"A terrible piece of bad metaphysics"? Towards a history of abstraction in nineteenth- and early twentieth-century probability theory, mathematics and logic

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CHAPTER 7

John Venn on the foundations of symbolic logic: a non-conceptualist Boole

0. Introduction: Venn, Mill and Boole on induction and deduction

The English logician, philosopher and fellow of Gonville and Caius College, Cambridge, John Venn (1834-1923) is mostly remembered for his creation of what, since the publication of Clarence Irving Lewis' *Survey of Symbolic Logic* of 1918, has become known as 'Venn diagrams'.[1] Despite the fact that these 'cogwheels of the mind', which were first put forward in Venn's paper of 1880 entitled 'On the diagrammatic and mechanical representation of propositions and reasonings',[23] are still widely used in mathematical education (elementary set theory) and as illustrations of (set) connections in, for example, logic, linguistics and probability theory,[4] almost nothing has been written on the particular approach to so-called 'algebra of logic, or 'symbolic logic' as Venn himself preferred to call it, of which they were the graphical expression. This is all the more surprising because Venn, being a critical successor of George Boole (1815-1864) standing 'at the apogee of nineteenth century algebraic logic'[5], occupied a highly interesting position in the Renaissance of logic in nineteenth-century Britain.[6] Furthermore, the fact that, however close his views were to current

1 See *Lewis 1918*, chapter 3.

2 *Venn 1880b*.

3 Edward notes that Venn first hit upon the idea of a 'Venn diagram' in a series of lectures at Caius College which he began in 1869 (Edwards 2004, p. 35). See also *Venn 1880c* and *Venn 1880d*.

4 For accounts on 'visual logic' see, for example, *Hammer & Shin 2007*; *Moktefi & Shin 2012*; *Shin 1994*.

5 *Van Evra 2008*, p. 514.

6 *See Gabbay & Woods 2008*.

logical theory, Venn remained committed to traditional or Aristotelian logic makes that a reading of his *Symbolic Logic* (*1881*) can add an intriguing new perspective to the (current) debate about the question of whether Boole's 'algebra of logic' (*calculus ratiocinator*) or Gottblob Frege's (1848-1925) 'mathematical logic' (*lingua characteristica universalis*) parented modern logic.[7]

Venn's logical oeuvre is characterized by the attempt to combine the two philosophical developments spurred by the publication of Richard Whately's (1787-1863) *Elements of Logic* of 1826,[8] namely the algebraic generalization of traditional logic found in the work of Boole[9] and the empiricist reduction of ('real') logical inference to induction of John Stuart Mill.[10] Although this fundamental distinction between deductive and inductive logic is reflected in his three major works – separating, as it does, *Venn 1881* from the *Logic of Chance* (*1866*)[11] and *Principles of Inductive or Empirical Logic* (*1889*) –, theoretically speaking, Venn rejected both Boole's idea that the whole of inductive logic can be incorporated within deductive (syllogistic) logic as well as Mill's idea that deductive (syllogistic) logic must be dismissed in favor of an inductive logic. Instead, Venn put forward an alternative approach to logic that occupies the eclectic middle ground between these two general views.

To begin with, Venn holds, *contra* Boole, that (the deductive form of) the traditional syllogism is 'circuitous and artificial; the actual inference [being] to all intents and purposes inductive'[12] and, *contra* Mill,[13] that there are more complicated cases of deductive reasoning not appearing in the traditional syllogism which contain non-trivial (or 'ampliative') inferences. As Venn has it:

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7 See *Peckhaus 2014* and *Van Heijenoort 1967* for this historico-theoretical distinction.

8 See *McKerrow 1987*; *Peckhaus 2009* and *Van Evra 1984*.

9 *Boole 1847*; *Boole 1854*.

10 *Mill 1843/1958*. For accounts on Mill's induction see, for example, *Anschutz 1949*; *Jackson 1941*; *Walsch 1962*.

11 In direct opposition to (mathematical) tradition, Venn considered probability theory to be a specific portion of the system of (material) logic. The central aim of *Venn 1866* was to redefine the foundations of the theory after it has been incorporated within this very system. See *Kiliç 1999* and *Verburgt 2014a* for a detailed analysis of Venn's work on probability theory.

12 *Venn 1889*, p. 378.

13 See *Nelson 1925*; *Botting 2014*.

> The common account of [the syllogistic] process seems a perfectly tenable one; that is, the objection that it involves [a] *petitio principii*, and only states as a conclusion what we must have known as a premise, is not valid [for] it is quite possible to suppose any one setting out from the two premises as his real starting point, and reaching a conclusion which [...] may be something distinctly new. [But] whenever [the] original data on which the major premise was based are so close to hand that a moment's reflection will suffice to revive them [I] think Mill's account of the syllogistic process is the simplest and the best. (*Venn 1889*, pp. 337-378)

Venn, *pace* Mill, embraced the simple cases of inference within traditional (syllogistic) logic – or, for that matter, recognized them as parts of a syllogistic process –,[14] while arguing, *contra* Boole, that 'the really characteristic elements of [...] Inductive Logic are of a [...] non-formal nature'.[15] This does not mean that Venn has any Millian scruples – for the 'creative part' of inductive inference that is pulverized when induction is put into syllogistic form is precisely what goes unrecognized in Mill's 'mechanical' definition of induction.[16] Venn,

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14 Venn wrote that it was his contention 'that the really characteristic elements of the hypothetical of common life and of Inductive Logic are of a non-formal nature. Those characteristic are partly material and partly psychological, and therefore the determination to force these [inductive] propositions through the forms of our [algebraical] system will naturally, so to say, crush all the life out of them. Such a loss as this is inevitable. Whenever we substitute anything resembling machine work for hand or head work, we find that though the former possesses vast superiority of power there are always some delicacies of performance in which it exhibits comparative failure. So it is here' (*Venn 1881*, p. 341).

15 *Venn 1881*, p. 243.

16 Venn was of the opinion that Mill's definition of induction ('Induction is that act of the mind by which, from a certain definite number of things or observations, we make an inference extending to an indefinite number of them') does not account for original inferences. Venn writes that in so far as Mill assumes 'that the data, viz. the limited number of things from which the formula starts, and on which it is grounded, are already clearly recognized' he omits 'all practical difficulty which may have existed as to discovering and recognizing our [data]' (*Venn 1866*, p. 194). Referring to the work of the idealist (!) William Whewell, Venn argues that '[t]he objects from which our inference started as its basis must have been selected; and since this selection was neither made at random nor performed for us by others, there must have been some principle of selection in our minds' (*Venn 1866*, p. 194). Taken together, Venn accused Mill of neglecting the creative part of the process of induction (see *Laudan 1971*; *Strong 1955*; *Walsch 1962*). See *Verburgt 2014b* for a detailed discussion of the connection between Mill and Venn.

thus, seems to hold that an 'inductive syllogism' must be interpreted as a syllogism coming from an inductive process that itself involves an 'unformalizable' act of the mind. At least, that is what would allow for the possibility of simultaneously upholding the following two ideas about a (Boolean) 'calculus of deductive reasoning':

> [I]nasmuch as it is [a] Formal Science, the resolve to fit it in with the problems of Induction, or to regard it as an introduction to the Principles of Science in general, seems to me a grave error, and to result merely in the attempt to combine heterogeneous materials. (*Venn 1881*, p. xxvi, p. 34)

[T]he determination to force [inductive] propositions through the forms of our system will naturally, so to say, crush all the life out of them [but] [s]uch a loss as this is inevitable. (*Venn 1881*, p. 341)

The complex cases of inference that account for the fact that, on the one hand, the traditional syllogism does not commit a *petitio principii* and, on the other hand, there are processes of reasoning involving non-trivial (deductive) inference for which it holds that there s nothing corresponding to it in traditional (syllogistic) logic, explain Venn's interest in the work of Boole. Venn was of the opinion that 'algebra of logic' 'should be regarded as a Development or Generalization [of] Formal, viz. Aristotelian or Scholastic Logic'[17] such that it becomes 'more suitable for the treatment of complicated problems and broad generalizations [of reasoning]'.[18] This position committed him to a twofold task which Venn himself considered original to his *Venn 1881*, namely the

> Thorough examination of the Symbolic Logic as a whole, that is, its relation to ordinary Logic and ordinary thought and language [and] the establishment and explanation of every general symbolic expression and rule on purely logical principles, instead of looking mainly to its formal justification. (*Venn 1881*, p. xxix).

17 *Venn 1881*, p. xxvii, p. xxvi, f. 1.
18 *Venn 1881*, p. 165, f. 1.

The aim of the present paper is to provide a detailed analysis of the way in which this was carried out (section 1) and to reflect on the meaning of, on the one hand, Venn's statement that Boole, despite his 'many and serious omissions'[19], agreed with the non-mathematical character of logic and, on the other hand, Venn's own non-conceptualist reinterpretation of 'algebra of logic' (section 2) for an understanding of the rise of modern logic during the nineteenth-century (section 3).

1. From traditional predicates to non-conceptualist compartments – from traditional to symbolic logic

Venn had become acquainted with Boole's *The Laws of Thought* (*1854*) through his tutor Isaac Todhunter as early as in the year 1858. But it was only 'untill after the third or fourth study of the book, in 1878/1879'[20] that Venn felt confident enough to reflect on it and bring it into contact with his own general views on logic. After having devoted 'what might seem, a disproportionate [amount] of time to one peculiar development of Logic',[21] namely (Millian) material logic and probability theory, and preparing lectures for the intercollegiate scheme at Caius College during the 1870s, it was in 1876 that Venn published a critical discussion of 'Boole's system of logic'.[22] The substance of the chapters of what would become *Venn 1881*[23] appeared in some articles of the year 1880.[24]

Because Venn did not publish any work prior *Venn 1866* and his first two articles only appeared in 1876,[25] the theoretical path leading to *Venn 1881* cannot be described.[26] Nevertheless, when it is observed that the purpose of Venn's 'On

19 *Venn 1881*, p. xxi.
20 Venn quoted in *Cook 2005*, p. 338.
21 Venn quoted in *Cook 2005*, p. 338.
22 *Venn 1876b*.
23 The book went through three editions of the which the first appeared in 1881 (*Venn 1881*), the second in 1894 (*Venn 1894*) and the third, which was a reprint of *Venn 1894*, in 1974 (Venn 1974).
24 *Venn 1880a*; *Venn 1880b*; *Venn 1880c*; *Venn 1880d*; *Venn 1888e*.
25 *Venn 1876a*; *Venn 1876b*.
26 For detailed accounts of the development of symbolic logic see, for example, *Howson 1997/2005*; *Langer 1967*; *Shearman 1906*.

the forms of logical propositions'[27] of 1879 was that of finding a scheme of propositional import that could form the basis for the 'widest extension possible of our [ordinary] logical processes by the aid of symbols'[28] they could be reconstructed – and it is to this task that the following sub-sections are dedicated.

