Centrality and rapidity dependence of inclusive jet production in $\sqrt{s_{\text{NN}}} = 5.02$ TeV proton-lead collisions with the ATLAS detector


DOI
10.1016/j.physletb.2015.07.023

Publication date
2015

Document Version
Final published version

Published in
Physics Letters B

License
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Citation for published version (APA):
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ABSTRACT

Measurements of the centrality and rapidity dependence of inclusive jet production in $\sqrt{s_{NN}} = 5.02$ TeV proton–lead ($p + \text{Pb}$) collisions and the jet cross-section in $\sqrt{s} = 2.76$ TeV proton–proton collisions are presented. These quantities are measured in datasets corresponding to an integrated luminosity of 27.8 nb$^{-1}$ and 4.0 pb$^{-1}$, respectively, recorded with the ATLAS detector at the Large Hadron Collider in 2013. The $p + \text{Pb}$ collision centrality was characterised using the total transverse energy measured in the pseudorapidity interval $-4.9 < \eta < -3.2$ in the direction of the lead beam. Results are presented for the double-differential per-collision yields as a function of jet rapidity and transverse momentum ($p_T$) for minimum-bias and centrality-selected $p + \text{Pb}$ collisions, and are compared to the jet rate from the geometric expectation. The total jet yield in minimum-bias events is slightly enhanced above the expectation in a $p_T$-dependent manner but is consistent with the expectation within uncertainties. The ratios of jet spectra from different centrality selections show a strong modification of jet production at all $p_T$ at forward rapidities and for large $p_T$ at mid-rapidity, which manifests as a suppression of the jet yield in central events and an enhancement in peripheral events. These effects imply that the factorisation between hard and soft processes is violated at an unexpected level in proton–nucleus collisions. Furthermore, the modifications at forward rapidities are found to be a function of the total energy only, implying that the violations may have a simple dependence on the hard parton–parton kinematics.

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1. Introduction

Proton–lead ($p + \text{Pb}$) collisions at the Large Hadron Collider (LHC) provide an excellent opportunity to study hard scattering processes involving a nuclear target [1]. Measurements of jet production in $p + \text{Pb}$ collisions provide a valuable benchmark for studies of jet quenching in lead–lead collisions by, for example, constraining the impact of nuclear parton distributions on inclusive jet yields. However, $p + \text{Pb}$ collisions also allow the study of possible violations of the QCD factorisation between hard and soft processes which may be enhanced in collisions involving nuclei.

Previous studies in deuteron–gold ($d + \text{Au}$) collisions at the Relativistic Heavy Ion Collider (RHIC) observed such violations, manifested in the suppressed production of very forward hadrons with transverse momenta up to 4 GeV [2–4]. Studies of forward di-hadron angular correlations at RHIC also showed a much weaker dijet signal in $d + \text{Au}$ collisions than in $p p$ collisions [4,5]. These effects have been attributed to the saturation of the parton distributions in the gold nucleus [6–8], to the modification of the nuclear parton distribution function [9], to the higher-twist contributions to the cross-section enhanced by the forward kinematics of the measurement [10], or to the presence of a large nucleus [11]. The extended kinematic reach of $p + \text{Pb}$ measurements at the LHC allows the study of hard scattering processes that produce forward hadrons or jets over a much wider rapidity and transverse momentum range. Such measurements can determine whether the factorisation violations observed at RHIC persist at higher energy and, if so, how the resulting modifications vary as a function of particle or jet momentum and rapidity. The results of such measurements could test the competing descriptions of the RHIC results and, more generally, provide new insight into the physics of hard scattering processes involving a nuclear target.

This paper reports the centrality dependence of inclusive jet production in $p + \text{Pb}$ collisions at a nucleon–nucleon centre-of-mass energy $\sqrt{s_{NN}} = 5.02$ TeV. The measurement was performed using a dataset corresponding to an integrated luminosity of 27.8 nb$^{-1}$ recorded in 2013. The $p + \text{Pb}$ jet yields were

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compared to a nucleon–nucleon reference constructed from a measurement of jet production in pp collisions at a centre-of-mass energy $\sqrt{s} = 2.76$ TeV using a dataset corresponding to an integrated luminosity of 4.0 $\text{pb}^{-1}$ also recorded in 2013. Jets were reconstructed from energy deposits measured in the calorimeter using the anti-$k_t$ algorithm with radius parameter $R = 0.4$ [12].

The centrality of $p + \text{Pb}$ collisions was characterised using the total transverse energy measured in the pseudorapidity $\eta$ interval $-4.9 < \eta < -3.2$ in the direction of the lead beam. Whereas in nucleus–nucleus collisions centrality reflects the degree of nuclear overlap between the colliding nuclei, centrality in $p + \text{Pb}$ collisions is sensitive to the multiple interactions between the proton and nucleons in the lead nucleus. Centrality has been successfully used at lower energies in $d + \text{Au}$ collisions at RHIC as an experimental handle on the collision geometry [2,13,14].

A Glauber model [15] was used to determine the average number of nucleon–nucleon collisions, $\langle N_{\text{coll}} \rangle$, and the mean value of the overlap function, $T_{pA}(b) = \int_{-\infty}^{\infty} \rho(b,z) dy$, where $\rho(b,z)$ is the nucleon density at impact parameter $b$ and longitudinal position $z$ in each centrality interval. Per-event jet yields, $(1/N_{\text{coll}})(d^2N_{\text{jet}}/dp_Tdy)$, were measured as a function of jet centre-of-mass rapidity, $y^*$, and transverse momentum, $p_T$, where $N_{\text{jet}}$ is the number of jets measured in $N_{\text{coll}}$ $p + \text{Pb}$ events analysed. The centrality dependence of the per-event jet yields was evaluated using the nuclear modification factor,

$$R_{pA} \equiv \frac{1}{T_{pA}} \frac{(1/N_{\text{coll}})(d^2N_{\text{jet}}/dp_Tdy)}{(d^2N_{\text{jet}}/dp_Tdy)|_{\text{cent}}}|_{\|} \text{ cent,}$$

for a given centrality selection “cent”, where $(d^2N_{\text{jet}}/dp_Tdy)$ is determined using the jet cross-section measured in pp collisions at $\sqrt{s} = 2.76$ TeV. The factor $R_{pA}$ quantifies the absolute modification of the jet rate relative to the geometric expectation. In each centrality interval, the geometric expectation is the jet rate that would be produced by an incoherent superposition of the number of nucleon–nucleon collisions corresponding to the mean nuclear thickness in the given class of $p + \text{Pb}$ collisions.

