricing assumptions (complete markets allowing for arbitrage-free price determination) do not hold due to the complicated dynamics of underlying gas price processes. Second, this real option problem has a more complex option structure than typically considered in the academic literature: the option to withdraw or to inject gas can be exercised many times on the amount requested by the option holder; but the amount of withdrawal or injection is constrained by the decisions in earlier periods. Third, the Dutch TTF market was only initiated in 2003 and its corresponding option market has not become active until 2011. Thus the illiquid and thinly traded futures and options market leads to the failure of market completeness, which is a fundamental assumption required by the classical option pricing theory. As a result, option pricing methods based on no arbitrage conditions (risk neutral pricing) lose their applicability and we need an effective approach to deal with this issue.

Also, the method for valuing and estimating extrinsic values used in most of the existing literature appears to be too simplified, not taking the investors’ active hedging behavior over spot market into consideration. For instance, as one can see from historical data, if an investor observes a stable TTF market for a long period, she is apparently less interested in bidding for storage capacity, or in other words, she values capacity less. This characteristic feature of capacity pricing is missing from most of current literature and industry applications. Examples are the relatively regular failures of capacity auctions held by GasTerra in 2011 (both March 28, 2011 and November 29, 2011) and in early 2013. Further studies therefore are needed to investigate the investors’ bidding and pricing behavior.

Another issue concerns the impact of volatility on the valuation of the storage capacity, because of the option feature of storage capacity contracts. So far, only scenario analysis has been performed and results are usually given separately for high or low but always constant volatilities. In this chapter, we allow for a more complicated model structure by taking into account the time-varying dynamics of volatilities. We apply both multivariate GARCH models and GAS (Generalized autoregressive score) models, so as to capture the changing of volatility of gas spot prices. Introducing non-trivial volatility risk factors once again leads
to a violation of the assumptions needed for the applicability of risk neutral pricing methods, so we use a full fledged stochastic dynamic optimization approach to embed optimal investor behavior into the pricing analysis. We use Least Square Monte Carlo (Longstaff and Schwartz [2001]) to reduce the dimensionality problem endemic in Dynamic Programming problems down to manageable proportions, but in a different manner than Boogert and De Jong [2008] have proposed. Details will be discussed in Section 4.5. Furthermore, the invalidity of preference-free pricing in our setting makes it necessary to explicitly consider the impact of the degree of risk aversion of investors (capacity buyers, in our case) on valuation. This has the added benefit of allowing us to infer the implied risk aversion of capacity buyers by observing realized auction prices.

Our methodology is able to produce valuations for storage capacity that approximate the realized auction prices quite well. Our findings suggest that the investor assigns a higher value to the storage capacity when facing a riskier market. This can be explained by the fact that in a more volatile market, there is more demand for an effective hedging product. The revealed risk aversion of the investor is far from zero, which again suggests the importance of parameterizing the risk aversion of individual investor. Also, we compare the values of several possible storage contracts with various injection/withdrawal capacity constraints. In accordance with intuition, we find that storage with higher injection/withdrawal rate (faster storage) results in higher contract values.

This chapter is organized as follows. Section 4.2 reviews studies that have been conducted on pricing storage capacity and on GAS models. An extensive analysis of the spot market and historical auction prices can be found in Section 4.3 and Section 4.4. Section 4.5 discusses the setup of the model for one unit of storage (one SBU). Section 4.6 deals with the statistical models for gas prices, while Section 4.7 discusses the option pricing implications. Section 4.8 concludes.

4.2 Literature Review

4.2.1 Pricing Storage Capacity

De Jong [2015] discusses and compares four commonly used valuation approaches to valuing gas storage, namely, intrinsic, rolling intrinsic, basket of spreads, and spot trading approaches. These methods are not mutually exclusive, they all share certain similarities. Intrinsic and rolling intrinsic methods focus mainly on optimal trading in forward markets, with the latter allowing for recalculating optimal strategies over time and for adjusting the trading strategy as new information flows in. However, as indicated in De Jong [2015], applications of these two methods overlook the fact that the required extensive forward contracts
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

are often not traded in the forward markets, resulting in an upwards bias in estimated storage values. The basket of spreads method can be seen as a simplified rolling intrinsic approach. The number of feasible spreads is limited due to the restrictions embedded in a storage capacity problem. This results in a possible downward bias of this approach, in contrast to the intrinsic and rolling intrinsic approaches. Finally, the spot trading method includes the hedging behaviors of spot trading on a frequent basis. Secomandi and Seppi [2014] use a different classification based on relevant underlying assets, including spot price models (e.g. Boogert and De Jong [2008]), futures term structure models (e.g. Lai et al. [2010]), and equilibrium asset pricing models. This is a more general classification than the one given by De Jong [2015] since they included the equilibrium pricing models. The reason of the unpopularity of the equilibrium asset pricing models among practitioners lies in their low tractability. These classifications highlight the two main aspects of modeling and pricing storage capacity: the dynamics of underlying asset prices and the option pricing techniques used. We discuss the literature on both aspects separately.

**Underlying processes** Jaillet et al. [2004], Secomandi [2010] and Boogert and De Jong [2008] all assume that the underlying asset follows a mean-reversion process with constant volatility. Moreover, Thompson et al. [2009] add jumps to constant volatility models. Similarly, Carmona and Ludkovski [2008] study the optimal switching regimes between electricity and gas markets by employing an Ornstein-Uhlenbeck process with jumps. In addition, based on Parsons [2013], Henaff et al. [2013] studies the hedging problem with interactions between spot market, futures market and storage capacity, where a two-factor model is considered for the future prices and a GARCH model with jumps is used for spot prices. Deviating from the above models, Boogert and De Jong [2011] and Parsons [2013] decompose the long-term and short-term behavior of the observed price dynamics and describe them separately in a two-factor model.

**Option pricing techniques** Both Jaillet et al. [2004] and Secomandi [2010] apply trinomial lattice techniques to solve for the capacity prices. However, this method suffers from the curse of dimensionality and therefore requires heavy computations. Another popular technique is the Least Square Monte Carlo method where continuation values are approximated by functions of concurrent state variables, with their parameters estimated from earlier simulations. Boogert and De Jong [2008] modify the LSMC method in their valuation of storage capacity and test the basis functions and convergence for LSMC. They formulate the basis functions based on both storage space and gas prices, where grids of storage space are created for each period. This is an efficient adjustment for the use of LSMC but by using the grids without considering any restrictions on the injection/withdrawal rate or the gas-in-storage
prior to certain periods, it is likely to lead to an upward bias, overestimating the value of the storage contracts. To solve the stochastic control problem, both Thompson et al. [2009] and Carmona and Ludkovski [2008] also use (a modified) version of the LSMC method for its simplicity, flexibility and fast convergence properties. We have also settled on LSMC, also with modifications necessitated by the structure of our pricing problem; we elaborate on our specific approach below.

