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Appendix A from P. Branco et al., "Eco-Evolutionary Dynamics of Ecological Stoichiometry in Plankton Communities" (Am. Nat., vol. 192, no. 1, p. E1)

Nutrient Mass Balance and Model Equations

Nutrient Mass Balance

According to equation (10), the total amount of nutrient i in our model ecosystem, T_i, includes the freely available nutrient in the environment, R_i, as well as the nutrient contained in phytoplankton and zooplankton. Differentiation of equation (10) with respect to time yields

$$\frac{dT_i}{dt} = \frac{dR_i}{dt} + \frac{dA}{dt}Q_i + \frac{dQ_i}{dt}A + \frac{dZ}{dt}q_i. \tag{A1}$$

Substituting equations (1), (4), (8), and (9) into equation (A1), we get

$$\begin{aligned}\frac{dT_i}{dt} = &-f_i(R_i, Q_i)A + dAQ_i + mZq_i\\ &+ g(A, Q_\mathrm{N}, Q_\mathrm{P})Z(Q_i - e(Q_\mathrm{N}, Q_\mathrm{P})q_i)\\ &+ [(\mu(Q_\mathrm{N}, Q_\mathrm{P}) - d)A - g(A, Q_\mathrm{N}, Q_\mathrm{P})Z]Q_i\\ &+ (f_i(R_i, Q_i) - \mu(Q_\mathrm{N}, Q_\mathrm{P})Q_i)A + [(e(Q_\mathrm{N}, Q_\mathrm{P})g(A, Q_\mathrm{N}, Q_\mathrm{P}) - m)Z]q_i.\end{aligned} \tag{A2}$$

We note that the terms on the right-hand side of equation (A2) cancel each other out. Hence, $dT_i/dt = 0$, so that the total amount of nutrient i in the ecosystem remains constant over time. In other words, our model ecosystem is a closed system with respect to nutrients.

Model Equations

Below we give a concise overview of all equations in the model.

Differential equations:

$$\frac{dR_i}{dt} = -f_i(R_i, Q_i)A + dAQ_i + mZq_i + g(A, Q_\mathrm{N}, Q_\mathrm{P})Z(Q_i - e(Q_\mathrm{N}, Q_\mathrm{P})q_i),$$

$$\frac{dA}{dt} = \mu(Q_\mathrm{N}, Q_\mathrm{P})A - dA - g(A, Q_\mathrm{N}, Q_\mathrm{P})Z,$$

$$\frac{dQ_i}{dt} = f_i(R_i, Q_i) - \mu(Q_\mathrm{N}, Q_\mathrm{P})Q_i,$$

$$\frac{dZ}{dt} = (e(Q_\mathrm{N}, Q_\mathrm{P})g(A, Q_\mathrm{N}, Q_\mathrm{P}) - m)Z,$$

$$\varepsilon\frac{dp_\mathrm{N}}{dt} = BV\frac{\partial w}{\partial p_\mathrm{N}},$$

where $i = \mathrm{N}, \mathrm{P}$. Functions:

$$\begin{aligned}
f_i(R_i, Q_i) &= f_{\max,i}\left(\frac{R_i}{R_i + K_i}\right)\left(\frac{Q_{\max,i} - Q_i}{Q_{\max,i} - Q_{\min,i}}\right),\\
\mu(Q_\mathrm{N}, Q_\mathrm{P}) &= \mu_{\max}\min\left(1 - \frac{Q_{\min,\mathrm{N}}}{Q_\mathrm{N}}, 1 - \frac{Q_{\min,\mathrm{P}}}{Q_\mathrm{P}}\right),\\
g(A, Q_\mathrm{N}, Q_\mathrm{P}) &= \frac{a\alpha(Q_\mathrm{N}, Q_\mathrm{P})A}{1 + ah\alpha(Q_\mathrm{N}, Q_\mathrm{P})A},\\
\alpha(Q_\mathrm{N}, Q_\mathrm{P}) &= \exp\left\{-[(Q_\mathrm{N}/Q_\mathrm{P}) - (q_\mathrm{N}/q_\mathrm{P})]^2/2(1/S)^2\right\},\\
e(Q_\mathrm{N}, Q_\mathrm{P}) &= \min\left(\frac{Q_\mathrm{N}}{q_\mathrm{N}}, \frac{Q_\mathrm{P}}{q_\mathrm{P}}\right),\\
T_i &= R_i + AQ_i + Zq_i,\\
f_{\max,\mathrm{N}} &= p_\mathrm{N}F_{\max,\mathrm{N}},\\
f_{\max,\mathrm{P}} &= p_\mathrm{P}F_{\max,\mathrm{P}},
\end{aligned}$$

