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Synchronously pumped laser without inversion in cadmium

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We report a lasing without inversion (LWI) experiment on the basis of coherent population trapping, realized in low-pressure cadmium vapor. A single-pass inversionless amplification by more than a factor of 10 has been measured, a significant improvement over previous experiments. In addition, we report synchronously pumped laser oscillation without inversion. Using a simple model to fit the gain/absorption curves as a function of magnetic field, we have obtained values for the populations in the system, showing that there is no inversion in the atomic basis. Combined with a simple optical method to measure absolute densities of a gas, this model results in a completely optical method to determine the absolute populations and density in the system.

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I. INTRODUCTION

The topic of lasing without inversion (LWI) has attracted a lot of attention for quite some time, because of the possible applications in short-wavelength lasing. Due to the \( \omega^3 \) dependence of the Einstein \( A \) coefficient, transitions with larger energy splitting generally have faster decay processes which makes it more difficult to create inversion using conventional incoherent pumping. The study of atomic coherence in laser schemes has led to many suggestions for LWI schemes. In this paper we report a laser oscillation experiment in a coherent population trapping scheme.

Since the original theoretical work [1–3] and amplification experiments [4–6] many theoretical contributions have followed and new experiments showing actual lasing without inversion have been reported [9,10]. Studies on lasing without inversion [11–14] show that two kinds of schemes can be identified:

1. Schemes based on coherent population trapping where a hidden inversion can be revealed by a basis transformation. Most of the experimental work has concentrated on these coherent population trapping schemes but so far only amplification without inversion has been reported [4–8].

2. Schemes that are inversionless in the sense that there is no atomic basis for which inversion can be found. Laser action has been reported in a \( V \)-type level structure for \( \text{Rb} \) [9] and in a \( \Lambda \)-level structure for \( \text{Na} \) [10]. The \( V \) scheme has recently been discussed in terms of the quantum Zeno effect [15].

The experiments reported in this paper belong to the first type of schemes and are an extension of previous work on inversionless amplification in \( \text{Cd} \) [6]. In that experiment single nanosecond pulses were used. In the present experiment the use of pulse trains and shorter optical pulses (picosecond pulses) has made inversionless laser oscillation possible in the same cadmium system.

II. INVERSIONLESS AMPLIFICATION AND LASING IN CADMIUM

We consider inversionless amplification and laser action between the \( ^3S_1 \) state and the \( ^3P_1 \) state in cadmium; see Fig.

1. By inversionless we mean that at all times \( N_{^3S_1} < N_{^3P_1} \), where \( N \) denotes the dimensionless population fraction of a level, irrespective of the distribution over its magnetic sublevels. In a coherently pumped two-level system inversionless lasing is possible on the Rabi-side bands; see Ref. [16] and references therein. Inversionless lasing between two incoherently pumped levels is impossible. A similar strong statement cannot be made for systems that involve more than two levels. In the present experiment we use the Zeeman substructure of the \( ^3P_1 \) state to create an amplification condition for a pulsed optical field. We will use the notation \( | \rangle \), \( |0\rangle \), \( |+\rangle \) for the \( m_J=-1,0,1 \) sublevels of the \( ^3P_1 \) state, and \( |\mu\rangle \) for the \( m_J=0 \) sublevel of the \( ^3S_1 \) state. Simultaneous excitation of the magnetic sublevels \( |\rangle \) and \( |+\rangle \) of the \( ^3P_1 \) state [see Fig. 2(a)] generates a coherence \( \rho_{++} \) between these levels, responsible for the cancellation of absorption (induced transparency), which makes inversionless amplification possible.

![FIG. 1. Wavelengths and lifetimes of atomic cadmium states involved in the experiment. The population of the upper \( ^3S_1 \) state is monitored by detecting the fluorescence decay to the \( ^3P_2 \) state. The wavelength of this fluorescence, 508 nm, is the longest wavelength in the experiment and is easily selected by filtering.](image)
but not to the pump pulses. The total population of the 3 state and magnetic sublevels of the 3 state leads to a coherent superposition, |\phi_N\rangle = (1/\sqrt{2}) (|\downarrow\rangle - |\uparrow\rangle). In the basis of superposition states |\phi_N\rangle couples only to the pumping pulses and not to the probe or laser field; |\phi_C\rangle couples to the probe (or laser field) but not to the pump pulses. The total population of the 3 prepares a Larmor precession between |\phi_N\rangle and |\phi_C\rangle states.