1.1 'Non-conceptualist formalism'

Venn's article of 1876 entitled 'Consistency and real inference'[29]sets out from the statement that in so far as the proposition 'by analysis leads to terms, and by synthesis to arguments'[30] it follows that 'what holds of the proposition holds equally throughout the entire field of logic'.[31] It is on the basis of this statement that Venn uses the common definition of the proposition as being 'a statement in words of a judgment about things'[32] to distinguish between the 'conceptualist' and 'material' accounts of logic. The first defines the judgment as consisting of concepts, or 'mental representations', standing in a certain relation to each other such that logic is concerned with 'the mental order or stratum of things'.[33] The latter defines the judgment as consisting of propositions of which the terms refer to things such that logic, at least in Venn's reformulation of material logic, is concerned with the judgment of the human mind about a world of phenomena resulting from objective and subjective assumptions of reasoning. In other words, where the material approach to logic emphasizes the import, or truth or falsehood, of propositions the conceptualist approach to logic, not unlike pure mathematics, occupies itself with the 'pure form' of mental concepts.

Venn wishes to dismiss the conceptualist notion of 'perfect induction' – presented, as it is, in the form of an 'inductive syllogism'[34] in which induction 'is

27 *Venn 1879*.
28 *Venn 1879*, p. 337.
29 *Venn 1876a*.
30 *Venn 1876a*, p. 43.
31 *Venn 1876a*, p. 43.
32 *Venn 1876a*, 44.
33 *Venn 1876a*, 44.
34 For instance in his Elementary Lessons in Logic of 1870, William Stanley Jevons (1835-1882) gives the following example of such an inductive syllogism: 'Mercury, Venus, the Earth etc. all move round the sun from West to East. Mercury, Venus, the Earth etc. are all the known planets. Therefore all the known planets move round the sun from West to East' (*Jevons 1870*/1888, pp. 214-215).

performed formally and necessarily' – not only for its neglect of the Millian, or material, side of induction, but also for formalizing the 'Whewellian', or mental, side of induction. As said, Venn himself is deeply committed to the view that inductive logic, being 'partly material and partly psychological'[35], is of a non-formal nature. Importantly, despite this close connection between 'conceptualism' and 'formalism', Venn distinguishes them as follows:

> Formal and Conceptualist, [...] are frequently used as [...] synonyms [but] [t]hese terms are obviously distinct in their original significations. "Formal" has reference to the *limits* of the subject rather than its actual nature. It reminds us that we are confining ourselves to those mental processes [...] which are independent of the particular subject matter, that is [...] which follow from the mere form of expression. "Conceptualist" [...] refers rather to the nature of our subject than to its limits; it reminds us that we are occupying ourselves with the consideration of concepts [...] as distinguished from external phenomena. (*Venn 1876a*, p. 46).

Three things must be inferred from this passage. Firstly, that in so far as the reasonings expressing inductive inferences refer to a particular subject matter they are unsuitable for 'formalization'. Secondly, that it is possible, in principle, to set out to 'formalize' the structure or form of non-inductive expressions all the while eschewing conceptualism. And thirdly, that these expressions are connected to a mental process different from the one involved in the second, 'Whewellian', part of induction. For even if both processes are to a certain degree independent from 'facts' or 'things', it holds that where the first follows from the structure of an expression the second does not due to its 'non-mechanical' character. Taken together, Venn has enabled himself to formalize the structure of the mental processes involved in deductive expressions without committing himself to the 'conceptualist' definition of these expressions in terms of 'concepts'.

35 *Venn 1881*, p. 341.

1.1.2 The material import of formal compartments

At this point it must be re-emphasized that the possibility of the symbolic extension of traditional logic first suggested itself in the context of the *formalization* of the process of the division of denotative terms – which, strictly speaking, does not belong to inductive logic.[36] In Venn's own words,

> the traditional logical process of Division [...] did not lead to much result [since] it was hampered in execution owing to the fact that though it professed to be, and really aimed at being, a formal process, it nevertheless attended sufficiently to the dictates of common sense to endeavour to render itself [...] useful [...] And in a treatise on Induction, it [is] somewhat of a departure from strict consistency to touch upon such a procedure at all. It [is] however desirable to do this, partly because the continuity of technical language in the subject rendered it necessary to give some explanation of the [...] meaning of the term [and] partly because, by seeing what and where were the main deficiencies of the old treatment we may be better able to see where improvement is needed [...] What we [may] do is to see what comes of the attempt to develop [...] the formal [side] separately. (*Venn 1889*, p. 318).

In traditional logic the process of division or dichotomy is dealt with along the lines of predication, that is, it aims to reach the particular class or individual of a proposition by breaking up every class into two divisions or contradictories.[37] Because the subdivisions are themselves not subjected to further analysis and, on the other, it is clear that 'there is not any very rigid adherence displayed to a truly formal dichotomy of the *X* and not-*X* kind'.[38] The field of symbolic logic is opened up by pursuing exactly this course, namely by dealing 'with nothing but formal contradictories'[39] and introducing 'every alternative of which the form will admit'.[40] Thus,

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36 See *Venn 1889*, pp. 318-319.
37 See *Venn 1889*, chapter 12.
38 *Venn 1889*, p. 319.
39 *Venn 1889*, p. 319.
40 *Venn 1889*, p. 319.

> [s]uppose that we start with a class *S*, and that we are concerned with three attributes which shall serve as the bases of division, viz. *X*, *Y* and *Z*. We divide *S* into *X*, and not-*X*. We then proceed to subdivide both of these by the introducing of *Y*, thus obtaining four classes. Introduce the third attribute *Z*, and make the same division again, and we get eight resultant classes. And this we might continue doing with any number of such dividing attributes. (*Venn 1889*, p. 319)

Venn points out that this methodological improvement of the traditional treatment of division in respect of its completeness is premised on the thoroughgoing, albeit purely instrumental, redefinition of the proposition as consisting of (formal) class-compartments rather than actual classes. The precise formulation of this theory of the proposition is presented, by Venn, in his 'On the forms of logical proposition' of 1879.[41] Before discussing the details of this article it must be emphasized that the substitution of actual classes for formal class-compartments allowing for the further development of the formal side of the 'old problem of Division'[42] is accompanied by a development 'to the utmost [of] the material side'.[43]

> What we start with [...] is a framework of class-compartments, the number of these being determined by the number of class-terms involved in the proposition or group of propositions [...] If the propositions involved, as is the case with the common syllogism, three terms, say *X*, *Y* and *Z*, we should have eight such compartments before us [such that] all the possible combinations [are] exhausted. Now this being so, it may be shown – and this forms the basis of the whole Theory of Symbolic Logic – that every significant universal proposition must necessarily destroy some one or more of the possible classes [and] [c]onversely, whatever may be the number and description of empty compartments, there must be some corresponding proposition which will unambiguously express the facts. (*Venn 1889*, p. 320)

41 *Venn 1879*.
42 *Venn 1889*, p. 321.
43 *Venn 1889*, p. 321.

For example, 'All X is Y' amounts to the statement that 'there is no X that is not Y' – i.e. that there is no class as X, not-Y. Venn thus puts 'symbolic logic' forward as an extension of traditional logic in which propositions are formalized by defining them as consisting of class-compartments which themselves have an import in so far as they can be materially interpreted.

1.1.3 The 'compartmental view' of propositions

It is in his 'On the forms of logical proposition'[44] that this approach is given the name of the 'compartmental view' and in which this particular view is explicitly presented as the one to be preferred above the traditional 'predication view' and conceptualist[45] 'class inclusion and exclusion view' for the purpose of 'securing the widest extension [of Logic] possible'.[46]

The 'predication view' adopts the conventional 'method of asserting or denying attributes of a subject, that is, of the whole or part of a subject; whence they naturally yield four forms – the universal and particular, affirmative and negative'[47]. If this view is a more or less technical clarification of the ordinary pre-logical modes of linguistic expression, these forms represent 'the most primitive and natural modes in which thought begins to express itself'.[48] Taken together, even though this view and its forms are not suitable for 'very complicated reasonings', it is 'not likely to be surpassed […] for the expression and improvement of ordinary thought and speech'.[49]

In order to be able to account for the purely formal side of the process of division, Venn observes that it is necessary to dismiss the traditional habit of 'translating the subject in respect of extension and the predicate in respect of intension'[50] in favor of a scheme in which both subject and predicate are interpreted in respect of their (denotative) extension. Before describing in detail

44 *Venn 1880a*.
45 Venn writes that when in the class inclusion and exclusion view the predicate is quantified it may as well be termed the Hamiltonian view (see *Venn 1880a*, p. 340).
46 *Venn 1880a*, p. 337, p. 345. See also *Venn 1889*, chapter 9.
47 *Venn 1880a*, p. 337. See also *Venn 1889*, chapter 8.
48 *Venn 1880a*, pp. 337-338.
49 *Venn 1880a*, p. 338.
50 *Venn 1889*, p. 228.

the 'compartmental view', Venn introduces the 'class inclusion and exclusion view' as one in which the proposition is not regarded 'as made up of a subject determined by a predicate', but instead 'as assigning the relations, in the way of mutual inclusion and exclusion, of two classes to one another'.[51] The relations between two classes can take on the following forms; the one can either coincide with the other ('All *X* is all *Y*'), include it ('All *X* is some *Y*'), be included by it ('Some *X* is all *Y*'), partially include and partially exclude it ('Some *X* is some *Y*'), or entirely exclude it ('No *X* is any *Y*').[52] Venn's main criticism of this view closely resembles his dismissive characterization of conceptualism in his 'Consistency and real inference' of 1876.[53] Firstly, on the conceptualist view of the process of division it is confined to its formal side. Although it is true that division by dichotomy is formally valid, conceptualism precludes the possibility of establishing whether or not the classes are occupied – and 'unless we had the material information that some of them did possess [e.g.] *X* and some did not we [are] led to the absurdity of a class which was without any members to compose it'.[54] Secondly, conceptualism makes 'no reference to belief [for] belief cannot but have some degree of reference to external objects [instead of] the […] data of thought';[55] for the 'class inclusion and exclusion view' it holds that when its forms express 'the relations of class inclusion and exclusion […] we only need, or can find place for, five. Regard them as expressing to some extent out uncertainty about these class relations, and we want more than eight[56]'.[57]

These two criticisms are circumvented on the 'compartmental' view; it allows for the expression of doubt in so far as it establishes whether propositions assert or deny 'the existence of things corresponding to a certain term or combination

51 *Venn 1880a*, p. 338.

52 See *Venn 1880a*, p. 339 and *Venn 1889*, p. 228.

53 *Venn 1876a*.

54 *Venn 1889*, p. 320.

55 *Venn 1876a*, p. 46.

56 These eight forms follow from the extension of the five-fold arrangement by means of the so-called 'quantification of the predicate' – such that it also includes 'Any *X* is not some *Y*', 'Some *X* is not any *Y*' and 'Some *X* is not some *Y*'. According to Venn, these new three forms are 'superfluous, or ambiguous equivalents for one or more of the first five' (*Venn 1880a*, p. 341).

57 *Venn 1880a*, p. 343.

of terms'[58] by implication. For this reason, Venn sometimes even calls this third view the 'existential' view, which

> is still distinctly a *class*, rather than a predication, view; but instead of regarding the mutual relation of two or more classes in the way of inclusion and exclusion, it substitutes a complete classification of all the subdivisions which can be yielded by putting any number of classes together, and indicates whether any one or more of these classes is occupied; that is, whether things exist which possess the particular combination of attributes in question. (*Venn 1880a*, pp. 345-346)

Thus, in the case of the two class terms *X* and *Y* there are four possible compartments 'for everything which exists must certainly possess both the attributes marked by *X* and *Y*, or neither of them, or one and not the other. This is the *range of possibilities* from which that of *actualities may fall short*; and the difference between these [...] is just what it is the function of the proposition to describe'[59]. For example, 'No *X* is *Y*' is interpreted as unconditionally denying the existence of the class *XY* and 'Some *X* is *Y*' as conditionally affirming the existence of the class *XY*.

