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## Lower Critical Field $H_{c1}$ and Barriers for Vortex Entry in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Crystals

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The penetration field  $H_p$  of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  crystals is determined from magnetization curves for different field sweep rates  $dH/dt$  and temperatures. The obtained results are consistent with theoretical reports in the literature about vortex creep over surface and geometrical barriers. The frequently observed low-temperature upturn of  $H_p$  is shown to be related to metastable configurations due to barriers for vortex entry. Data of the true lower critical field  $H_{c1}$  are presented. The low-temperature dependence of  $H_{c1}$  is consistent with a superconducting state with nodes in the gap function. [S0031-9007(98)07346-3]

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High-temperature superconductors (HTSC's) are strongly type II, and as such their  $H$ - $T$  phase diagram involves complete flux expulsion (Meissner state) at low fields and magnetic field penetration in the form of quantized flux lines or vortices (mixed state) at higher fields. The onset of the mixed state, where vortex penetration becomes energetically favorable, is defined as the lower critical field  $H_{c1} \propto 1/\lambda^2$  (here,  $\lambda$  is the London penetration depth). From the temperature dependence of  $H_{c1}$ , important information can be gained, particularly, regarding the symmetry of the superconducting state, since the appearance of gap nodes strongly modifies the  $T$  dependence of the superfluid density and thereby the penetration depth  $\lambda(T)$ .

The experimental determination of  $H_{c1}$  has been a challenging and controversial problem since the beginning of HTSC research. Not only the values reported for the first flux penetration field  $H_p$  are scattered over a wide field range, but in strongly layered superconductors with  $H$  perpendicular to the planes, striking features have been observed, such as a marked upturn of  $H_p$  for temperatures  $T \lesssim T_c/2$  [1–6]. The origin of the positive curvature of  $H_p$  at low temperatures has been the matter of various speculations. Different mechanisms have been proposed: Bean-Livingston surface barriers [1,3,4,6], bulk pinning [2,6], a modification of the character of the field penetration in layered structures [5], a low-temperature enhancement of the superconducting order parameter in the normal layers [7], etc.

For type-II superconductors, there are at least two kinds of barriers which hinder the system from reaching a thermodynamic equilibrium state: (i) Surface and geometrical barriers [8–10] governing vortex penetration into the superconductor; (ii) bulk pinning barriers governing vortex motion in the superconductor. For the determination of  $H_{c1}$ , the relevant barriers are those which govern vortex entry into the sample. Of particular interest in this context is the phenomenon of vortex creep over surface and geo-

metrical barriers. From the theoretical point of view this subject has been discussed in Refs. [10] and [11]; however, to our knowledge no systematic experimental study has been carried out so far.

In this Letter we investigate the dependence of  $H_p$  on the magnetic field sweep rate  $dH/dt$  of isothermal magnetization curves for the strongly layered  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  (Bi2212) material. This has been done for crystals with different cross sections along the  $c$  axis. For specimens with ellipsoidal cross sections geometrical barriers can be neglected [10,12], so that the relevant barriers for vortex entry are expected to be surface barriers. Indeed, for our specimen with an ellipsoidal-like cross section, the  $H_p$  vs  $dH/dt$  dependence obtained at high sweep rates is well described by vortex creep over surface barriers [11]. However, for decreasing  $dH/dt$  rates, these barriers are observed to collapse. The subsequent saturation of  $H_p$  at lower rates indicates that the system is in equilibrium with respect to surface barriers. This is interpreted as strong evidence that we have reached the lower critical field  $H_{c1}$  in our measurements. The obtained low-temperature dependence of  $H_{c1}$  is in good agreement with recent microwave absorption measurements of the penetration depth  $\lambda$  [13,14]. On the other hand, for specimens with rectangular cross sections, no significant vortex creep over the relevant barriers for vortex entry could be observed, in consistency with theoretical results for geometrical barriers [10]. The low-temperature upturns of  $H_p$  observed for specimens of either cross section are then explained in terms of measurements where the system is out of equilibrium with respect to barriers for vortex entry.