The experiment is divided in three steps. The first, linearly polarized excitation pulse [pulse 1, in Fig. 2(b)] prepares a coherent superposition |\phi_N\rangle = (1/\sqrt{2}) (|\downarrow\rangle - |\uparrow\rangle) of the Zeeman sublevels of the 3 state. We call the orthogonal superposition state |\phi_C\rangle = (1/\sqrt{2}) (|\downarrow\rangle + |\uparrow\rangle); see Fig. 2(b). Initially only |\phi_N\rangle is populated but the 3 states start to perform a Larmor precession in a magnetic field; the 3 population oscillates back and forth between |\phi_N\rangle and |\phi_C\rangle.

After a delay time a second pulse is applied, orthogonal polarization excites part of the available |\phi_N\rangle population to the upper 3 state. Only the magnetic sublevel m J = 0 of the upper state is populated. Immediately after the second pulse a third pulse arrives with polarization parallel to the first pulse. This third pulse probes the population difference between the 3 state and |\phi_C\rangle and can be either attenuated or amplified.

### III. REVEALING THE HIDDEN INVERSION

The role of coherences in a three-level laser scheme can easily be described using the semiclassical density matrix formalism [17]. Using the Bloch equations we will show how a coherence between |\downarrow\rangle and |\uparrow\rangle can overcome the lack of inversion between the 3 and 3 states. Consider the propagation of a short optical probe pulse (pulse 3 in the experiment) with a carrier frequency \(\omega_0\) resonant with the |u\rangle\rightarrow|\downarrow\rangle transition, \(\omega_0\). Since the polarization is linear and the bandwidth of this pulse is larger than the Larmor frequency splitting, the field interacts with both the |u\rangle\rightarrow|\downarrow\rangle and |u\rangle\rightarrow|\uparrow\rangle transition coherently; see Fig. 2(a). We will use the notation \(\Omega_3(z,t)\) for the pulse envelope expressed as a Rabi frequency \(\Omega_3(z,t)\) is slowly varying compared to the carrier frequency \(\omega_0\), and 2\(\Delta\) for the Larmor frequency splitting between levels |\downarrow\rangle and |\uparrow\rangle. Ignoring the decay of the 3 state (\(\tau_{\text{pulse}} \gg \tau_1\)) the Bloch equations for the upper-state population and coherences are (in the rotating-wave approximation and after transforming to the rotating frame)

\[
\frac{\partial \rho_{uu}}{\partial t} = -\frac{i}{2\sqrt{2}} \Omega_3 [\text{Im}(e^{-i\Delta t \rho_{u-}}) + \text{Im}(e^{i\Delta t \rho_{u+}})],
\]

The factor \(1/\sqrt{2}\) appearing in front of \(\Omega_3\) is a consequence of the decomposition of linearly polarized light into a superposition of left-handed and right-handed circularly polarized light. When the probe pulse \(\Omega_3(z,t)\) arrives the populations \(\rho_{uu},\rho_{-+},\rho_{++}\) and the coherences \(\rho_{++},\rho_{--}\) have been given an initial value by the pump pulses. Although the pump pulses also create coherences \(\rho_{uu}\) and \(\rho_{uu}\), the probe beam cannot interact with these coherences since the pump pulse 2 and the probe pulse 3 make a small angle and are therefore not phase matched. So we assume \(\rho_{uu}(t=0) = \rho_{uu}(t=0) = 0\). This three-level system is inversionless, \(\rho_{uu}\) is smaller than \(\rho_{-+}\) or \(\rho_{++}\). The probe field is amplified if its interaction with the atomic system reduces the upper state population, \(\rho_{-+}\). The equations show that the transfer of upper-state population into gain of the optical pulse is proportional to \(\text{Im}(e^{-i\Delta t \rho_{uu}} + e^{i\Delta t \rho_{uu}})\). Without preparation of an initial coherence \(\rho_{-+}\) the coherences \(\rho_{uu}\) and \(\rho_{uu}\) are created from the population terms proportional to \(\rho_{uu}\) and \(\rho_{uu}\), which only contribute to gain if there is inversion. If, however, a coherence \(\rho_{+}\) is prepared before the pulse interacts with the system, gain becomes possible even without inversion. For a short probe pulse, we have \(e^{\pm i\Delta t} = 1\) so that a real and negative initial coherence \(\rho_{-+}\) will lead to gain without inversion if (assuming \(\Omega_3\) is real and positive)

\[|\rho_{-+}| > |\rho_{uu} - \rho_{-+}|, \quad |\rho_{uu} - \rho_{++}|,\]

meaning that the lack of inversion in Eq. (2) and Eq. (3) is overcome by the coherence term. This is a general signature of all LWI schemes.