Results are also presented for the central-to-peripheral ratio,

$$R_{CP} \equiv \frac{1}{R_{\text{coll}}} \frac{(1/N_{\text{coll}})(d^2N_{\text{jet}}/dp_Tdy)}{(d^2N_{\text{jet}}/dp_Tdy)|_{\text{per}}} \text{ cent,}$$

where $R_{\text{coll}}$ represents the ratio of $N_{\text{coll}}$ in a given centrality interval to that in the most peripheral interval, $R_{\text{coll}} \equiv |N_{\text{cent}}|/|N_{\text{per}}|$. The $R_{CP}$ ratio is sensitive to relative deviations in the jet rate from the geometric expectation between the $p + \text{Pb}$ event centralities. The $R_{pA}$ and $R_{CP}$ measurements are presented as a function of inclusive jet $y^*$ and $p_T$.

For the 2013 $p + \text{Pb}$ run, the LHC was configured with a 4 TeV proton beam and a 1.57 TeV per-nucleon Pb beam that together produced collisions with $\sqrt{s_NN} = 5.02$ TeV and a rapidity shift of the centre-of-mass frame of 0.465 units relative to the ATLAS rest frame. The run was split into two periods, with the directions of the proton and lead beams being reversed at the end of the first period. The first period provided approximately 55% of the integrated luminosity with the Pb beam travelling to positive rapidity and the proton beam to negative rapidity, and the second period provided the remainder with the beams reversed. The analysis in this paper uses the events from both periods of data-taking and $y^*$ is defined so that $y^* > 0$ always refers to the downstream proton direction.

2. Experimental setup

The measurements presented in this paper were performed using the ATLAS inner detector (ID), calorimeters, minimum-bias trigger scintillator (MBTS), and trigger and data acquisition systems [16]. The ID measures charged particles within $|\eta| < 2.5$ using a combination of silicon pixel detectors, silicon microstrip detectors, and a straw-tube transition radiation tracker, all immersed in a 2 T axial magnetic field [17]. The calorimeter system consists of a liquid argon (LAr) electromagnetic (EM) calorimeter covering $|\eta| < 3.2$, a steel/scintillator sampling hadronic calorimeter covering $|\eta| < 1.7$, a LAr hadronic calorimeter covering $1.5 < |\eta| < 3.2$, and two LAr electromagnetic and hadronic forward calorimeters (FCal) covering $3.2 < |\eta| < 4.9$. The EM calorimeters use lead plates as the absorbers and are segmented longitudinally in shower depth into three compartments with an additional presampler layer in front for $|\eta| < 1.8$. The granularity of the EM calorimeter varies with layer and pseudorapidity. The middle sampling layer, which typically has the largest energy deposit in EM showers, has a $\Delta\eta \times \Delta\phi$ granularity of 0.025 × 0.025 within $|\eta| < 2.5$. The hadronic calorimeter uses steel as the absorber and has three segments longitudinal in shower depth with cell sizes $\Delta\eta \times \Delta\phi = 0.1 \times 0.1$ for $|\eta| < 2.5$ and $0.2 \times 0.2$ for $2.5 < |\eta| < 4.9$. The two FCal modules are composed of tungsten and copper absorbers with LAr as the active medium, which together provide ten interaction lengths of material. The MBTS detects charged particles over 2.1 $< |\eta| < 3.9$ using two hodoscopes of 16 counters each, positioned at $z = \pm 3.6$ m.

The $p + \text{Pb}$ and pp events used in this analysis were recorded using a combination of minimum-bias (MB) and jet triggers [18]. In $p + \text{Pb}$ data-taking, the MB trigger required hits in at least one counter in each side of the MBTS detector. In pp collisions the MB condition was the presence of hits in the pixel and microstrip detectors reconstructed as a track by the high-level trigger system. Jets were selected using high-level jet triggers implemented with a reconstruction algorithm similar to the procedure applied in the offline analysis. In particular, it used the anti-$k_t$ algorithm with $R = 0.4$, a background subtraction procedure, and a calibration of the jet energy to the full hadronic scale. The high-level jet triggers were seeded from a combination of low-level MB and jet hardware-based triggers. Six jet triggers with transverse energy thresholds ranging from 20 GeV to 75 GeV were used to select jets within $|\eta| < 3.2$ and a separate trigger with a threshold of 15 GeV was used to select jets with $3.2 < |\eta| < 4.9$. The triggers were prescaled in a fashion which varied with time to accommodate the evolution of the luminosity within an LHC fill.

3. Data selection

In the offline analysis, charged-particle tracks were reconstructed in the ID with the same algorithm used in pp collisions [19]. The $p + \text{Pb}$ events used for this analysis were required to have

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1. ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the z-axis along the beam axis. The x-axis points from the IP to the centre of the LHC ring, and the y-axis points upward. Cylindrical coordinates (r, $\phi$) are used in the transverse plane, $\phi$ being the azimuthal angle around the beam pipe. The pseudorapidity is defined in laboratory coordinates in terms of the polar angle $\theta$ as $\eta = -\ln \tan(\theta/2)$. During 2013 $p + \text{Pb}$ data-taking, the beam directions were reversed approximately half-way through the running period, but in presenting results the direction of the proton beam is always chosen to point to positive $\eta$.

2. The jet rapidity $y^*$ is defined as $y^* = 0.5 \ln \frac{E + p_y}{E - p_y}$, where $E$ and $p_y$ are the energy and the component of the momentum along the proton beam direction in the nucleon–nucleon centre-of-mass frame.