with $p_\mathrm{N} + p_\mathrm{P} = 1$, and

$$w = \frac{1}{A}\frac{dA}{dt}.$$

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Appendix B from P. Branco et al., "Eco-Evolutionary Dynamics of Ecological Stoichiometry in Plankton Communities" (Am. Nat., vol. 192, no. 1, p. E1)

Eco-Evolutionary Dynamics in an Open Ecosystem with Respect to Nutrients

One of the assumptions of our model is that it is a closed ecosystem with respect to nutrients. To investigate the robustness of our model predictions, we relax this assumption to consider an open ecosystem with respect to nutrients. In the open model ecosystem, we add a new term to equation (9) describing nutrient inflow and outflow from the ecosystem:

$$\begin{aligned}\frac{dR_i}{dt} &= \delta(R_{\text{in},i} - R_i) - f_i(R_i, Q_i)A + dAQ_i + mZq_i \\ &\quad + g(A, Q_\text{N}, Q_\text{P})Z(Q_i - e(Q_\text{N}, Q_\text{P})q_i),\end{aligned} \tag{B1}$$

where δ is the rate of inflow and outflow of nutrient i, and $R_{\text{in},i}$ is the nutrient supply. The population dynamics of phytoplankton and zooplankton then read as follows:

$$\frac{dA}{dt} = \mu(Q_\text{N}, Q_\text{P})A - dA - \delta A - g(A, Q_\text{N}, Q_\text{P})Z, \tag{B2}$$

$$\frac{dZ}{dt} = (e(Q_\text{N}, Q_\text{P})g(A, Q_\text{N}, Q_\text{P}) - m - \delta)Z, \tag{B3}$$

where δ describes the outflow rate of individuals from the ecosystem.

The results show that our model predictions for an open ecosystem are qualitatively similar to those for a closed ecosystem (compare figs. 6 and B1 and figs. 7 and B2).

