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Dynamic R&D with spillovers: A comment

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ABSTRACT

Cellini and Lambertini [2009. Dynamic R&D with spillovers: competition vs cooperation. J. Econ. Dyn. Control 33, 568–582] study a dynamic R&D game with spillovers. This comment demonstrates that, contrary to what is claimed in their paper, the game is not state redundant and the open-loop Nash equilibrium is not subgame perfect.
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1. Introduction

Dynamic models are popular in modern industrial organization. They allow to model firms smoothing their investments over a long time, as well as reacting to each other's past actions. Cellini and Lambertini (2009), CL in what follows, presented a continuous-time generalization of the seminal static R&D model of d'Aspremont and Jacquemin (1988). Their paper compares R&D incentives of firms that compete on R&D to those that cooperate on R&D. They claim that in a dynamic model the conflict between individual and social incentives does not necessarily arise, unlike the situation for the static model.

In particular, their analysis consists of three steps: they characterize the open-loop Nash equilibrium, they claim to prove that it is subgame perfect, and then they analyze the steady-state allocation. Their proof of subgame perfectness rests on the claim that the closed-loop equilibrium collapses to the open-loop equilibrium.

The aim of this comment is to show that the second step of their analysis, embodied in their Lemma 1, is incorrect. First we shall show that the proof of this lemma is flawed, and subsequently we give a simple argument why the statement of the lemma cannot hold either. That is, we show that the open-loop equilibrium is not subgame perfect and the game is not state redundant or perfect. The solution analyzed in their paper therefore reduces to the open-loop situation, where firms commit to the entire investment schedule at the beginning of the game.

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2. The model

We quickly summarize the model of CL. Time $t \geq 0$ is continuous. There are two firms that compete in a market with market demand

$$p(t) = A - q_1(t) - q_2(t). \tag{1}$$

Firms decide simultaneously how much to produce (q_i) and how much R&D effort to exert (k_i). Instantaneous production costs are $C_i(t) = c_i(t) q_i(t)$, $i = 1,2$, where c_i is the marginal cost of firm i. Marginal costs evolve over time as

$$\dot{c}_i(t) = c_i(t)\big(-k_i(t) - \beta k_j(t) + \delta\big), \tag{2}$$

where, as always, $j \neq i$, where $0 \leq \beta \leq 1$ is the level of spillover, and where $\delta \geq 0$ is the technology depreciation rate. R&D costs (Γ_i) are quadratic,

$$\Gamma_i(k_i) = b k_i^2, \tag{3}$$

with $b > 0$, and the instantaneous profit of firm i is therefore

$$\pi_i(t) = \big(A - q_i(t) - q_j(t) - c_i(t)\big) q_i(t) - b k_i(t)^2. \tag{4}$$

Total discounted profits are

$$\Pi_i = \int_0^\infty \pi_i(t) e^{-\rho t} dt, \tag{5}$$

where $\rho > 0$ is a constant discount rate that is equal for both firms. The optimal control problem of firm i is to find controls q_i^* and k_i^* that maximize the profit functional Π_i subject to the state equations (2) and the initial conditions $c_i(0) = c_{i0}$.

3. The open-loop Nash equilibrium is not subgame perfect

3.1. Subgame perfectness and time consistency

Introduce the notation $u_i(t)$ for an open-loop strategy. Recall that an open-loop Nash equilibrium $(u_1^*(t), u_2^*(t))$ of this differential game is *subgame perfect*, or *strongly time consistent*, if for every time $T > 0$, we can change the strategies $u_i^*(t)$ for times $0 \leq t < T$ at will, as long as the resulting strategies are still admissible, and the resulting strategies still provide an open-loop Nash equilibrium for $t \geq T$.

The equilibrium is *time consistent*, or *weakly time consistent*, if after having played up to time T according to the strategies $u_i^*(t)$, the players are given the option to reconsider their strategies for the remainder of the time, and the strategies $(u_1^*(t), u_2^*(t))$, restricted to $t \geq T$, still form an open-loop Nash equilibrium.

3.2. First argument

CL claim, in their Lemma 1, that the open-loop equilibrium of this game is subgame perfect.

We contest this. Our argument runs as follows: Fershtman (1987) showed that to be subgame perfect, a Nash equilibrium in open-loop strategies has to be an equilibrium in feedback strategies; in particular the open-loop equilibrium strategies have to be independent of initial conditions. For infinite horizon games like the present one, where the only explicit time dependence is exponential discounting, the set of feedback strategies is necessarily invariant under time-shifts (Basar and Olsder, 1999). That is, if $t \mapsto u^*(t)$ is an equilibrium strategy tuple, then so is $t \mapsto u^*(\tau + t)$, for each $\tau > 0$. Moreover, if $u^*(t)$ tends to a limit u_∞^* as $t \to \infty$, which is the case in the present game, then $u(t) = u_\infty^*$ is also an equilibrium feedback strategy tuple, which is moreover independent of both time and initial conditions; that is, it is a real constant. This follows from letting τ tend to infinity.