3. An exception is the third (outermost) sampling layer, which has a segmentation of $0.2 < |\eta| < 1.7$. 
a reconstructed vertex containing at least two associated tracks with $p_T > 0.1$ GeV, at least one hit in each of the two MBTS hodoscopes, and a difference between times measured on the two MBTS sides of less than 10 ns. Events containing multiple $p + Pb$ collisions (pileup) were suppressed by rejecting events having two or more reconstructed vertices, each associated with reconstructed tracks with a total transverse momentum scalar sum of at least 5 GeV. The fraction of events with one $p + Pb$ interaction rejected by this requirement was less than 0.1%. Events with a pseudorapidity gap (defined by the absence of clusters in the calorimeter with more than 0.2 GeV of transverse energy) of greater than two units on the Pb-going side of the detector were also removed from the analysis. Such events arise primarily from electromagnetic or diffractive excitation of the proton. After accounting for event selection, the number of $p + Pb$ events sampled by the highest-luminosity jet trigger (which was unprescaled) was 53 billion. The event selection criteria described here were designed to select a sample of $p + Pb$ events to which a centrality analysis can be applied and for which meaningful geometric parameters can be determined.

The $pp$ events used in this analysis were required to have a reconstructed vertex, with the same definition as the vertices in $p + Pb$ events above. No other requirements were applied.

4. Centrality determination

The centrality of the $p + Pb$ events selected for analysis was characterised by the total transverse energy $\Sigma E_T^{Pb}$ in the FCal module on the Pb-going side. The $\Sigma E_T^{Pb}$ distribution for minimum-bias $p + Pb$ collisions passing the event selection described in Section 3 is presented in Fig. 1. Following standard techniques [20], centrality intervals were defined in terms of percentiles of the $\Sigma E_T^{Pb}$ distribution after accounting for an estimated inefficiency of $(2 \pm 2)\%$ for inelastic $p + Pb$ collisions to pass the applied event selection. The following centrality intervals were used in this analysis, in order from the most central to the most peripheral: 0–10%, 10–20%, 20–30%, 30–40%, 40–60%, and 60–90%, with the 60–90% interval serving as the reference for the $R_{CP}$ ratio. Events with a centrality beyond 90% were not used in the analysis, since the uncertainties on the composition of the event sample and in the determination of the geometric quantities are large for these events.

A Glauber Monte Carlo (MC) [15] analysis was used to calculate $R_{coll}$ and $T_{PA}$ for each centrality interval. First, a Glauber MC program [21] was used to simulate the geometry of inelastic $p + Pb$ collisions and calculate the probability distribution of the number of nucleon participants $N_{part}$, $P(N_{part})$. The simulations used a Woods–Saxon nuclear density distribution and an inelastic nucleon–nucleon cross-section, $\sigma_{NN}$, of $70 \pm 5$ mb. Separately, PYTHIA 8 [22,23] simulations of 4 TeV on 1.57 TeV pp collisions provided a detector-level $\Sigma E_T^{Pb}$ distribution for nucleon–nucleon collisions, to be used as input to the Glauber model. This distribution was fit to a gamma distribution.

Then, an extension of the wounded-nucleon (WN) [24] model that included a non-linear dependence of $\Sigma E_T^{Pb}$ on $N_{part}$ was used to define $N_{part}$-dependent gamma distributions for $\Sigma E_T^{Pb}$, with the constraint that the distributions reduce to the PYTHIA distribution for $N_{part} = 2$. The non-linear term accounted for the possible variation of the effective FCal acceptance resulting from an $N_{part}$-dependent backward rapidity shift of the produced soft particles with respect to the nucleon–nucleon frame [25]. The gamma distributions were summed over $N_{part}$ with a $P(N_{part})$ weighting to produce a hypothetical $\Sigma E_T^{Pb}$ distribution. That distribution was fit to the measured $\Sigma E_T^{Pb}$ distribution shown in Fig. 1 with the parameters of the extended WN model allowed to vary freely. The best fit, which contained a significant non-linear term, successfully described the $\Sigma E_T^{Pb}$ distribution in data over several orders of magnitude. From the results of the fit, the distribution of $N_{part}$ values and the corresponding $[N_{part}]$ were calculated for each centrality interval. The resulting $R_{coll}$ and $T_{PA}$ values and corresponding systematic uncertainties, which are described in Section 8, are shown in Table 1.

5. Monte Carlo simulation

The performance of the jet reconstruction procedure was evaluated using a sample of 36 million events in which a measured $\sqrt{s} = 5.02$ TeV pp hard-scattering events were overlaid with minimum-bias $p + Pb$ events recorded during the 2013 run. Thus the sample contains an underlying event contribution that is identical in all respects to the data. The simulated events were generated using PYTHIA 8 [22] (version 6.425, AUET2B tune [26], CTEQ6L1 parton distribution functions [27]) and the detector effects were fully simulated using GEANT4 [28,29]. These events were produced for different $p_T$ intervals of the generator-level (“truth”) $R = 0.4$ jets. In total, the generator-level spectrum spans $10 < p_T < 10^2$ GeV. Separate sets of 18 million events each were generated for the two different beam directions to take into account any $z$-axis asymmetries in the detector. For each beam direction, the four-momenta of the generated particles were longitudinally boosted by a rapidity of $\pm 0.465$ to match the corresponding beam conditions. The events were simulated using detector conditions appropriate to the two periods of the 2013 $p + Pb$ run and reconstructed using the same algorithms as were applied to the experimental data. A sep-
erate 9-million-event sample of fully simulated 2.76 TeV PYTHIA pp hard scattering events (with the same version, tune and parton distribution function set) was used to evaluate the jet performance in \(\sqrt{s} = 2.76\) TeV pp collisions during 2013 data-taking.

6. Jet reconstruction and performance

The jet reconstruction and underlying event subtraction procedures were adapted from those used by ATLAS in Pb+Pb collisions, which are described in detail in Refs. [30,31], and are summarised here along with any substantial differences from the referenced analyses.

An iterative procedure was used to obtain an event-by-event estimate of the underlying event energy density while excluding contributions from jets to that estimate. The modulation of the underlying event energy density to account for potential elliptic flow was not included in this analysis. Jets were reconstructed from the anti-kt algorithm with \(R = 0.4\) applied to calorimeter cells grouped into \(\Delta\eta \times \Delta\phi = 0.1 \times 0.1\) towers, with the final jet kinematics calculated from the background-subtracted energy in the cells contained in the jet. The rate of jets reconstructed from the underlying event fluctuations of soft particles was negligible in the kinematic range studied and therefore no attempt to reject them was made. The mean subtracted transverse energy in pp collisions was 2.4 GeV (1.4 GeV) for jets with \(|y^*| < 1\) (\(|y^*| > 3\)). In pp collisions, this procedure simply subtracts the underlying event pedestal deposited in the calorimeter which can arise, in part, from the presence of additional pp interactions in the same crossing (in-time pileup).