The crystals under investigation have been grown with different techniques and have different shapes. The specimen with an ellipsoidal-like cross section along the  $c$  axis has been grown in a  $\text{ZrO}_2$  crucible [15] and is approximately  $1 \times 1.3 \times 0.05 \text{ mm}^3$  in size. The specimens with rectangular cross sections along the  $c$  axis have been grown with the traveling solvent floating zone method

[16] and are slightly larger in size. The experiment is performed in a noncommercial SQUID magnetometer where the sample is stationary in the pickup coil. The field  $H$  is supplied by a superconducting coil working in a non-persistent mode. For the magnetization curve measurements, the sample is zero field cooled from above  $T_c$  and stabilized at a fixed temperature (the residual field of the cryostat is  $<10$  mOe). Further, the field  $H$  is applied at a fixed rate  $dH/dt$ . For  $H_p$ , we select the field at which a deviation from Meissner shielding occurs (see the inset of Fig. 3, shown below).

Figure 1 shows the dependence of  $H_p$  on the field sweep rate  $dH/dt$  at three characteristic temperatures for the specimen with an ellipsoidal-like cross section. A sharp step in the  $H_p$  vs  $dH/dt$  curves is observed at all three temperatures. At high sweep rates, the curves display a finite slope which is most pronounced at  $T = 11$  and 17 K and to a lesser degree at  $T = 61$  K. These slopes are in good agreement with the theoretical results of Burlachkov *et al.* [11] for vortex creep over surface barriers (see the analysis below). Proceeding from the creep regime towards lower rates, the sharp drop signals a collapse of the surface barriers (or equivalently a heat pulse) which leads to a rapid flux entry, possibly in terms of flux jumps. Indeed, an avalanche type of flux penetration into a type-II superconductor has been reported by Durán *et al.* [17]. On decreasing the sweep rates even further, the flux penetrates into the sample through the occurrence of rare events when the applied magnetic field reaches  $H_{c1}$ . We thus identify the saturated value of  $H_p$  with the lower critical field  $H_{c1}$  and use it to obtain information about the nature of the superconducting state.

Analogous measurements have been done on thin specimens with rectangular cross sections. According to Zeldov *et al.* [10], for such specimens, geometrical barriers build up over a distance  $s$  from the sample edges, where  $s$  is

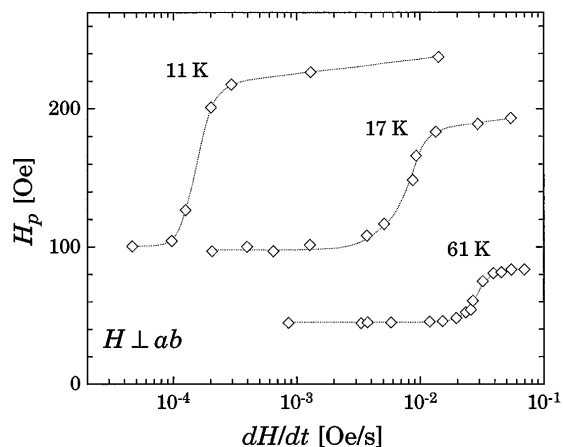


FIG. 1. For the specimen with an ellipsoidal-like cross section, the first flux penetration field  $H_p$  vs the applied magnetic field sweep rate  $dH/dt$  is displayed for different temperatures. The dotted lines are guides to the eyes. The data are scaled with the demagnetization factor  $N = 0.96(\pm 15\%)$ .

the sample thickness. The energy required to overcome such a barrier is macroscopic  $\sim \varepsilon_0 s$  so that vortex creep over geometrical barriers is expected to be very weak [here  $\varepsilon_0 = (\Phi_0/4\pi\lambda)^2$  and  $\Phi_0$  is the unit flux]. As a matter of fact, we were not able to detect significant vortex creep over geometrical barriers within the experimental time scale  $10^{-5} \leq dH/dt \leq 10^{-1}$  Oe/s.

