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Abstract

We study cartel stability when firms maintain collusion only if it is more profitable than competition by a sufficiently large margin. Accounting for an (endogenous) cartel margin suggests new unambiguous comparative statics of changes in market characteristics on the scope for stable cartels. The margin increases their effect on the gain from collusion, relative to the gain from deviation. More specifically, we find that when there is a (small) cartel margin, both lower industry marginal cost and less product differentiation can increase cartel stability. The common conjecture that collusion is more prevalent in homogeneous goods and low cost industries—which has no basis in existing cartel theory—is canonically true when firms require even only a small cartel margin. Implications for competition policy include a focus in enforcement on standardized product-low input cost industries. In merger control, efficiencies may increase the risk of coordinated effects.

JEL-codes: K21, L13, L41
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If you are going to kill, then kill an elephant;
If you are going to steal, make sure it's a treasure. Indian Proverb

1 Introduction

The conditions under which collusion can be sustainable are well known to depend on such characteristics as the number and relative market shares of the firms active in the market, their frequency of interaction and price adjustment, the threat of entry and prospective demand. In a framework of repeated interaction, Friedman (1971) and Abreu (1986, 1988) formalized that cartel members have a critical discount factor against which they value the future stream of cartel profits just equal to the instantaneous gain from defection, followed by the implementation of a punishment strategy. The common approach in cartel theory, at least since Tirole (1988), is to consider a cartel agreement internally stable when for each member its actual discount factor is above its critical value, and unstable below. Any change in market structure characteristics that lowers the (binding) critical discount factor increases the scope for cartels in that market, in the sense that they are sustainable for a wider range of actual discount factors.

This knife-edge configuration of cartel stability remains silent on *by how much* firms may be better off colluding than not. Yet it is reasonable to consider a cartel 'more stable' when its members' incentives to adhere to the collusive agreement exceed those to defect by a larger margin. Changes in the circumstances of the conspiracy may affect cartel stability differently when a cartel is stable by a margin—which itself may also vary with these changes, but typically differently than the cartel profits.

In this paper, we study the comparative statics of cartel stability when collusion is more profitable than competition by a margin. We bring the analysis of the comparative statics effects when there is a cartel margin under conventional cartel stability theory by incorporating the margin in the derivation of the critical discount factor. The premise is that firms maintain a cartel only if each period they are sufficiently better off colluding. The margin is a threshold value that firms require on top of instantaneous cartel profits for participation in the cartel agreement.

The cartel margin can have different sources. It may be needed to compensate for moral disutility from participating in the illegal conspiracy, or to insure against uncertain events such as the sudden appearance of a maverick firm or sharpened exposure to antitrust damages actions, which fan the distrust between coconspirators. More concretely, the cartel margin can cover the expected consequences of cartel participation: the cost of colluding—of monitoring, communication and managerial effort to repair internal tensions—plus the liabilities—of fines, damages and reputation loss—in the event of break-down—by natural causes or death by antitrust, including a leniency application. It can also reflect risk aversion towards stochastic demand and costs.

The cartel margin affects the determination of the critical discount factor as a reduction of the difference between defection and cartel profits. We find that a sufficiently large cartel margin provides new unambiguous comparative statics of variations in market characteristics on cartel stability where these are otherwise not available. In particular in cases where the gains from collusion and deviation move in the same direction, so that there is *a priori* ambiguity on the comparative statics, we obtain unambiguous comparative statics for a wide range of margin sizes. The intuition is that an increase in the margin increases the effect that the gain from collusion has on cartel stability—which is positive—relative to the gain from deviation—which is negative. When the cartel margin is sufficiently large, the effect on the gain from collusion dominates.

In particular, we show in a wide range of circumstances that with a cartel margin, collusion is more stable with lower marginal cost of production and more homogeneous products. While this is commonly conjectured, it lacks in fact a solid theoretical foundation. In standard cartel theory, the overall level of marginal costs is immaterial to cartel stability, because common changes in marginal costs do not affect the classic critical discount factor(s)—as reiterated in Section 3. However for any positive margin the critical discount factor decreases in the marginal cost level, provided the margin itself does not decrease too strongly in marginal cost. On the effects of product differentiation on collusion there is no consensus in the theoretical literature—reviewed in Section 4. Results are highly case specific without, but with a sufficiently high cartel margin that does not itself increase too strongly in product homogeneity, the critical discount factor increases unambiguously in the level of product differentiation—provided competition is not too strong to begin with.

These findings have implications for competition policy. Cartels should be expected in lower marginal cost sectors, in response to industry-cost reducing innovations or a common input cost drop. For merger control, we raise a new possible concern for coordinated effects when the parties claim merger-specific efficiencies in compensation for the unilateral anticompetitive effects of a merger. If one or both of the merging firms critically determine the binding critical discount factor, a reduction in their marginal cost of production increases the scope for collusion unambiguously for any nonnegative cartel margin. This aspect of the efficiency defense does not seem to be recognized by the competition authorities. On the contrary, the 2010 U.S. *Horizontal Merger Guidelines* state that:

> "In a coordinated effects context, incremental cost reductions may make coordination less likely or effective by enhancing the incentive of a maverick to lower price or by creating a new maverick firm."[1]

[1] *Op.cit.*, page 33. The 2004 *European Guidelines on the Assessment of Horizontal Mergers under the Council Regulation on the Control of Concentrations between Undertakings* similarly hold at recital 82 that: "In the context of coordinated effects, efficiencies may increase the merged entity's incentive to increase production and reduce prices, and thereby reduce its incentive to coordinate

Instead, we find that merger-specific efficiencies can *increase* the risk of coordinated effects if the merging firms were the companies that could not commit to collusion with a margin pre-merger—also if the merger increases heterogeneity in marginal cost of production across all firms.

Our results also suggest that there is more scope for collusion when products are more homogeneous. Competition authorities already work on this conjecture and are critical, for example, to industry standardization. Levenstein and Suslow (2006) report that of discovered cartels, a substantial part was active in fairly homogeneous goods industries. The *Merger Guidelines* hold on product differentiation that it diminishes business stealing effects, due to better anticipation of responses, to conclude that:

> "A market typically is more vulnerable to coordinated conduct if (...) products in the relevant market are relatively homogeneous."[2]

We contribute that this is canonically (and only truly "typically") true when firms require to gain from coordination by a sufficiently large margin, so that the effect of increased product homogeneity on the incentives of firms to compete is not generally dominated by the increased incentives to collude when products are more homogeneous.

In the main text, we analyze a general model specification, so as not to unnecessarily burden the notation. It can encompass various specific models that fit the motivations for the existence of a cartel margin given above. Section 5 shows how the cartel margin can be interpreted as the expected consequences of cartel participation when there is a risk of cartel break-down that leads to liabilities and costs of colluding as long as the cartel continues. In that setting, the cartel margin nests the exogenous cost of colluding introduced in Thomadsen and Rhee (2007) and Colombo (2013), which study the effect of product differentiation on cartel formation to find that a decrease in product differentiation increases cartel stability—as we do, providing the parameter ranges. We provide the necessary and sufficient conditions under which any cartel margin affects the sign of the comparative statics.

The remainder of this paper is organized as follows. Section 2 presents a general result on the comparative statics of market structure changes when there is an endogenous cartel margin. In sections 3 and 4 this finding is subsequently applied to changes in industry marginal cost of production and product differentiation. Numerical exercises with (non-)linear demand provide illustrations of the material effects and insight into the parameter ranges for which results are unambiguous. In Section 5 illustrates how the general model encompasses specific cartel models with enforcement. Section 6 concludes.

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its market behaviour with other firms in the market. Efficiencies may therefore lead to a lower risk of coordinated effects in the relevant market."

[2] *Op.cit.*, page 26. The European Commission's *Horizontal Mergers Guidelines* state the same at recital 45: "It is also easier to coordinate on a price for a single, homogeneous product, than on hundreds of prices in a market with many differentiated products."