Following the above jet reconstruction, a small correction, typically a few percent, was applied to the transverse momentum of those jets which did not overlap with a region excluded from the background determination and thus were erroneously included in the initial estimate of the underlying event background. Then, the jet energies were corrected to account for the calorimeter energy response using an \(\eta\)- and \(p_T\)-dependent multiplicative factor that was derived from the simulations [32]. Following this calibration, a final multiplicative in situ calibration was applied to account for differences between the simulated detector response and data. The measured \(p_T\) of jets recoiling against objects with an independently calibrated energy scale (such as Z bosons, photons, or jets in a different region of the detector) was investigated. The in situ calibration, which typically differed from unity by a few percent, was derived by comparing this \(p_T\) balance in pp data with that in simulations in a fashion similar to that used previously within ATLAS [33].

The jet reconstruction performance was evaluated in the simulated samples by applying the same subtraction and reconstruction procedure as was applied to data. The resulting reconstructed jets with transverse momentum \(p_T^{\text{jet}}\) were compared with their corresponding generator jets, which were produced by applying the anti-kt algorithm to the final-state particles produced by PYTHIA, excluding muons and neutrinos. Each generator jet was matched to a reconstructed jet, and the \(p_T\) difference between the two jets was studied as a function of the generator jet transverse momentum, \(p_T^{\text{gen}}\), and generator jet rapidity \(y^*\), and in the six pp and Pb Pb event centrality intervals.

The reconstruction efficiency for jets having \(p_T^{\text{jet}} > 25\) GeV was found to be greater than 99%. The performance was quantified by the means and standard deviations of the \(\Delta p_T/p_T\) (\(= p_T^{\text{rec}}/p_T^{\text{gen}} - 1\)) distributions, referred to as the jet energy scale closure and jet energy resolution respectively. The closure in pp events was less than 2% for \(p_T^{\text{gen}} \geq 25\) GeV jets and was better than 1% for \(p_T^{\text{gen}} > 100\) GeV jets. At low \(p_T^{\text{gen}}\), the energy scale closure and resolution exhibited a weak \(p_T\) dependence, with differences in the closure of up to 1% and differences in the resolution of up to 2% in the most central 0–10% events relative to the 60–90% peripheral events. At high jet \(p_T\), the response was centrality independent within sensitivity. In pp events, the closure was less than 1% in the entire kinematic range studied.

In order to quantify the degree of \(p_T\)-bin migration introduced by the detector response and reconstruction procedure, response matrices were populated by recording the \(p_T\) values of each generator–reconstructed jet pair. Separate matrices were constructed for each \(y^*\) interval and \(p_T\) centrality interval used in the analysis. The \(p_T\) bins used were chosen to increase with \(p_T\) such that the width of each bin was \(\approx 0.25\) of the bin low edge. Using this binning, the proportion of jets with reconstructed \(p_T\) in the same bin as their true \(p_T\) monotonically increased with truth \(p_T\) and was 50–70%.

7. Data analysis

A combination of minimum-bias and jet triggered \(p_T\) Pb Pb events were selected for analysis as described in Section 2. The sampled luminosity (defined as the luminosity divided by the mean luminosity-weighted prescale) of the jet triggers increased with increasing \(p_T\) threshold. Offline jets were selected for the analysis by requiring a match to an online jet trigger. The efficiency of the various triggers was determined with respect to the minimum-bias trigger and to lower threshold jet triggers. For simplicity, each \(p_T\) bin used jets selected by only one jet trigger. In a given \(p_T\) bin, jets were selected by the highest-threshold jet trigger for which the efficiency was determined to be greater than 99% in the bin. No additional corrections for the trigger efficiency were applied.

The double-differential per-event jet yields in \(p_T\) Pb Pb collisions were constructed via

\[
\frac{1}{N_{\text{evt}}} \frac{d^2N^{\text{jet}}}{dp_T dy^*} = \frac{1}{N_{\text{evt}}} N_{\text{evt}}^{\text{jet}} \frac{\Delta p_T \Delta y^*}{\Delta p_T \Delta y^*},
\]

where \(N_{\text{evt}}\) is the total (unprescaled) number of MB \(p + Pb\) events sampled, \(N_{\text{jet}}\) is the yield of jets corrected for all detector effects and the instantaneous trigger prescale during data-taking, and \(\Delta p_T\) and \(\Delta y^*\) are the widths of the \(p_T\) and \(y^*\) bins. The centrality-dependent yields were constructed by restricting \(N_{\text{evt}}\) and \(N_{\text{jet}}\) to come from \(p + Pb\) events within a given centrality interval. The double-differential cross-section in \(p + Pb\) collisions was constructed via

\[
\frac{d^2\sigma}{dp_T dy^*} = \frac{1}{L_{\text{int}}} \frac{N_{\text{jet}}^{\text{jet}}}{\Delta p_T \Delta y^*},
\]

where \(L_{\text{int}}\) is the total integrated luminosity of the jet trigger used in the given \(p_T\) bin. The \(p_T\) binning in the \(p + Pb\) cross-section was chosen such that the \(x_T = 2p_T/\sqrt{s}\) binning between the \(p + Pb\) and pp datasets is the same.

Both the per-event yields in \(p + Pb\) collisions and the cross-section in \(p + Pb\) collisions were restricted to the \(p_T\) range where the MC studies described in Section 6 show that the efficiency for a truth jet to remain in the same \(p_T\) bin is \(\geq 50\%\). This \(p_T\) range was rapidity dependent, with the lowest \(p_T\) bin edge used ranging from 50 GeV in the most backward rapidity intervals studied to 25 GeV in the most forward intervals.

The measured \(p + Pb\) and pp yields were corrected for jet energy resolution and residual distortions of the jet energy scale which result in \(p_T\)-bin migration. For each rapidity interval, the yield was corrected by the use of \(p_T\)-dependent (and, in the \(p + Pb\) case, centrality-dependent) bin-by-bin correction factors \(C(p_T, y^*)\) obtained from the ratio of the reconstructed to the truth jet \(p_T\) distributions for jets originating in a true \(y^*\) bin, according to
\[ C(p_T, y^*) = \frac{N_{\text{jet}}^\text{true}(p_T, y^*)}{N_{\text{jet}}^\text{reco}(p_T, y^*)}, \]  

(5)

where \( N_{\text{jet}}^\text{true} / N_{\text{jet}}^\text{reco} \) is the number of truth jets in the given \( p_T \) \( (p_T^\text{reco}) \) bin in the corresponding MC samples.