If it is this 'compartmental' scheme that has to be employed for the purpose of an extension of traditional logic, the view itself 'could never have been realised by any one who had not a thorough grasp of those mathematical conceptions [of Boole] which [e.g.] [William] Hamilton [(1788-1856)] unfortunately both lacked and despised'.[60] [61] Because

58 *Venn 1889*, p. 229.

59 *Venn 1880a*, p. 346, my emphasis.

60 *Venn 1880a*, p. 345.

61 It may be remarked that Venn and Hamilton were in agreement on the fact that mathematics deals with necessary inferences, but for contrasting reasons. Where Venn had it that any discipline, view, author (such as Hamilton) etc. supporting the very idea of a necessary inference was committed to the psychologically-inspired conceptualist idea of the existence of laws of mind, Hamilton argued that the mathematical treatment of necessary inferences merely belonged to that part of 'practical' logic, instead of 'theoretical' logic' (analyzing, as it does, the 'laws of thought'), that dealt with necessary, instead of contingent, matter. For an account of Hamilton's standpoint vis-à-vis mathematics see, for example, *Hamilton 1852*; *Stirling 1865*; *Laita 1979*.

when we quit the traditional arrangement and enumeration of propositions we must call for a far more thorough revision than that exhibited on the ['class inclusion and exclusion view']. Any system which merely exhibits the mutual relations of two classes to one another is not extensive enough. We must [...] conceive, and invent a notation for, all the possible combinations which any number of class terms can yield; and then find some mode of symbolic expression which shall indicate which of these various compartments are empty or occupied, by the implications involved in the given propositions. This is not so difficult as it might sound, since [Boole has shown that] the resources of mathematical notation are quite competent to provide a simple and effective symbolic language for the purpose'. (*Venn 1880a*, p. 345)

2. Non-mathematical 'symbolic logic': the transformation of Boole's 'mathematical treatment of logic'

In the introduction to *Venn 1881*, Venn writes that he is of the opinion that *Boole 1854* embodied a systematic account of the higher generalization of logic that, in its purely formal shape, was complete and almost perfect.[62] The fact that the bulk of *Venn 1881* consists of the task of refinement, had given rise to *communis opinio* which holds that the book is to be approached as a popularizing defense of the system of Boole.[63] It is undoubtedly impossible to appreciate Venn's book without taking into account its deep connection with that of Boole, but the claim that Venn's 'symbolic logic' is not more than another name for 'algebra of logic' is seriously misguided.

Venn himself puts forward the statement that Boole's 'algebra of logic' 'still lies open to a good deal of explanation and justification'[64] and contains 'many and serious omissions'[65] in order to emphasize that *Venn 1881* is not occupied 'with going over [...] the same ground as [Boole] or others have traversed',[66]

62 See *Venn 1881*, pp. xxix-xxx.

63 See, for example, *Barnett 2009*, pp. 14-20; *Craik 2007*, p. 334; *Hailperin 1976/1986*, p. 126; *Van Evra 2008*, p. 509.

64 *Venn 1881*, p. xxix.

65 *Venn 1881*, p. xxix.

66 *Venn 1881*, p. xxix.

but is 'intended to be an independent study of the subject [...] and in no sense a commentary [...] upon Boole'.[67] And Venn introduces the features characteristic to his own account, namely those of the inclusion of the relations of the extension of logic 'to ordinary Logic and ordinary thought and language [and] the establishment [...] of every general symbolic expression and rule on purely logical principles',[68] as being original to him. It is widely agreed that Venn's unprecedented treatment of the contributions of post-Leibnizian logicians such as Johann Adreas Segner (1704-1777), Gottfried Ploucquet (1716-1790), Johann Heinrich Lambert (1728-1777) and Georg Jonathan von Holland (1742-1784))[69] was a milestone in the nineteenth-century reflection on the historical roots of the new idea of the application of mathematical signs to logic. But it is not often noted that this treatment enabled Venn not only to show that Boole had been unaware of his predecessors and that his 'actual originality [is] by no means so complete as is commonly supposed',[70] but also to situate both Boole and himself within the (post-Leibnizian) current of the Aristotelian tradition that brought to use mathematical symbolism for purely logical purposes.

Venn's dismissal of the name of 'algebra of logic' in favor of 'symbolic logic' is thus not a mere quarrel over words. It rather suggests that the aim of *Venn 1881* is that of the creation of a non-mathematical and non-conceptualist system that completely generalizes traditional syllogistic logic, that is, a system which neither reduces logic to mathematics or redefines it as the formal analysis of 'mental concepts'. If Venn was of the opinion that Boole sometimes 'forgot' that his system satisfied the first criterion, Venn straightforwardly rejected what he found 'fanciful and of little value'[71] in the work of Boole, namely its

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67 *Venn 1881*, p. xxx.
68 *Venn 1881*, p. xxix.
69 See, for example, Capozzi & Roncaglia 2009; Gray 1978; Lenzen 2008; Peckhaus 2012.
70 *Venn 1881*, p. xxviii.
71 *Venn 1876b*, p. 490.

conceptualist ambitions.[72] Put anachronistically, given that Venn's 'symbolic logic' was an attempt to reformulate Boole's 'algebra of logic' along the lines of these viewpoints, his contribution to modern abstract logic is that of demonstrating that 'algebraic' logic, when understood as a generalizing extension of traditional logic, is not modern. To the much debated question whether Boole is 'really the father of modern logic',[73] Venn would have answered not only that Boole was actually the latest offspring of the Aristotelian tradition,[74] but also that the 'mathematical dress' of his work sometimes led him to overlook his own place in the history of logic. It is in this sense that Venn's reformulation of Boole's 'algebra of logic' can be understood as a highly sympathetic, albeit foundationally critical, reminder of this very fact.

2.1 Venn's Boole: 'un-mathematizing' mathematical logic

Given his equation of 'conceptualism' with 'pure mathematics' and his presenting of 'algebra of logic' as an extension of traditional logic, it should come as no surprise that Venn dedicated much effort to showing that the mathematical symbolism taken up to realize and describe the 'compartmental' scheme of propositions is nothing more than a 'convenient abbreviation' of well-known logical conceptions and processes. This also allows him to claim that there is nothing in Boole's system 'which can properly be called mathematical [since] it is simply a generalization of very familiar logical principles'.[75] Taken together, Venn aims to demonstrate that

> the nature of the characteristic processes of Boole's method [...] are at bottom *logical*, *not mathematical*, but they are stated in such a highly

72 Venn writes that one of his criticisms concerns Boole's 'views about the constitution of the human intellect, a subject upon which he considered that the mathematical form which his system assigned to the laws of thought threw much light. [This] is decidedly interesting [and] suggestive [...] but on the whole I must confess that it seems fanciful and of little value. Great as were Boole's deductive powers [...] he does not seem to have possessed much of that, certainly rare, metaphysical faculty which distinguished amongst elementary truths those which are really axiomatic' (Venn, 1876, p. 490). It is highly likely that Venn thought of the romantic idealist John Grote (1813-1866) as one of those figures with the suggested 'metaphysical faculty'. See footnote 141.

73 See *MacHale 1985*; *Peckhaus 2000*; *Quine 1955*.

74 See *Corcoran 2003*.

75 *Venn 1880e*, p. 248.

generalized symbolical form, and with such a mathematical dress upon them, that the reader may work through them several times before the conviction begins to dawn upon him that he had any *previous acquaintance* with them. (*Venn 1876b*, p. 484, my emphasis)

Although Boole himself did not arrive at the method of using mathematical symbolism to express logical relations from, what Venn calls, 'a logical path',[76][77] Venn argues that Boole, contrary to the then prevalent opinion about him, did not regard logic 'as a branch of mathematics [but] simply applied mathematical rules to logical problems'[78]. It is certainly true that Boole conceived the idea of a 'Calculus of Logic' under the influence of George Peacock (1791-1858) and Duncan Farquharson Gregory (1813-1844) on the nature of algebra from whose work he learned not only that 'there could be an algebra of entities which were not numbers in any ordinary sense [but also] that an algebra could be developed as an abstract calculus capable of various interpretations'.[79] But Venn also argued that Boole was prevented from recognizing that nothing can be done by his algebraic methods 'which could not equally be done by the [...] method [of] the old logic [...] and all that *it* can do can be done even by unassisted common sense'[80] – which, to be sure, was 'Boole's own view'[81] – because of his caring too much for 'the power and completeness of his rules'.[82] The legitimacy of this claim may of course be put in doubt; for it is far from obvious that this is what the algebraist Boole hinted at when he wrote that 'the ultimate laws of Logic – those alone upon which it is possible to construct a science of Logic – are mathematical in their form and expression, though not belonging to the mathematics of quantity [i.e. arithmetic'.[83] It must be clear that Venn's 'symbolic

76 *Venn 1880a*, p. 484.

77 Venn makes a distinction between arriving at the method by 'generalising the simple logical conceptions [...] and when [having] clothed them in their highly abstract symbols pulling down and throwing away the scaffolding which had led [one] there' (*Venn 1880a*, p. 484) and by beginning 'with pure formulae, and manipulat[ing] and condition[ing] them until they could fairly represent the rules and results of processes of thought' (*Venn 1880a*, p. 484).

78 *Venn 1876b*, p. 480.

79 Kneale 1956, p. 53. See, in this context, also Despeaux 2007; *Green 1994*; *Hailperin 1986*, part 1; *Verburgt 2014c*; *Verburgt 2014c*.

80 *Venn 1876b*, p. 486.

81 *Venn 1881*, p. xvii.

82 *Venn 1876b*, p. 487.

83 Boole quoted in *Venn 1881*, p. xviii, f. 1.

logic' is more than a mere renaming of Boole's 'algebra of logic' in so far as it is premised on both a fundamental theoretical interpretation of the connection between logic and mathematics as well as the assumption that Boole would have been in agreement with this particular interpretation.

Venn's presentation of the *Venn 1881* as a work advocating 'the system introduced by Boole'[84] seems to be a straightforward sign of his commitment to the '"mathematical" treatment of Logic',[85] or 'algebraic' camp in logic[86] and dismissal of those anti-mathematical logicians[87] who had pronounced 'a work which makes large use of [mathematical] symbols [...] to be mathematical and not logical'.[88] But a more detailed reading of the 'Introduction' of the book does, indeed, reveal that Venn first and foremost aimed at the separation of mathematics from logic – carried out by means of the establishment of an overarching 'Science of Symbols' – which allowed him not only to prove the anti-mathematical logicians wrong in their rejection of the employment of mathematical symbols in logic, but also to formulate, under the header of a 'symbolic logic', a non-conceptualist generalization of logic which makes use of mathematics without introducing it into logic. Taken together, if the general aim of Venn's 'symbolic logic' was to provide a systematic account of the way in which mathematical notions can be used to complete traditional (syllogistic) logic along the lines of the compartmental treatment of (the division of terms of) propositions, this took the form of demonstrating that all the elements of the Boolean mathematical analysis of logic stand 'on a purely logical basis'[89] – i.e. that they can be put forward in entire independence of mathematics such that they fall 'well within the [...] appreciation of any ordinary logician'.[90]

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84 *Venn 1881*, p. 22.
85 *Venn 1889*, p. 230.
86 The members of this 'mathematical' or 'algebraical' camp include figures such as Boole (even though not on Venn's interpretation), Augustus De Morgan (1806-1871) and William Rowan Hamilton (1805-1865).
87 The members of this 'anti-mathematical' camp include figures such as William Spalding (1809-1859) and Thomas Spencer Baynes (1823-1887).
88 *Venn 1881*, p. ix.
89 *Venn 1881*, p. xiii.
90 *Venn 1881*, p. xiv.