The LWI scheme presented here contains hidden inversion as can be revealed by a simple basis transformation. This leads to a simpler picture of the physical processes. We transform to a basis of superposition states of the magnetic sublevels |\downarrow\rangle and |\uparrow\rangle [see Fig. 2(b)]

\[
|\phi_C\rangle = \frac{1}{\sqrt{2}} (|\downarrow\rangle + |\uparrow\rangle), \quad |\phi_N\rangle = \frac{1}{\sqrt{2}} (|\downarrow\rangle - |\uparrow\rangle).
\]

Horizontally polarized light of 479 nm (pulse 3) interacts with the coupled state |\phi_C\rangle, and will have no interaction with the noncoupled state, |\phi_N\rangle, while vertically polarized light of 479 nm (pulse 2) interacts with only the noncoupled state |\phi_N\rangle. In the new basis the Bloch equations for the three-level system |u\rangle, |\downarrow\rangle, |\uparrow\rangle reduce to two-level equations for the |u\rangle and |\phi_C\rangle states:

\[
\frac{\partial \rho_{uu}}{\partial t} = -\Omega_3 \text{Im}(\rho_{uc}),
\]
These are simply the usual Bloch equations for two states, \(|\psi\rangle\) and \(|\phi_c\rangle\), coupled by an optical field \(\Omega(z, t)\). The probe can only be amplified if an initial inversion \(\rho_{uu} - \rho_{cc}\) is prepared by the pump pulses. Note that such initial inversion is indeed created in our experiment. The population of the \(^3P_1\) state is hidden in the \(|\phi_n\rangle\) state, which is decoupled from the probe field.

In further analysis of the gain and laser oscillation experiments we will use this simple two-level nature of the coupling of probe field (or laser field) to the atomic system.

**IV. EXPERIMENTAL SETUP**

The setup consists of two synchronously pumped pulsed dye lasers operating at 652 nm and 479 nm, at a repetition rate of 10 Hz. We use a mode-locked Nd:YAG pump laser producing a train of approximately 8 pulses of duration 40 ps, with 7.1 ns between the pulses. Part of the pulse train is amplified to 18 mJ per train, the remainder passes a pulse slicer and is amplified as a single pulse to 15 mJ per pulse. The frequency doubled and tripled pulse trains at 532 and 355 nm are used to synchronously pump two dye oscillators [18] at 652 and 479 nm, respectively. Both lasers have a 1-mm intracavity etalon that limits the bandwidth of the pulses to 30 GHz.

The single pulse produced by the Nd:YAG pump laser is also frequency doubled and tripled and is used to amplify one pulse out of the dye oscillator pulse trains. The 652-nm laser uses DCM dye dissolved in methanol for the oscillator, a transverse amplifier, and a longitudinally pumped amplifier, and gives about 60 \(\mu\)J. The output is frequency doubled to excite the 326-nm transition to the \(^5S_2 \rightarrow ^3P_1\) state and is referred to as pulse 1, with an energy of approximately 1 \(\mu\)J. The 479-nm laser uses Coumarine 102 dissolved in methanol. The pulse energy after the single transverse amplifier is approximately 10 \(\mu\)J. This second pulse (479 nm) has a fixed delay with respect to the first excitation pulse (326 nm) of 3 (10 ns). The third, “probe” pulse (479 nm) is split off from the second pulse, reduced in intensity by three crossed polarizers, and is spectrally filtered to a bandwidth of 2 GHz using a 1-mm etalon with 95% reflectivity. Its polarization is perpendicular to the second, “pump” pulse. The pulse duration of the amplified dye-laser pulses is 35 ps.

The atomic vapor of the enriched cadmium-112 isotope is contained in a 10-cm-long quartz cell, which is heated inside an oven to 225 °C. The density of the atomic vapor is adjusted by changing the temperature of the cell. A magnetic field is supplied by two coils of 1-m diameter in a Helmholtz configuration, giving a homogeneous magnetic field over the length of the cell. The field is directed along the propagation axis of the beams; see Fig. 3. A photomultiplier tube (PMT 1) monitors the fluorescence through a 1-cm-diameter pipe, from the atoms in the middle of the 10-cm cell. The photomultiplier tube is placed outside the large coils to avoid disturbance by the magnetic field.