Since the determination of the correction factors \( C(p_T, y^*) \) is sensitive to the shape of the jet spectrum in the MC sample, the response matrices used to generate them were reweighted to provide a better match between the reconstructed distributions in data and simulated events. The spectrum of generator jets was weighted jet-by-jet by the ratio of the reconstructed spectrum in data to that in simulation. This ratio was found to be approximately linear in the logarithm of reconstructed \( p_T \). A separate reweighting was performed for the \( p + \) Pb jet yield in each centrality interval, resulting in changes of \(-10\%\) from the original correction factors before reweighting. The resulting corrections to the \( p + \) Pb and pp yields were at most 30\%, and were typically \(-10\%\) for jets with \( p_T > 100 \text{ GeV} \). These corrections were applied to the detector-level yield \( N_{\text{jet}}^\text{reco} \) to give the particle-level yield via

\[ N_{\text{jet}} = C(p_T, y^*)N_{\text{jet}}^\text{reco}. \]  

(6)

A \( \sqrt{s} = 5.02 \text{ TeV} \) pp reference jet cross-section was constructed through the use of the corrected 2.76 TeV pp cross-section and a previous ATLAS measurement of the \( x_T \)-scaling between the inclusive jet cross-sections at \( \sqrt{s} = 2.76 \text{ TeV} \) (measured using 0.20 pb\(^{-1}\) of data collected in 2011) and 7 TeV (measured using 37 pb\(^{-1}\) of data collected in 2010) [34]. In this previous analysis, the \( \sqrt{s} \)-scaled ratio \( \rho \) of the 2.76 TeV cross-section to that at 7 TeV was evaluated at fixed \( x_T \).

\[ \rho(x_T; y^*) = \left( \frac{2.76 \text{ TeV}}{7 \text{ TeV}} \right)^3 \frac{d^2\sigma/\sqrt{s}/dp_T dy^*}{d^2\sigma/7 \text{ TeV}/dp_T dy^*}. \]  

(7)

where \( d^2\sigma/\sqrt{s}/dp_T dy^* \) is the pp jet cross-section at the given centre-of-energy \( \sqrt{s} \), and the numerator and denominator are evaluated at the same \( x_T \) (but different \( p_T = x_T\sqrt{s}/2 \)). Equation (7) can be rearranged to define the cross-section at \( \sqrt{s} = 7 \text{ TeV} \) in terms of that at 2.76 TeV times a multiplicative factor and divided by \( \rho \).

The \( \sqrt{s} = 5.02 \text{ TeV} \) pp cross-section at each \( p_T \) and \( y^* \) value was constructed by scaling the corrected \( \sqrt{s} = 2.76 \text{ TeV} \) pp cross-section measured at the equivalent \( x_T \) according to

\[ \frac{d^2\sigma}{dp_T dy^*}(2.76 \text{ TeV}) = \rho(x_T; y^*) \frac{d^2\sigma}{dp_T dy^*}(5.02 \text{ TeV}). \]  

(8)

where the power \(-\ln(2.76/5.02)/\ln(2.76/7) \approx -0.643\) interpolates between 2.76 TeV and 7 TeV to 5.02 TeV using a power-law collision energy dependence at each \( p_T \) and \( y^* \). Since the jet energy scale and \( x_T \)-interpolation uncertainties are large for the pp data at large rapidities \( |y^*| > 2.8 \), a \( \sqrt{s} = 5.02 \text{ TeV} \) reference is not constructed in that rapidity region.

The pp jet cross-section at \( \sqrt{s} = 2.76 \text{ TeV} \) measured with the 2013 data was found to agree with the previous ATLAS measurement of the same quantity [34] within the systematic uncertainties.

8. Systematic uncertainties

The \( R_{CP} \) and \( R_{p+Pb} \) measurements are subject to systematic uncertainties arising from a number of sources: the jet energy scale and resolution, differences in the spectral shape between data and simulation affecting the bin-by-bin correction factors, residual inefficiency in the trigger selection, and the estimates of the geometric quantities \( R_{coll} \) (in \( R_{CP} \)) and \( T_{pA} \) (in \( R_{p+Pb} \)). In addition to these sources of uncertainty, which are common to the \( R_{CP} \) and \( R_{p+Pb} \) measurements, \( R_{pA} \) is also subject to uncertainties from the \( x_T \)-interpolation of the \( \sqrt{s} = 2.76 \text{ TeV} \) pp cross-section to the \( \sqrt{s} = 5.02 \text{ TeV} \) centre-of-mass energy and from the integrated luminosity of the pp dataset.

Uncertainties in the jet energy scale and resolution influence the correction of the \( p + \) Pb and pp jet spectra. The uncertainty in the scale was taken from studies of the in situ calorimeter response and systematic variations of the jet response in simulation [32], as well as studies of the relative energy scale difference between the jet reconstruction procedure in heavy-ion collisions and the procedure used by ATLAS for inclusive jet measurements in 2.76 TeV and 7 TeV pp collisions [34,35]. The total energy scale uncertainty in the measured \( p_T \) range was \( \lesssim 4\% \) for jets in \( |y^*| < 2.8 \), and \( \lesssim 7\% \) for jets in \( |y^*| > 2.8 \). The sensitivity of the results to the uncertainty in the energy scale was evaluated separately for ten distinct sources of uncertainty. Each source was treated as fully uncorrelated with any other source, but fully correlated with itself in \( p_T \), \( \eta \), and \( \sqrt{s} \). The uncertainty in the resolution was taken from in situ studies of the dijet energy balance [36]. The resolution uncertainty was generally \( < 10\% \), except for low-\( p_T \) jets where it was \( < 20\% \). The effects on the \( R_{CP} \) and \( R_{p+Pb} \) measurements were evaluated through an additional smearing of the energy of reconstructed jets in the simulation such that the resolution uncertainty was added to the original resolution in quadrature.