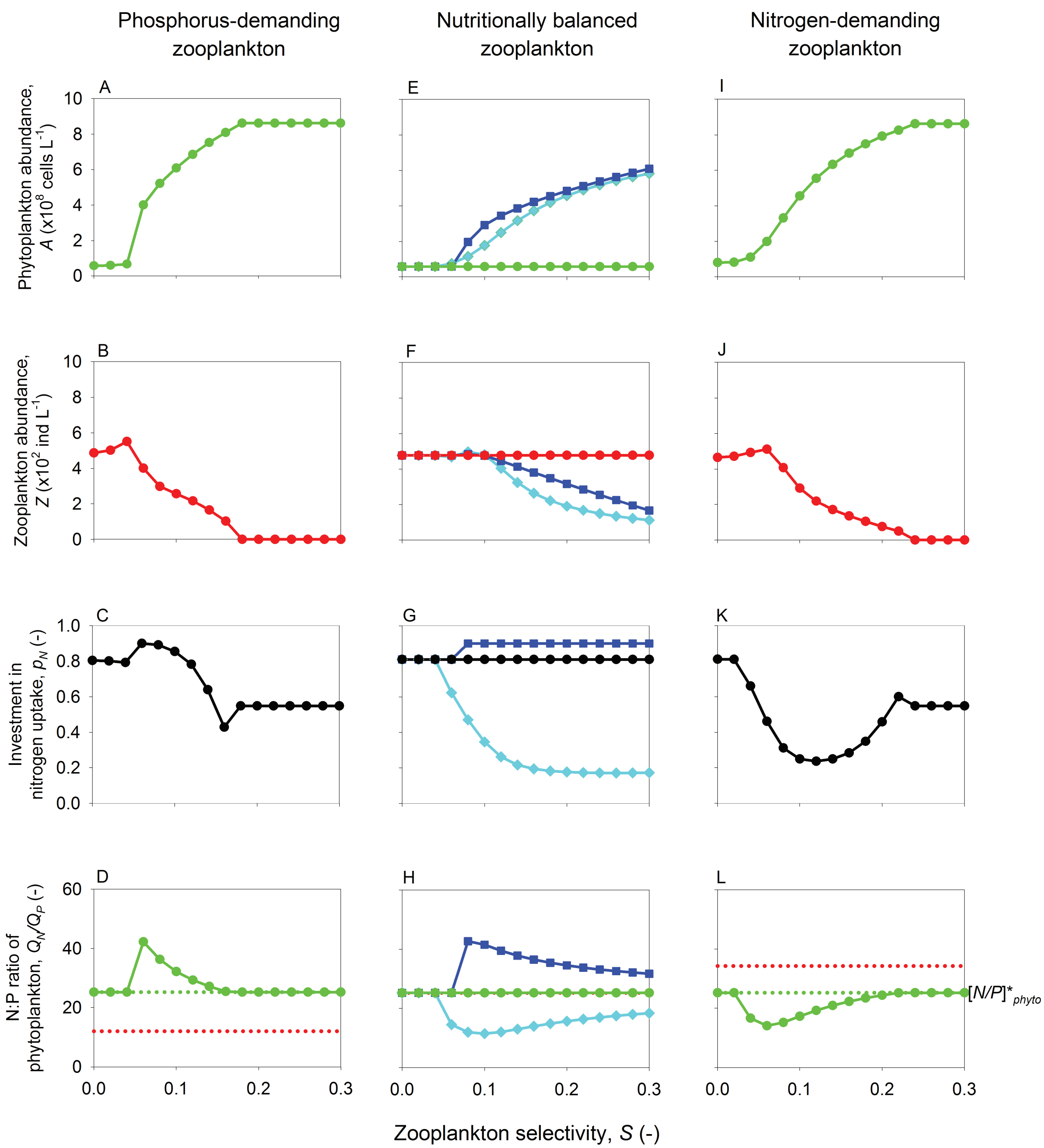

Figure B1: Bifurcation analysis along a gradient of zooplankton selectivity, assuming an open ecosystem with respect to nutrients. Bifurcation diagrams are shown for a community with phosphorus-demanding (*A*–*D*), nutritionally balanced (*E*–*H*), and nitrogen-demanding (*I*–*L*) zooplankton. *A*, *E*, *I*, Population abundance of phytoplankton. *B*, *F*, *J*, Population abundance of zooplankton. *C*, *G*, *K*, Investment of phytoplankton in nitrogen uptake. *D*, *H*, *L*, Cellular N : P ratio of phytoplankton (solid lines), as functions of zooplankton selectivity. Multiple solid lines within the same panel represent alternative stable states. The dotted green and red horizontal lines in *D*, *H*, and *L* indicate the body N : P ratios at which phytoplankton and zooplankton, respectively, are colimited by nitrogen and phosphorus. Parameter values are the same as in figure 6, with a nutrient inflow and outflow rate of $\delta = 0.05$ day^{-1}.