**Incomplete market assumption** Despite the extensive discussions on complex mathematical modeling, the existence and consequences of market incompleteness are effectively overlooked or trivialized by most of the literature by simply assuming zero risk premia or by setting the physical probabilities equal to the risk-neutral adjusted ones. For instance, Thompson et al. [2009] state that the embedded risks are not fully hedgeable but they simply use real probabilities to avoid the incomplete market issue; Parsons [2013] argues that neither side of the market has bargaining power, therefore risk premium is too small that the physical probabilities become equivalent to the risk-neutral probabilities. None of these simplifying assumptions is satisfactory as we will show below.

### 4.2.2 GAS Models

The importance of correctly modeling underlying asset prices is widely accepted in the literature. For example, by allowing simple trading strategy for managing the storage with fixed spot price thresholds for actions including injection, withdrawal, and doing nothing, Secomandi [2010] indicates that the dynamics of stochastic spot prices have major effects on the optimal decision policies. He also emphasizes the importance of correctly modeling the uncertainty embedded in the underlying processes and shows how the valuations differ for high and low volatility assumptions. However, most of earlier research ignores the complex volatility structure into consideration. We extend the existing applications and take one step away from GARCH models by employing and econometrically testing GAS models, in order to characterize some unique features of the time series.

GAS models have recently been developed by Creal et al. [2013], Harvey [2013]. Creal et al. [2013] first propose a model framework called Generalized autoregressive score (GAS) Model, which updates the dynamics based on the scaled score of the loglikelihood. This GAS model is able to fully exploit the information provided by the loglikelihood function. Koopman et al. [2015] investigate the forecasting performance of GAS models comparing to state space models. Their Monte Carlo study confirms that GAS models outperform many observation-driven models in terms of predictive accuracy; and they perform closely to correctly specified parameter-driven models in terms of forecasting errors. Creal et al. [2011] consider a multivariate GAS model for times series, particularly with fat tails, where they
develop a novel method to estimate the time-varying correlations and volatilities, a method we also use. Our estimation and simulation results similarly show that the multivariate t-GAS model slightly outperforms GARCH/DCC models in terms of loglikelihood values, and provides more accurate out-of-sample predictions. We have introduced univariate student’s t GAS model in Chapter 2, as a first attempt. In this chapter, we extend it to a bivariate t-GAS model, in order to further investigate the time-varying volatilities and time-varying correlations between two correlated time series.

4.3 Data Analysis

4.3.1 Statistical Descriptions

Initiated in 2003, TTF has quickly grown into one of the biggest gas markets in Europe in term of daily trading volumes. Figure 4.3.1 plots the daily spot prices for day-ahead, month-ahead, quarter-ahead, season-ahead, and year-ahead data covering the period from early 2005 to early 2015. The day-ahead price is defined as the price for a standard unit of natural gas (one megawatt hour) to be delivered the next working day. The month-ahead price is therefore the price that will be paid for delivering one megawatt hour through each working day of the next calendar month.

Statistical descriptions of daily spot prices and daily returns are shown in Table 4.1a and 4.1b respectively. It can be seen that the average spot price is increasing in the maturity of the contract. The correlations of the spot prices between a year-ahead contract and other contracts are shown in the last column of Table 4.1a, and the correlation coefficient increases as the differences of the delivery periods are smaller. This suggests the existence of price persistence in the data. From Table 4.1b, we observe that the standard deviation is declining with the maturity of the contract. Thus the data exhibit mean reversion.

Our study focuses the dynamics between Day-ahead and Month-ahead return data. To further investigate the features of the time series, we plot the spot return time series in Figure 4.3.2a and the spot squared return time series in Figure 4.3.2b. In both figures, clustering of volatility is clearly visible. Therefore we continue our analysis with GARCH and GAS models in the later sections. Furthermore, note that the time series presents certain features of heavy tails, therefore robust GAS model is expected to incorporate these characteristics.

---

2 Quarters: Q1 = January to March, Q2 = April to June, Q3 = July to September, Q4 = October to December.
3 Seasons: Winter = October to March, Summer = April to September.
4 Source: Continental Gas Snapshot Methodology by ICIS.
4.3. Data Analysis

Table 4.1: Statistical descriptions of TTF daily spot prices and daily returns

(a) Statistical descriptions of daily spot prices

<table>
<thead>
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<th>Statistics</th>
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<th>Month</th>
<th>Quarter</th>
<th>Season</th>
<th>Year</th>
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<tr>
<td>Sample size</td>
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<td>2616</td>
<td>2616</td>
<td>2616</td>
<td>2616</td>
</tr>
<tr>
<td>Mean</td>
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<td>20.8477</td>
<td>22.0978</td>
<td>23.3084</td>
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<tr>
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Correlation

<table>
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(b) Statistical descriptions on returns on daily spot prices

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<th>Statistics</th>
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<th>Quarter</th>
<th>Season</th>
<th>Year</th>
</tr>
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<tr>
<td>Sample size</td>
<td>2615</td>
<td>2615</td>
<td>2615</td>
<td>2615</td>
<td>2615</td>
</tr>
<tr>
<td>Mean ($\times 10^{-3}$)</td>
<td>0.1990</td>
<td>0.2036</td>
<td>0.2081</td>
<td>0.1364</td>
<td>0.1807</td>
</tr>
<tr>
<td>Std.Dev.</td>
<td>0.0694</td>
<td>0.0297</td>
<td>0.0294</td>
<td>0.0282</td>
<td>0.0197</td>
</tr>
</tbody>
</table>

Correlation

<table>
<thead>
<tr>
<th></th>
<th>Day</th>
<th>Month</th>
<th>Quarter</th>
<th>Season</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>1.0000</td>
<td>0.2661</td>
<td>0.2018</td>
<td>0.1610</td>
<td>0.1449</td>
</tr>
<tr>
<td>Month</td>
<td>1.0000</td>
<td>0.3574</td>
<td>0.2541</td>
<td>0.3580</td>
<td></td>
</tr>
<tr>
<td>Quarter</td>
<td></td>
<td>1.0000</td>
<td>0.2695</td>
<td>0.3063</td>
<td></td>
</tr>
<tr>
<td>Season</td>
<td></td>
<td></td>
<td>1.0000</td>
<td>0.3501</td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td></td>
<td></td>
<td></td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

4.3.2 Seasonality

It is widely accepted that seasonality plays an essential role in energy spot markets due to the high sensitivity of gas demand and supply to temperature. Figure 4.3.3 gives a quick view on the seasonality in spot prices and returns for both day-ahead and month-ahead time series, including the mean and the standard deviation for each month.

For day-ahead and month-ahead data, the average highest (lowest) spot price over a year happens in December and October (August and April); while the most (least) volatile periods are March and September (January and January) respectively. The level and volatility of spot price for day-ahead is relatively high (low) during the winter (summer) period, which can be explained by the high (low) demand for gas in cold (warm) days. However, the price level of month-ahead time series is high and volatile in the autumn (September and October) before entering the winter and the price stays relatively high until spring. It can be explained that the gas providers are preparing for the high demand in the winter and start buying gas forwards (e.g. month-ahead contracts); while when spring comes, the gas providers feel less...
The vertical dashed lines denote auctions and the dots label the auction prices achieved through.