We proceed with a brief discussion of vortex creep over surface barriers. As shown by Burlachkov *et al.* [11], the surface barrier for pancake vortices is given by  $U \approx \varepsilon_0 d \ln(0.76H_c/H)$ , where  $H_c$  is the thermodynamic critical field and  $d$  the interlayer distance. During the time  $t$ , thermal creep allows vortices to overcome barriers of size  $U(t) \approx T \ln(t/t_0)$ , where  $t_0$  is a “microscopic” time scale [18]. Equating the two expressions one obtains the time dependence of the penetration field for the pancake-vortex regime,

$$H_p(t) \approx H_c (t/t_0)^{-T/\varepsilon_0 d}. \quad (1)$$

With the definition  $T_0 = \varepsilon_0 d / \ln(t/t_0)$  Eq. (1) becomes  $H_p \approx H_c \exp(-T/T_0)$ . At higher temperatures creep proceeds via half-loop excitations of vortex lines [11]. The creation of a half-loop saddle configuration involves an energy  $U \approx \pi \varepsilon \varepsilon_0^2 c \ln^2(j_0/j) / 2\Phi_0 j$ , where  $j_0$  is the depairing current density and  $\varepsilon = (m/M)^{1/2} < 1$  is the anisotropy parameter. With a similar analysis as above, the time dependence of  $H_p$  for the vortex-line regime takes the form

$$H_p(t) \approx H_c \frac{\pi}{2\sqrt{2}} \frac{\varepsilon \varepsilon_0 \xi}{T \ln t/t_0} \ln^2\left(\frac{H_c}{H_p}\right), \quad (2)$$

where  $\xi$  is the coherence length. According to Ref. [11], this expression gives a temperature dependence  $H_p \propto (T_c - T)^{3/2}/T$ . The crossover between the two regimes occurs when the size of the half-loop excitation is of the order of the interlayer distance  $d$ . This is expected to occur at a temperature  $T^* \approx T_0(T^*) \ln(d/\varepsilon\xi)$ .

According to (1), creep of pancake vortices over surface barriers results in a linear dependence of  $\ln(H_c/H_p)$  on  $\ln(t/t_0)$  with a slope  $T/\varepsilon_0 d$ . This dependence is given in Fig. 2(a) using the data at high cycling rates in Fig. 1. For  $T = 11$  and 17 K, the values of  $T/\varepsilon_0 d$  obtained from the fits are in good agreement with the values estimated from the penetration depth  $\lambda_{ab}(T)$  (see Table I). On the other hand, for  $T = 61$  K the above analysis for pancake vortices is not satisfactory since the  $T/\varepsilon_0 d$  value obtained from the fit is 1 order of magnitude smaller than the calculated value. According to (2), for creep of vortex lines over surface barriers  $(H_c/H_p) \ln^2(H_c/H_p)$  vs  $\ln(t/t_0)$  is linear with a slope  $2\sqrt{2}T/\pi\varepsilon\varepsilon_0\xi$ . For the  $T = 61$  K data at high cycling rates in Fig. 1, such a representation is given in Fig. 2(b). The slope  $2\sqrt{2}T/\pi\varepsilon\varepsilon_0\xi$  obtained from the fit is  $15 \pm 3$ , in good agreement with the value 16, estimated with the help of the Ginzburg-Landau formula for the coherence length  $\xi(T)$ , with