2 Cartel Stability by a Margin

Consider n identical firms in repeated interaction, each discounting with the common factor $\delta \in [0,1)$. Let $\pi^N(\mathbf{x})$ be the per-period competitive static Nash profit as a function of some generic market structure variables $\mathbf{x}$ that directly affect profit, such as the degree of horizontal or vertical product differentiation or any other characteristics affecting demand, the number of competitors and production cost characteristics. A firm's per-period profit if all firms collude is $\pi^C(\mathbf{x})$. The firm that deviates obtains as profit $\pi^D(\mathbf{x})$. Assume that each profit is continuous and differentiable in each market structure variable $x \in \mathbf{x}$. Naturally, $\pi^N(\mathbf{x}) \leq \pi^C(\mathbf{x}) \leq \pi^D(\mathbf{x})$.

The cartel margin is an absolute per-period required payoff $M(\mathbf{x}) \geq 0$, that depends on $\mathbf{x}$ typically differently than the profits—also allowing for relative specifications, for instance as a percentage of competitive profits or a rate of return. Each period, each cartel member requires that

$$\pi^C(\mathbf{x}) \geq \pi^N(\mathbf{x}) + M(\mathbf{x}).$$

Naturally, no cartel can ever be sustained if its members required a margin that is larger than the per-period gain from colluding, so that $M(\mathbf{x}) < \pi^C(\mathbf{x}) - \pi^N(\mathbf{x}) = \overline{M}(\mathbf{x})$, which puts an upper bound on the margin.

Firms play an infinitely repeated game with grim trigger punishment strategy: upon defection by one, all firms revert back to competition forever after. Hence, for each firm the present discounted value of collusion and deviation are

$$V^C(\mathbf{x}) = \sum_{t=0}^{\infty} \delta^t \left(\pi^C(\mathbf{x}) - M(\mathbf{x})\right) \quad \text{and} \quad V^D(\mathbf{x}) = \pi^D(\mathbf{x}) + \sum_{t=1}^{\infty} \delta^t \pi^N(\mathbf{x}).$$

Collusion is internally stable as long as $V^C(\mathbf{x}) \geq V^D(\mathbf{x})$, which holds if

$$\delta \geq \delta^*(\mathbf{x}, M(\mathbf{x})) = \frac{\pi^D(\mathbf{x}) - \pi^C(\mathbf{x}) + M(\mathbf{x})}{\pi^D(\mathbf{x}) - \pi^N(\mathbf{x})}. \tag{1}$$

The critical discount factor $\delta^*(\mathbf{x}, M(\mathbf{x}))$ is between 0 and 1 as long as $M(\mathbf{x}) < \overline{M}(\mathbf{x})$. The cartel margin reduces the difference between defection and cartel profits. Other things equal, it negatively affects cartel stability: $\partial\delta^*/\partial M > 0$.[3]

The effect of (global or local) changes in one or more of the market structure variables $\mathbf{x}$ on cartel stability can now be evaluated based on how they affect δ^*. If δ^* decreases (increases), it increases (decreases) the scope of discount factors for which collusion is stable. A change in $\mathbf{x}$ can affect cartel stability in two ways, by changing, relative to competition: (i) the net gain from colluding $\pi^C(\mathbf{x}) - \pi^N(\mathbf{x}) - M(\mathbf{x}) = \Delta\pi^C(\mathbf{x}) - M(\mathbf{x})$, which affects cartel stability positively, and (ii) the gain from

[3] See Section 5 for foundational specifications of this general model setup with costs of colluding, a probability of cartel break-down, liability for fines and antitrust damages.

deviating $\pi^D(\mathbf{x}) - \pi^N(\mathbf{x}) = \Delta\pi^D(\mathbf{x})$, which affects cartel stability negatively. As long as the net gain from collusion and the gain from deviation move in different directions with a change in $\mathbf{x}$, the combined effect on cartel stability is *a priori* unambiguous. This is, however, rarely the case for all possible values of the relevant market structure variables, not even the number of firms.

When $\Delta\pi^C(\mathbf{x})$ and $\Delta\pi^D(\mathbf{x})$ change in the same direction with a change in $\mathbf{x}$, the effect of the change in the market structure variable on cartel stability is *a priori* ambiguous. We find that if there is a cartel margin above a critical value $\underline{M}(\mathbf{x})$, new unambiguous comparative statics result in this case.

Theorem *When $\Delta\pi^C(\mathbf{x}) - M(\mathbf{x})$ and $\Delta\pi^D(\mathbf{x})$ both increase (decrease) in x, for any $M(\mathbf{x}) \in (\underline{M}(\mathbf{x}), \overline{M}(\mathbf{x})) \neq \varnothing$, $\partial\delta^*(\mathbf{x}, M(\mathbf{x}))/\partial x < 0$ (> 0).*

Proof. In the proof we suppress $\mathbf{x}$ in the notation where possible. Solving for the derivative of δ^* with respect to x in case $M < \overline{M}$ provides

$$\frac{\partial\delta^*}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\pi^D - \pi^C + M}{\pi^D - \pi^N}\right)$$

if $M < \overline{M}$ and $\partial\delta^*/\partial x = 0$ if $M \geq \overline{M}$. Define $\underline{M}(\mathbf{x})$ as the lower bound value of M at which $\partial\delta^*/\partial x = 0$ where $M < \overline{M}$, which solves as

$$\underline{M}(\mathbf{x}) = \left(\pi^D - \pi^N\right)\frac{\partial\left(\pi^D - \pi^C + M\right)}{\partial x}\left(\frac{\partial\left(\pi^D - \pi^N\right)}{\partial x}\right)^{-1} - \left(\pi^D - \pi^C\right).$$

Whenever the gain from deviation increases in x, that is $\partial\Delta\pi^D/\partial x > 0$, it follows that $\partial\delta^*/\partial x > 0$ when $M < \underline{M}$ and $\partial\delta^*/\partial x < 0$ when $M > \underline{M}$. And when the gain from deviation decreases in x, that is $\partial\Delta\pi^D/\partial x < 0$, it follows that $\partial\delta^*/\partial x < 0$ when $M < \underline{M}$ and $\partial\delta^*/\partial x > 0$ when $M > \underline{M}$. From the definitions of $\underline{M}$ and $\overline{M}$ it subsequently follows that

$$\frac{\partial\delta^*}{\partial x}\begin{cases} > 0 & \text{if } M < \{\underline{M}, \overline{M}\} \\ = 0 & \text{if } M = \underline{M} < \overline{M} \\ < 0 & \text{if } M \in (\underline{M}, \overline{M}) \\ = 0 & \text{if } M \geq \overline{M} \end{cases},$$

if $\partial\Delta\pi^D/\partial x > 0$ and

$$\frac{\partial\delta^*}{\partial x}\begin{cases} < 0 & \text{if } M < \{\underline{M}, \overline{M}\} \\ = 0 & \text{if } M = \underline{M} < \overline{M} \\ > 0 & \text{if } M \in (\underline{M}, \overline{M}) \\ = 0 & \text{if } M \geq \overline{M} \end{cases},$$

if $\partial\Delta\pi^D/\partial x < 0$. The Theorem describes the situation where $M \in (\underline{M}, \overline{M})$. What remains to be shown is therefore that $\underline{M} < \overline{M}$, so that this situation can actually occur. The necessary and sufficient condition for this can be derived as follows

$$\begin{aligned}
\underline{M} &< \overline{M} \\
\pi^C - \pi^N &> \left(\pi^D - \pi^N\right) \frac{\partial\left(\pi^D - \pi^C + M\right)}{\partial x} \left(\frac{\partial\left(\pi^D - \pi^N\right)}{\partial x}\right)^{-1} - \left(\pi^D - \pi^C\right) \\
\pi^D - \pi^N &> \left(\pi^D - \pi^N\right) \frac{\partial\left(\pi^D - \pi^C + M\right)}{\partial x} \left(\frac{\partial\left(\pi^D - \pi^N\right)}{\partial x}\right)^{-1} \\
1 &> \left(\frac{\partial\left(\pi^D - \pi^N\right)}{\partial x} - \frac{\partial\left(\pi^C - \pi^N - M\right)}{\partial x}\right) \left(\frac{\partial\left(\pi^D - \pi^N\right)}{\partial x}\right)^{-1} \\
0 &< \frac{\partial\left(\Delta\pi^C - M\right)}{\partial x} \left(\frac{\partial\Delta\pi^D}{\partial x}\right)^{-1},
\end{aligned}$$

which holds whenever $\Delta\pi^C - M$ and $\Delta\pi^D$ both in- or decrease in x. ■