2.2 *The 'principle of the transfer of signification'*

It is remarkable to observe that *Venn 1881* does not contain any explicit reference to the development, in the work of algebraists such as Charles Babbage (1791-1871),[91] Peacock and Gregory, of either symbolical algebra in general or the so-called 'calculus of operations' in specific.[92] For if it is what had led Boole to formulate his 'algebra of logic', it also resembles the 'principle of the transfer of signification' which Venn introduced to justify the use of mathematical symbols for logical purposes.

In a passage worth quoting in full, Venn explains that

> [t]he prevalent objections to employing mathematical symbols rest upon an entire misapprehension of their nature and existent range of interpretation [...] The objectors who protest against the introduction "of relations of number and quantity" into logic, and who reject the employment of the sign (+) "unless there exists exact analogy between mathematical addition and logical alternation", cannot [...] get rid of the notion that mathematics in general are [sic] of the nature of elementary arithmetic. We must [...] remind the reader of the wide range of interpretation which already exists within the domain of mathematics: how [...] the sign (×) has [...] extended its interpretation; beginning with true multiplication of integers, it has embraced fractions and negative quantities [...] [s] o that [...] $A \times B$ may [also] mean [...] a certain rotation of a line through an angle. (*Venn 1881*, 89-90).[93]

Similarly, to the anti-mathematical logician's objection that the mathematical logician is 'meddling with [the] actual *laws of operations*'[94] of these signs, Venn replied that if it is true 'that we do not use [the] mathematical signs consist-

91 See Fisch 1999, section 2.

92 See, for example, *Allaire & Bradley 2002*; *Fisch, 1999*; *Koppelman 1971*; *Pycior 1981*; *Verburgt 2014c*; *Verburgt 2014d* for accounts on the development of symbolical algebra and the calculus of operations.

93 It may here be noted that this is exactly how D.F. Gregory introduced and then extended symbolical algebra in his 'On the real nature of algebra' of 1840 (*Gregory 1840*).

94 *Venn 1881*, p. 91.

ently, that is, [if] we [do] put special restrictions upon their laws of operation which are not admitted in mathematics',[95] this can be justified when it is recognized that the same is admitted in mathematics itself. In fact, compared with the so-called 'mathematical freedom' initiated in the work of Peacock and Gregory and first executed in that of William Rowan Hamilton in the form of four-dimensional 'quaternions'[96] [97] the symbolic logician 'really shows the caution [and] timidity [of] an amateur'.[98] It is, of course, somewhat remarkable that Venn made use of new approaches to mathematics in order to be able to maintain that 'far from [mathematics] being introduced *into* Logic [...] we [merely] propose to carry out [...] a precisely similar extension of signification of symbols to that [...] in mathematics'.[99] But it was fundamental for his observation that the anti-mathematical sentiment among the logicians of his time was due to their lack of recognition of the 'principle of the transfer of signification' – which is 'very difficult for any one to grasp who has not acquired some familiarity with mathematical formulae'.[100] It is in this sense that Venn did for traditional logic what the British algebraists did for pure mathematics, namely to come to grips with the 'anxiety' for formal generality.[101]

The application of the 'principle of transfer of signification' to logic – dependent, as it is, on some, merely practical, acquaintance with mathematics – remained of an instrumental character in so far as the generalization afforded by it 'might conceivably [and will probably] be [...] attained within the province of Logic alone'.[102] Or, to put it in more historical terms, there is 'no reason [...] why [a symbolic logic] should not have been developed [...] before that of mathematics had made any start at all'.[103] This explains why Venn repeatedly

95 *Venn 1881*, p. 91.

96 *Hamilton 1844*; *Hamilton 1847*; *Hankins 2004*; *Lewis 2005*. In brief, 'quaternions' are numbers that do not commute under multiplication.

97 These developments may be said to have started evolving when algebra seized to be the study of operations on real numbers – or, in more positive terms, at the moment that operations other than the traditional ones performed on 'mathematical entities' other than real numbers were put forward were being studied. For an account of the complex history of these developments see *Verburgt 2014d*.

98 *Venn 1881*, p. 91, f. 2.

99 *Venn 1881*, p. 231, p. xii.

100 *Venn 1881*, p. x.

101 See Gray *2004* for the notion of 'anxiety' as historiographical concept.

102 *Venn 1881*, p. xii.

103 *Venn 1881*, p. xii.

emphasizes 'the *present accidental dependence* of the Symbolic Logic upon Mathematics'[104] and it is suggestive of what he himself considers the central contribution of *Venn 1881*; the explanation of a completely generalized logic in entire independence of mathematics.[105]

2.3 The mathematical symbols of logical classes and operations

After repeating the conclusions of his article of 1880 entitled 'On the forms of logical proposition'[106] in chapter 1 ('On the forms of logical proposition'),[107] Venn commences *Venn 1881* by distinguishing, in chapter 2 ('Symbols of classes and of operations'),[108] between signs standing classes of things and symbols representing the operations performed on these signs. Further developing his statement, found in his article of 1880 entitled 'Symbolic logic',[109] that 'whereas the common logic [merely] uses symbols for *classes* [...] we shall make equal use of symbols for *operations upon these classes*',[110] Venn makes the fundamental observation that

> a class may be almost always described as the result of an operation, namely of an operation of selection. The individuals which compose the class have been somehow taken from amongst others, or they would not be conceived as being grouped together into what we call a class. Similarly what we call operations always result in classes [such that] [t]he mere signs of operations never occur by themselves, but only in their applications, so that [...] we never encounter them except as yielding a class, and in fact almost indistinguishably merged into a class. (*Venn 1881*, p. 32)

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104 *Venn 1881*, p. ix.

105 If Venn acknowledges that this 'is one of the main objects which I have had before me in writing this book' (*Venn 1881*, p. xiii), he also observes that, in contrast to the more philosophical mathematicians, no logician had ever discussed the symbolic language of logic.

106 *Venn 1880a*.

107 *Venn 1881*, chapter 1.

108 *Venn 1881*, chapter 2.

109 *Venn 1880e*.

110 *Venn 1880e*, p. 248.

Venn repeatedly emphasizes that in order to develop 'symbolic logic' in a systematic fashion it must be separated from 'the problems of Induction [and] the Principles of Science'[111]. It thus may seems as though Venn, in this passage, fails to distinguish between the material 'operations' of selection and the formal 'operation' – thereby introducing 'symbolic logic' issues which, as he himself claims, are to be excluded from it. But what Venn here hints at is not only the fact that even though 'symbolic logic' first suggested itself in the context of the *formal* generalization of the process of division of the denotation side of traditional (see section 1.1.2), it is always to be accompanied by *material* considerations of classification, namely because 'conceptualism' is unable to account for the very process of division (see section 1.1.3 and 1.1.4). Venn also suggests that the operations performed on the classes of necessity result in (sub-)classes of which it is possible to ascertain their material import (see section 2.4).

2.3.1 Addition, subtraction and multiplication

In the first chapters of *Venn 1881*, the general signs x, y, z (etc.) take the place of (denotative) classes consisting of 'the concrete [individual] subjects and predicates of [the] propositions',[112] the mathematical symbols '+', '−', '×' and '÷' are used to express the (logical) operations of addition, subtraction, multiplication and division, respectively. It is of some interest to compare Boole's definition of classes and operations as formulated in *Boole 1854*: 'The literal symbols represent [...] things as *subjects of conceptions*. [The] signs of operation [...] stand for those operations of *the mind* by which the conceptions of things are combined'.[113] Where Boole thus allows the construction of his system to begin with formal definitions of a given set of signs and operations which are only later interpreted as having particular meaning, Venn wants to secure their logical meaning from the outset. In other words, Venn does

> not want [...] to concern [himself] with mathematical relations and symbols [...] but with logical relations and their appropriate symbolic representations. Doubtless we shall [...] find that [this] symbolic statement [...] may be conveniently carried out by the use of symbols borrowed

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111 *Venn 1881*, p. 34.
112 *Venn 1881*, p. 33.
113 *Boole 1854*, p. 27, my emphasis.

> from mathematics. But this is a very different thing from starting with these, and trusting to being able to put some logical interpretation upon, say, [addition] and [subtraction], or upon the signs indicative of multiplication and division. (*Venn 1881*, p. 38)

This much can be observed in the case of the specific course that Venn proposes to adopt for the construction of his system of 'symbolic logic'. For after examining the 'distinct ways in which classes or class terms practically have to be combined with one another for *logical purposes*',[114] his treatment proceeds by discussing in each case, firstly, 'the various words and phrases which are *popularly* employed to express these combinations', secondly, whether 'they may not be *briefly* [...] *conveyed* by help of such symbols as those of mathematics'.[115] Already at this point does Venn insists 'that our procedure is to be logical and not mathematical'[116], that is, even if 'we shall be grateful [f]or suggestions [...] coming whether from mathematics or from any other source',[117] the determination and justification of the processes 'must be governed solely by the requirements of logical and common sense'.[118] Put differently, the mathematical symbolism mere function is that of being 'an afterthought to express [that which] which we are already [...] accustomed'[119] and they are to have 'no conventional *interpretation* other than that which has been [...] originally assigned to them'.[120] This procedure is that of what Venn calls the 'symbolic justification' of the transformation of the everyday expression of the logical operations performed on classes into their expression in terms of mathematical symbols in which the meaning of the first is 'symbolically [set] straight'[121] by the latter. Thus, the first operation is

> so naturally represented by the sign for addition [+] [that] [n]o [...] formal or symbolic justification for it seems to be called for [...]. The [second operation] [...] is expressed with equal convenience by aid of

114 *Venn 1881*, p. 38.
115 *Venn 1881*, p. 38, my emphasis.
116 *Venn 1881*, p. 38.
117 *Venn 1881*, p. 38.
118 *Venn 1881*, pp. 38-39.
119 *Venn 1881*, p. 68.
120 *Venn 1881*, p. 63, my emphasis.
121 *Venn 1881*, p. 36.