The first excitation pulse (326 nm) and the third (probe) pulse (479 nm) have the same linear polarization and they overlap spatially. The second excitation pulse (479 nm) has orthogonal polarization and makes a small angle with the other pulses allowing us to spatially separate it from the probe pulse after passing the cell. A small prism disperses the overlapping first excitation pulse and probe pulse. The probe is detected by a second photomultiplier tube (PMT 2) after passing two pinholes and a polarizer to eliminate stray light of the second pump pulse. All three beams have a diameter of 2 mm and are copropagating inside the cell. The interaction volume is 0.3 cm³.

**V. ABSOLUTE GAS-DENSITY MEASUREMENT**

The cadmium density inside the vapor cell is measured by a simple optical method. Consider an absorbing medium in a thin long cell of length \(L\). The fluorescence intensity from absorbing atoms can be detected at position \(z\), where \(z\) is the path length that the exciting beam has traveled through the medium, \(0 < z < L\). The fluorescence intensity at position \(z\), \(I_f(\rho, z)\) is proportional to the product of the atomic density, \(\rho\), and the local light intensity. In the rate-equation regime this local intensity depends on \(z\) through Beer’s law, so that

\[
I_f(\rho, z) \approx \rho I_0 e^{-\sigma_\lambda z},
\]

with \(I_0\) the input intensity, and \(\sigma_\lambda\) the stimulated emission cross section of the atom-light interaction. The fluorescence intensity at position \(z\) as a function of the density reaches a maximum for a density such that \(z\) equals the 1/e decay length: \(\rho = (\sigma_\lambda z)^{-1}\). At lower densities the decrease of available atoms reduces the fluorescence, at higher densities it is increased absorption of incoming light that reduces the fluorescence. The maximum signal can be found by tuning \(\rho\) (changing the temperature of the vapor cell) and the density is simply calculated from just \(z\) and \(\sigma_\lambda\).
In our experiment the bandwidth of the pump pulses (30 GHz) is large compared to the inhomogeneous broadening ($\Delta \nu_{D1} = 1.4$ GHz) so that Beer’s law does not apply. The first pulse is weak, i.e., it excites only a small fraction of the ground state population to the $^3P_1$ state. In this case of weak, resonant excitation the absorption is easily calculated in the frequency domain [19,20]. The position-dependent fluence is

$$F(z) = \int_{-\infty}^{\infty} I(v,z) dv,$$  

with $F(0)$ the initial fluence of about 1 $\mu J / (\pi \text{mm}^2)$. Each frequency component of the intensity decays exponentially:

$$I(v,z) = e^{-\sigma_1(v) \rho L} I(v,0),$$  

with $I(v,0)$ the spectral fluence distribution at $z=0$, and $\sigma_1(v)$ the absorption cross section. The absorption cross section $\sigma_1(v)$ for a Doppler broadened line is [21]

$$\sigma_1(v) = \sqrt{\frac{\ln 2}{16 \pi^3 \Delta \nu_D}} \exp \left( - \frac{4 \ln 2 \nu^2}{\Delta \nu_D^2} \right),$$

with $A_{21}$ the Einstein $A$ coefficient and $\Delta \nu_D$ the full width at half maximum (FWHM) Doppler width. The density of excited $^3P_1$ atoms is now easily calculated from the spatial decay of the fluence:

$$n(z,\rho) = \frac{dF(z)}{e_1 \rho L} = \int_{-\infty}^{\infty} \sigma_1(v) \rho e^{-\sigma_1(v) \rho L} I(v,0) dv,$$  

with $e_1$ the photon energy at 326 nm. The function $n(z,\rho)$ has a maximum as a function of $\rho$, similar to the light intensity given in Eq. (8). In the actual experiment we optimized the fluorescence signal of the $^3P_1$ state at $z=L/2$ by adjusting the temperature. The resulting atomic density for $z=L/2=5 \text{ cm}$ is $\rho = 3.1 \times 10^{13} \text{ cm}^{-3}$, based on $\lambda_1 = 326 \text{ nm}$, and $A_{21} = (2.4 \mu s)^{-1}$. The $^3P_1$ excited-state population density in the middle of the cell as a function of atomic density is given in Fig. 4(a). The fluence of pulse 1 as a function of the position in the cell is depicted by the solid line in Fig. 4(b), the dotted line represents Beer’s law of absorption for the same cross section and density. The excited-state population fraction as function of position in the cell is given in Fig. 4(c). [Both fluence and population fraction are calculated for the optimized atomic density obtained from Fig. 4(a).] Since the fraction of excited states decays almost exponentially along the cell, we will from now on assume pure exponential decay of the population fraction in the experiment:

$$\frac{N_{^3P_1}(z)}{N_{^3P_1}(0)} = e^{-2z/L}.$$  

Based on the temperature of the oven of 225 °C, we expect a vapor pressure of $2.6 \times 10^{-3}$ mbar or an atomic density of $\rho = 3.8 \times 10^{13} \text{ cm}^{-3}$. This is in good agreement with...