The resulting systematic uncertainties on \( R_{CP} \) (\( \delta R_{CP} \)) and \( R_{p+Pb} \) (\( \delta R_{p+Pb} \)) were evaluated by producing new response matrices in accordance with each source of the energy scale uncertainty and the resolution uncertainty, generating new correction factors, and calculating the new \( R_{CP} \) and \( R_{p+Pb} \) results. Each energy scale and resolution variation was applied to all rapidity bins and to both the \( p + \) Pb and pp response matrices simultaneously. The uncertainty on \( R_{CP} \) and \( R_{p+Pb} \) from the total energy scale uncertainty was determined by adding the effects of the ten energy scale uncertainty sources in quadrature. Since the correction factors for the \( p + \) Pb spectra in different centrality intervals were affected to a similar degree by variations in the energy scale and resolution, the effects tended to cancel in the \( R_{CP} \) ratio, and the resulting \( \delta R_{CP} \) were small. The resulting \( \delta R_{p+Pb} \) values were somewhat larger than the \( \delta R_{CP} \) values due to the relative centre-of-mass shift between the \( p + \) Pb and pp collision systems. The centrality dependence of the energy scale and resolution uncertainties in \( p + \) Pb events was negligible.

To achieve better correspondence with the data, the simulated jet spectrum was reweighted to match the spectral shape in data before deriving the bin-by-bin correction factors as described above. To determine the sensitivity of the results to this reweighting procedure, the slope of the fit to the ratio of the detector-level spectrum in data to that in simulation was varied by the fit uncertainty, and the correction factors were recomputed with this alternative weighting. The resulting \( \delta R_{p+Pb} \) and \( \delta R_{CP} \) from the nominal values were included in the total systematic uncertainty.

As the jet triggers used for the data selection were evaluated to have greater than 99\% efficiency in the \( p_T \) regions where they are used to select jets, an uncertainty of 1\% was chosen for the centrality selected \( p + \) Pb yields and the pp cross-section in the range \( 20 < p_T < 125 \text{ GeV} \). This uncertainty was taken to be uncorrelated between the centrality-selected \( p + \) Pb yields and the pp cross-section, resulting in a 1.4\% uncertainty on the \( R_{CP} \) and \( R_{p+Pb} \) measurements.

The geometric quantities \( R_{coll} \) and \( T_{pA} \) and their uncertainties are listed in Table 1. These uncertainties arise from uncertainties in the geometric modelling of \( p + \) Pb collisions and in modelling the \( N_{part} \) dependence of the forward particle production measured by \( \Sigma E_T^p \). In general, the uncertainties were asymmetric.
Uncertainties in $R_{\text{coll}}$ were largest for the ratio of the most central to the most peripheral interval (0–10%/60–90%), where they were +17/−6%. The smallest in the 40–60%/60–90% ratio, where they were +4/−3%. Uncertainties in $T_{\text{pp}}$ were largest in the most central (0–10%) and most peripheral (60–90%) centrality intervals, where the upper or lower uncertainty was as high as 10%, and smaller for intervals in the middle of the $p + \text{Pb}$ centrality range, where they reached a minimum of +3/−2% for the 20–30% interval.

The $x_t$-interpolation of the $\sqrt{s} = 2.76$ TeV pp jet cross-section to 5.02 TeV is sensitive to uncertainties in $\rho(x_t, y^*)$, the $\sqrt{s}$-scaled ratio of jet spectra at 2.76 and 7 TeV. Following Eq. (8), the uncertainty in the interpolated pp cross-section ($\delta\sigma/5.02$ TeV) at fixed $x_t$ is related to the uncertainty in $\rho$ ($\delta\rho$) via $\delta\sigma/5.02$ TeV/$\sigma/5.02$ TeV = 0.643($\delta\rho/\rho$), where $\delta\rho$ was taken from Ref. [34]. The values of $\delta\rho$ ranged from 5% to 23% in the region of the measurement and were generally larger at lower $x_t$ and at larger rapidities.

The integrated luminosity for the 2013 pp dataset was determined by measuring the interaction rate with several ATLAS subdetectors. The absolute calibration was derived from three van der Meer scans [37] performed during the pp data-taking in 2013 in a fashion similar to that used previously within ATLAS [38] for pp data-taking at higher energies. The systematic uncertainty on the integrated luminosity was estimated to be 3.1%.

The uncertainties from the jet energy scale, jet energy resolution, reweighting and $x_t$-interpolation are $p_T$ and $y^*$ dependent, while the uncertainties from the trigger, luminosity, and geometric factors are not. The total systematic uncertainty on the $R_{\text{ppb}}$ measurement ranges from 7% at mid-rapidity and high $p_T$ to 18% at forward rapidities and low $p_T$. In most $p_T$ and rapidity bins, the dominant systematic uncertainty on $R_{\text{ppb}}$ is from the $x_t$-interpolation. The $p_T$ and $y^*$-dependent systematic uncertainties on $R_{\text{CP}}$ are small. Near mid-rapidity or at high $p_T$, they are 2%, rising to approximately 12% at low $p_T$ in forward rapidities. Thus, in most of the kinematic region studied, the dominant uncertainty on $R_{\text{CP}}$ is from the geometric factors $R_{\text{coll}}$.

9. Results

Fig. 2 presents the fully corrected per-event jet yield as a function of $p_T$ in 0–90% $p + \text{Pb}$ collisions, for each of the jet centre-of-mass rapidity ranges used in this analysis. At mid-rapidity, the yields span over eight orders of magnitude.

The jet nuclear modification factor $R_{\text{ppb}}$ for 0–90% $p + \text{Pb}$ events is presented in Fig. 3 in the eight rapidity bins for which the pp reference was constructed. At most rapidities studied, the $R_{\text{ppb}}$ values show a slight ($\approx 10\%$) enhancement above one, although many bins are consistent with unity within the systematic uncertainties. At mid-rapidity, the $R_{\text{ppb}}$ values reach a maximum near 100 GeV. No large modification of the total yield of jets relative to the geometric expectation (under which $R_{\text{ppb}} = 1$) is observed. The data in Fig. 3 are compared to a next-to-leading order perturbative QCD calculation of $R_{\text{ppb}}$ with the EPS09 parameterisation of nuclear parton distribution functions [9], using CT10 [39] for the free proton parton distribution functions and following the procedure for calculating jet production rates in $p + \text{Pb}$ collisions described in Refs. [1,40]. The data are slightly higher than the calculation, but generally compatible with it within systematic uncertainties.