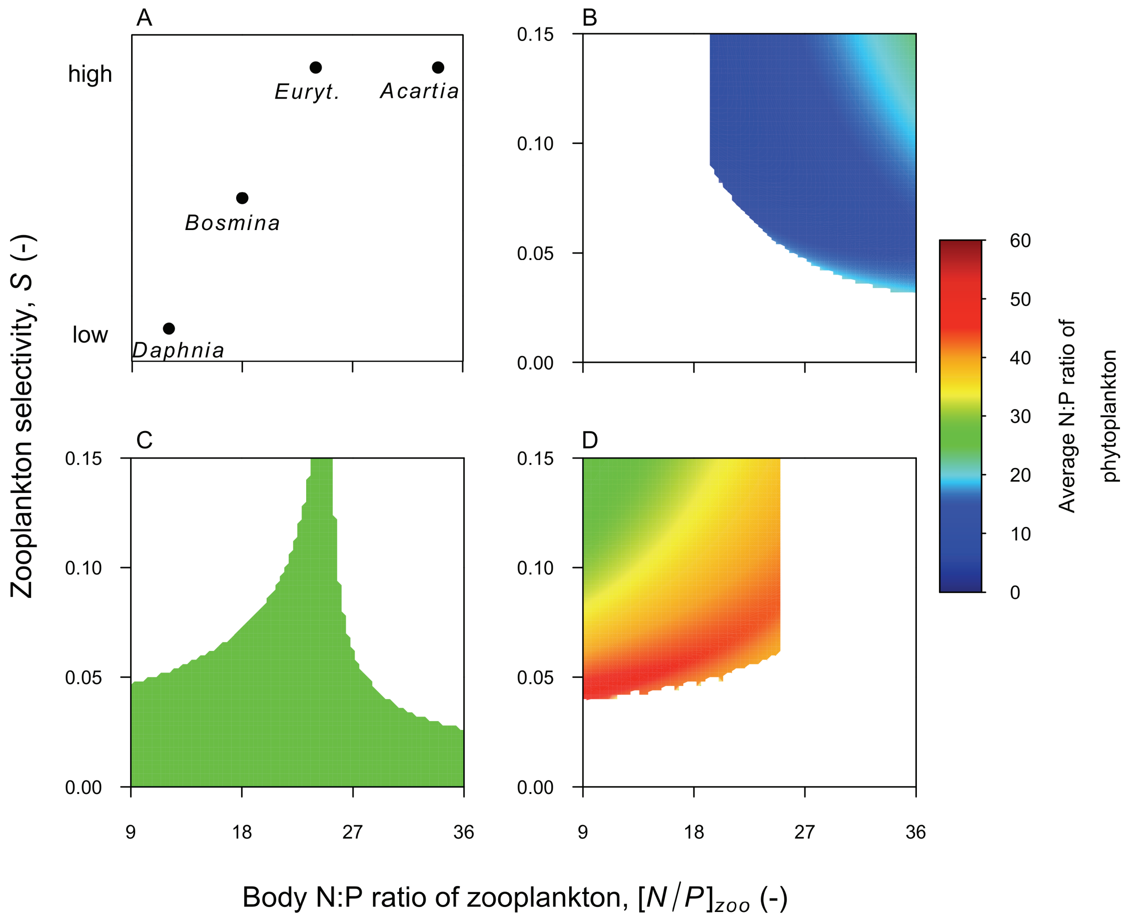

Figure B2: Effect of body N : P ratio and grazing selectivity of zooplankton on the N : P ratio of phytoplankton, assuming an open ecosystem with respect to nutrients. *A*, Body N : P ratio and grazing selectivity of four ecologically relevant zooplankton taxa; Euryt. = *Eurytemora*. *B–D*, Alternative stable states with low (*B*), optimum (*C*), and high (*D*) N : P ratios of phytoplankton predicted as functions of the body N : P ratio and grazing selectivity of zooplankton. Parameter values used in *B–D* are the same as in figure 7, with a nutrient inflow and outflow rate of $\delta = 0.05$ day^{-1}.