The Theorem says that if the net gain from collusion and the gain from deviation both increase (decrease) in market structure variable x, then for any margin stability unambiguously increases (decreases) in x. To see why the cartel margin assures this, note that (1) can be rewritten as

$$\delta^*\left(\mathbf{x}, M\left(\mathbf{x}\right)\right) = \frac{\Delta\pi^D\left(\mathbf{x}\right) - \Delta\pi^C\left(\mathbf{x}\right) + M\left(\mathbf{x}\right)}{\Delta\pi^D\left(\mathbf{x}\right)},$$

so that

$$\frac{\partial\delta^*\left(\mathbf{x}, M\left(\mathbf{x}\right)\right)}{\partial\Delta\pi^C\left(\mathbf{x}\right)} = -\frac{1}{\Delta\pi^D\left(\mathbf{x}\right)} \quad \text{and} \quad \frac{\partial\delta^*\left(\mathbf{x}, M\left(\mathbf{x}\right)\right)}{\partial\Delta\pi^D\left(\mathbf{x}\right)} = \frac{\Delta\pi^C\left(\mathbf{x}\right) - M\left(\mathbf{x}\right)}{\Delta\pi^D\left(\mathbf{x}\right)^2}.$$

The value of $M\left(\mathbf{x}\right) > 0$ thus does not affect the comparative effect of $\Delta\pi^C\left(\mathbf{x}\right)$ on δ^*, but unambiguously decreases the comparative effect of $\Delta\pi^D\left(\mathbf{x}\right)$ on δ^*. For $M\left(\mathbf{x}\right)$ sufficiently large, therefore, the comparative effect of x on $\Delta\pi^C\left(\mathbf{x}\right)$ will dominate the comparative effect of x on $\Delta\pi^D\left(\mathbf{x}\right)$ in determining the joint comparative effect of x on δ^*. An increase in $M\left(\mathbf{x}\right)$ increases the effect that the gain from collusion has on cartel stability (which is positive) relative to the gain from deviation (which is negative).

Note that $\underline{M}\left(\mathbf{x}\right)$ can be positive or negative. When $\underline{M}\left(\mathbf{x}\right) > 0$, once $M\left(\mathbf{x}\right)$ surpasses $\underline{M}\left(\mathbf{x}\right)$, there is a reversal in the comparative statics of x on δ^*. When $\underline{M}\left(\mathbf{x}\right) < 0$, no sign change to $\partial\delta^*/\partial x$ can occur for any $M\left(\mathbf{x}\right) < \overline{M}\left(\mathbf{x}\right)$. This can also not happen when $\underline{M}\left(\mathbf{x}\right) > \overline{M}\left(\mathbf{x}\right)$, which is so in the *a priori* unambiguous case when $\Delta\pi^C\left(\mathbf{x}\right)$ and $\Delta\pi^D\left(\mathbf{x}\right)$ change in different directions. The cartel margin changes the relative effects that the gains from collusion and deviation have on the critical discount factor and therefore only affects the sign of the comparative statics in the ambiguous cases. As a result, all unambiguous effects of market structure on cartel stability remain unambiguous also with $M(\mathbf{x}) > 0$ required—but can change in magnitude in the size of $M(\mathbf{x})$. Since $\overline{M}(\mathbf{x}) = \Delta\pi^C(\mathbf{x})$, the upper bound increases in market structure variables that increase collusive profits more than competitive profits—such as product substitutability—and decreases for those that decrease collusive profits more than competitive profits—such as marginal cost.

In a wide class of cases—including those considered in the next two sections—$\Delta\pi^C(\mathbf{x})$ and $\Delta\pi^D(\mathbf{x})$ move in the same direction. If $M(\mathbf{x})$ also changes in the market structure variable x considered, $\Delta\pi^C(\mathbf{x}) - M(\mathbf{x})$ and $\Delta\pi^D(\mathbf{x})$ continue to move in the same direction only when $M(\mathbf{x})$ either change is the opposite direction of $\Delta\pi^C(\mathbf{x})$ or less. In the first case, the endogeneity of M amplifies the comparative statics, both an in(de)crease in $\Delta\pi^C(\mathbf{x})$ and a de(in)crease in $M(\mathbf{x})$ in(de)crease cartel stability. In the second case, M also changing with x dampens the comparative statics. The effect of changes in x on M should then not be too large for the Theorem to hold, in the sense that

$$\left|\frac{\partial\pi^C(\mathbf{x})}{\partial x} - \frac{\partial\pi^N(\mathbf{x})}{\partial x}\right| > \left|\frac{\partial M(\mathbf{x})}{\partial x}\right|. \tag{2}$$

When firms are heterogeneous so that static Nash, collusive and deviation payoffs differ between them, each firm i has its own actual and critical discount factor, where each condition $\delta_i \geq \delta_i^*$ needs to hold in order for a complete cartel to be internally stable. As a result, the cartel member with the tightest participation constraint determines the critical discount factor. The comparative statics result in the Theorem holds for each firm individually, provided the conditions do, and therefore in particular also for the critical cartel member.

In the following two sections, we study for two *a priori* ambiguous variations, in marginal cost of production and product differentiation, how material the effects on comparative static are, and how restrictive lower and upper bounds $\underline{M}$ and $\overline{M}$.

3 Changes in Marginal Cost

The marginal costs of production of firms are key drivers of prices and in many markets can vary considerably for all producers over time, for example due to seasonality in the availability of inputs, financial market fluctuations or price developments in commodity, energy and labour markets. In the cartel stability literature, however, the industry level of marginal cost is generally considered irrelevant for cartel stability: it has no effect on the classic critical discount factor. The effects of heterogeneity in cost structures across firms on the stability of collusive agreements between them has been studied extensively, including in Bae (1987), Harrington (1991), Rothschild (1999) and Miklos-Thal (2011).

Little or no work has been done on the relevance of the overall marginal cost level for collusion. An exception is Lambertini and Sasaki (2001), who argue that an overall decrease in marginal cost decreases cartel stability in a model with optimal punishment that relies upon the ability of firms to charge prices strictly below marginal cost. This argument applies only, however, in cases where full collusion on monopoly conduct cannot be sustained and only once marginal cost is so low that penal pricing becomes constrained by non-negativity.

To obtain more insight in the effects of a cartel margin on the sustainability of collusion, consider a market in which n symmetric firms labelled $i = 1, ..., n$ produce horizontally differentiated products. Firm i serves inverse demand

$$p_i = (a - bq_i - \gamma b \sum_{j \neq i} q_j)^{\rho}, \tag{3}$$

in which a and b are positive parameters, $\gamma \in (0, 1]$ reflects product differentiation—where products are more homogeneous when γ is close to 1—and $\rho \in (0, \infty]$ is a curvature parameter—where demand is concave for $\rho < 1$, convex for $\rho > 1$ and linear for $\rho = 1$.[4]

All firms produce with the same constant returns to scale production technology, giving them each marginal cost c, and no fixed cost, so that the profits of firm i are $\pi_i = (p_i - c)q_i$ for all $i = 1, ..., n$. In the following, we consider competition in quantities—qualitatively equivalent results obtain for price competition and are provided in an appendix. We impose that $(p_i, q_i) \geq 0$ for all i and take $c < a^{\rho}$ to ensure that prices and quantities are positive in equilibrium. Initially, we consider M independent of c.

For linear demand ($\rho = 1$), a change in marginal cost has an *a priori* ambiguous effect on cartel stability: both $\Delta\pi^C(\mathbf{x})$ and $\Delta\pi^D(\mathbf{x})$ decrease in c, for all relevant parameters values. We therefore find that the Theorem applies with $\underline{M}(\mathbf{x}) = 0$, so that the smallest positive margin suffices to obtain new unambiguous comparative statics.