Figure 4.3.1: TTF daily spot prices (euro/MWh) and historical auction prices (euro/SBU)
Figure 4.3.2: Spot Returns and Spot Squared Returns of TTF Day-ahead and Month-ahead Data

(a) Spot Returns of TTF Day-ahead and Month-ahead

(b) Spot Squared Returns of TTF Day-ahead and Month-ahead
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

Figure 4.3.3: Seasonality

(a) Day-ahead

(b) Month-ahead
pressured to buy products on TTF market and the month-ahead price falls.

Table 4.2: Seasonality Tests on Day-ahead and Month-ahead Spot Prices/Returns

<table>
<thead>
<tr>
<th>Month</th>
<th>Day-ahead</th>
<th></th>
<th>Month-ahead</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spot Price</td>
<td>Price Return</td>
<td>Spot Price</td>
<td>Price Return</td>
</tr>
<tr>
<td>Dec</td>
<td>21.8998***</td>
<td>0.0005</td>
<td>22.9422***</td>
<td>-0.0029</td>
</tr>
<tr>
<td>Jan</td>
<td>-0.0572</td>
<td>-0.0015</td>
<td>-0.9877*</td>
<td>0.0001</td>
</tr>
<tr>
<td>Feb</td>
<td>-1.5373***</td>
<td>-0.0033</td>
<td>-3.2412***</td>
<td>-0.0010</td>
</tr>
<tr>
<td>Mar</td>
<td>-1.4445***</td>
<td>-0.0002</td>
<td>-3.6333***</td>
<td>0.0039</td>
</tr>
<tr>
<td>Apr</td>
<td>-2.6608***</td>
<td>-0.0046</td>
<td>-3.8597***</td>
<td>0.0022</td>
</tr>
<tr>
<td>May</td>
<td>-2.6758***</td>
<td>0.0013</td>
<td>-3.4407***</td>
<td>0.0041</td>
</tr>
<tr>
<td>Jun</td>
<td>-2.8741***</td>
<td>-0.0003</td>
<td>-3.7248***</td>
<td>0.0028</td>
</tr>
<tr>
<td>Jul</td>
<td>-2.6251***</td>
<td>-0.0010</td>
<td>-3.3896***</td>
<td>0.0042</td>
</tr>
<tr>
<td>Aug</td>
<td>-3.1393***</td>
<td>0.0015</td>
<td>-3.1206***</td>
<td>0.0057**</td>
</tr>
<tr>
<td>Sep</td>
<td>-1.6454***</td>
<td>-0.0022</td>
<td>-0.5338</td>
<td>0.0099***</td>
</tr>
<tr>
<td>Oct</td>
<td>-1.6704***</td>
<td>0.0038</td>
<td>0.4623</td>
<td>0.0036</td>
</tr>
<tr>
<td>Nov</td>
<td>-0.3887</td>
<td>0.0032</td>
<td>0.2715</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

***, **, * represent 1%, 5%, 10% significance level respectively.

Unfortunately the spot price time series contains a unit root, therefore we instead focus on the return series of day-ahead and month-ahead data, which are stationary according to the results of Phillips-Perron tests.

Table 4.2 illustrates the results of formal tests for seasonality in spot prices and returns for both day-ahead and month-ahead data. Seasonality effect is presented significantly for both spot day-ahead and month-ahead prices. Despite the test rejects the significance effects of seasonality in the daily price returns for day-ahead, seasonality still exists in the return series of month-ahead data: significantly higher average spot returns are found in August and September comparing to the average return in December. We have to take this into account in our modeling and a simple solution would be described as follows. First eliminate the seasonality effect and model the adjusted time series with standard GARCH or GAS models. Second, add back the seasonality effect to the simulated future data series.

4.4 Auction Analysis

The storage capacity auctions conducted by GasTerra are carried out over at least two rounds and at most five rounds. For each round, bidders have to submit their bids including both the price for one Standard Bundled Unit (euro per SBU) and the amount of SBUs required. After the first round, a start price for the second round will be announced, which serves as
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

A minimum bidding price for the second round. Given this information, bidders may adjust their bidding prices in the next round, provided that only upward adjustment is allowed. Those who decide not to raise their bids above the minimum price of the subsequent round will be considered out of market. Unfortunately, we only have the final weighted average price paid by successful bidders, but not the auction data for each round of auctions in detail.

The first gas storage auction in the Netherlands was held by GazTerra on March 28, 2011, followed by ten auctions since then. Both Figure 4.3.1 and Table 4.3 present the historical auction prices for GazTerra Storage Service, obtained from ICE-ENDEX. More specifically, Table 4.3 shows the weighted average price of each auction held, where a price of zero means an auction failure. In addition, the last column of Table 4.3 gives the information on the highest bid even if the auction failed. The auction price is defined as euros per SBU.

Table 4.3: Historical Auction Prices From GazTerra Storage Service (Data source: ICE-ENDEX)

<table>
<thead>
<tr>
<th>Auction Date</th>
<th>Auction Price (euro/SBU)</th>
<th>Highest bid if failed (euro/SBU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28-Mar-2011</td>
<td>0</td>
<td>2.96</td>
</tr>
<tr>
<td>26-Apr-2011</td>
<td>4.82</td>
<td></td>
</tr>
<tr>
<td>27-Apr-2011</td>
<td>4.11</td>
<td></td>
</tr>
<tr>
<td>29-Nov-2011</td>
<td>0</td>
<td>3.01</td>
</tr>
<tr>
<td>15-Feb-2012</td>
<td>6.28</td>
<td></td>
</tr>
<tr>
<td>28-Nov-2012</td>
<td>3.63</td>
<td></td>
</tr>
<tr>
<td>13-Feb-2013</td>
<td>0</td>
<td>1.47</td>
</tr>
<tr>
<td>21-Nov-2013</td>
<td>2.10</td>
<td></td>
</tr>
<tr>
<td>12-Feb-2014</td>
<td>2.58</td>
<td></td>
</tr>
<tr>
<td>19-Nov-2014</td>
<td>2.87</td>
<td></td>
</tr>
<tr>
<td>04-Feb-2015</td>
<td>2.57</td>
<td></td>
</tr>
</tbody>
</table>

A natural question that follows is how the auction results are related to the market. One widely accepted belief is that the bidders consider the winter-summer difference as the main elements of the storage prices. In order to understand whether the statement is true or false, we compute the correlations between auction prices and the gas market daily spot prices/returns in Table 4.4, including the average daily spot price/return and standard deviation of daily price/return in the past 1/3/6 month(s), as well as the winter-summer price/return difference in the previous year and in the next year. One surprising observation is that, opposite to the popular opinion mentioned before, the correlations between auction prices and the winter-summer price/return difference are relatively low, for both cases of winter-summer price difference of the previous year and the following year.
Table 4.4: Correlation between auction prices and gas market daily spot prices/returns