In the present game, this would imply that the state variables are constant — as a consequence of Eq. (16) below — and hence that every state is a steady state. However, CL have showed that under open-loop Nash equilibrium dynamics, there are at most three steady states. This constitutes a contradiction.

A second, more detailed argument is given in Section 3.5.

3.3. First-order optimality conditions

It follows that the proof of Lemma 1 cannot be correct: we shall try to point out its flaws.

CL use the memoryless closed-loop information structure (cf. Basar and Olsder, 1999): a strategy is memoryless closed-loop, if it conditions the action of the player on the current state and time, as well as on the initial state. That is, in the present game a closed-loop strategy u_i^* of player i is of the form

$$u_i^* = \left(q_i^*(t, c_{10}, c_{20}, c_1(t), c_2(t)), k_i^*(t, c_{10}, c_{20}, c_1(t), c_2(t))\right),$$

where $c_{i0} = c_i(0)$ for $i = 1,2$.

The necessary optimality conditions for firm i can be derived from the Maximum Principle, assuming that player j has announced a closed-loop strategy $u_j^*(t, c_{10}, c_{20}, c_1(t), c_2(t))$. Denoting the costates that firm i associates with the states c_i and c_j as λ_{ii} and λ_{ij}, the current-value Hamiltonian of firm i is

$$\begin{aligned} H_i(\lambda_{ii}, \lambda_{ij}, c_i, c_j, q_i, q_j^*, k_i, k_j^*) &= (A - q_i - q_j^* - c_i)q_i - bk_i^2 \\ &\quad - \lambda_{ii} c_i (k_i + \beta k_j^* - \delta) - \lambda_{ij} c_j (k_j^* + \beta k_i - \delta). \end{aligned} \tag{6}$$

Here we use the convention that arguments t are suppressed, as in $c_i = c_i(t)$, and that the arguments of the closed-loop strategy functions like $q_i^*(t, c_{10}, c_{20}, c_1, c_2)$ are suppressed as well. The non-starred variables q_i and k_i indicate mere variables.

If the maximum is obtained for an interior solution, the first-order conditions with respect to the controls q_i and k_i are

$$\frac{\partial H_i}{\partial q_i} = A - 2q_i - q_j^*(c_1, c_2) - c_i = 0 \quad \Leftrightarrow \quad q_i^* = \frac{A - q_j^* - c_i}{2}, \tag{7}$$

$$\frac{\partial H_i}{\partial k_i} = -2bk_i - \lambda_{ii} c_i - \beta \lambda_{ij} c_j = 0 \quad \Leftrightarrow \quad k_i^* = -\frac{\lambda_{ii} c_i + \beta \lambda_{ij} c_j}{2b}. \tag{8}$$

The first costate equation reads

$$\dot{\lambda}_{ii} = \rho \lambda_{ii} - \frac{\partial H_i}{\partial c_i} - \frac{\partial H_i}{\partial q_i}\frac{\partial q_i^*}{\partial c_i} - \frac{\partial H_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial c_i} - \frac{\partial H_i}{\partial k_i}\frac{\partial k_i^*}{\partial c_i} - \frac{\partial H_i}{\partial k_j^*}\frac{\partial k_j^*}{\partial c_i} \tag{9}$$

$$= \rho \lambda_{ii} - \frac{\partial H_i}{\partial c_i} - \frac{\partial H_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial c_i} - \frac{\partial H_i}{\partial k_j^*}\frac{\partial k_j^*}{\partial c_i}. \tag{10}$$

The third and fifth terms on the RHS in (9) are enveloped out using the first-order conditions (7) and (8). Evaluating the derivatives, we obtain

$$\dot{\lambda}_{ii} = q_i^* + \lambda_{ii}(k_i^* + \beta k_j^* + \rho - \delta) + q_i^* \frac{\partial q_j^*}{\partial c_i} + (\beta \lambda_{ii} c_i + \lambda_{ij} c_j)\frac{\partial k_j^*}{\partial c_i}. \tag{11}$$

The second costate equation reads

$$\dot{\lambda}_{ij} = \rho \lambda_{ij} - \frac{\partial H_i}{\partial c_j} - \frac{\partial H_i}{\partial q_i}\frac{\partial q_i^*}{\partial c_j} - \frac{\partial H_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial c_j} - \frac{\partial H_i}{\partial k_i}\frac{\partial k_i^*}{\partial c_j} - \frac{\partial H_i}{\partial k_j^*}\frac{\partial k_j^*}{\partial c_j} \tag{12}$$

$$= \rho \lambda_{ij} - \frac{\partial H_i}{\partial c_j} - \frac{\partial H_i}{\partial q_j^*}\frac{\partial q_j^*}{\partial c_j} - \frac{\partial H_i}{\partial k_j^*}\frac{\partial k_j^*}{\partial c_j}. \tag{13}$$