Corollary 1 *When demand is linear ($\rho = 1$), for any $M \in (0, \overline{M}) \neq \varnothing$, $\partial\delta^*/\partial c > 0$.*

Proof. Optimal quantities for static Nash, collusion and deviation are

$$q_i^N = \frac{a - c}{b(2 + (n-1)\gamma)}, \quad q_i^C = \frac{a - c}{2b(1 + (n-1)\gamma)} \quad \text{and} \quad q_i^D = \frac{(a - c)(2 + (n-1)\gamma)}{4b(1 + (n-1)\gamma)}.$$

The associated profits are multiplicatively separable into the common scaling term

[4] This demand specification nests *inter alia* those used in Deneckere (1983; 1984), Wernerfelt (1989), Ross (1992), Tyagi (1999) and Osterdal (2003).

$(a-c)^2/b$ and a profit term depending only on γ and n, indicated with $\sim$:

$$\begin{aligned}
\pi^N &= \frac{(a-c)^2}{b}\frac{1}{(2+(n-1)\gamma)^2} = \frac{(a-c)^2}{b}\widetilde{\pi}^N(\gamma,n) \\
\pi^C &= \frac{(a-c)^2}{b}\frac{1}{4(1+(n-1)\gamma)} = \frac{(a-c)^2}{b}\widetilde{\pi}^C(\gamma,n) \quad \text{and} \\
\pi^D &= \frac{(a-c)^2}{b}\frac{(2+(n-1)\gamma)^2}{16(1+(n-1)\gamma)^2} = \frac{(a-c)^2}{b}\widetilde{\pi}^D(\gamma,n).
\end{aligned}$$

Scaled profits rank strictly as $\widetilde{\pi}^N(\gamma,n) < \widetilde{\pi}^C(\gamma,n) < \widetilde{\pi}^D(\gamma,n)$.

Note that indeed:

$$\begin{aligned}
\frac{\partial\left(\pi^C - \pi^N - M\right)}{\partial c} &= -\frac{2(a-c)}{b}\left(\widetilde{\pi}^C(\gamma,n) - \widetilde{\pi}^N(\gamma,n)\right) < 0 \quad \text{and} \\
\frac{\partial\left(\pi^D - \pi^N\right)}{\partial c} &= -\frac{2(a-c)}{b}\left(\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)\right) < 0,
\end{aligned} \tag{4}$$

so that the Theorem applies.

Combining the profit functions in (1),

$$\delta^* = \frac{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^C(\gamma,n)}{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)} + \frac{b}{(a-c)^2}\frac{M}{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)},$$

so that

$$\frac{\partial\delta^*}{\partial c} = \frac{2b}{(a-c)^3}\frac{M}{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)} > 0 \quad \text{for any } M \in \left(0,\overline{M}\right), \tag{5}$$

where $\overline{M} = (a-c)^2\left(\widetilde{\pi}^C - \widetilde{\pi}^N\right)/b$ is strictly positive and decreasing in c, as established. ■

Note that for $M = 0$ the analysis reduces to standard cartel stability theory and $\partial\delta^*/\partial c = 0$ from derivation (5) in the proof.

The cartel margin may itself vary with marginal costs. M can increase in c, with a higher industry cost level implying higher prices and affected turnover, on the basis of which public fines are typically determined.[5] M can decrease in c as well, for example when lower marginal costs trigger a maverick firm to stir up competition, or if a higher cartel overcharge increases the probability of detection and antitrust damages.[6] Including $M(c)$ changes condition (4) into

$$\frac{\partial\left(\pi^C - \pi^N - M(c)\right)}{\partial c} = -\frac{2(a-c)}{b}\left(\widetilde{\pi}^C(\gamma,n) - \widetilde{\pi}^N(\gamma,n)\right) - \frac{\partial M(c)}{c},$$

[5] United States Sentencing Commission (2016), *United States Sentencing Guidelines*, §2R1.1(d)(1); Commission of the European Communities (2006), *Guidelines on the method of setting fines imposed pursuant to Article 23(2)(a) of Regulation No 1/2003*, recital 13.

[6] This role of the maverick firm is relied upon in the U.S. *Merger Guidelines* quoted in Section 1. The cartel overcharge for linear demand (3) is $p^C - p^N = (a-c)\frac{(1+\gamma n)(n-1)\gamma}{(2+2(n-1)\gamma)(2+(n-1)\gamma)}$, which decreases in c.

which is negative as long as $\partial M/\partial c > -2(a-c)(\widetilde{\pi}^C - \widetilde{\pi}^N)/b$. That is, the net effect of changes in c on M can be of either sign but should not be too negative for the Theorem to apply.

For non-linear demand ($\rho \lesseqgtr 1$), closed-form solutions for the profits and critical discount factor are not available. Numerical analyses show, however, that the effects of the cartel margin on the comparative statics remain and are material.[7] Figure 1 provides δ^* (left-side panels) and ranges for $\underline{M}$ and $\overline{M}$ (right-side panels) for the case of symmetric Cournot duopoly ($n = 2$) with homogeneous products ($\gamma = 1$) and concave ($\rho = 0.5$), linear ($\rho = 1$) and convex ($\rho = 2$) demand, with $a = 2$, $b = 0.5$ and $c \in [0, 1]$.

PLACE FIGURE 1 HERE.

The panels on the left show that when there is a cartel margin ($M > 0$), the critical discount factor unambiguously increases in marginal cost—apart from the extreme case in the upper-left panel where demand is highly concave ($\rho = 0.5$) and M and c are very small. The relationship remains for higher numbers of firms (n), more product differentiation (lower γ), and any of the other parameters. Changes in the slope of δ^* are major, which implies that the effect on comparative statics of M are material.

In the panels on the right in Figure 1, the dark-gray area is the relevant region for M. The lower bound $\underline{M}$ is almost always negative or (near) zero, and so does not bind, except from the extreme case mentioned: in the upper-right panel, in the lower-left corner, in which case $\underline{M}$ is still very close to zero. These panels also show that the margins chosen in the comparative statics ($M \in [0, 0.03]$) are low relative to upper bound $\overline{M}$, particularly at low levels of c and more convex demand (higher values of ρ). These relationships remain for higher n, lower γ and variations in the other model parameters.

4 Changes in Product Differentiation

Theoretical work on the effects of product differentiation on cartel stability is more developed than on industry cost, but has not generated consensus. When products are closer substitutes, competition is stronger, which means that firms have more to gain from colluding, making adherence to the cartel agreement more attractive, but also are able to more easily attract customers away from rival firms when deviating, which undermines cartel stability.

Deneckere (1983; 1984) finds in a duopoly model with multi-product demand and grim trigger punishment strategies that increased product substitutability decreases

[7] Calculations are programmed in MATLAB® using grid-search algorithms.

cartel stability in case of quantity competition, and has ambiguous effects in case of price competition. Wernerfelt (1989) shows how in this setting the effect of product substitutability becomes ambiguous when firms use the optimal carrot-and-stick punishment strategy proposed by Abreu (1986, 1988)—although Osterdal (2003) argues that Deneckere's result remains for the number of firms sufficiently large. Tyagi (1999) shows that Deneckere's quantity competition results crucially depend on the curvature of demand—where increased product substitutability instead increases cartel stability when demand is sufficiently convex.

Similarly ambiguous results obtain for spatial product differentiation. Ross (1992) shows for duopoly price competition and grim trigger punishment that when transportation costs are linear, product substitutability increases cartel stability. Chang (1991) finds the opposite when transportation costs are quadratic. Haeckner (1996) shows in this setting that when firms use the optimal carrot-and-stick punishment, the collusive price decreases when product substitutability increases. Miklos-Thal (2008) establishes for duopoly price competition that conclusions crucially depend on which punishment strategy is used: in case of optimal carrot-and-stick punishment, product substitutability tends to undermine collusion, while in case of grim trigger punishment, collusion may be more stable if products are more homogeneous. Extensions beyond duopoly price competition are not readily available for spatial specifications.

We analyze the effects of product differentiation in the model introduced in the previous section, which is a generalization of Deneckere (1983) that allows for n-firms and a varying demand curvature. Again we consider M independent of γ first. For the case of linear demand, we obtain that when the cartel margin is large enough, collusion becomes more stable when product differentiation decreases: the Theorem applies, provided that competition is not too strong to begin with.