<table>
<thead>
<tr>
<th>Correlation between auction price and gas market daily spot price</th>
<th>Auction price</th>
<th>Auction price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(when failed, 0)</td>
<td>(when failed, the highest bid)</td>
</tr>
<tr>
<td>Winter-Summer price difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the previous year</td>
<td>-0.0062</td>
<td>-0.0687</td>
</tr>
<tr>
<td>in the next year</td>
<td>0.1691</td>
<td>-0.1713</td>
</tr>
<tr>
<td>Average daily spot price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the past 1 month</td>
<td>-0.1217</td>
<td>-0.1981</td>
</tr>
<tr>
<td>in the past 3 months</td>
<td>-0.1925</td>
<td>-0.3192</td>
</tr>
<tr>
<td>in the past 6 months</td>
<td>-0.1573</td>
<td>-0.3227</td>
</tr>
<tr>
<td>Std. Dev. of daily spot price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the past 1 month</td>
<td>0.5769</td>
<td>0.7365</td>
</tr>
<tr>
<td>in the past 3 months</td>
<td>0.3224</td>
<td>0.5125</td>
</tr>
<tr>
<td>in the past 6 months</td>
<td>0.2110</td>
<td>0.2092</td>
</tr>
<tr>
<td>Winter-Summer return difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the previous year</td>
<td>0.2082</td>
<td>0.2349</td>
</tr>
<tr>
<td>in the next year</td>
<td>0.0191</td>
<td>0.3228</td>
</tr>
<tr>
<td>Average daily return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the past 1 month</td>
<td>-0.0663</td>
<td>-0.0440</td>
</tr>
<tr>
<td>in the past 3 months</td>
<td>-0.0750</td>
<td>-0.1016</td>
</tr>
<tr>
<td>in the past 6 months</td>
<td>0.2588</td>
<td>0.2862</td>
</tr>
<tr>
<td>Std. Dev. of daily return</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in the past 1 month</td>
<td>0.6040</td>
<td>0.7989</td>
</tr>
<tr>
<td>in the past 3 months</td>
<td>0.4663</td>
<td>0.7692</td>
</tr>
<tr>
<td>in the past 6 months</td>
<td>0.5184</td>
<td>0.7629</td>
</tr>
</tbody>
</table>
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

In all the figures in Table 4.4, the standard deviations of the spot returns in the past 1/3/6 month(s) are the most highly and positively correlated to the auction prices (with highest bid prices considered given auction failure). Since observed volatility (standard deviation) is the standard benchmark for investors to assess risks, the result implies that when the observed market risk is high, the storage buyers assign high value to the storage capacity. This is reasonable since storage capacity is an effective tool to hedge market risk, given that the corresponding market on futures and options is poorly developed. This point is missing from most of the literature.

Moreover, the average daily price in the past 1/3/6 month(s) is negatively correlated to the auction price with failed prices considered. In other words, if the average price in the past few months is high, the storage buyers tend to bid less for the future storage; and vice versa. One possible explanation is that a low market price implies a high market supply, thus the potential storage investors have easy access to the gas products and do not need storage capacity for insurance reasons. Note that if the price of a failed auction is given as zero, the correlation between average daily price and auction price is still low and negative.

In brief, both the low correlation between winter-summer difference and the storage capacity prices, and the high correlation between the market volatility and the storage capacity prices have led to the invalidation of the commonly accepted belief. Therefore, we need to develop a new model to accommodate and explain the counter-intuitive observations.

4.5 Decision Strategy

This section presents the model for pricing storage capacity. A key feature of our approach involves parameterizing key factors affecting the investors’ valuation for the storage capacity. The capacity storage service provided by GasTerra is an example of slow storage, constrained by both the storage space and the injection/withdrawal capacity. We assume there is a 2-day delay of the execution after the decision is made, and no further action can be undertaken before the execution of the previous action is finished. An indirect tradeoff of withdrawal or injection is that the larger amount is ordered, the longer periods are locked from new action being enforced. Therefore, in order to effectively take advantage of the storage capacity, we assume the capacity holder makes a decision with monthly frequency. Thus the capacity holder is able to continue with their injection/withdrawal in the following period or revise their initial long-term plan based on new market information.

---

5 A fast storage does not have the injection/withdrawal capacity constraint, which is therefore simpler to deal with regarding modeling and optimization.
4.5 Decision Strategy

4.5.1 Value Functions

Consider the pricing of a standard contract of one SBU, with the maturity of one year\(^6\) (i.e. \(T = 1\)). Define the maximum capacity or storage space \(C^{max}\), minimum space \(C^{min}\) and the current usage level \(C_t\), therefore we have \(C_t \in [C^{min}, C^{max}]\). For a GasTerra product, \(C^{min} = 0\) kWh and \(C^{max} = 1440\) kWh. After observing the current storage usage level \(C_t\) and the current market gas price \(P^D_t\) (day-ahead) and \(P^M_t\) (month-ahead) at time \(t\) \((t \in [0, T])\), the capacity holder makes a decision of injection \((A^I)\), or withdrawal \((A^W)\), or doing nothing \((A^N)\). Define \(\pi\) as the action set \(\pi(C_t, P^D_t, P^M_t) = \{A^I_t, A^W_t, A^N_t\}\).

Here, the injection capacity is \(r^I = 0.3333\) kWh/h and withdrawal capacity is \(r^W = 1.0\) kWh/h. Note that for an SBU offered by GasTerra, it takes 180 days to inject an empty capacity space until its maximum capacity and it takes at least 60 days (approximately 69 days) to deplete a full storage.

Costs for injection and withdrawal are \(c^I = 0.00042\) euros/kWh and \(c^W = 0.00003\) euros/kWh respectively, and it is never optimal to withdraw and to inject at the same time due to the positive costs related to both actions. The injection factor \(b^I\) is constant and equals 1.0, and the withdrawal factor \(b^W\) is fixed in the contract and defined as (also see Figure 4.9.1)

\[
b^W_t = b^W_t(C_t) = \min\left(1, \frac{1}{2160} C_t + 0.6\right)
= \begin{cases} 
1 & \text{if } 864\text{kWh} \leq C_t \leq C^{max} \\
\frac{1}{2160} C_t + 0.6 & \text{if } C^{min} \leq C_t < 864\text{kWh}
\end{cases}
\]

The immediate payoff function at time \(t\) for injection can be written as

\[
v^I_t = - \left(\min\left(P^D_t, P^M_t\right) + c^I\right) A^I_t;
\]

and similarly, the withdrawal payoff function at \(t\) is (the gas extracted can be sold one day after the completion of withdrawal)

\[
v^W_t = \left(\max\left(P^D_t, P^M_t\right) - c^W\right) A^W_t.
\]

Naturally, the payoff function for \(A^N\) is simply

\[
v^N_t = 0.
\]

\(^6\)We consider a one-year contract offered by GasTerra covering the period from 1 April up to 31 March (incl.) the following year.
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

At the final date \( T \), the action injection is not allowed, therefore the payoff function is

\[ v^W_T = \left( \max \left( P^D_T, P^M_T \right) - c^W \right) C_T \]

where a fine \( c^W > 0 \) has to be paid out for any gas in storage \( C_T \) left at the final contract date.