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Appendix C from P. Branco et al., "Eco-Evolutionary Dynamics of Ecological Stoichiometry in Plankton Communities" (Am. Nat., vol. 192, no. 1, p. E1)

Ecological Dynamics of Phytoplankton-Zooplankton Interactions

To assess how evolution affects the phytoplankton-zooplankton interactions, for comparison we also analyze an ecological version of our model where investment of phytoplankton in nitrogen uptake is balanced and does not evolve (i.e., $p_N = 0.5$). Similar to our analysis of the eco-evolutionary dynamics of phytoplankton-zooplankton interactions, we contrast the results of our ecological model for two different zooplankton types: phosphorus-demanding "cladocerans" with a low N:P ratio (fig. C1) and nitrogen-demanding "copepods" with a high N:P ratio (fig. C2).

For phosphorus-demanding zooplankton (i.e., cladocerans), the model predicts that nitrogen-poor ecosystems will lead to a stable equilibrium with low phytoplankton abundances, where zooplankton cannot find sufficient food to survive (fig. C1*A*). Phytoplankton do not evolve, and their investment in nitrogen versus phosphorus uptake remains balanced (fig. C1*B*), yielding a low N:P ratio at which their growth rate is severely nitrogen limited (fig. C1*C*). For ecosystems with intermediate nitrogen levels, the model predicts a stable equilibrium with high phytoplankton and low zooplankton abundances (fig. C1*D*), where phytoplankton growth is moderately phosphorus limited (fig. C1*F*). Further nitrogen enrichment leads to a stable equilibrium with high phytoplankton abundance, where zooplankton again cannot survive (fig. C1*G*). This time, however, zooplankton extinction is caused by the high N:P ratio of phytoplankton (fig. C1*I*), which deviates substantially from the body N:P ratio of zooplankton. These results clearly differ from the corresponding eco-evolutionary version of our model, which predicts slow phytoplankton-zooplankton oscillations induced by trait cycles and survival of the zooplankton population at high nitrogen levels (fig. 4).

The ecological dynamics of a community with nitrogen-demanding zooplankton differ from those with phosphorus-demanding zooplankton. In particular, nitrogen enrichment does not cause the extinction of nitrogen-demanding zooplankton and yields phytoplankton-zooplankton oscillations at intermediate and high nitrogen availability (fig. C2*D*, C2*G*). In other words, nitrogen enrichment improves the nutritional quality of phytoplankton for nitrogen-demanding zooplankton and thereby enhances zooplankton growth. These results are similar to the corresponding eco-evolutionary version of our model, which also predicts that nitrogen enrichment yields phytoplankton-zooplankton oscillations (fig. 5).