Corollary 2 *When demand is linear ($\rho = 1$) and $\{n, \gamma\}$ are sufficiently low, for any $M \in (\underline{M}, \overline{M}) \neq \varnothing$, $\partial\delta^*/\partial\gamma < 0$.*

Proof. From the Theorem, it is required to show that the gains from collusion and deviation both increase in γ. Using the solutions in the proof of Corollary 1,

$$\frac{\partial\left(\pi^C - \pi^N - M(\gamma)\right)}{\partial\gamma} = -\frac{(a-c)^2}{b}\left(\frac{n-1}{4\left(1+(n-1)\gamma\right)^2} - \frac{2(n-1)}{\left(2+(n-1)\gamma\right)^3}\right) - \frac{\partial M(\gamma)}{\partial\gamma}, \tag{6}$$

which, since $\partial M/\partial\gamma = 0$, is positive when $\left(2+(n-1)\gamma\right)^3 < 8\left(1+(n-1)\gamma\right)^2$, which solves as $n < \left(1+\sqrt{5}+\gamma\right)/\gamma$. This condition holds for values of n for γ sufficiently small, from $n \leq 4$ when $\gamma \to 1$ to unbounded n for $\gamma \to 0$.

Analogously,

$$\frac{\partial\left(\pi^D - \pi^N\right)}{\partial\gamma} = -\frac{(a-c)^2}{b}\left(\frac{\left(2+(n-1)\gamma\right)(n-1)}{8\left(1+(n-1)\gamma\right)^3} - \frac{2(n-1)}{\left(2+(n-1)\gamma\right)^3}\right)$$

is positive when $(2+(n-1)\gamma)^4 < 16\,(1+(n-1)\,\gamma)^3$, which holds for values of n for γ sufficiently small, from $n \leq 11$ when $\gamma \to 1$ to unbounded n for $\gamma \to 0$.

The expression of the lower bound is

$$\underline{M} = \frac{128b\,(1+(n-1)\,\gamma)\,(2+(n-1)\,\gamma)^2\left(16+24\,(n-1)\,\gamma+8\,(n-1)^2\,\gamma^2-(n-1)^3\,\gamma^3\right)}{(a-c)^2\gamma^2\left(8+8\,(n-1)\,\gamma+(n-1)^2\,\gamma^2\right)^4},$$

which increases in γ and n for $\{n,\gamma\}$ small enough: from $n \leq 8$ when $\gamma \to 1$ to unbounded n for $\gamma \to 0$. As established, the upper bound $\overline{M}$ increases in γ under the same condition that $n < \left(1+\sqrt{5}+\gamma\right)/\gamma$. ■

When the cartel margin is sufficiently high, collusion becomes unambiguously more stable following a decrease in product differentiation, as long as there are not too many firms in the market—where the upper bound on the number of firms increases when products are more heterogeneous. The reason why competition should not be too strong is that collusion should not be too attractive a proposition to make it more interesting, relative to defection, with more product homogeneity. Exactly how restrictive the lower and upper bounds on the cartel margin are, we study numerically.

There are opposite possible effects of changes in product substitutability on the cartel margin. Arguably, it is easier to coordinate a cartel and monitor deviations when products are more homogeneous—which are the mechanisms behind the statements quoted from the horizontal merger guidelines in Section 1. If $\partial M/\partial\gamma < 0$, condition (6) holds for more values of n and γ, further increasing stability. Yet, expected liabilities may increase in product homogeneity as well, for example when agencies or plaintiffs find it easier to discover and successfully prosecute collusion when products are more similar. $\partial M/\partial\gamma > 0$ goes against derivative (6) being positive, but changes the comparative statics only for a very strong endogenous net increase of M in γ. The Theorem applies for all net effects of changes in γ on M, except too large an increase.

For non-linear demand ($\rho \lesseqgtr 1$), closed-form solutions for the profits and critical discount factor are again not available. Figure 2 provides δ^* (left-hand panels) and ranges for $\underline{M}$ and $\overline{M}$ (right-hand panels) for the case of symmetric Cournot duopoly ($n = 2$) and different demand curvatures, with $a = 2$, $b = 0.5$ and $c = 0$.

PLACE FIGURE 2 HERE

The panels on the left in Figure 2 show that without a cartel margin, an increase in product substitutability decreases cartel stability, as found by Deneckere (1983). However, when demand is convex the opposite may hold, as shown by Tyagi (1999), although the effect is marginal for the specifications here analyzed. When there is a sufficiently large margin ($\underline{M} > 0$), an increase in product substitutability unambiguously increases cartel stability. The effect on the slope also appears to be material.

From the panels on the right, it shows that the lower bound $\underline{M}$ is not very restrictive for any degree of product differentiation, nor is the upper bound $\overline{M}$ often binding, at least not for low degrees of product differentiation. These conclusions remain when adjusting for any of the model parameters apart from the number of firms.

PLACE FIGURE 3 HERE

In the numerical exercises, for a larger number of firms than four, there are values of γ sufficiently close to 1 for which lower bound $\underline{M}$ exceeds upper bound $\overline{M}$, and there exists no margin effect. Figure 3 shows for linear demand ($\rho = 1$) how sufficiently low values of γ maintain a space for M for which increased product substitutability increases cartel stability when $n = 5$. The right-side panel shows that $\underline{M}$ increases stronger than $\overline{M}$, so that the lower bound at some point surpasses the upper bound for γ sufficiently close to 1. The light-gray area is nevertheless substantial, which remains the case when the number of firms becomes larger, even though the area shrinks in n.

5 A Model of Cartel Enforcement

The general model encompasses various more specific models that fit motivations for the existence of a cartel margin given in the introduction. Consider a basic model of cartel stability with antitrust enforcement, in which there is a per-period risk of cartel break-down $\mu \in [0, 1)$, followed by liabilities of size L, while as long as the cartel continues, with probability $1 - \mu$, there are costs of colluding C. Suppressing $\mathbf{x}$ in the notation,

$$\begin{aligned} V^C &= (1-\mu)\left(\pi^C - C + \delta V^C\right) + \mu\left(\pi^C - L + \delta V^N\right) \text{ and} \\ V^D &= \pi^D + \frac{\delta}{1-\delta}\pi^N. \end{aligned}$$

Using the closed-form solution for V^C and rearranging terms to solve for δ provides $V^C \geq V^D$ if

$$\delta \geq \delta^* = \frac{\pi^D - \pi^C + (1-\mu)\,C + \mu L}{\pi^D - \pi^N}\frac{1}{1-\mu},$$

which is identical to (1) for

$$M = C + \left(\pi^D - \pi^C + L\right)\frac{\mu}{1-\mu}.$$

Hence, changes in M and its lower and upper bounds depend on market structure variables $\mathbf{x}$ through π^D and π^C, while obviously also C, L and μ can be functions of $\mathbf{x}$. Exactly how will depend on the level of these variables, specifics of demand and the collusive strategy. Note that C simply equals M if it were the only aspect of cartel enforcement modelled—that is if $\mu = L = 0$, which replicates Thomadsen and Rhee (2007) and Colombo (2013). For the fuller models, simulations with demand (3) for all reasonable parameter values show that variations in μ and L keep M well within bounds. In fact, the upper- and lower-bound almost perfectly track $\underline{M}$ and $\overline{M}$ in Figure 1 for marginal costs changes and in Figure 2 for product differentiation.

The Theorem applies only when, following a change in market structure variable x, M moves either in the opposite direction of $\Delta\pi^C(\mathbf{x}) = \pi^C(\mathbf{x}) - \pi^N(\mathbf{x})$ or less in the same direction—see condition (2) in Section 2. In this model

$$\frac{\partial M}{\partial x} = \frac{\partial C}{\partial x} + \left(\frac{\partial \pi^D}{\partial x} - \frac{\partial \pi^C}{\partial x} + \frac{\partial L}{\partial x}\right)\frac{\mu}{1-\mu} + \frac{\pi^D - \pi^C + L}{(1-\mu)^2}\frac{\partial \mu}{\partial x}.$$

For $x = c$, it generally holds that $\Delta\pi^C$ decreases in x and the Theorem applies as long as

$$\frac{1}{1-\mu}\frac{\partial \Delta\pi^C}{\partial c} < \frac{\partial C}{\partial c} + \left(\frac{\partial \Delta\pi^D}{\partial c} + \frac{\partial L}{\partial c}\right)\frac{\mu}{1-\mu} + \frac{\pi^D - \pi^C + L}{(1-\mu)^2}\frac{\partial \mu}{\partial c},$$

where the left-hand side is negative. As long as C, L and μ are unaffected by c, this condition simplifies to

$$\frac{\partial \Delta\pi^C}{\partial c} < \mu\frac{\partial \Delta\pi^D}{\partial c},$$

which are both negative. It holds as long as μ is sufficiently small, since generally the gain from deviation decreases quicker in c than the gain from collusion. If additionally C, L and/or μ are affected by c, the comparative statics may go out of bounds. However, for the predominant such dependence, which is that M increases in c through affected turn-over increasing fines and antitrust damages L, the condition is reinforced. It can accommodate a higher probability of entry by a maverick firm in low cost industries.