The optimal value function at time \( t \) is \( V_t \left( P^D_t, P^M_t, C_t \right) \). Given a discount rate \( \beta \), the dynamic program for \( t \in [0, T) \) can be formulated as,

\[
V_t \left( P^D_t, P^M_t, C_t \right) = \max_{A_j \in \pi \left( C_t, P^D_t, P^W_t \right)} \left[ w_t \left( A_j, P^D_t, P^M_t, C_t \right) \right]
\]

\[
w_t \left( A_j, P^D_t, P^M_t, C_t \right) = v_j^t + \beta E^Q \left[ V_{t+1} \left( P^D_t, P^M_t, C_t + A_j \right) \right]
\]

where \( j \in \{ I, W, N \} \).

4.5.2 Utility Function

As we have discussed before, preference free pricing does not exist in incomplete markets, so we have to parametrize the risk aversion of the investor to deal with the market incompleteness. In the previous chapters we adopted CARA (constant absolute risk aversion) utility function. However, the problem considered in this chapter is fundamentally different. Since the auction price is only given as the weighted average for one SBU, it is reasonable to assume a CRRA (constant relative risk aversion) utility function. As a result, we assume the investor has a utility function as below:

\[
u(x) = \frac{x^{1-\alpha} - 1}{1 - \alpha}
\]

where \( \alpha > 0 \) is her risk aversion parameter. Note that when \( \alpha \Rightarrow 1 \), this utility function approaches a log utility in the limit and can be replaced by logarithmic utility for \( \alpha = 1 \).

4.5.3 Modified LSMC Method

Boogert and De Jong [2008] adopt LSMC for capacity pricing by using grids of gas-in-storage volumes to form basis functions. However, as we have explained in the previous section, the gas-in-storage is highly dependent on previous actions and the grids may not be achievable, which may result in an overestimation of prices due to the ignorance of constraints. Thus, instead of assigning grids of storage levels for each period, we modify LSMC in the following manner. First we compute the optimized storage strategy for each simulated day-ahead and month-ahead pair. Second, similar to LSMC, start from the final
date $T$ and compute the final cash flows. Third, regress the final cash flows on basis functions formulated from simulated day-ahead and month-ahead prices as well as gas-in-storage and injection/withdrawal amount at time $T - \Delta t$; and the fitted value from this regression is the continuation value. Fourth, compare the continuation value with the value of a project when no further action would be taken afterward, and work backward.

4.6 Estimation Results

We model the clustering volatility via both GARCH and GAS models, based on both normality and student t assumptions. The results are reported in Appendix 4.9.2.

Table 4.5: Estimation Results for the Bivariate t-GAS Model

<table>
<thead>
<tr>
<th>Bivariate t-GAS</th>
<th>Seasonality Eliminated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$ (Variance)</td>
<td>1.3743***</td>
</tr>
<tr>
<td></td>
<td>1.4263***</td>
</tr>
<tr>
<td>$\omega$ (Correlation)</td>
<td>0.2529***</td>
</tr>
<tr>
<td>$A$</td>
<td>0.0779***</td>
</tr>
<tr>
<td></td>
<td>0.0304***</td>
</tr>
<tr>
<td></td>
<td>0.0065***</td>
</tr>
<tr>
<td>$B$</td>
<td>0.9985***</td>
</tr>
<tr>
<td></td>
<td>0.9980***</td>
</tr>
<tr>
<td></td>
<td>0.9978***</td>
</tr>
<tr>
<td>$\nu$</td>
<td>3.2935***</td>
</tr>
<tr>
<td>Loglikelihood</td>
<td>-2345.2</td>
</tr>
<tr>
<td>AIC</td>
<td>4710.4</td>
</tr>
<tr>
<td>BIC</td>
<td>4769.1</td>
</tr>
</tbody>
</table>

***, **, * represent 1%, 5%, 10% significance level respectively.

It is evident from both Table 4.6 and Table 4.7 that both the day-ahead and month-ahead data series are heavy-tailed, thus Student t assumption is preferred over a Gaussian one. When comparing t-GARCH models with t-GAS models, the latter yields a higher loglikelihood level, which implies a better fit of the data. Take the results for day-ahead data for example. From Table 4.6, we conclude that the t-GAS model fits the data better than t-GARCH one, with rejecting a loglikelihood ratio test $LR = 2 \times (-2144 + 2164) = 40 > \chi^2 (1)$. Furthermore, the estimated degrees of freedom $\nu$ is 3.7238 with the t-GAS model and is significant with 1% level, which proves the heavy-tailed feature in the day-ahead return series. Further discussions in model selections and result comparisons between GARCH and GAS models are presented in Appendix 4.9.2.
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

Table 4.5 demonstrates the estimation results from a Bivariate t-GAS model. As can be seen, the estimated degree of freedom is 3.2935, which again confirms the fat-tails featured in the data.

4.7 Results for Storage Capacity Prices

4.7.1 Comparison of two auctions for the same product

The hedging plan can be described as following: the investor compares the day-ahead and month-ahead price on a monthly basis, then buys and injects at the low price, and withdraws and sells at the high price. Note that injection and withdrawal are mutually exclusive actions, so the investor has to make a decision one way or the other at any given time.

We compare our calculations for two auctions held respectively on February 4, 2015 and November 19, 2014. Intuitively, the prices for the two auctions should be close to each other given that the auction target for both auctions is exactly the same contract. In fact, given other conditions the same, the price for the first auction should be expected to be slightly lower than the second since the latter is closer to the maturities of the options brought in by storage capacity. However, the realized auction prices not only differ from each other, with 2.87 euros/SBU for November 19, 2014, and 2.57 euros/SBU for February 4, 2015 respectively, but the second auction price is actually lower in spite of the shorter time to maturity.

Our modeling results for capacity prices are shown in Figure 4.7.1. The red dot presents the realized auction price on February 4, 2015 and the solid red line is the estimated price based on our model given different risk aversion parameters of potential storage capacity buyers; the orange dashed line and dot present estimated price and realized auction price for the auction held on November 19, 2014. Note that the contracts for both auctions are the same, i.e. one standard SBU contract covering the period of April 1, 2015 to March 31, 2016. For each calculation of the capacity prices, we use the market information until the date of the auction.