![FIG. 4. (a) Population density of the $^3P_1$ state as a function of atomic density at $z=L/2=5 \text{ cm}$, assuming a pulse energy of 1 $\mu J$. The position of the maximum value, $\rho = 3.1 \times 10^{13} \text{ cm}^{-3}$ depends only on the cross section $\sigma_1(v)$. (b) Loss of fluence of pulse 1 as a function of position in the cell. The solid line is calculated for the experiment. The dotted line would be the result if Beer’s law would apply, i.e., if the pulse were narrow band. (c) Position dependence of the excited $^3P_1$ population fraction. The solid line was calculated for the experiment, the dotted line gives a pure exponential decay proportional to $e^{-2z/L}$. This justifies our assumption that both excited-state population fractions $N_{^3P_1}(z)$ and $N_{^3S}(z)$ decrease from their initial value at $z=0$ by a factor $e^{-1}$ halfway into the cell, and a factor $e^{-2}$ at $z=L$.](image)

![FIG. 5. The fluorescence intensity of the $^3P_1$ state in Cd in a magnetic field of $B = 6.25 \times 10^{-5} \text{ T}$. This is a typical quantum beat picture showing a well defined coherence between the $^3P_1$ sublevels. Larmor frequencies from 2.4 MHz (Earth magnetic field) up to 140 MHz have been measured. The lifetime of the $^3P_1$ state is 2.4 $\mu s$.](image)
time in our experiment is only 10 ns, Fig. 5 shows that the populations and coherences are preserved long enough for the gain experiments.

VI. PUMPING THE LASER TRANSITION

The second excitation pulse, promoting population from |\phi_N\rangle to |\mu\rangle, arrives after a 10-ns delay with respect to the first pulse. During this delay the \(3P_1\) population oscillates between |\phi_N\rangle and |\phi_C\rangle. Thus the amount of population pumped into the \(3S_1\) state by pulse 2 depends on the phase \(\theta_L\) of the Larmor precession in the \(3P_1\) state. For example, if the |\phi_N\rangle state is empty when pulse 2 arrives, there simply is no population to be transferred to the upper state. Thus, for a fixed delay, the population that is excited to the \(3S_1\) level depends on the Larmor frequency and varies as a cosine-squared function of the magnetic field,

\[
N_u = N_{u,\text{max}} \cos^2(\theta_L),
\]

with \(N_{u,\text{max}}\) the maximum amount of \(3S_1\) population. In the experiment, \(N_u\) is monitored by measuring the fluorescence of the \(3S_1\) \(\rightarrow\) \(3P_2\) transition at 508 nm; see Fig. 1. The fluorescence intensity is directly proportional to the population of the \(3S_1\) state and shows the expected cosine squared dependence as a function of magnetic field; see Fig. 6(a).

The influence of shot-to-shot fluctuations in intensity was reduced by averaging over 100 shots per magnetic field value.

In contrast to pulse 1, pulse 2 cannot be considered weak. The cross section for the second pump field (and probe pulse) at 479 nm is much larger than for the 326 nm transition, because it involves an optically allowed transition. In addition, the pulse energy at 479 nm is larger. For a bandwidth-limited pulse this would mean that an arbitrary fraction of the \(3P_1\) population could be transferred to the upper state, depending on the area of the pulse. However, the pulses in our experiment are not Fourier limited. The FWHM product of pulse duration (35 ps) and bandwidth (30 GHz) is

2.4 times larger than the smallest possible product (0.44) for a Gaussian pulse. Calculations based on the Bloch equations show that pulses with these characteristics and a fluence of a few times the equivalent 2\(\pi\) fluence excite within narrow margins (2\%) exactly 50\% to the excited state. This result is quite robust against variation of both the exact fluence and the duration-bandwidth product; see Fig. 7(a). The populations before and after interaction with pump pulse 2 are schematically indicated in Fig. 7(b) and Fig. 7(c), respectively.