For $x = \gamma$, $\Delta\pi^C$ generally increases in x and the Theorem applies as long as

$$\frac{1}{1-\mu}\frac{\partial \Delta\pi^C}{\partial \gamma} > \frac{\partial C}{\partial \gamma} + \left(\frac{\partial \Delta\pi^D}{\partial \gamma} + \frac{\partial L}{\partial \gamma}\right)\frac{\mu}{1-\mu} + \frac{\pi^D - \pi^C + L}{(1-\mu)^2}\frac{\partial \mu}{\partial \gamma},$$

where the left-hand side is positive. Again, with C, L and μ unaffected by γ, it holds for μ small enough. If indeed coordination and monitoring is easier when the goods are more homogeneous, so that C decreases in γ, the condition is reinforced. It can accommodate that cartel detection may be more likely in sectors in which products are more alike. In a wide set of the most reasonable circumstances, our unambiguous comparative statics results apply.

6 Concluding Remarks

We show the imperative of accounting for a required payoff beyond the common collusive gain in cartel stability analysis. When the net gain from collusion and the gain from deviation both increase (decrease) in a market structure variable—so that unambiguous comparative statics results are not *a priori* available—then a sufficiently large margin assures that cartel stability unambiguously increases (decreases) in this market structure variable. Some comparative statics results of variations in market characteristics are reversed by the introduction of a margin.

In particular do we find that both lower marginal cost and reduced product differentiation increase cartel stability when cartel members require a margin that does not change too much in these market structure parameters. These comparative statics effects are material in common (non-)linear demand systems, both in quantity competition and price competition. Beliefs that collusion may be more attractive and/or easy in more homogenous goods and low cost industries, we show can be based in theory for even a small margin. That lower cost and more homogeneous goods industries are more prone to collusion is relevant for focus in competition policy. In merger control, when firms are heterogenous, invoking the efficiency defense may imply an increased risk of coordinated effects.

Market structure changes that affect the cartel margin in the same direction as the net gain from colluding reduce the magnitude of the comparative statics we point out—or even change the sign when the effect on the margin is too large. The empirical implications are immediate from the general break-down of effects in condition (2) for any specific foundational model. The conditions for unambiguous comparative statics reasonably hold in the models we analyzed of cartel enforcement with costs of colluding, a probability of cartel break-down, liability for fines and antitrust damages.

We do note that some more specific models can become quite involved. For example in a model with stochastic demand, the margin can be expressed as compensation for the uncertain fluctuations, including a risk premium if cartel members are risk averse. However, the point of these models is that cartel prices are to be adjusted below monopoly levels in order to stabilize the cartel when the incentive compatibility constraint binds, as in Rotemberg and Saloner (1986). The critical discount factor is effectively equated to the actual discount factor continuously by adjusting prices in that case. This prevents comparative statics on the critical discount factor—except in case the monopoly profit can be sustained.

Possible extensions of the general model include asymmetry between firms and alternative punishment strategies, such as temporary Nash reversion or optimal carrot-and-stick combinations. While our qualitative findings hold if firms have heterogeneous discount factors or profits, the comparative statics will have different magnitudes. Different demand systems, for example iso-elastic demand or spatial product differentiation, also change the trade-offs quantitatively.

Finally, note that our results apply only for market characteristics that directly affect profits. Many relevant variables do, including the number of firms, cost and demand characteristics. However, some circumstances that are known to affect cartel stability are not readily incorporated in the profit function of firms, such as the frequency of interaction or entry barriers. Instead these factors affect the calculation of the present discounted values of the payoff streams from collusion and deviation. Combinations of these two types of relevant market structure variables will have their own comparative statics results, depending on case specifics.

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A Price Competition

All results reported in the text hold qualitatively the same for Bertrand competition. Demand from (3) of firm i is

$$q_i = \alpha - \beta_1 p_i^{\frac{1}{\rho}} + \beta_2 \sum_{j \neq i} p_j^{\frac{1}{\rho}},$$

in which

$$\alpha = \frac{a}{b(1+(n-1)\gamma)}, \quad \beta_1 = \frac{1-(n-2)\gamma}{b(1-\gamma)(1+(n-1)\gamma)} \quad \text{and} \quad \beta_2 = \frac{\gamma}{b(1-\gamma)(1+(n-1)\gamma)}.$$

This specification allows for proofs of both corollaries in case of price competition, as well as similar numerical illustrations.

Proof of Corollary 1 in case of price competition. Optimal prices for static Nash, collusion and deviation in case of Bertrand competition are

$$\begin{aligned}
p_i^N &= \frac{(a+c)(1-\gamma)+(n-1)\gamma c}{2+(n-3)\gamma} \\
p_i^C &= \frac{a+c}{2} \\
p_i^{D,nb} &= \frac{(a+c)(2+(n-3)\gamma)+2(n-1)\gamma c}{4(1+(n-2)\gamma)} \\
p_i^{D,b} &= \frac{(2\gamma-1)a+c}{2\gamma},
\end{aligned}$$

where $p_i^{D,nb}$ is the optimal deviation price in case the non-negativity constraint $q_{-i} \geq 0$ does not bind and $p_i^{D,b}$ in case it does bind. The non-negativity constraint binds for $\gamma > \left(n-3+\sqrt{n^2-1}\right)/(3n-5)$, which is found by solving for γ in $p_i^{D,nb} = p_i^{D,b}$. Associated profits are again multiplicatively separable into the common scaling term $(a-c)^2/b$ and a profit term depending only on γ and n:

$$\begin{aligned}
\pi^N &= \frac{(a-c)^2}{b} \frac{(1-\gamma)(1+(n-2)\gamma)}{(1+(n-1)\gamma)(2+(n-3)\gamma)^2} = \frac{(a-c)^2}{b} \widetilde{\pi}^N(\gamma, n) \\
\pi^C &= \frac{(a-c)^2}{b} \frac{1}{4(1+(n-1)\gamma)} = \frac{(a-c)^2}{b} \widetilde{\pi}^C(\gamma, n) \\
\pi^{D,nb} &= \frac{(a-c)^2}{b} \frac{(2+(n-3)\gamma)^2}{16(1-\gamma)(1+(n-1)\gamma)(1+(n-2)\gamma)} = \frac{(a-c)^2}{b} \widetilde{\pi}^{D,nb}(\gamma, n). \\
\pi^{D,b} &= \frac{(a-c)^2}{b} \frac{2\gamma-1}{4\gamma^2} = \frac{(a-c)^2}{b} \widetilde{\pi}^{D,b}(\gamma, n),
\end{aligned}$$

for which $\widetilde{\pi}^N(\gamma, n) < \widetilde{\pi}^C(\gamma, n) < \{\widetilde{\pi}^{D,nb}(\gamma, n), \widetilde{\pi}^{D,b}(\gamma, n)\}$.

Combining the profits in (1) provides

$$\delta^* = \frac{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^C(\gamma,n)}{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)} + \frac{b}{(a-c)^2}\frac{M}{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)},$$

so that

$$\frac{\partial\delta^*}{\partial c} = \frac{2b}{(a-c)^3}\frac{M}{\widetilde{\pi}^D(\gamma,n) - \widetilde{\pi}^N(\gamma,n)} > 0 \quad \text{for any } 0 < M < \overline{M}.$$

■

We omit the figures of numerical exercises under Bertrand competition, but these provide conclusions that are similar to those under Cournot competition. The critical discount factor still unambiguously decreases when marginal cost decreases, with the exception of extreme case where demand is highly concave and c and M are very low. The relationship remains for higher numbers of firms (n), more product differentiation (lower γ), and any of the other parameters. Changes in the slope of δ^* are major, which implies that the effect on comparative statics of M are material. Lower bound $\underline{M}$ is also regularly non-positive, so that it does not bind.