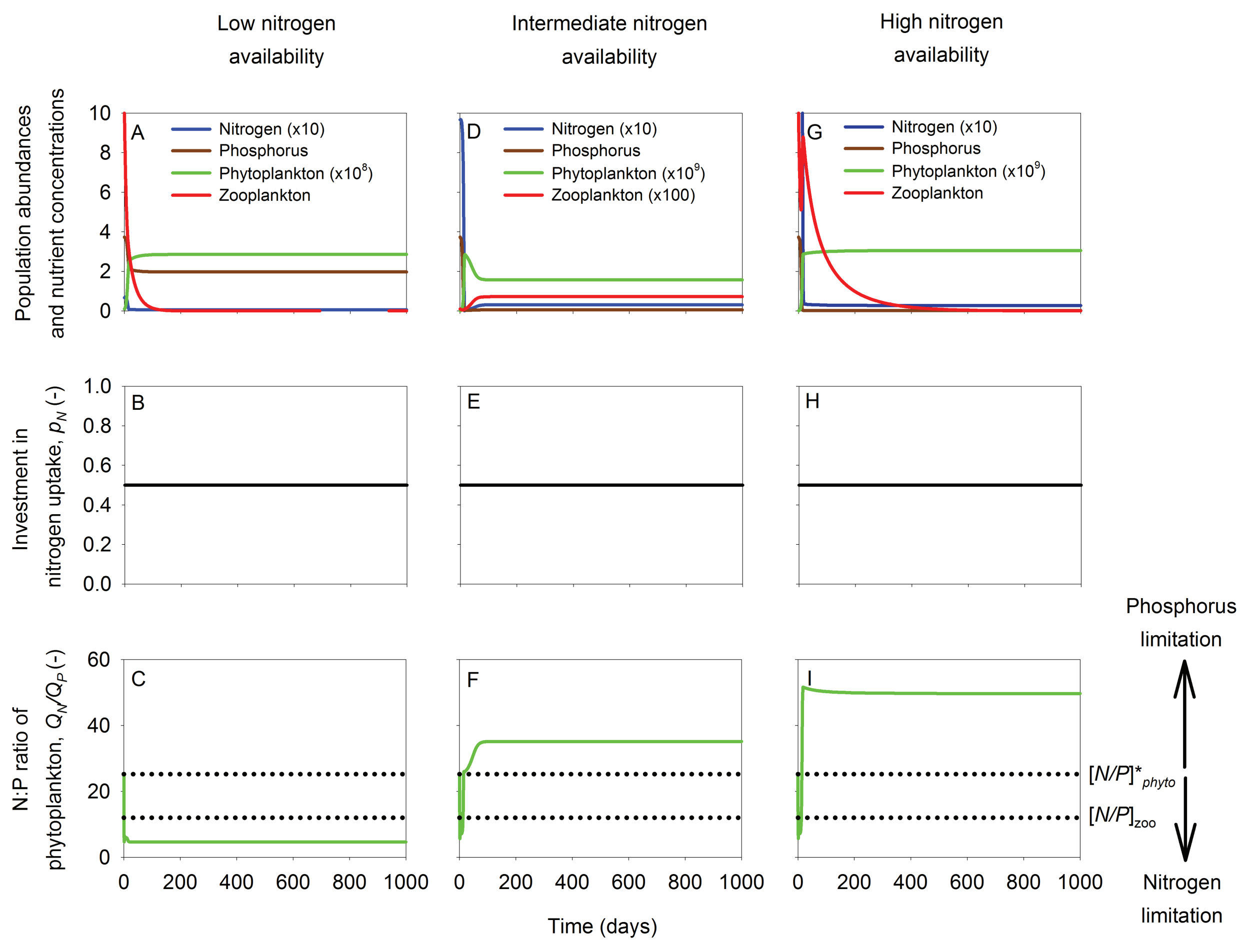

Figure C1: Ecological dynamics of phytoplankton-zooplankton interactions with phosphorus-demanding zooplankton ("cladocerans"). The graphs compare three ecosystems, with low (*A*–*C*), intermediate (*D*–*F*), and high (*G*–*I*) nitrogen availability. *A*, *D*, *G*, Dynamics of phytoplankton, zooplankton, and dissolved nitrogen and phosphorus concentrations. *B*, *E*, *H*, Investment of phytoplankton in nitrogen uptake. *C*, *F*, *I*, Cellular N∶P ratio of phytoplankton (green line); the dotted horizontal lines indicate the body N∶P ratios at which phytoplankton and zooplankton are colimited by nitrogen and phosphorus. Parameter values are the same as in figure 4, but with non-evolving phytoplankton ($V = 0$).