Several observations stand out. First for given risk aversion, bidders valued the contract less during the second auction than during the first; the solid line lies everywhere below the slotted line in 4.7.1. To see why, look at Figure 4.7.2: there we show that between the two auction dates, both the market price and the market volatility changed, the price went up somewhat but volatility went down. Table 4.4 shows that both price and volatility are positively correlated with the auction price, which makes sense given the call option characteristics of the contract. But the volatility effect apparently dominated, which can be seen from Figure 4.7.1: the estimated prices for November 19, 2014 dominate those for February 4, 2015, for all risk aversion levels. Higher volatility leads to higher option values,
as is to be expected given the convexity of the payouts of the implied options embedded in the storage contract. If the storage capacity is an effective hedging contract, it is reasonable that the bidder is more keen on a hedging product when facing a more risky market.

Second, the revealed risk aversion of the average capacity buyer on February 4, 2015 is lower than the one on November 19, 2014\(^7\). The implied risk aversion of the successful bidders is around 2.3 for the auction taking place on November 19, 2014, and 1.8 for the second auction on February 4, 2015. This can be understood from the incomplete markets structure of the problem. In a complete market setting risk aversion heterogeneity of bidders has no impact on the price (preference free pricing or risk neutral valuation) because additional risk will be compensated for at the market price of risk. However with markets incompleteness that neutrality breaks down and shifts in the composition of the pool of bidders can have an impact on the price. The lower value given risk aversion (the solid line lies below the slotted line in 4.7.1) has as a consequence that only less risk averse investors are drawn into the auction the second time around, with as a result a lower average implied degree of risk aversion. This can be further explained by Figure 4.7.3, which ranks all potential bidders in order of increasing risk aversion from left to right. As their value of storage capacity goes down from \(V_H\) to \(V_L\), more risk averse bidders will be driven out of the market. As a result,

\(^7\)The auction design is such that every successful bidder pays his/her bidprice, and the reported auction price is the average price paid, so the implicit risk aversion is a (complicated) weighted average of the risk aversion parameter of all successful bidders.
Figure 4.7.2: Statistics Comparison of Past 6-month Market Information Before Auction Date (November 19, 2014 and February 4, 2015)
the implied average risk aversion of bidders goes down.

Figure 4.7.3: Rank All Potential Bidders in Order of Increasing Risk Aversion

A third interesting observation is that the plot of capacity price as a function of risk aversion is declining faster in case of the auction taken on November 19, 2014 than the one taken on February 4, 2015. This suggests that investors with various risk aversion levels disagree more in valuing the capacity product when the market is more volatile (November 19, 2014).

### 4.7.2 Comparing two more auctions: November 21, 2013 and February 12, 2014

Figure 4.7.4: Risk Aversion vs. Storage Capacity Price (November 21, 2013 and February 12, 2014)
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

A similar line of reasoning explains the results stemming from comparing another set of auction results, those of November 21, 2013 and February 12, 2014, as shown in Figure 4.7.4. Note that the market risk is higher for the second auction in this case, see Figure 4.7.5. In this case, higher volatility causes the second auction schedule to lie above the first, so more investors are drawn into the auction and the average implied coefficient of risk aversion is higher the second time around.

We only compare these two pairs of auctions because at least one auction failed for the other pairs.

4.7.3 Injection/Withdrawal Rate vs. Capacity Price

Figure 4.7.6: Capacity Price Given the Injection/Withdrawal Rate $X$ times the Capacity Rate of a Standard SBU

We have calculated the price for one standard SBU as specified in the GasTerra gas storage auction, which is a slow storage as explained in Section 4.5. In this section we do not assume a fixed rate, but we investigate how the injection/withdrawal capacity affects the value of the storage contract. Assume six different contracts with the injection/withdrawal rate equal to $X$ times the original contract on November 19, 2014. The benchmark case is $X = 1$, which is the same contract as we considered before (the second auction in Section 4.6). If $X > 1$, it implies a faster storage and naturally results in a higher value of the contract since the capacity holder would have more flexibility within the contract periods; and vice versa.
Figure 4.7.5: Statistics Comparison of Past 6-month Market Information Before Auction Dates (November 21, 2013 and February 12, 2014)
This impact of capacity rates is demonstrated in Figure 4.7.6. For instance, when the capacity rate is only half as the benchmark \( X = 0.5 \), the capacity price would drop to 1.38 euros per (nonstandard) SBU given the implied risk aversion \( \alpha = 2.48 \), nearly half the price that the benchmark implies (2.57 euros/SBU); while when \( X = 2 \), the price increases to 4.14 euros per (nonstandard) SBU with \( \alpha = 2.48 \), about 1.61 times of the benchmark price. Therefore we conclude that the value of the contract is increasing with the capacity rate, however, the marginal effect of capacity rate is decreasing.

4.8 Conclusion

In this chapter, we have developed a model for pricing gas storage capacity, a model that focuses on the interactions between the gas spot market, price volatility and gas storage capacity. We explicitly model the time-varying volatilities and correlations between TTF spot day-ahead and month-ahead returns. Due to the heavy-tailed feature of the data, we adopt the recently proposed GAS models. Given a feasible hedging strategy, we then adopt and modify the Least Square Monte Carlo method for pricing purposes. Several interesting findings are worth mentioning.

First, by comparing two auctions with the same target contract, we find that the investors are willing to pay higher prices for storage contracts when the market is more volatile, given the same risk aversion level. This stems from the insurance feature of a capacity contract, a feature that is missing from most of the existing literature. A second feature of our results is inextricably linked to the incomplete markets nature of our set up: the price is affected by the pool of bidders. A higher value given any degree of risk aversion draws in more investors, changing the implied average risk aversion of the successful pool of bidders. The implied risk aversion is in all auction cases considered not zero, invalidating the assumption of risk-neutrality typically made in the literature. The point to note is that the revealed risk aversion of the bidders is far from zero, contradicting the assumption of risk neutrality made in the literature on capacity pricing. Apparently market incompleteness is essential and cannot be overlooked. Third, our results also suggest that investors disagree more on the storage capacity prices when the market is riskier. Finally, a high injection/withdrawal capacity rate would increase the contract value, which shows that fast storage is more valuable than slow storage, given everything else.
4.9 Appendix

4.9.1 A Standard GasTerra Contract

An SBU for a standard GasTerra Product (source: iceendex.com) is specified as follows.

1. Available storage space: 1440 kWh

2. Injection cost: EUR 0.00042 per kWh

3. Injection factor: 1.0

4. Withdrawal capacity: 1.0 kWh/h

5. Withdrawal cost: EUR 0.00003 per kWh

6. Withdrawal factor: if the gas-in-storage is between 0 kWh and 864 kWh, it increases linearly from 0.6 to 1; if the gas-in-storage is between 864 kWh and 1440 kWh, it equals 1. See Figure 4.9.1.