> the symbol (−) so that $x - y$ will stand for the class that remains when x has had all the y's left out of it. The only point here that seems to call for symbolic justification is the ascertainment of the fact that the well known mathematical rule, about *minus* twice repeated producing *plus*, is secured in Logic [...] The third logical operation [...] may be represented by the sign of multiplication. The analogy here is by no means so close as in the preceding cases, but the justification of our symbolic usage must still be regarded as resting on a simple question [...] i.e. do we in the performance of the process [...] and in the verbal statements of it, act under the same laws of operation in each case, logical and mathematical alike? (*Venn 1881*, p. 51, p. 53, pp. 54-55)

The 'symbolic justification' of the third operation of multiplication is less straightforward than that of the first two operations of addition and subtraction in so far as it only satisfies the laws of operation governing the process corresponding to the mathematical sign of multiplication up to a certain point. For if 'we have to make our way [...] through various grammatical obstacles [and] the idiosyncrasies of language'[122] so as to make the commutative and distributive law prevail in logic, 'we depart from mathematical usage, or rather restrict the generality of its laws'[123] in the case of the rules dealing with comparative terms. Where in mathematics xx or x^2 is different from x, in logic x must equal xx, xxx, etc. such that every logical statement is reduced to the first degree.

Up to this point, Venn's system of 'symbolic logic' thus includes

> a direct operation *closely analogous* to the addition of ordinary arithmetic and algebra, and suitably symbolized by the familiar sign (+) [...] The inverse operation to the above, and therefore *closely analogous* to subtraction, and which may be suitably symbolized by the sign (−) [...] A direct operation *very remotely analogous* to multiplication [...] which [can] be expressed by the usual signs for that process, viz. (×) or simple juxtaposition of the terms. (*Venn 1881*, p. 67, my emphasis)

122 *Venn 1881*, p. 55.
123 *Venn 1881*, p. 56.

2.3.2 Division

Because the fourth operation of division (÷) is the operation which, in its 'not-yet-symbolized' form, was the one which first introduced the very idea of further formalizing the process of the division of the denotative terms of propositions and which, in its symbolized form, is 'the most fundamental and important with which we shall have to concern ourselves',[124] it is perhaps somewhat unfortunate for Venn to acknowledge that it 'suggests itself [purely] by way of the symbols'.[125] Venn admits that it was for exactly this reason that the 'common people feel no want of [it], and that even the logician has not yet found a place for it'.[126] If this explains why Venn would consider its discussion in chapter 13 of *Venn 1889* extraneous to such a traditional treatise on logic, it accounts for the fact that Venn himself had dismissed the operation in his first article on Boole of 1876.

And also in chapter 3 of *Venn 1881* ('Symbols of operation (continued)')[127] Venn forces himself to proceed with caution, for he writes that

> [w]e [must] conceive the [symbol ÷] conveying the following hint to us: Look out and satisfy yourselves on logical grounds whether there be not an inverse operation to [×]. We do not say that there is such, though we strongly suggest it. If however you can ascertain its existence, then there is one [mathematical symbol] at your service appropriate to express it. (*Venn 1881*, p. 68)

Or, in words drawn from his 'Symbolic logic' of 1880:[128]

> We do not say, Adopt the sign of division and see if you can make any logical use of it. There is no need to take the initiative [...] from the mathematicians. What we say instead is this: Keeping strictly to the field of logic, see if there is an *inverse* operation to that class restriction [or operation] which we denote by the multiplication sign. If there is, then

124 *Venn 1881*, p. 189.
125 *Venn 1881*, p. 67.
126 *Venn 1880e*, p. 254.
127 *Venn 1881*, chapter 3.
128 *Venn 1880e*.

> we have a sign ready at hand to denote it [...] We shall thus keep wholly to the sphere of logic, and though borrowing a sign for convenience from another science, we shall put an interpretation entirely of our own upon it [....]. (*Venn 1880e*, p. 254)

Venn's provisory solution to the problem of the introduction of the operation of division into 'symbolic logic' is the following. On the one hand, if it is true that even though 'in common thought it does not [...] present itself and even in the common logic we find but faint traces of it [...] this [...] is [merely] owing to the fact that [...] it is by no means easy to see what it means without [the] aid [...] of our symbols'.[129] On the other hand, when the meaning of the inverse operation – i.e. that of multiplication – has been established, it may be maintained that also the fractional form is merely a substitute for an inverse operation tacitly and hardly ever made use of in ordinary language. Taken together, Venn suggests that 'any scheme of thought and language which can find a rational use for the symbolic form $\frac{x}{y}$ might [then] introduce such a form [...] into ordinary language in order to express its [new?] wants'.[130]

Having introduced four symbols for the (logical) operations (addition (+), subtraction (−), multiplication (×) and division (÷) and the sign of equality (=), Venn symbolically justifies one remaining mathematical logic, namely that of a 'function' – which, even though it is 'presumably, to the bulk of logicians, the most puzzling and deterrent of all the various mathematical adaptations'[131] made use of in *Venn 1881*, is nothing more than the general expression for, or the higher abstraction of, those logical classes of things found in traditional logic.[132] That is, it may, for example, stand for 'a compound class aggregated of many simple classes [or class-group] [or] it may be composed of two [class-groups] declared equal to one another, or (what is the same thing) their difference declared equal to zero, that is, a logical [class-]equation'.[133]

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129 *Venn 1880e*, p. 245.

130 *Venn 1881*, pp. 76-77.

131 *Venn 1881*, p. 296.

132 Put differently, '[w]e are doing absolutely nothing more than making use of a somewhat wider generalization of the same kind as those with which the ordinary logician is already familiar, and which form one of the main distinctions between his language and that of common life' (*Venn 1881*, p. 86).

133 *Venn 1881*, p. 87.

Taken together, Venn has put forward all the elements of his 'symbolic logic' which, in so far as these elements have been justified 'in entire independence of those of the mathematical calculus',[134] may now also be termed a 'logical calculus'.

2.4 Non-conceptualist 'symbolic logic'

Having arrived at a 'logical calculus', Venn first touched upon the main subject of his *Venn 1881*, namely that of the logical statements or 'equations' (or *propositions*) and their interpretation (or *reasonings*).[135] It is this treatment which is suggestive of the fact that Venn's system of 'symbolic logic' is not only of a non-mathematical, but also of a non-conceptualist nature – i.e. that it is characterized by a dismissal of both the reduction of logic to mathematics as well as the idea, as entertained by among others Boole, that logic is concerned with the mere (formal) logical existence of 'mental concepts'. For his discussion of the interpretation of logical statements is premised on the formulation of a criterion of ('material') existence (see section 2.4.2.1 and 2.4.2.2) on the basis of which the statements can be said to refer to a certain world called 'universe of discourse' (see section 2.4.2.3).

2.4.1 Division, continued: out-Booleing Boole?

But before entering the discussion of these two topics, it is helpful to once more recall the process of 'division' which, in its non-symbolized form, brought with it the very possibility of the extension of traditional logic – namely via the formalization of the division of the denotative terms of propositions – and which, in its symbolized form, 'is the most fundamental and import with which we shall [...] concern ourselves').[136] About the fact that any assignable class admits of dichotomy, or division into two parts x and not-x, Venn writes that 'one or other of these two parts may [fail] to be actually represented, but both cannot [fail]; [and] these may be regarded [as] compartments into one or other of which every individual must fall, and into one or both of which every class

134 *Venn 1881*, p. xiii.

135 See *Venn 1881*, chapter 10 & 11. Here, Venn notes that the division between logical equations and their interpretation 'may be said, roughly speaking, to correspond to that between Propositions and Reasonings in ordinary Logic' (*Venn 1881*, p. 222).

136 *Venn 1881*, p. 190.

must be distributable'.[137] Importantly, whereas 'common Logic' stops at this point, it is clear that

> we have [made] but a single step along a path where indefinite progress is possible. Each of these classes or compartments [...] produced equally admits of subdivision in respect of *y* [...] and so on without limit [...] Stating the results with full generality, we see that with *n* terms thus to combine and subdivide we have a complete list of 2^n ultimate classes' (Venn, 1881, p. 191). This dichotomous scheme [...] contains the [...] raw materials for the statement of every purely logical proposition. (*Venn 1881*, p. 191)

It was Boole who introduced the following 'perfectly general symbolic rule of operation'[138] that is capable of creating the dichotomous scheme in its entirety:

> Write 1 for *x* all through the given [statement], and multiply the result so obtained by *x*: then write 0 for *x* all through it, and multiply this result by [not]*x*. The sum of these two results is the full development of the given [statement] with respect to *x*. (*Venn 1881*, p. 197)[139]

Because Boole only gave the 'formal proofs'[140] of this rule, he left unexplained its logical interpretation and it was for this reason that he, allegedly, failed to acknowledge the restrictions under which it is to be employed. For in Venn's opinion, the rule is not to be applied to uninterpretable statements, 'either at

137 *Venn 1881*, p. 191.

138 *Venn 1881*, p. 196.

139 For Boole's introduction of the rule of operation for obtaining the development see *Boole 1854*, chapter 4. Venn gives the following example of the rule: 'Take [...] a group of class terms [such as] $x + y + xyz$, and suppose we develop this with respect to *x*. The first and third of these terms remain unchanged, for since they involve *x* they cannot yield any not-*x* part. The second term splits up into *xy* and [not-]*xy*. The whole expression thus becomes $x + y$ (x + [not-]x) + xyz. [T]he rule of formation [...] in this case [...] is this. Every term in the given [statement] which involves either *x* or [not-]*x* is left as it stands, and every term which does not involve *x* is multiplied by the factors *x* and [not-]*x*, i.e. is subdivided into these two parts, these being then added together to form the result [...] [T]o resolve the total class into its elementary parts and to retain all these parts before us, is [...] the very process which we are proposing to carry out' (*Venn 1881*, p. 196).

140 *Venn 1881*, p. 197, f.1.

first hand or in the process of passing through such [statements]'.[141] Although Venn himself suggests that Boole also held this opinion, he criticizes him for regarding the rule 'as a sort of engine potent enough to reduce to a series of intelligible logical terms [statements] which as given [...] had not a vestige of intelligible meaning to them'.[142] This somewhat ambiguous approach to Boole's position vis-à-vis division is, of course, characteristic of Venn's general strategy in coming to terms with the latter's work; what Venn claims, in this specific case, is that Boole's 'mysterious'[143] analogical justification of the rule with reference to the (geometrical) interpretation of $\sqrt{-x}$ in mathematics,[144] was 'actually' a slip preventing him from putting forward a proper logical interpretation.

It is this criticism of Boole which brings Venn not only to demonstrate that there is a symbolic justification for the specific procedure of division,[145] but also to consider the general question concerning the 'material reality' of the classes of statements, or 'equations', namely whether 'there [must] be things corresponding to the various class terms?'[146] Here,

> we step out of *formal considerations* into those which are *material* [since] [w]e must [...] have some kind of data to correct, or rather to limit, our necessary but hypothetical scheme of division. How are these data or conditions to be obtained[?] [T]hey are given by the premises of our [statement and] [t]hese [...] put material conditions or limitations on the purely formal considerations [...] and lead us in fact to all the conclusions which the argument admits of. (*Venn 1876b*, p. 481, my emphasis)

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141 *Venn 1881*, p. 201.
142 *Venn 1881*, p. 201.
143 *Venn 1881*, p. xxviii. See *Hailperin 1986*, chapter 1. Laita 1980 has put forward the suggestion that these kind of considerations were connected to the so-called 'extra-logical' sources of Boole's 'algebra of logic'.
144 See Hailperin 2000, pp. 68-72. For an account of the complex history of the geometric representation of $\sqrt{-x}$ in nineteenth-century British mathematics see, for instance, Rice 2001.
145 See *Venn 1881*, pp. 201-216.
146 *Venn 1881*, p. 216.