There are two differences with the numerical results under Cournot competition. On the one hand is the upper bound $\overline{M}$ around eight times higher in price competition, which means that the permissible area of M is larger. This results from the fact that static Nash competitive profits are zero under Bertrand competition with homogeneous goods, whereas they are positive under Cournot competition. On the other hand, however, the material effects on the comparative statics for any cartel margin are around five times lower under Bertrand than Cournot competition. This means that a higher cartel margin is required to achieve the same material effect on comparative statics. This is because also the gain from deviation is a lot larger under price than quantity competition, which requires a larger margin to offset.

Proof of Corollary 2 in case of price competition. From the Theorem it is required to show that the gains from collusion and deviation both increase in γ. Using the derivations in the proof of Corollary 1 in case of price competition, the gain from collusion is

$$\pi^C - \pi^N = \frac{(a-c)^2}{b}\frac{(n-1)^2\gamma^2}{4\left(1+(n-1)\gamma\right)\left(2+(n-3)\gamma\right)^2},$$

and its derivative with respect to γ

$$\frac{\partial\left(\pi^C - \pi^N\right)}{\partial\gamma} = -\frac{(a-c)^2}{b}\frac{(n-1)^2\gamma\left((n-1)(n-3)\gamma^2 - 2(n-1)\gamma - 4\right)}{4\left(1+(n-1)\gamma\right)^2\left(2+(n-3)\gamma\right)^3},$$

which is positive when $(n-1)(n-3)\gamma^2 - 2(n-1)\gamma - 4 < 0$, which solves to $n < \left(1 + 2\gamma + \sqrt{5 + 2\gamma + \gamma^2}\right)/\gamma$. This condition holds for values of n for γ sufficiently small, from $n \leq 5$ when $\gamma \to 1$ to unbounded n for $\gamma \to 0$.

The gains from deviation are:

$$\begin{aligned}
\pi^{D,nb} - \pi^{N} &= -\frac{(a-c)^2}{b}\frac{(n-1)^2\gamma^2\left(8+8(n-3)\gamma+\left(n^2-10n+17\right)\gamma^2\right)}{16(1-\gamma)(1+(n-1)\gamma)(1+(n-2)\gamma)(2+(n-3)\gamma)^2} \quad \text{and} \\
\pi^{D,b} - \pi^{N} &= \frac{(a-c)^2}{b}\frac{(2\gamma-1)(1+(n-1)\gamma)(2+(n-3)\gamma)^2-4\gamma^2(1-\gamma)(1+(n-2)\gamma)}{4\gamma^2(1+(n-1)\gamma)(2+(n-3)\gamma)^2}.
\end{aligned}$$

Taking the derivatives of these functions with respect to γ provides, in case lower bound $q_{-i} \geq 0$ non-binding

$$\frac{\partial\left(\pi^{D,nb}-\pi^{N}\right)}{\partial\gamma} = \frac{(a-c)^2}{b}\frac{(n-1)^2\gamma\left(32+\gamma F_1+\gamma F_2+\gamma^3F_3+\gamma^4F_4+\gamma^5F_5+\gamma^6F_6\right)}{16(1-\gamma)^2(1+(n-1)\gamma)^2(1+(n-3)\gamma)^2(2+(n-3)\gamma)^3},$$

in which

$$\begin{aligned}
F_1 &= 80n-208 \\
F_2 &= 64n^2-368n+496 \\
F_3 &= 14n^3-154n^2+490n-478 \\
F_4 &= -2n^4+22n^3-62n^2+34n+40 \\
F_5 &= 14n^4-118n^3+366n^2-482n+220 \\
F_6 &= n^5-16n^4+88n^3-218n^2+247n-102.
\end{aligned}$$

Since $32+\sum_{i=1}^{6}\gamma^iF_i > 0$ for all values of $n \geq 2$ and $\gamma \in (0,1]$, the derivative is always positive.

Similarly, in case lower bound $q_{-i} \geq 0$ binding

$$\frac{\partial\left(\pi^{D,b}-\pi^{N}\right)}{\partial\gamma} = \frac{(a-c)^2}{b}\frac{8+\gamma F_1+\gamma F_2+\gamma^3F_3+\gamma^4F_4+\gamma^5F_5+\gamma^6F_6}{2\gamma^3(1+(n-1)\gamma)^2(2+(n-3)\gamma)^3},$$

in which

$$\begin{aligned}
F_1 &= 28n-60 \\
F_2 &= 38n^2-176n+186 \\
F_3 &= 25n^3-191n^2+443n-309 \\
F_4 &= 8n^4-93n^3+365n^2-571n+299 \\
F_5 &= n^5-19n^4+118n^3-318n^2+381n-163 \\
F_6 &= -n^5+11n^4-48n^3+102n^2-103n+39.
\end{aligned}$$

Since $8+\sum_{i=1}^{6}\gamma^iF_i > 0$ for all values of $n \geq 2$ and $\gamma \in (0,1]$, this derivative is also always positive. ■

Numerical analyses of various demand variations again lead to results comparable to those under Cournot competition. Without a cartel margin, we obtain the ambiguous relationship between product differentiation and cartel stability found in Deneckere (1983), where cartel stability initially decreases but at some point increases when products become more homogeneous. However, introducing a sufficiently large cartel margin makes that product substitutability unambiguously increases cartel stability. The effects on the slope again appear to be material, as in the case of Cournot competition.

The same two material differences between quantity and price competition as in the case of changing marginal cost are found. The upper bound on M lies about eight times higher in case of Bertrand than Cournot competition, resulting from the much lower static Nash competitive profit. The required size of the margin to obtain similar effects on the comparative statics is about five times higher. Also in case of product differentiation, the relationship is non-monotonous with no cartel margin, as in Deneckere (1983), but becomes monotonous once the cartel margin is sufficiently large. The results require also that competition is not too strong to begin with.