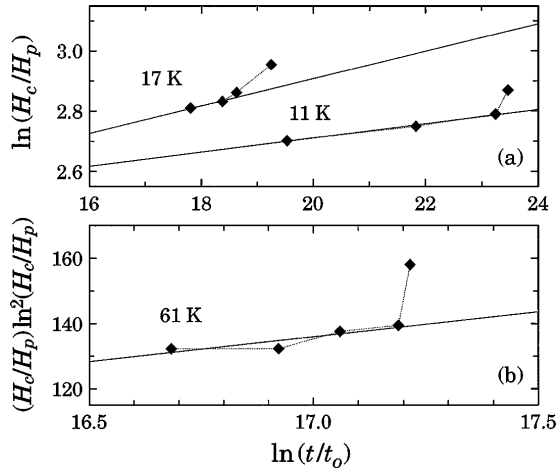


FIG. 2. (a)  $T = 11$  and  $17$  K data of the upper plateau in Fig. 1 in a different representation. Here we choose the time origin  $t = 0$  at  $H = H_{c1}$ . The critical field  $H_c(T) \approx H_c(0) (1 - T/T_c)$  is calculated with  $\lambda_{ab}(0)$  as obtained from the  $H_{c1}$  data in Fig. 3 and the coherence length  $\xi(0) \approx 25$  Å. We assumed  $t_0 \approx 10^{-6}$  s. (b)  $T = 61$  K data at high sweep rates in Fig. 1 in a convenient representation. The lines in (a) and (b) are fits according to Eqs. (1) and (2), respectively.

$\xi(0) \approx 25$  Å and an anisotropy parameter  $\varepsilon \approx 1/100$ . From these considerations we conclude that the behavior of  $H_p$  at high sweep rates is determined by creep of pancake vortices (at low temperatures) and vortex half-loops (at higher temperatures) over surface barriers. The estimated value of the activation barrier for vortex entry is  $U \approx 300$  K for temperatures  $T$  between 10 and 20 K and  $U \approx 1200$  K for  $T \approx 60$  K (here we set the time origin  $t = 0$  when the applied field  $H$  reaches  $H_{c1}$  and assume  $t_0 \approx 10^{-6}$  s).

In Fig. 3 we make use of the curves in Fig. 1 to determine  $H_p(T)$  with three different criteria: (i)  $H_p$  data (○) are taken at low sweep rates so as to guarantee that they lie in the regime where the system is in equilibrium with respect to surface barriers [these data represent  $H_{c1}(T)$ ], (ii)  $H_p$  data (\*) are taken at high rates so that they always lie in the creep regime, and (iii) the field  $H$  is swept at the constant rate  $dH/dt = 8 \times 10^{-3}$  Oe/s for all temperatures. In this case, the values of  $H_p$  (▽) lie on the  $H_{c1}$  curve for  $T \geq 60$  K, they go through the steplike crossover for temperatures  $30 \leq T \leq 60$  K, and, finally, for  $T \leq 30$  K they lie in the creep regime. From

TABLE I.  $T/\varepsilon_0 d$  values calculated with the penetration depth  $\lambda_{ab}(T)$  as obtained with formula  $H_{c1} = (\Phi_0/4\pi\lambda_{ab}^2) \ln \kappa$  from the  $H_{c1}$  data in Fig. 3 and values of  $T/\varepsilon_0 d$  obtained from the fits in Fig. 2(a).

	$T = 11$ K	$T = 17$ K
$T/\varepsilon_0 d$ (calculated)	$\frac{1}{34}$	$\frac{1}{21}$
$T/\varepsilon_0 d$ (from fit)	$\frac{1}{43} \pm 30\%$	$\frac{1}{22} \pm 30\%$

a comparison of the curves in Fig. 3, it follows that the low-temperature upturn of  $H_p$  results from measurements where the system is out of equilibrium with respect to surface barriers.