2.4.2 Reconciling the foundations of traditional logic with its symbolic extension: the 'principle of negative existential commitment' and the 'universe of discourse'

Venn begins his treatment, in chapter 6 of *Venn 1881*, on the import of statements or 'equations' with the central remark that '[m]any logicians, if not a majority of them, have [...] passed [this] subject by entirely [...] This is [...] owing to the prevalent acceptance [...] of the Conceptualist theory of Logic, to which [it is] unfortunately somewhat alien'.[147] It is insightful to observe that Venn himself introduces the subject by emphasizing the difficulties involved in the following 'vexing question in the common ['predication'] theory'[148] of the proposition: Given that propositions such as the universal affirmative ('All *X* is *Y*') and universal negative ('No *X* is *Y*') seem to imply the existence of *X*'s and *Y*'s it is the case that 'before [one] can make any assertion whatever, [one] must make sure not only that both subject and predicate are represented in reality, but also that they are *not* represented'.[149] Venn's argument is that the 'compartmental' view rather than the conceptualist 'class inclusion and exclusion' view of the proposition allows for the widest possible extension of traditional logic by the aid of symbols precisely in so far as it embodies the solution to this 'vexing' problem. This situation is described, by Venn, in the form of two postulates that 'I [Venn] state explicitly [...] because they are not familiar to logicians, if indeed they have ever been definitely enunciated':[150]

> (1) That we must be supposed to know the nature and limits of the universe of discourse with which we are concerned, whether we state it or not [...] (2) That we must become furnished with some criterion of existence and reality suitable to that universe. That is, all our assertion and denial must admit [...] of verification. (*Venn 1881*, p. 128)

These two postulates accompanying the 'compartmental' view of the proposition allow Venn to reconcile the existential demands of traditional logic with the formal nature of its symbolic extension. As will become clear in what follows, this is, essentially, the case because they provide him with an argument for the combining of the formal and material character of the process of the division of

147 *Venn 1881*, p. 126.
148 *Venn 1880e*, p. 260.
149 *Venn 1880e*, p. 260.
150 *Venn 1881*, p. 128.

classes of statements or 'equations' such that their ('Boolean') uninterpretability is excluded by means of, firstly, a ('negative') reversal of the ('Millian') use of their referring to, secondly, a world constructed from their premises. Before discussing the notions of the 'universe of discourse' (section 2.4.2.3) and the principle of what may be called 'negative existential commitment' (section 2.4.2.2) themselves it is helpful to briefly draw attention to Venn's logical and conventional, rather than ontological or formal, definition of 'existence'.

2.4.2.1 'Existence'

At many points in his later oeuvre, Venn emphasizes that he wishes to discuss the issue of 'existence' entirely on logical grounds. This means that he dismisses Mill's 'ontological' and, for example, Boole's 'formal' definition. For where the first – confining itself to the ontological distinction between existence and non-existence – is not even able to account for the 'formal' side of the process of division of denotative terms, the second – not allowing itself to refer to external objects – cannot account for the 'material' conditions that are to limit the 'formal' scheme resulting from the performance of the operation of division upon classes, i.e. for ('symbolic') reasoning. Given that the first criticism concerns the ('prior' or 'external') issue of the creation out of an aspect of 'material logic' of 'symbolic logic' and the second relates to the ('posterior' or 'internal') issue of the logical justification of the elements of 'symbolic logic', it should follow that Venn's entertains a twofold approach to ''existence' on logical ground'. But it is in fact the case that the 'criterion of existence and reality'[151] which Venn, in *Venn 1881*, introduces as being suitable for the interpretative import of symbolic propositions (i.e. statements or 'equations') is premised on a (quasi-idealist)[152] definition of 'existence', put forward in *Venn 1889*. The sole, albeit fundamental, difference between Venn's two accounts is that this definition, in *Venn 1889*, is the collective term for the 'objective' and 'subjective' foundational assumptions of inductive ('material') logic, and that,

151 *Venn 1881*, p. 128.

152 Notwithstanding the accepted characterization of Venn as a Millian empiricist, in his logical work Venn also took, both directly and indirectly, inspiration from the oeuvre of the British idealist William Whewell (1794-1866) – concerning his views on induction – and Grote – concerning his general philosophy of logic and language. See Collini 1975; Dewey 1974; Gibbins 1998; Gibbins 2006; MacDonald 1996; Whitmore 1927. This point is further developed in *Verburgt 2014b*.

in *Venn 1881* it functions as the name for the 'universe' resulting from the interpretation of the statements or 'equations' of deductive ('symbolic') logic. Thus, in *Venn 1889*, Venn notes that the logician must regard existence as 'supplied by [the reader] himself, or be gathered from the intention of the speaker, or from the context in which the statement occurs [such that] in every proposition [...] the distinction between reality and unreality, between existence and non-existence, is in some signification or other taken for granted [or] already admitted and appreciated'.[153] In his *Venn 1881*, Venn, after writing that 'all our assertion [e.g. 'All *A* is *B*'] and denial ['No *A* is *B*'] must admit [...] of verification [...] without any necessary digression into metaphysics',[154] remarks that the limit of 'what is meant by 'all' [or 'no'] [...] is a part of the [totality of the] data and therefore to be postulated by the logician [and] not a formal principle'.[155]

Taken together, the 'principle of negative existential commitment' and the notion of a 'universe of discourse' can be approached as a translation of a certain 'material' definition of 'existence' such that, on the one hand, it becomes amenable to 'symbolic logic' and, on the other, 'symbolic logic' can properly be said to form a part of traditional logic in so far as it satisfies the criterion of being 'about' the world of ordinary experience.

2.4.2.2 The 'principle of negative existential commitment'

The fundamental problem involved in the formulation of a criterion of existence for 'symbolic logic' is that of upholding not only the material import, or logical interpretability, of symbolic statements or 'equations', but also that of the idea that these statements or 'equations' do not afford positive knowledge about the world. Interestingly, it is precisely in combining these two points that Venn finds the solution to both this problem as well as the abovementioned 'vexing' problem of traditional logic. For

> if we adopt the simple [suggestion] that *the burden of implication of existence is shifted from the affirmative to the negative form*; that is, that it is not the existence of the subject or the predicate (in affirmation) which is

153 *Venn 1889*, p. 231.
154 *Venn 1881*, p. 128.
155 *Venn 1881*, p. 187.

> implied, but the non-existence of any subject which does *not* possess the predicate, we shall find that […] all difficulty vanished. (*Venn 1881*, p. 141).

In other words, where the rules of the syllogism of traditional logic imply that it is not possible 'to assert […] or deny anything about *X* or *Y* unless we are certain […] that there are things which are *X* and *Y* [and] things which are not *X*, and not *Y*' and in 'symbolic logic' 'we […] deal with a number of propositions simultaneously, involving perhaps a dozen or more of terms',[156] it holds that it is simply 'impossible to tolerate a system in which either assertion or denial were permitted to carry along with it the ['positive'] implication of the existence of things corresponding to the subject and predicate'.[157] It is the principle which shifts the burden of implication from the affirmative to the negative such that it is only the non-existence of any subject which does not possess the predicate that is implied – i.e. the 'principle of negative existential commitment' – which allows Venn to write the following:

> Take the proposition 'All *x* is *y*'. There being two class terms here, there are […] four ultimate classes […] *xy*, *x* not-*y*, *y* not-*x*, and not-*x* not-*y*. Now what […] the proposition 'All *x* is *y*' [does] is not to assure us as to any one of these classes […] being *occupied*, but to assure us of one of them being *unoccupied*. [For] [w]hether there be any *x*'s or *y*'s we cannot tell for certain, but we […] feel quite sure that there are no such things existing as '*x* which is not *y*' […] The [statement] 'Al *x* is *y*' […] for the purposes in hand is much better written, 'No *x* is not-*y*' […] that is, it empties the compartment *x* not-*y*'. (*Venn 1881*, p. 141, p. 142)

The general claim expressed by the 'principle of negative existential commitment' is that in respect of what statements affirm they are to be regarded as 'conditional' and in respect of what they deny statements are to be regarded as 'absolute'. Thus, if the universal affirmative ('All *x* is *y*', or 'No *x* is not-*y*') absolutely declares the non-existence, or 'materially destroys' (!) the existence, of things which are the combination of *x* and *y*, it 'positively but conditionally [declares] that if there are such things as *x*, then all the *x*'s are *y*' – and, for com-

156 *Venn 1881*, p. 139.
157 *Venn 1881*, p. 140.

pleteness' sake, it 'does not tell us whether there is any *y* at all; or, if there be, whether there is also any *x* [...] carefully limiting itself (so far [...] as existence of the things is concerned) to the single negation of there being any *x* not-*y*'.[158]

Venn follows Boole in proposing that it is the particular proposition which accounts for the familiar form 'Some *x* is *y*'. If the formula $xy = 0$ expresses that a class is absent and $xy = 1$ that it is present to the exclusion of all else, the intermediate form $xy = v$ – 'where *v* is to stand for a class of which we merely know [...] that it is intermediate between 1 and 0, viz. between all and nothing'[159] – expresses that '*xy* is something' or 'there is *xy*'.[160] Because particular propositions by themselves can neither destroy nor establish a class, and therefore have 'no categorical information to give the world',[161] they cannot be symbolized.[162] Or, in Venn's own words, in so far as the particular propositions 'Some *X*'s are *Y*'s' and 'Some *X*'s are not *Y*'s' 'extinguish no class and establish no class [they] slip in between [...] the sort of propositions [...] which yield two alternatives only [and] this we cannot [...] represent symbolically'.[163] This is an uncomfortable statement – and for several reasons. Firstly, the point is not that particular propositions do *not* affirm anything about the world, but rather that the way in which they *actually do* cannot be expressed by means of the 'principle of negative existential commitment'. Secondly, it follows that particular propositions – not having the same 'characteristic of really establishing something'[164] as universal statements – are 'about' a kind of (positively implied) existence unsuitable to 'symbolic logic'.[165] This, thirdly, can be seen from Venn's argument that the former cannot be deduced from the

158 *Venn 1881*, p. 142.
159 *Venn 1881*, p. 144.
160 Boole described the symbol v as representing the 'operation of selecting all elements, V, of a nonempty subset of appropriate terms' (*Green 1994*, p. 51).
161 *Venn 1881*, p. 161.
162 See *Venn 1881*, chapter 7.
163 *Venn 1881*, p. 161, p. 163.
164 *Venn 1881*, p. 161.
165 Venn writes that 'we shall not [...] make much of the symbolic treatment of particular propositions [...] What Symbolic Logic works upon by preference is a system of dichotomy, of *x* and not-*x*, *y* and not-*y*, and so forth. The sort of propositions therefore that suits us best are those which yield two alternatives only, such as individual propositions: - *A* is *B*, *A* is not-*B*, and so on. But the particular proposition, in its common acceptation, slips in between these two and says '*Some* of the *A*'s are, or are not, *B*' (*Venn 1881*, p. 163).