7. A Late Storage Fee has to be paid out for any gas remaining in Gas-in-Storage at the end of the contract Period.
4. Asset Pricing in Incomplete Markets: Valuing Gas Storage Capacity

Figure 4.9.1: Withdrawal Factor, storage service agreement 2015-2016, ICE-ENDEX

### 4.9.2 GARCH vs. GAS

Table 4.6: Estimate Results for Day-ahead Daily Logarithm Return Data

<table>
<thead>
<tr>
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<th>Gaussian GAS</th>
<th>t-GARCH</th>
<th>t-GAS</th>
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<tr>
<td>omega</td>
<td>12.3954</td>
<td>-0.8901***</td>
<td>5.8559</td>
<td>-0.8943***</td>
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<td>A</td>
<td>0.2285***</td>
<td>0.1493***</td>
<td>0.1568***</td>
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<tr>
<td>B</td>
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<td>0.9244***</td>
<td>0.9985***</td>
<td>0.9546***</td>
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<tr>
<td>ν</td>
<td>3.5305</td>
<td>3.7238</td>
<td></td>
<td></td>
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<tr>
<td>Log-likelihood</td>
<td>-2399.465</td>
<td>-13657.141</td>
<td>-2164.468</td>
<td>-2144.376</td>
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*, **, *** represent a significance level of 10%, 5%, and 1% respectively.

Table 4.7: Estimate Results for Month-ahead Daily Logarithm Return Data

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<th>Gaussian GAS</th>
<th>t-GARCH</th>
<th>t-GAS</th>
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<td>0.4414</td>
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<td>A</td>
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<td>0.0293***</td>
<td>0.0780***</td>
<td>0.0851***</td>
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<td>B</td>
<td>0.9934***</td>
<td>0.9829***</td>
<td>0.9973***</td>
<td>0.9865***</td>
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<tr>
<td>ν</td>
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<td>2.6297</td>
<td></td>
<td></td>
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<td>Log-likelihood</td>
<td>-1103.944</td>
<td>-140115.742</td>
<td>-405.425</td>
<td>-402.205</td>
</tr>
</tbody>
</table>

*, **, *** represent a significance level of 10%, 5%, and 1% respectively.
Moreover, as shown in Figure 4.3.1 and Table 4.1, the high correlation between the day-ahead and month-ahead time series could not be simply overlooked. Therefore, we extend the univariate t-GAS models into a bivariate t-GAS model. Let $y_{1t}$ and $y_{2t}$ be the logarithm daily returns of day-ahead and month-ahead respectively, and we have

$$y_t = \epsilon_t \text{ or } \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

where $\epsilon_t$ follows a bivariate student t distribution, whose covariance matrix is

$$\Sigma_t = \begin{pmatrix} \sigma^2_{1t} & \rho_t \sigma_{1t} \sigma_{2t} \\ \rho_t \sigma_{1t} \sigma_{2t} & \sigma^2_{2t} \end{pmatrix}$$

and degrees of freedom is $\nu$. Following Creal et al. [2011], we choose the factor

$$f_t = \begin{pmatrix} \sigma^2_{1t} \\ \sigma^2_{2t} \\ \text{vech}(Q_t) \end{pmatrix}$$

where

$$R_t = \Delta_t^{-1} Q_t \Delta_t^{-1} = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix}$$

is the correlation matrix and $Q_t$ is symmetric and positive definite. To guarantee the properties of the correlations matrix, (e.g. symmetric, positive definite, and $|\rho_t| < 1$), the elements of the diagonal matrix $\Delta_t$ is chosen as the square root of the diagonal elements of $Q_t$. The factor follows an autoregressive process as

$$f_{t+1} = \omega + A_1 s_t + B_1 f_t$$

where the scaled score function $s_t$ is defined as

$$s_t = S_t \nabla_t \text{ and } \nabla_t = \frac{\partial \log p(y_t|f_t, F_{t-1}; \theta)}{\partial f_t}$$

with $S_t$ being the inverse of the information matrix.
Summary

Methods to assess real option value embedded in economic projects have been around for a long time, but have by and large been dismissed in practice because real world problems quickly lead to what is called the curse of dimensionality. Their solution requires solving quintessentially non-linear stochastic dynamic optimization problems, and the numerical problems solving those become rapidly insurmountable as the problem’s dimensionality increases. However, we demonstrate that a dimension reduction approach for the valuation of American options can also be successfully applied to the stochastic dynamic programming (SDP) problems that arise in complex high dimensionality real option problems.

Together with Prof. Sweder van Wijnbergen, we analyze several practical applications that have arisen recently due to the emergence of new energy market. In Chapters 2 and 3 we consider the valuation of two connected off shore gas fields in the presence of price uncertainty with time-varying variable volatility, intertemporally correlated uncertainty and even model ambiguity concerning the reservoir size of the two connected gas fields (Chapter 3). Our analysis finds very substantial payoffs to explicitly modeling asymmetries in variances, substantial option values in the presence of unhedgeable risks (although in that case we show them to depend on preferences) and the importance for decision making of taking into account (declining) model ambiguity. We show that traditional valuation approaches will consistently underestimate the value of project flexibility and in general lead to overly conservative investment decisions in the presence of time dependent stochastic structures.

Chapter 4 focuses on gas storage capacity, a different gas product embedded with option properties. Our model is able to replicate the prices that the investors are willing to pay for storage capacities. We also provide explanations of investors’ behavior and their bidding reaction with respect to the dynamics of spot market.

In this thesis we have discussed several applications of real option theory in the energy sector. It is worth mentioning that before applying any option evaluation methods, additional analytical procedures need to be carefully executed. The investor first has to reformulate the development plan into a strategic one, which exploits all the inherent managerial flexibilities.
embedded in the investment project. Next, in order to determine an optimal investment strategy, the investor has to consider mainly three aspects: the dynamics of the underlying asset returns, the constraints on the investment strategy, and the value of her strategy. Each aspect significantly affects the final decisions and has to be carefully taken into account.
Hoewel er al heel lang methodes bestaan om de reële optie-waarde van economische projecten te berekenen, heeft de praktijk deze methodes tot nu toe links laten liggen. De reden hiervoor is dat toepassing van deze methodes al heel snel tot dimensionaliteitsproblemen leidt. Het oplossen van dergelijke problemen vergt namelijk het oplossen van typische niet-lineaire stochastisch dynamische optimalisatie problemen. De numerieke problemen bij het oplossen worden heel snel onoverkomelijk wanneer de dimensionaliteit van het probleem toeneemt. In ons onderzoek laten we echter zien dat een benadering die gebruikt wordt voor het waarderen van Amerikaanse opties, en bestaat uit het reduceren van het aantal dimensies, ook toegepast kan worden op stochastisch dynamische problemen (SDP), die ontstaan bij het oplossen van complexe reële optie problemen met een hoog aantal dimensies.