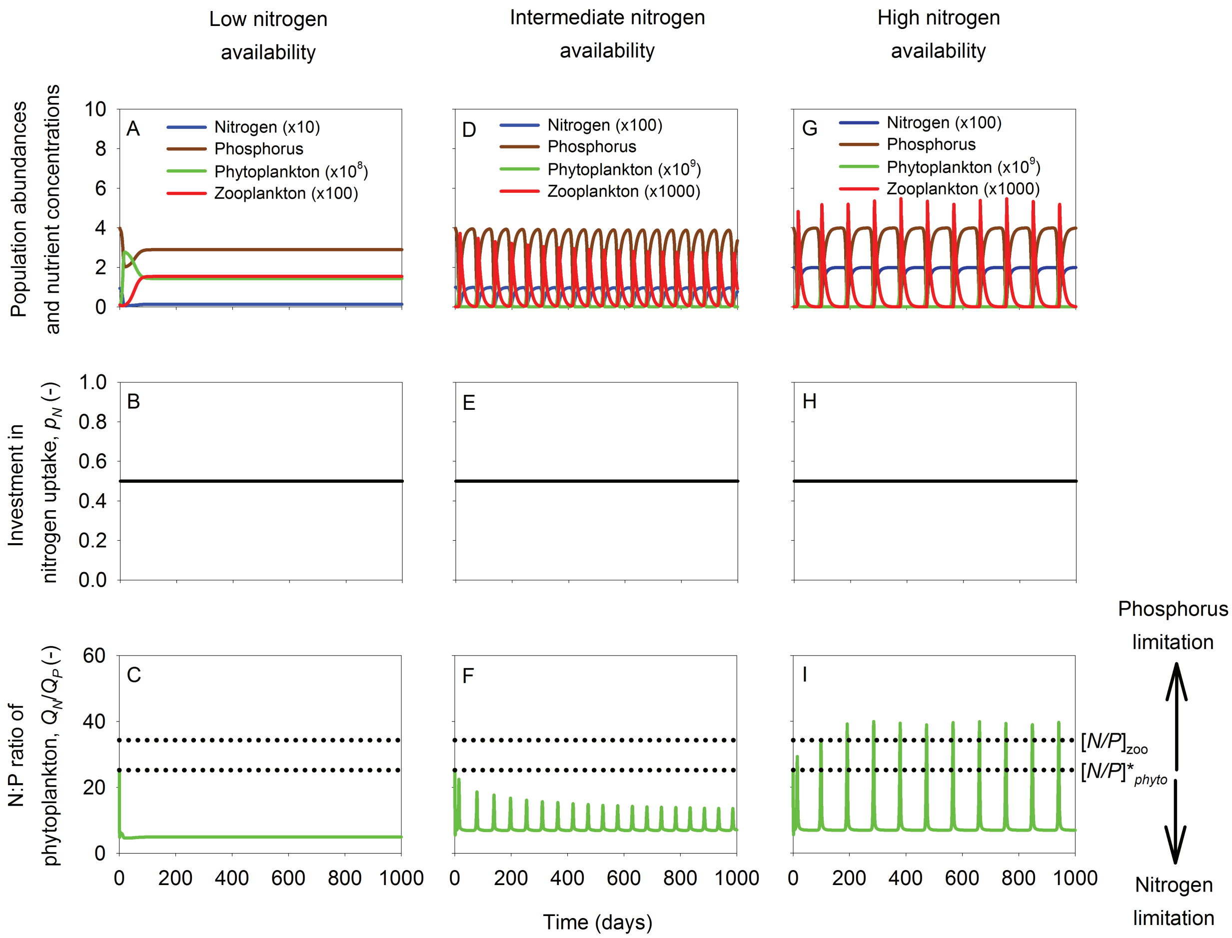

Figure C2: Ecological dynamics of phytoplankton-zooplankton interactions with nitrogen-demanding zooplankton ("copepods"). The graphs compare three ecosystems, with low (*A–C*), intermediate (*D–F*), and high (*G–I*) nitrogen availability. *A*, *D*, *G*, Dynamics of phytoplankton, zooplankton, and dissolved nitrogen and phosphorus concentrations. *B*, *E*, *H*, Investment of phytoplankton in nitrogen uptake. *C*, *F*, *I*, Cellular N : P ratio of phytoplankton (green line); the dotted horizontal lines indicate the body N : P ratios at which phytoplankton and zooplankton are colimited by nitrogen and phosphorus. Parameter values are the same as in figure 5, but with nonevolving phytoplankton ($V = 0$).