latter for 'if the [latter] destroys [*x*not-*y*] and the [former] merely saves [*xy*], and if these two classes are entirely distinct [...] then the two propositions clearly *do not come into contact* with one another at any point'.[166] In other words, 'Some *X* is *Y*' can only be inferred from 'All *X* is *Y*' provided that the existence of *X* and *Y* is affirmed unconditionally[167] – and this, to be sure, is at odds with Venn's treatment. Fourthly, in so far as Venn recognizes that at this point his 'symbolic logic' comes 'into serious conflict with well grounded [opinion]',[168] he forces himself to argue that his system does not deal with particular propositions proper, but only with their 'universalized' form: '[W]e can very often [and should] succeed [...] in determining the 'some' so that instead of saying vaguely that 'Some [*X*] is [*Y*] we can put it more accurately by stating that 'The [*X*] which is [*Z*] is [*Y*]', when of course the proposition instantly becomes universal'.[169]

Many of Boole's successors were motivated to (re)solve the difficulty of symbolically representing particular propositions[170] – and Venn is no exception. It was his opinion that these propositions 'in their common acceptation, are of a somewhat temporary and unscientific character. Science seeks for the universal, and will not be fully satisfied until it has attained it'.[171] This view, on which *v* stands for a class 'intermediate between 1 and 0'[172] such that it 'shall not equal *nothing*',[173] differs greatly from that of Boole on which *v*, 'speak[ing] of it as if it was like any other class term',[174] functions as an 'indeterminate' (class) symbol that is 'yet to be interpreted'. Venn thus wants to universalize particular propositions, or, put differently, give them a universal interpretation, while

166 *Venn 1881*, p. 167, my emphasis.
167 See *Venn 1881*, pp. 168-169.
168 *Venn 1881*, p. 167.
169 *Venn 1881*, p. 169. Venn continues this passage by noting that '[p]ropositions which resist such treatment and remain incurably particular are comparatively rare; their hope and aim is to be treated statistically, and so to be admitted into the theory of Probability' (*Venn 1881*, pp. 169-170). It is in his famous *Venn 1866* that Venn deals with these kind of particular propositions. For a discussion of this treatment against the background of the whole of Venn's theory of material and symbolic logic see Verburgt (2014a).
170 See *Green 1994*.
171 *Venn 1881*, p. 169.
172 *Venn 1881*, p. 144.
173 *Venn 1881*, p. 161.
174 *Venn 1881*, p. 162.

Boole, especially in his later works,[175] attempted to give universal statements a material, or existential import. That Venn considered Boole's approach to be a much wider departure from 'popular speech' and 'common logic' may come as no surprise[176] for it introduces a symbol that, possibly, does not represent anything interpretable in logic. More in general, it is exemplary for

> the boldness [...] with which [Boole] carries on his processes through stages which have no logical or other significance whatever – that is, which admit of no possible interpretation – provided only that they terminate in an interpretable result [...] If it be asked whether, and how far, such a step is capable of being justified, it is difficult to know what to answer [...] Boole justifies himself by maintaining that a single valid employment of such a step enables the mind to recognise it as intuitive and axiomatic [but] I [Venn] apprehend [that it] will need the occasional support afforded by some kind of contact with experience. (*Venn 1876b*, p. 485)

Where Venn, in this passage from the year 1876, straightforwardly dismisses as 'fanciful'[177] Boole's appeal to the ability of the human intellect to intuit the validity of steps without logical significance – just as he characterizes the statement of Boole that mathematical laws are really laws of thought expressed in mathematical form[178] – he has not yet arrived at a formulation of the particular 'kind of contact with experience' that is to take its place as a criterion for their validity. But it is clear that Boole's approach to the symbolic representation of particular propositions does not suit the 'principle of negative existential commitment' first put forward some four years later in 1880.[179] For if Venn's universalization of these propositions is premised on reforming them such that they, on the hand, yield only two alternatives, namely, on the other hand, (positive) conditional affirmation (e.g. 'There is *xy*') and (negative) absolute negation ('e.g. 'There are such things as *x*'s which are not *y*'), Boole's interpreting of particular propositions seems to presuppose their unconditional or absolute affirmation and denial. Given that Boole, by means of his commitment to con-

175 See Grattan-Guinness 2000; Van Evra 1977.
176 See *Venn 1881*, p. 166-167.
177 *Venn 1876b*, p. 490.
178 See *Venn 1876b*, pp. 490-491.
179 See *Venn 1880e*.

ceptualism, has already prevented himself from allowing to refer to external objects, Venn considers this conclusion to be not only 'very awkward',[180] but also a clear symptom of Boole's failure to account for the need to put 'material' conditions on the 'formal' scheme resulting from the operation of division.

2.4.2.3 The 'universe of discourse'

The notion of a 'universe of discourse' forms the second[181] postulate which Venn puts forward in order to solve traditional logic's 'vexing problem' of existence.[182] Because this 'universe', being 'entirely a question of the application of our formulae [and] not of their symbolic statement',[183] only arises at the moment that statements or 'equations' are interpreted on the basis of the 'principle of negative existential commitment', Venn has it that the notion is strictly speaking extra-logical. In other words,

> the settlement of the Universe [is] a question of application merely [and] it can never be indicated by our symbols, for these [...] know nothing of any kind of limit except what is purely formal. When it is asked, What are the limits of not-*x*? the symbolic answer is invariably the same, 'all that is excluded from *x* is taken up by not-*x*'. It is only when we go on to enquire what is meant by 'all' that the question of a limit comes in, and this is a practical matter involving the interpretation of our data. (*Venn 1881*, p. 187)

Two remarks may be made about this passage. Firstly, if Venn writes that the 'settlement' of the 'universe of discourse' – or, for that matter, the very question of a limit – is a practical matter, he does put a formal restriction on the possible 'scope' of this universe. That is, in so far as his strict class view of the nature of propositions must dismiss so-called 'infinite' or indefinite terms (or classes) and

180 *Venn 1881*, p. 162.
181 The first thus being the 'principle of negative existential commitment' which 'furnishe[s] [us] with [a] criterion of existence and reality suitable to that universe [such that] all our assertion and denial [...] admit[s] of verification' (*Venn 1881*, p. 128).
182 Recall that this problem was that that 'before [one] can make any assertion whatever, [one] must make sure not only that both subject and predicate are represented in reality, but also that they are not represented' (*Venn 1880e*, p. 260).
183 *Venn 1881*, p. 184.

propositions (or statements), there is a 'necessity of some restriction upon the extent of the class which we take into account'.[184] In a passage worth quoting in full, Venn explains that

> [v]erbally, of course it is easy enough to say that we must either assert that *A* is *B*, or deny that it is *B*, or [...] assert that *A* is not-*B*; and we may readily admit that there is some [...] difference of signification between these cases. [But] [o]n any rigid class view [...] it is impossible to extract more than two divisions; for, that to exclude a thing from a boundary is to include it somewhere outside that boundary, that to deny that any thing has a given attribute is to assert that it has not that attribute, seems indubitably clear. [T]he idea underlying the distinction is this. When we deny that *A* is *B* we think of *A* as a whole, and *B* as an attribute and therefore as a whole, so that the judgment is finite in both terms. But when we say that *A* is not-*B* and try to consider this not-*B* as an attribute, we have forced upon our notice the vague amplitude of its extent; and therefore, when we do not make appeal to a *limited universe*, we must recognize that the judgment is in respect of its predicate an infinite or indefinite one. (*Venn 1881*, p. 183)

Secondly, it is the boundaries between the '1' (all) and '0' (nothing), or '*X*' and 'not-*X*', of this limited (symbolic) universe which are 'in every respect open to our own [free and arbitrary] choice'[185] – such that 'the real extent of that sum total of things which makes up our Universe' is 'a part of the data and therefore [...] not a formal principle'.[186]

184 *Venn 1881*, p. 183.
185 *Venn 1881*, p. 185.
186 *Venn 1881*, p. 189.

Venn acknowledges that it was De Morgan who originated the term 'universe of discourse' in his *Formal Logic* of 1847.[187] But it must be noted that by representing this 'universe' by unity (i.e. '1') Venn seems to introduce exactly that confusion against which De Morgan had warned,[188] namely that where 'in a ['universe of discourse'] of individuals, the greatest class we can form out of these individuals happens to coincide with [this 'universe'] [...] when we are relating classes to each other, the greatest class, 1, is not [the 'universe of discourse'], but [...] an element in [it], just like any other class'.[189] Venn, for his part, insists that De Morgan's general restriction that no simple class-term is to be equated to '1' (or '0') would be 'suicidal'.[190] The reason for this statement also undermines the idea that Venn committed an unmistakable error. For it is clear that the acceptance of a 'universe (of discourse)' that is somehow independent from the classes which it includes – i.e. whose existence is purely formal – is premised on the conceptualist approach to (symbolic) logic which Venn wishes to undermine when writing that the possibility of the 'complete *material* destruction' (Venn, 1881, p. 145) is what the symbolic classes 'must always be prepared to face as something which may *at any moment* be declared to be their lot' (Venn, 1881, p. 146). And this is exactly what the 'principle of negative existential commitment' and the notion of the 'universe of discourse' enable him to do.

187 Although the origin of the phrase 'universe of discourse' has often been found in *De Morgan 1847* (e.g. *Kneale & Kneale 1962*, p. 408), this book only included references to the 'universe of possible objects' (*De Morgan 1847*, p. 55), 'universe of names' (*De Morgan 1847*, p. 149) and 'universe of (a) proposition(s)' (*De Morgan 1847*, p. 153). It was in his article of 1846 entitled 'On the structure of the syllogism' that De Morgan used the term 'universe of discourse' as a new technical term (see *De Morgan 1846*, p. 380). Boole was one of the first to take up the term in *Boole 1854* (e.g. *Boole 1854*, p. 42).

188 See *Langer 1967*, pp. 170-171.

189 *Langer 1967*, p. 171.

190 *Venn 1881*, p. 162, f. 1.

3. Venn and the history of (British) logic: an afterword

The central goal of this paper was to provide a detailed account of Venn's presentation of the foundations of symbolic logic. This concluding section purports to bring to bear its main findings upon an assessment of the place of Venn in the history of nineteenth-century British and modern logic.