The central-to-peripheral ratio $R_{\text{CP}}$ for jets in $p + \text{Pb}$ collisions is summarised in Fig. 4, where the $R_{\text{CP}}$ values for three centrality intervals are shown in all rapidity ranges studied. The $R_{\text{CP}}$ ratio shows a strong variation with centrality relative to the geometric expectation, under which $R_{\text{CP}} = 1$. The jet $R_{\text{CP}}$ for 0–10%/60–90% events is smaller than one at all rapidities for jet $p_T > 100$ GeV and at all $p_T$ at sufficiently forward (proton-going, $y^* > 0$) rapidities. Near mid-rapidity, the 40–60%/60–90% $R_{\text{CP}}$ values are consistent with unity up to 100–200 GeV, but indicate a small suppression at higher $p_T$. In all rapidity intervals studied, $R_{\text{CP}}$ decreases with increasing $p_T$ and in increasingly more central collisions. Furthermore, at fixed $p_T$, $R_{\text{CP}}$ decreases systematically at more forward rapidities. At the highest $p_T$ in the most forward rapidity bin, the 0–10%/60–90% $R_{\text{CP}}$ value is $\approx 0.2$. In the backward rapidity direction (lead-going, $y^* < 0$), $R_{\text{CP}}$ is found to be enhanced by 10–20% for low-$p_T$ jets.

Fig. 5 summarises the jet $R_{\text{ppb}}$ in central, mid-central and peripheral events in all rapidity intervals studied. The patterns observed in the centrality-dependent $R_{\text{ppb}}$ values are a consequence of the near-geometric scaling of the minimum-bias $R_{\text{ppb}}$ values along with the strong modifications of the central-to-peripheral ratio $R_{\text{CP}}$. At sufficiently high $p_T$, $R_{\text{ppb}}$ in central events is found to be suppressed ($R_{\text{ppb}} < 1$) and in peripheral events to be enhanced ($R_{\text{ppb}} > 1$). Generally, these respective deviations from the geometric expectation (under which $R_{\text{ppb}} = 1$ for all centrality intervals) increase with $p_T$ and, at fixed $p_T$, increase as the rapidity becomes more forward. Thus, the large effects in $R_{\text{CP}}$ are consistent with a combination of modifications that have opposite sign in the centrality-dependent $R_{\text{ppb}}$ values but have little effect on the centrality-inclusive (0–90%) $R_{\text{ppb}}$ values. At backward-going rapidities ($y^* < 0$) the $R_{\text{ppb}}$ value for low-$p_T$ jets in all centrality intervals is consistent with unity within the uncertainties.
Given the observed suppression pattern as a function of jet rapidity, in which the suppression in \( R_{\text{CP}} \) at fixed \( p_T \) systematically increases at more forward-going rapidities, it is natural to ask if it is possible to find a single relationship between the \( R_{\text{CP}} \) values in the different rapidity intervals which is a function of jet kinematics alone. To test this, the \( R_{\text{CP}} \) values in each rapidity bin were plotted against the quantity \( p_T \times \cosh(y^*) \approx E \), where \( y^* \) is the centre of the rapidity bin and \( E \) is the total energy of the jet. In relativistic kinematics, the total energy of a particle is given by \( E = m_T \cosh(y^*) \), where the transverse mass \( m_T = \sqrt{m^2 + p_T^2} \).

In the kinematic range studied, the mass of the typical jet is sufficiently small relative to its transverse momentum that approximating the transverse mass, \( m_T \), with the \( p_T \) is reasonable. The 0–10% (0–90%, 0–20%, 80–90%) \( R_{\text{CP}} \) versus \( p_T \times \cosh(y^*) \) is shown for all ten rapidity ranges in Fig. 6. When plotted against this variable, the \( R_{\text{CP}} \) values in each of the five forward-going rapidities \( (y^* > 0.8) \) fall along the same curve, which is approximately linear in the logarithm of \( E \). This trend is also observed in the two most forward of the remaining rapidity intervals \( (-0.3 < y^* < 0.3) \), but the \( R_{\text{CP}} \) values at backward rapidities \( (y^* < -0.3) \) do not follow this trend. This pattern is also observed in other centrality intervals, albeit with a different slope in \( \log(E) \) for each centrality interval.

These patterns suggest that the observed modifications may depend on the initial parton kinematics, such as the longitudinal momentum fraction of the parton originating in the proton, \( x_p \). In particular, a dependence on \( x_p \) would explain why the data follow a consistent trend vs. \( p_T \times \cosh(y^*) \) at forward rapidities (where jet production at a given jet energy \( E \) is dominated by \( x_p \sim E/\sqrt{s}/2 \) partons in the proton) but do not do so at backward rapidities (where the longitudinal momentum fraction of the parton originating in the lead nucleus, \( x_{p\text{b}} \), as well as \( x_p \) are both needed to relate the jet and parton kinematics).

By analogy with Fig. 6 where the \( R_{\text{CP}} \) values are plotted versus \( p_T \times \cosh(y^*) \), the \( R_{p\text{b}} \) values in the four most forward-going bins studied are plotted against this variable in Fig. 7. The \( R_{p\text{b}} \) values in central and peripheral events are shown separately. Although the systematic uncertainties are larger on \( R_{p\text{b}} \) than on \( R_{\text{CP}} \), the observed behaviour for jets with \( p_T > 150 \) GeV is consistent with the nuclear modifications depending only on the approximate total jet energy \( p_T \times \cosh(y^*) \). In central (peripheral) events, the \( R_{p\text{b}} \) values at forward rapidities are consistent with a rapidity-independent decreasing (increasing) function of \( p_T \times \cosh(y^*) \). Thus, the single trend in \( R_{\text{CP}} \) versus \( p_T \times \cosh(y^*) \) at forward rapidities appears to arise from opposite trends in the central and peripheral \( R_{p\text{b}} \), both a single function of \( p_T \times \cosh(y^*) \).
10. Conclusions

This paper presents the results of a measurement of the centrality dependence of jet production in $p+{\text{Pb}}$ collisions at $\sqrt{s_{\text{NN}}} = 5.02$ TeV over a wide kinematic range. The data were collected with the ATLAS detector at the LHC and correspond to 27.8 nb$^{-1}$ of integrated luminosity. The centrality of $p+{\text{Pb}}$ collisions was characterised using the total transverse energy measured in the forward calorimeter on the Pb-going side covering the interval $-4.9 < \eta < -3.2$. The average number of nucleon–nucleon collisions and the mean nuclear thickness factor were evaluated for each centrality interval using a Glauber Monte Carlo analysis.