The pulse energy of pulse 2 (10 \(\mu\)J) is much larger than the total energy that can be absorbed by the excited \(3P_1\) atoms, \(\rhoV N_{3P_1} \epsilon a \approx 0.1 \mu\)J. Therefore the intensity of pulse 2 remains constant and we can assume the same position dependence of the \(3S_1\) population in the cell as for the \(3P_1\) population, illustrated in Eq. (13) and Fig. 4.

VII. GAIN MEASUREMENTS

In the following we describe how the absolute populations \(\rho N_{3S_1}\) and \(\rho N_{1P}\) can be determined, relying only on relative measurements, by a probe experiment using a third pulse, coupling the |\mu\rangle \leftrightarrow |\phi_C\rangle transition.

The probe field (pulse 3) arrives immediately after the second excitation pulse (\(\Delta \tau = 0.2\) ns), and probes the |\mu\rangle \leftrightarrow |\phi_C\rangle transition. The probe is amplified or absorbed according to the sign of the population difference on this transition,

\[
\Delta N_{uc}(z, \theta_L) = N_{u,\text{max}}(z) \cos^2(\theta_L) - N_{3P_1}(z) \sin^2(\theta_L).
\]

(15)

We discuss the gain for three different precession angles of the population in the \(3P_1\) state, \(\theta = 0, \pi/4, \pi/2\).

(i) \(\theta = 0\), \(\Delta N_{uc}(0) = N_{u,\text{max}}(0)\): maximum gain, the optically coupled |\phi_C\rangle state is empty.

(ii) \(\theta = \pi/4\), \(\Delta N_{uc}(0) = \frac{1}{2}(N_{u,\text{max}}(0) - N_{3P_1}(0))\): absorption because \(N_{u,\text{max}}(0) < N_{3P_1}(0)\).

(iii) \(\theta = \pi/2\), \(\Delta N_{uc}(0) = -N_{3P_1}(0)\): maximum absorption, the \(3S_1\) state is empty.

For instance, if the magnetic field is 1.2 mT, then \(\theta = \pi/2\). When pulse 2 arrives the population in the \(3P_1\) state has completed one-half oscillation cycle in the superposition state basis leaving |\phi_N\rangle empty. Therefore no population can
The density giving the peaks are narrower than the valleys; see Fig. 6.

To derive the gain for pulse 3 we need to determine the fluence fraction following calculation. which provides nonlinear gain, as will be explained in the following calculation.

The density giving the peaks are narrower than the valleys; see Fig. 6.

The observed gain curve can be reproduced using a model modulated the absorption in Eq. (2) and the overlap factor.

The density giving the peaks are narrower than the valleys; see Fig. 6.

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The observed gain curve can be reproduced using a model modulated the absorption in Eq. (2) and the overlap factor.
The round-trip gain factor is

\[ \text{gain} = \frac{G_{\text{rt}}}{1} \]

where the equality of copropagating and counterpropagating gain in the direction of the pump beams is equal to the travel direction of the pump beams, the single-pass gain of the laser reduces in the upper state population of the laser. Thus the observed amplification from a single photon energy to the saturation energy, can be achieved in only a few round trips. Assuming a Gaussian envelope of the pump pulses (with 5 pulses above 25% of the peak value), a round-trip gain factor of \( G_{\text{rt}} = 230 \) will lead to a total gain of \( 10^{10} \) for the strongest peak under the envelope. Although only five pulses exceed 25% of the peak value, the preceding weak pulses (below 25%) play an important role in the buildup of the laser oscillations. The \( ^3S_1 \) population needed for a round-trip gain \( G_{\text{rt}} = 230 \) can be calculated using Eq. (20) and Eq. (17), giving \( N_{u, \text{max}}(0) = 8.5 \times 10^{-3} \). On the other hand the maximum gain measurement resulted in an upper-state population fraction of \( N_{u, \text{max}}(0) = 9 \times 10^{-3} \). The value found in the lasing experiment is consistent with this interval and supports our assumptions concerning the small number of pulses needed to reach the low saturation energy.

**IX. CONCLUSION**

We have performed both amplification and laser-oscillation experiments in cadmium vapor. There is no inversion between any two levels in the system. Transforming to a basis of coupled and noncoupled states reveals a hidden inversion and allows us to make a two-level analysis of the experiment. A method based on a local maximum in fluorescence signal was used to calculate the atomic density. Using this density as an input parameter in a model for the gain, we extracted the population fractions of the laser levels involved and verified that there is no inversion in the atomic basis. Inversionless single-pass gains were measured. In addition, we obtained synchronously pumped inversionless laser operation.

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