3.1 Venn as a successor of Boole

When seen against the background of the well-known discussion about the significance of Boole's 'algebra of logic' or *calculus ratiocinator*,[191] in comparison to Gottlob Frege's (1848-1925) 'mathematical logic' or *characteristica*,[192] for the development of modern logic,[193] it is interesting to observe what Venn himself considered to be the main contribution of *Venn 1881* to the fundamental task of the symbolic transformation of (syllogistic) logic into a propositional 'calculus of deductive reasoning'. The book aimed at a revision of Boole's system that was to demonstrate that, in contrast to the revisions of formal logic proposed in the work of (the pre-Boolean conceptualist) William Hamilton (1788-1856) and (the post-Boolean conceptualist) William Stanley Jevons (1835-1882), the 'algebra of logic' 'should be regarded as a Development or Generalization'[194] of traditional logic. Following the oft forgotten 'philosophical school' in logic,[195] Venn not only put forward the general argument that in so far as the logic of Aristotle provided the 'topics' to be dealt with by the 'algebra of logic' it belonged to this philosophical tradition, but also the more detailed claim that the mathematics employed to symbolically express logical relationships is merely a tool in the hands of the logician-philosopher (or 'amateur symbolic algebraist' (!)). Although Boole, as Venn repeatedly emphasized, would have agreed with the general argument,[196] Venn criticized him for his 'many and serious omissions' – the supplying of which is put forward as an indication of

191 See, for example, *Irving 1918*, chapter 1-3; *Kneale 1956*; *Green 1994*; *Gasser 2000.*
192 See, for example, *Beaney & Reck 2005*; Tichy 1988.
193 See, for example, *Jetli 2011*; *Peckhaus 2000*; *Peckhaus 2004*; *Van Heijenoort 1967*; *Van Heijenoort 1992.*
194 *Venn 1881*, p. xxvii.
195 *See Peckhaus 1999.*
196 *See Corcoran 2003*; *Peckhaus 2003*, pp. 5-6; *Nambiar 2000.*

'what may be supposed to be characteristic and original'[197] in *Venn 1881*. And all these telling omissions from the side of Boole – accounting, as they do, for the fact that Venn published 'an independent study [...] and in no sense a commentary [...] upon Boole'[198] – are connected to the neglect of, firstly, comparing the system of 'algebra of logic' to the traditional system of ordinary logic and, secondly, establishing all of the algebraic expressions on purely logical, rather than formal, principles. By showing that as a logical system Boole's system embodied the complete generalization of the 'science of Formal, viz. Aristotelian or Scholastic Logic',[199] Venn simultaneously dismissed the attempt of moving beyond syllogistic logic – or, in his words, to 'cut ourselves loose from the familiar forms of speech'[200] – by either quantifying the predicate (Hamilton and Jevons)[201] or introducing relational terms (De Morgan, Charles Sanders Peirce (1839-1914)) and rejected the claim that the 'algebra of logic' is a mathematical system.[202]

Put somewhat anachronistically, Venn would also have disagreed with those commentators, such as Bertrand Russell (1872-1970),[203] who characterize Boole as the 'father' of modern logic and this for the simple reason that, on the one hand, he conceived of his work as being a contribution to the *calculus ratiocinator* and, on the other, this process of accomplishing logical deductions by manipulation of the algebraic formulae was another name for the generalization of traditional syllogistic logic. Furthermore, if Venn was in agreement with the fact that Boole's work occupies an important place in the history of logic, he would have disputed the claim that Boole was the originator of mathematical logic[204] for the equally simple reason that his was not a mathematization of logic at all. Taken together, Venn applauded Boole for using mathematical algebra to complete traditional logic and not for surpassing it by means of making logic into a branch of mathematics. Venn would thus have taken the statement that 'Boole's quasi-mathematical system [can] hardly be regarded as a final and

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197 *Venn 1881*, p. xxix.
198 *Venn 1881*, p. xxx.
199 *Venn 1881*, p. xxvi.
200 *Venn 1881*, p. xxvii.
201 See *Bednarowski 1955*; *Fogelin 1976*.
202 See, for example, *Jevons 1864*.
203 See *Jager 1972*, chapter 3.
204 See, for example, *Feys 1954*.

unexceptionable solution of the problem of supplying a viable alternative to Aristotelian logic'[205] as its main strength, rather than the weakness accounting for the fact that many historians of logic have largely ignored it[206] philosophers of logic have repeatedly questioned its scientific value.[207]

3.2 Venn's anti-Fregeanism

Given that Venn was one of the six (!) reviewers of Frege's *Begriffschrift*[208] of 1879,[209] it is also possible to determine his position vis-à-vis the influential view which singles out Frege as the founder of modern logic.[210] In his very hostile review Venn pointed out that he does not recognize anything novel in 'Dr. Frege's scheme'.[211] The *Begriffschrift* is dismissed as an instance of 'an ingenious man working [...] in entire ignorance that anything of the kind had ever been achieved before'[212] for 'I [Venn] should suppose, from his [i.e. Frege] making no reference to [Boole], that he has not seen it [for] the merits which he claims as novel for his own [...] are common to every symbolic method'.[213] It was only in his *Symbolic Logic* that Venn came to acknowledge the fundamental difference between Boole and Frege – one that he described not in terms of the distinction between an 'algebra of logic' and a 'mathematical logic',[214] but with reference to the distinction between an extensional generalization (*calculus ratiocinator*) and an intensional generalization (*lingua characterica*) of traditional logic. In contrast to some modern commentators who either interpret this latter distinction as standing for two compatible approaches to logic[215] or define the second kind as constituting the universality of logic,[216] Venn was of the opinion that Frege's construction of universal language representing the structure of the conceptual content of expressions is 'a hopeless one'.[217]

205 *Stanley 1958*, p. 113.
206 See, for example, *Van Heijenoort 1967*.
207 See, for example, *Dummett 1959*; *Quine 1995*.
208 *Frege 1879* [1967].
209 See *Venn 1880f*; *Vilkko 1998*.
210 See, for example, *Quine 1955*.
211 *Venn 1880f*, p. 297.
212 *Venn 1881*, p. 415.
213 *Venn 1880f*, p. 297.
214 See *Grattan-Guinness 1988*.
215 See, for example, *Frege 1996*; *Peckhaus 2004*.
216 See, for example, *Van Heijenoort 1967*.
217 *Venn 1881*, p. 390.

The argument for this claim seems to be that the very attempt to either further develop the traditional process of division from the connotative side or 'interpret our terms in respect of their intension, instead of regarding them [...] extensively, viz. as merely representing classes'[218] is conceptualist *per se*. For Venn not only holds that connotation, being part of the formal process of definition, 'consists of, i.e. actually *is*, the attributes which we are [...] proposing to enumerate, and which we must [...] presume to be [...] present to the mind of every one who is fully informed of the meaning of the word in question [and] does not stand in need of any appeal to fresh experience'.[219] But he also has it that it must be presupposed, since this 'does not altogether coincide with facts'[220], that 'all people whom we take into account [i.e. the 'universe of reasoners'] are agreed as to what [...] the intension, connotation, or [...] meaning of a term [...] or rather 'concepts', in order to use an appropriate conceptualist expression, [is]'.[221] The problems to which these features of the Fregean account give rise become apparent as soon as it is attempted to put forward a rigid intensive interpretation of propositions.

For if the fact 'one group of attributes may include or exclude another, partially or wholly, just as a class of concrete objects may do [...] appears to afford four relations of the kind'[222] it is clear that the intensional scheme cannot account for the possibility of one group coinciding with another and yet being recognized as distinct. Hence, whether or not this possibility is admitted 'by those who speak a pure Conceptualist speech'[223], feels confident to conclude that these logicians are unable to distinguish the not necessarily identical proposition 'All *X* is all *Y*' from the proposition 'All *X* is all *X*' expressing absolute identity. It is situation that perverts, so to say, the whole scheme of intensionally interpreted propositions such that it follows that, apart from 'verbal, necessary, analytic, or

218 *Venn 1881*, p. 390.
219 *Venn 1889*, p. 309.
220 *Venn 1881*, p. 391.
221 *Venn 1881*, p. 391.
222 *Venn 1881*, p. 392.
223 *Venn 1881*, p. 392.

essential'[224] universal propositions, none of either the ordinary propositions[225] or processes[226] can be expressed symbolically without 'analysis', on the basis of 'information', of *X* and *Y*.

At least two remarks can be made about Venn's dismissive treatment of Frege's 'conceptualist' *Begriffschrift*. Firstly, Venn insist that in so far as in traditional logic 'all names [...] possess both intension and extension'[227] it holds that 'we may conceivably examine the mutual relations of terms, and through these develop a system of propositions and of reasonings, from either of these two sides' (Venn, 1881, p. 396). Secondly, his argument is that the intensional scheme – as it stands, for Venn claims that it has 'never been fairly tried at all' (Venn, 1881, p. 396) – fails precisely because it does not succeed in further developing traditional logic. Thirdly, given that Venn cannot but conceive that this is Frege's aim, he does not recognize the newness of the *Begriffschrift*. He characterizes as 'cumbruous and inconvenient' (Venn, 1880f, 297) the ('two-dimensional') notation which Frege adopted not only to circumvent some of the features inherent in that of Boole (and Venn),[228] but also to be able to create the logic of relations which enlarged, instead of generalized, traditional logic.

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224 *Venn 1881*, p. 394.

225 For example, 'suppose that the concepts [are] *AB* and *AC* [and] we see at once that we cannot with certainty conclude any ordinary proposition from this, in the absence of information as to the mutual relations of *B* and *C*. If *B* and *C* are really contradictory [...] then we conclude that No *P* is *Q*. If *B* includes *C* [...] then we know that All *Q* is *P*, and similarly if *C* includes *B*. If however *B* and *C* are really distinct in their meaning [...] then no proposition whatever can with certainty be elicited out of these concepts *AB* and *AC*' (*Venn 1881*, p. 394).

226 Venn writes that '[w]e shall see this best by examining [...] the now familiar [...] sign (+). If *A* and *B* are attributes or partial concepts, we should I presume agree in saying that *A* + *B* will signify those attributes 'taken together'. But when we go on to enquire what is meant by taking attributes together we perceive that this is a very different thing from taking classes together. The only consistent meaning surely is that we do this when we construct a new concept which contains them both. But this is clearly the analogue not of addition but of multiplication [such that] 'extensive multiplication' corresponds to 'intensive addition'' (*Venn 1881*, pp. 396-397).

227 *Venn 1881*, p. 396.

228 One of the most important reasons for which Frege had not applied the Boolean notation is that here the letters always refer to extensions of class-terms and never to individuals. Another major reason was of a more methodological character; the 'method of Boolean algebraists proceeds from concepts to judgments [...] whereas Frege's starting point was judgments themselves' (*Vilkko 1998*, p. 419) the content of which he expressed by means of the content stroke.

That is, Venn does neither realize that the notation of Frege's 'mathematical logic' embodies not the attempt to mathematize logic, but to logicize mathematics nor that it was exactly this (anti-psychologistic!) step that allowed for acknowledging the vagaries of ordinary language and declaring 'the death of Aristotelian logic'.[229]

The task of determining the place of Venn in the development of modern logic seems to be intimately connected with the seminal question of whether modern logic begins with Boole or Frege. At least this much is suggested by W.V. Quine when writing that 'I [Quine] have long hailed Frege as the founder of modern logic, and view Boole, De Morgan and Jevons as forerunners. John Venn [...] also belong[s] back with them, though coming on the scene only after the great event'.[230] This paper has, hopefully, made it clear that this statement is quite insensitive to what Venn himself considered the fundamental contribution of the *Symbolic Logic*, namely that of providing a non-conceptualist and non-mathematical reformulation of the 'algebra of logic' of Boole such that it becomes possible to place it squarely within the tradition of Aristotelian logic. Given that Venn was convinced that this aim reflected Boole's own opinion as to the meaning of the enlargement of traditional by means of symbols, the book can be read as a lengthy rebuttal of those accounts which see Boole as the father of modern logic. Because Venn does not recognize the *Begriffschrift* as a 'great event', the same, by and large, goes for those accounts that put forward Frege as a candidate for this paternal role. If it is in this sense that the whole of Venn's logical oeuvre crowns the late nineteenth-century attempt to save what, from hindsight, was already irretrievably lost, it also incorporates one of the most elaborate efforts, in the words of Otto Neurath, to combine an 'interest in logic with an interest in empiricism'[231] and to demonstrate, albeit unintentionally, the complexities involved in upholding this combination while recognizing and coming to terms with new developments.

229 *Jetli 2011*, p. 111.
230 *Quine 1995*, p. 254.
231 *Neurath 1996*, p. 325.

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