Results are presented for the nuclear modification factor $R_{p\text{b}}$, with respect to a measurement of the inclusive jet cross-section in $\sqrt{s} = 2.76$ TeV $pp$ collisions corresponding to 4.0 pb$^{-1}$ of integrated luminosity. The $pp$ cross-section was $x_\text{t}$-interpolated to 5.02 TeV using previous ATLAS measurements of inclusive jet production at 2.76 and 7 TeV. Results are also shown for the central-to-peripheral ratio $R_{CP}$. The centrality-inclusive $R_{p\text{b}}$ results for 0–90% collisions indicate only a modest enhancement over the geometric expectation. This enhancement has a weak $p_T$ and rapidity dependence and is generally consistent with predictions from the modification of the parton distribution functions in the nucleus, which is small in the kinematic region probed by this measurement.

The results of the $R_{CP}$ measurement indicate a strong centrality-dependent reduction in the yield of jets in central collisions relative to that in peripheral collisions, after accounting for the effects of the collision geometries. In addition, the reduction becomes more pronounced with increasing jet $p_T$ and at more forward (downstream proton) rapidities. These two results are reconciled by the centrality-dependent $R_{p\text{b}}$ results, which show a suppression in central collisions and enhancement in peripheral collisions, a pattern which is systematic in $p_T$ and $y^*$. The $R_{CP}$ and $R_{p\text{b}}$ measurements at forward rapidities are also reported as a function of $p_T \times \cosh((y^*))$, the approximate total jet energy. When plotted this way, the results from different rapidity intervals follow a similar trend. This suggests that the mechanism responsible for the observed effects may depend only on the total jet energy or, more generally, on the underlying parton–parton kinematics such as the fractional longitudinal momentum of the parton originating in the proton.

If the relationship between the centrality intervals and proton–lead collision impact parameter determined by the geometric models is correct, these results imply large, impact parameter-dependent changes in the number of partons available for hard scattering. However, they may also be the result of a correlation between the kinematics of the scattering and the soft interactions resulting in particle production at backward (Pb-going) rapidities [42,43]. Recently, the effects observed here have been hypothesised as arising from a suppression of the soft particle multiplicity in collisions producing high energy jets [44]. Independently, it has also been argued that proton configurations containing a large-$x$ parton interact with nucleons in the nucleus with a reduced cross-section, resulting in the observed modifications [45]. In any case the presence of such correlations would challenge the usual factorisation-based framework for describing hard scattering processes in collisions involving nuclei.

Acknowledgements

We thank CERN for the very successful operation of the LHC, as well as the support staff from our institutions without whom ATLAS could not be operated efficiently.

We acknowledge the support of ANPCyT, Argentina; YerPhI, Armenia; ARC, Australia; BMWFW and FWF, Austria; ANAS, Azerbaijan; SSTC, Belarus; CNPq and FAPESP, Brazil; NSERC, NRC and CFI, Canada; CERN; CONICYT, Chile; CAS, MOST and NSFC, China; COLCIENCIAS, Colombia; MSM CR, MPO CR and VSC CR, Czech Republic; DNRF, DNSRC and Lundbeck Foundation, Denmark; EPLANET, ERC and NSRF, European Union; IN2P3-CNRS, CEA-DSM/IRFU, France; GNSF, Georgia; BMBF, DFG, HGF, and AvH Foundation, Germany; GRST and NSRF, Greeece; IFI, MINERVA, GIF, I-CORE and Benoziyo Center, Israel; INFN, Italy; MEXT and JSPS, Japan; CNRST, Morocco; FOM and NWO, Netherlands; BRF and RCN, Norway; MNiSW and NCN, Poland; GRICES and FCT, Portugal; MEINE/IFA, Romania; MES of Russia and ROSATOM, Russian Federation; JINR; MSTD, Serbia; MSSR, Slovakia; ARRS and MIZŠ, Slovenia; DST/NRF, South Africa; MINECO, Spain; SRC and Wallenberg Foundation, Sweden; SER, SNSF and Cantons of Bern and Geneva, Switzerland; NSC, Taiwan; TAEK, Turkey; STFC, the
Fig. 6. Measured $R_{CP}$ values for $R = 0.4$ jets in $0-10\% p + Pb$ collisions. The panel on the left shows the five rapidity ranges that are the most forward-going, while the panel on the right shows the remaining five. The $R_{CP}$ values at each rapidity are plotted as a function of $p_T \times \cosh(y^*)$, where $y^*$ is the midpoint of the rapidity bin. Vertical error bars represent the statistical uncertainty while the boxes represent the systematic uncertainties on the jet yields. The shaded box at the left edge in the left panel and right edge in the right panel of the $R_{CP} = 1$ horizontal line indicates the systematic uncertainty on $R_{CP}$.

Fig. 7. Measured $R_{pb}$ values for $R = 0.4$ jets in $p + Pb$ collisions displayed for multiple rapidity ranges, showing $0-10\%$ events in the left panel and $60-90\%$ events in the right panel. The $R_{pb}$ at each rapidity is plotted as a function of $p_T \times \cosh(y^*)$, where $y^*$ is the midpoint of the rapidity bin. Vertical error bars represent the statistical uncertainty while the boxes represent the systematic uncertainties on the jet yields. The shaded box at the left edge of the $R_{pb} = 1$ horizontal line indicates the systematic uncertainties on $T_{AA}$ and the pp luminosity added in quadrature.

Royal Society and Leverhulme Trust, United Kingdom; DOE and NSF, United States of America.

The crucial computing support from all WLCG partners is acknowledged gratefully, in particular from CERN and the ATLAS Tier-1 facilities at TRIUMF (Canada), NDGF (Denmark, Norway, Sweden), CC-IN2P3 (France), KIT/GridKA (Germany), INFN-CNAF (Italy), NL-T1 (Netherlands), PIC (Spain), ASGC (Taiwan), RAL (UK) and BNL (USA) and in the Tier-2 facilities worldwide.

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