As shown in Fig. 3, for increasing temperatures the  $T$  dependence of  $H_p$  in the creep regime, (\*) and (▽) for  $T \leq 30$  K and (\*) for  $T \geq 30$  K, undergoes a crossover from an exponential behavior to a weak power law. This is in agreement with the results presented above and indicates that the crossover from creep of pancake vortices to creep of vortex half-loops over surface barriers takes place at  $T^* \approx 30$ – $40$  K [in consistency with the estimate  $T^* \approx T_0(T^*) \ln(d/\varepsilon\xi) \approx 40$  K obtained using the parameters for Bi2212 and  $\ln t/t_0 \approx 20$ ].

Making use of the correct  $H_{c1}$  data for  $H \perp ab$ , (○) in Fig. 3, we have investigated the temperature dependence of the penetration depth  $\lambda_{ab}$  using the expression  $H_{c1} = (\Phi_0/4\pi\lambda_{ab}^2) \ln \kappa$  (here we assume that  $\ln \kappa$  is  $T$  independent). For  $11 < T \leq 30$  K, we find a linear behavior  $\lambda_{ab}(T) - \lambda_{ab}(0) \propto T$  with slope  $d\lambda/dT \approx 10$  Å/K, where a linear extrapolation to the data gives a  $T = 0$  penetration depth  $\lambda_{ab}(0) \approx 2700$  Å. Our measurements thus confirm the linear low-temperature dependence of  $\lambda_{ab}$  as well as the slope  $d\lambda/dT$ , which have been recently determined with microwave absorption techniques on clean Bi2212 crystals [13,14]. A linear low-temperature dependence of  $\lambda_{ab}$ , as first observed by Hardy *et al.* [19] in YBCO crystals, is consistent with a superconducting state with line nodes on the Fermi surface [20], e.g., a  $d$ -wave symmetry of the order parameter.

Similar investigations of  $H_p$  have been done for  $H \parallel ab$ . The misalignment angle between  $H$  and the  $ab$

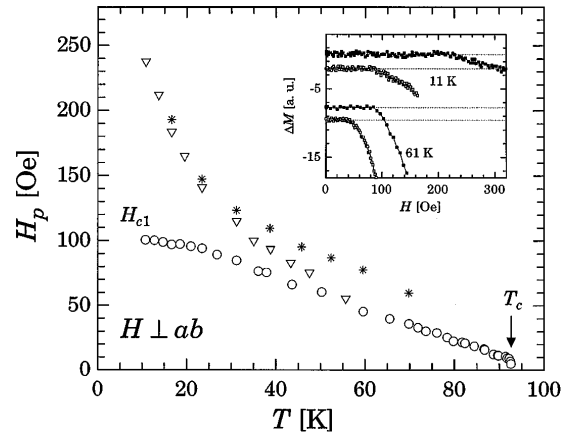


FIG. 3. Temperature dependence of the first flux penetration field  $H_p$  of the specimen with an ellipsoidal-like cross section for different field sweep rates: (○)  $dH/dt \leq 1 \times 10^{-4}$  Oe/s, (\*)  $dH/dt \geq 5 \times 10^{-2}$  Oe/s, and (▽)  $dH/dt = 8 \times 10^{-3}$  Oe/s. The inset shows the deviation from the Meissner slope for  $T = 11$  and  $61$  K. The upper curves (■) are measured at a rate  $dH/dt \geq 5 \times 10^{-2}$  Oe/s and the lower ones (□) at  $dH/dt \leq 1 \times 10^{-4}$  Oe/s. The data are scaled with the demagnetization factor  $N = 0.96(\pm 15\%)$ .