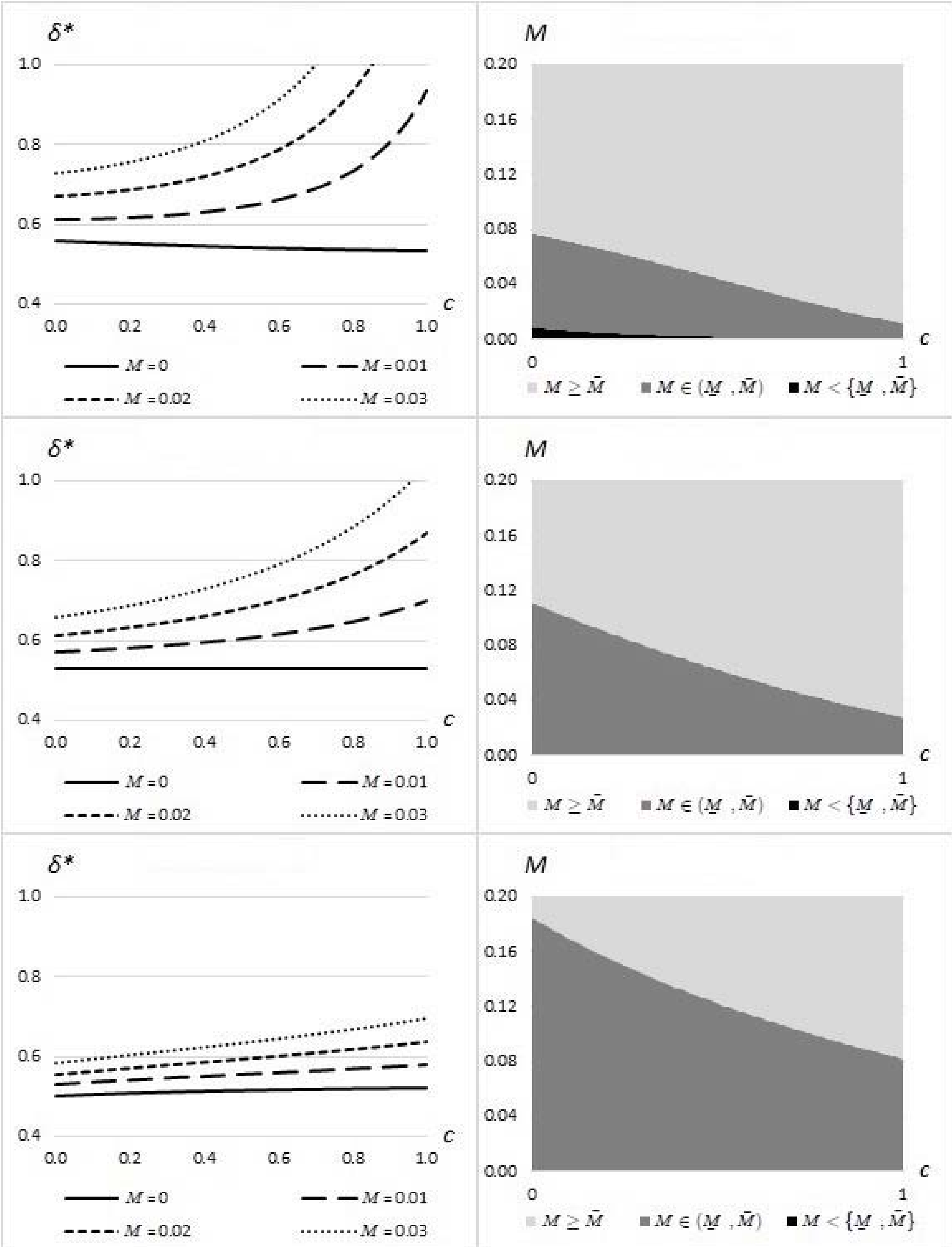

Figure 1: Critical discount factor (left-side panels) and upper and lower bounds (righ-side panels) as a function of marginal cost in case of: (t.t.b.) concave ($\rho = 0.5$), linear ($\rho = 1$) and convex ($\rho = 2$) demand.

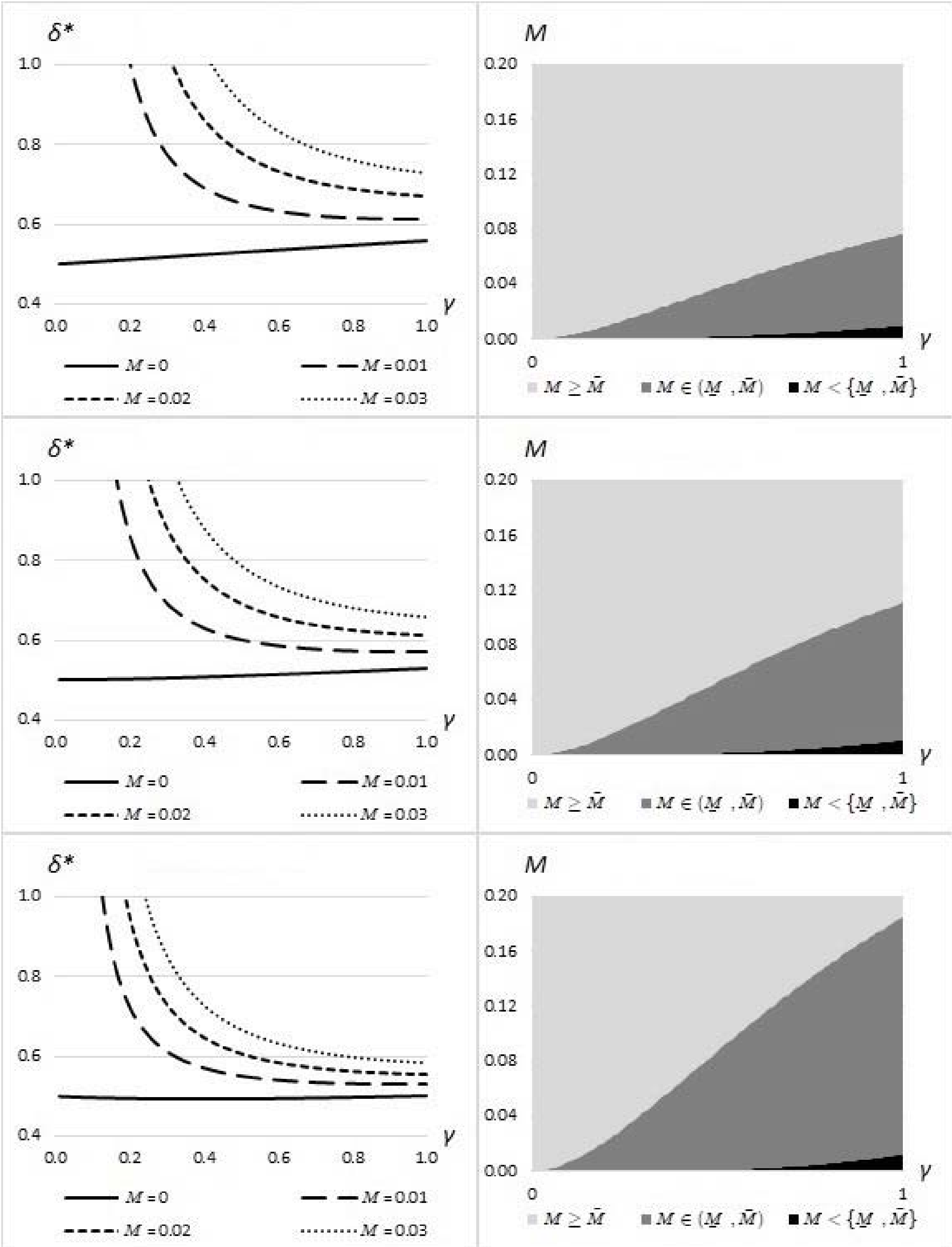

Figure 2: Critical discount factor (left-side panels) and upper and lower bounds (righ-side panels) as a function of product homogeneity in case of: (top to bottom) concave ($\rho = 0.5$), linear ($\rho = 1$) and convex ($\rho = 2$) demand.

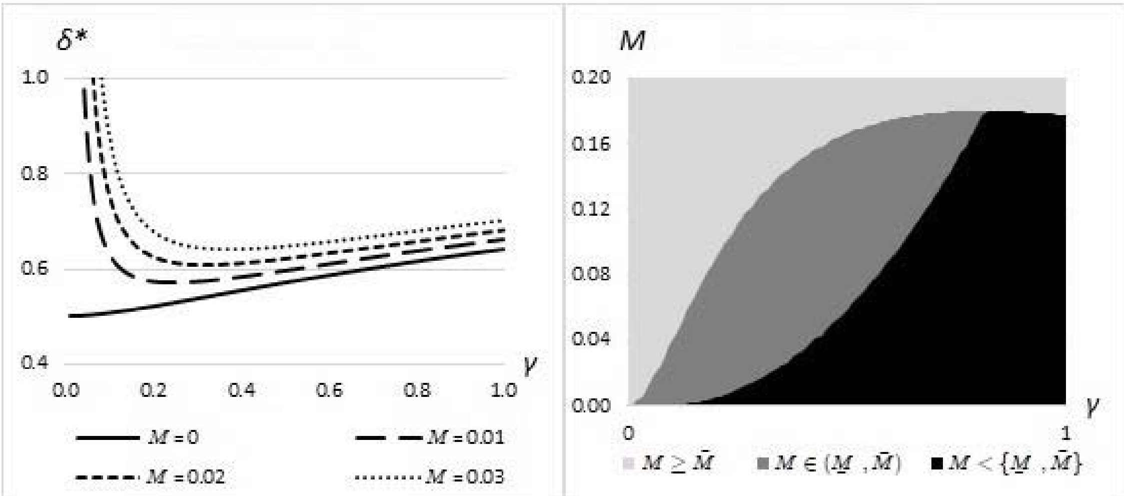

Figure 3: Critical discount rate (left-side panel) and bounds (right-side panel) as a functions of product homogeneity for linear demand ($\rho = 1$) and $n = 5$.