Samen met professor Van Wijnbergen analyseer ik verscheidene praktische toepassingen die zijn ontstaan door de opkomst van een nieuwe energiemarkt. In hoofdstuk 2 en 3 kijken we naar de waardering van twee onderling verbonden offshore gasvelden in de aanwezigheid van prijsonzekerheid met tijdsafhankelijke volatiliteit, intertemporeel gecorreleerde onzekerheid, en zelfs modeldubbelzinnigheid wat betreft de grootte van het reservoir van de twee verbonden gasvelden (hoofdstuk 3). Uit onze analyse blijkt dat de waardering van het project substantieel omhoog gaat door het expliciet modelleren van asymmetrie in de variantie, omdat de optie-waarde van het project (die bij andere waarderingsmethodes niet wordt meegenomen) substantieel is in de aanwezigheid van niet af te dekken risico’s (hoewel deze optie-waarde in dit geval blijkt af te hangen van de preferenties van de investeerders), en laat het belang zien van het rekening houden met (afnemende) modeldubbelzinnigheid voor het nemen van beslissingen. We laten zien dat de traditionele waardering strategieën de waarde van project flexibiliteit structureel onderwaarderen, en over het algemeen tot investeringsbeslissingen leidt die te conservatief zijn in de aanwezigheid van tijdsafhankelijke stochastische structuren.

In hoofdstuk 4 kijken we naar gasopslagcapaciteit, een ander gasproduct dat optiekarakteristieken vertoont. Ons model is in staat om de prijzen te reproduceren die investeerders
6. Samenvatting

bereid zijn te betalen voor opslagcapaciteit. We geven verschillende verklaringen voor het gedrag van investeerders, en hun biedingsgedrag met betrekking tot de dynamiek op de spot markt.

In dit proefschrift hebben we verscheidene toepassingen van reële optie theorie in de energie sector bestudeerd. Daarbij is het belangrijk om verscheidene analytische procedures zorgvuldig uit te voeren voordat men overgaat op toepassing van optie-evaluatie methodes. Een investeerder dient in de eerste plaats het ontwikkelingsplan te herformuleren in een strategisch plan dat in staat is om gebruik te maken van de inherente management flexibiliteit die besloten ligt in het investeringsproject. Om vervolgens de optimale investeringsstrategie te bepalen moet de investeerder naar drie aspecten kijken: de dynamiek van de onderliggende rendementen, de beperkingen die van toepassing zijn op de investeringsstrategie, en de waarde van de betreffende strategie. Elk van deze drie aspecten heeft namelijk een belangrijke invloed op de uiteindelijke investeringsbeslissing, en moet zorgvuldig worden meegewogen.


Mark Davis. Pricing weather derivatives by marginal value. 2001.


Andrew C Harvey and Tirthankar Chakravarty. *Beta-(e) garch*. University of Cambridge, Faculty of Economics, 2008.


The Tinbergen Institute is the Institute for Economic Research, which was founded in 1987 by the Faculties of Economics and Econometrics of the Erasmus University Rotterdam, University of Amsterdam and VU University Amsterdam. The Institute is named after the late Professor Jan Tinbergen, Dutch Nobel Prize laureate in economics in 1969. The Tinbergen Institute is located in Amsterdam and Rotterdam. The following books recently appeared in the Tinbergen Institute Research Series:

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<table>
<thead>
<tr>
<th>Page</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>601</td>
<td>R.J.D. POTTER VAN LOON, <em>Modeling non-standard financial decision making</em></td>
</tr>
<tr>
<td>602</td>
<td>G. MESTERS, <em>Essays on Nonlinear Panel Time Series Models</em></td>
</tr>
<tr>
<td>603</td>
<td>S. GUBINS, <em>Information Technologies and Travel</em></td>
</tr>
<tr>
<td>604</td>
<td>D. KOPÁNYI, <em>Bounded Rationality and Learning in Market Competition</em></td>
</tr>
<tr>
<td>605</td>
<td>N. MARTYNOVA, <em>Incentives and Regulation in Banking</em></td>
</tr>
<tr>
<td>606</td>
<td>D. KARSTANJE, <em>Unraveling Dimensions: Commodity Futures Curves and Equity Liquidity</em></td>
</tr>
<tr>
<td>607</td>
<td>T.C.A.P. GOSENS, <em>The Value of Recreational Areas in Urban Regions</em></td>
</tr>
<tr>
<td>608</td>
<td>Ł.M. MARĆ, <em>The Impact of Aid on Total Government Expenditures</em></td>
</tr>
<tr>
<td>609</td>
<td>C. LI, <em>Hitchhiking on the Road of Decision Making under Uncertainty</em></td>
</tr>
<tr>
<td>610</td>
<td>L. ROSENDAHL HUBER, <em>Entrepreneurship, Teams and Sustainability: a Series of Field Experiments</em></td>
</tr>
<tr>
<td>611</td>
<td>X. YANG, <em>Essays on High Frequency Financial Econometrics</em></td>
</tr>
<tr>
<td>612</td>
<td>A.H. VAN DER WEIJDE, <em>The Industrial Organization of Transport Markets: Modeling pricing, Investment and Regulation in Rail and Road Networks</em></td>
</tr>
<tr>
<td>614</td>
<td>C. DIETZ, <em>Hierarchies, Communication and Restricted Cooperation in Cooperative Games</em></td>
</tr>
<tr>
<td>615</td>
<td>M.A. ZOICAN, <em>Financial System Architecture and Intermediation Quality</em></td>
</tr>
<tr>
<td>616</td>
<td>G. ZHU, <em>Three Essays in Empirical Corporate Finance</em></td>
</tr>
<tr>
<td>617</td>
<td>M. PLEUS, <em>Implementations of Tests on the Exogeneity of Selected Variables and their Performance in Practice</em></td>
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<tr>
<td>618</td>
<td>B. VAN LEEUWEN, <em>Cooperation, Networks and Emotions: Three Essays in Behavioral Economics</em></td>
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<tr>
<td>619</td>
<td>A.G. KOPÁNYI-PEUKER, <em>Endogeneity Matters: Essays on Cooperation and Coordination</em></td>
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<tr>
<td>620</td>
<td>X. WANG, <em>Time Varying Risk Premium and Limited Participation in Financial Markets</em></td>
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<tr>
<td>621</td>
<td>L.A. GORNICKA, <em>Regulating Financial Markets: Costs and Trade-offs</em></td>
</tr>
<tr>
<td>622</td>
<td>A. KAMM, <em>Political Actors playing games: Theory and Experiments</em></td>
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<td>S. VAN DEN HAUWE, <em>Topics in Applied Macroeconometrics</em></td>
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<td>F.U. BRÅUNING, <em>Interbank Lending Relationships, Financial Crises and Monetary Policy</em></td>
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<td>J.J. DE VRIES, <em>Estimation of Alonso’s Theory of Movements for Commuting</em></td>
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<td>626</td>
<td>M. POPŁAWSKA, <em>Essays on Insurance and Health Economics</em></td>
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<td>X. CAI, <em>Essays in Labor and Product Market Search</em></td>
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