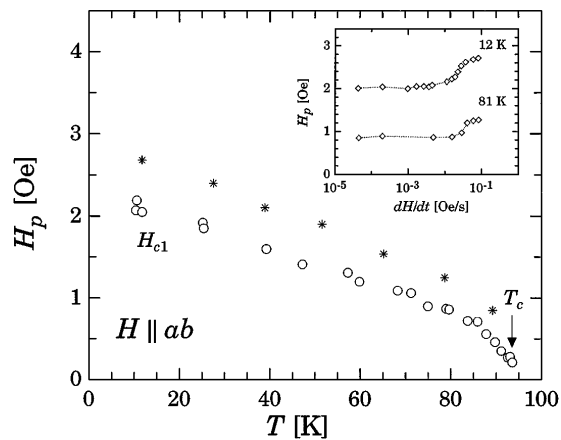


FIG. 4. Temperature dependence of  $H_p$  ( $H \parallel ab$ ) for a specimen with rectangular cross section and size  $2 \times 3.9 \times 0.05 \text{ mm}^3$  for different sweep rates: (\*)  $dH/dt \geq 6 \times 10^{-2} \text{ Oe/s}$  and (O)  $dH/dt \leq 6 \times 10^{-3} \text{ Oe/s}$ . The inset shows the  $H_p$  vs  $dH/dt$  dependence at  $T = 12$  and  $81 \text{ K}$ .

planes has been estimated to be smaller than  $2^\circ$  from measurements at magnetic fields in the range  $H_{p\parallel} \ll H < H_{p\perp}$  (see Ref. [21]; here  $H_{p\parallel}$  and  $H_{p\perp}$  are the parallel and perpendicular penetration fields). For this configuration geometrical barriers are irrelevant. As shown in Fig. 4, for temperatures  $T$  between 10 and 70 K,  $H_{c1}(T)$  has an approximately linear behavior. According to Ref. [22], for temperatures not so close to  $T_c$ , the correct description of  $H_{c1}$  in strongly layered materials is  $H_{c1} = \Phi_0 / (4\pi \lambda_{ab} \lambda_c) [\ln(\lambda_{ab}/d) + 1.12]$ . With this formula we calculated the penetration depth in  $c$ -direction  $\lambda_c$  using the previously determined  $\lambda_{ab}$  data. For  $T \leq 40 \text{ K}$ ,  $\lambda_c$  is approximately linear in  $T$  with a slope of  $0.1 \mu\text{m/K}$ . The extrapolated  $T = 0$  value is  $\lambda_c(0) \approx 15 \mu\text{m}$ . Figure 4 further shows  $H_p$  data at high sweep rates (\*). Contrary to the case for  $H \perp ab$ , no upturn of  $H_p$  is observed here for temperatures  $T \leq T_c/2$ . We attribute this difference to the absence of the strong pancake-vortex creep regime at low temperatures for  $H \parallel ab$ . From the  $H_p$  vs  $dH/dt$  dependence in the inset of Fig. 4, we obtain an activation barrier for vortex entry  $U \approx 200 \text{ K}$  at  $T = 12 \text{ K}$  and  $U \approx 1300 \text{ K}$  at  $T = 81 \text{ K}$ .

Close to  $T_c$  a downward bending of  $H_{c1}$  is observed for  $H \perp ab$  as well as  $H \parallel ab$  (see Figs. 3 and 4). This can be understood on the basis of an entropic downward renormalization of the vortex-line free energy due to fluctuations of the order parameter around its mean-field form [23]. The decrease in the free energy  $f_l = \varepsilon_l - Ts_l$  then leads to a drop in  $H_{c1}$  as  $T$  approaches  $T_c$  (here,  $\varepsilon_l$  is the line energy of the vortex excitation and  $s_l$  is the line entropy).

To conclude, isothermal magnetization curves were measured on Bi2212 crystals for configurations of neg-

ligible geometrical barriers. At very low sweep rates we found saturated values of the penetration field  $H_p$  which we interpret as the true lower critical field  $H_{c1}$ . The low-temperature dependence of  $H_{c1}$  is consistent with recent magnetic penetration depth measurements [13,14] suggesting a superconducting state with nodes in the gap function. Based on the results obtained for Bi2212, we argue that the frequently reported low-temperature upturns of  $H_p$  in strongly layered superconductors with  $H$  perpendicular to the planes [1–6] can be explained in terms of measurements where the system is out of equilibrium with respect to barriers for vortex penetration.

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