The third and fifth terms on RHS in (12) have again been enveloped out using the first-order conditions (7) and (8). Evaluating the derivatives, we obtain

$$\dot{\lambda}_{ij} = \lambda_{ij}\left(k_j^* + \beta k_i^* + \rho - \delta + c_j \frac{\partial k_j^*}{\partial c_j}\right) + \beta \lambda_{ii} c_i \frac{\partial k_j^*}{\partial c_j} + q_i^* \frac{\partial q_j^*}{\partial c_j}. \tag{14}$$

In a closed-loop Nash equilibrium, the conditions (7), (8), (11) and (14) need to hold for both firms, together with the state equation (2), the initial conditions $c_i(0) = c_{i0}$ and transversality conditions as $t \to \infty$.

Note that Eq. (7) for the quantities q_i^* does not involve any costates; in a Nash equilibrium in either closed-loop or feedback strategies, they form the system

$$q_i^* = \frac{A - q_j^* - c_i}{2}, \quad q_j^* = \frac{A - q_i^* - c_j}{2}, \tag{15}$$

which can be solved to obtain the optimal controls q_i^* as functions of the states

$$q_1^*(t, c_{10}, c_{20}, c_1, c_2) = \frac{A - 2c_1 + c_2}{3}, \quad q_2^*(t, c_{10}, c_{20}, c_1, c_2) = \frac{A - 2c_2 + c_1}{3}. \tag{16}$$

From this we obtain in particular that $\partial q_j^* / \partial c_j = -2/3$.

3.4. Cross-multipliers cannot always vanish

We can now examine the claim of CL that there is always a possible solution in this model for which the cross-multipliers λ_{ij} vanish identically for all t. Setting $\beta = 0$ and assuming that $\lambda_{ij}(t) = 0$ for all t, Eq. (14) implies that

$$0 = -\frac{2}{3}q_i^* \tag{17}$$

for all t, implying that there is never any production. But in the steady state given by CL, $q_i^* > 0$; this is a contradiction.

The incorrect conclusion is caused by two mistakes made in the derivation of Lemma 1, which is contained in Appendix A of CL. First, it is stated incorrectly that "Observe that (9) only contains firm i's state variable, so that in choosing the optimal output at any time during the game firm i may disregard the current efficiency of the rival" (CL, Appendix A). Their Eq. (9) is equivalent to our Eq. (7). This error possibly resulted from not writing out the argument of q_j^*.

Second, in the derivation of the all-important equation (14), that is Eq. (A.2) in CL, it is claimed that this equation is equal to the following equation, which is written in our notation as

$$\dot{\lambda}_{ij} = \rho\lambda_{ij} - \frac{\partial H_i}{\partial c_j} - \frac{\partial H_i}{\partial k_i}\frac{\partial k_i^*}{\partial c_j}. \tag{18}$$

Note the differences with our Eq. (14): the terms involving $\partial q_j^*/\partial c_j$ and $\partial k_j^*/\partial c_j$ are missing, while a term involving $\partial H_i/\partial k_i$ is present. But for an interior solution, this latter term vanishes because of the Maximum Principle.

3.5. Second argument

Here we give a more detailed argument, not relying on Fershtman (1987), showing that the claim of Lemma 1 of CL cannot hold.

In CL it has been shown that the game under open-loop strategies has at most three possible symmetric steady states, of which one is of a saddle type. Take initial cost levels $(c_1(0), c_2(0)) = (c_0, c_0)$, not equal to a steady state-value, and such that under the associated open-loop Nash equilibrium strategies $(u_1^*(t), u_2^*(t))$ the system tends to the symmetric steady state of a saddle type. Arguing by contradiction, let us assume that the open-loop Nash equilibrium is strongly time consistent.

Recall that in order to be subgame perfect, for any $T > 0$ the restriction of the $u_i^*(t)$ to $t \geq T$ should yield an open-loop Nash equilibrium for the trajectory starting at time T from the state $c_i(T)$.

We introduce the modified controls

$$\tilde{k}_i(t) = \begin{cases} \frac{\delta}{1+\beta} & \text{for } 0 \leq t \leq T, \\ k_i^*(t) & \text{for } t > T, \end{cases}$$

$$\tilde{q}_i(t) = q_i^*(t),$$

for $i=1,2$. It follows that the modified state evolutions satisfy $\dot{\tilde{c}}_i(t) = 0$ for $0 \leq t \leq T$: that is, $\tilde{c}_i(t)$ is constant on this time interval and $\tilde{c}_i(T) = \tilde{c}_i(0) = c_0$. The proof rests on the observation that the controls $u_i^*(t)$, and hence also the modified controls $\tilde{u}_i(t)$, tend exponentially to their steady-state values as $t \to \infty$; by subgame perfectness, $\tilde{u}_i(t)$ form an open-loop Nash equilibrium for the system starting at $\tilde{c}_i(T)$; hence $\tilde{c}_i(t)$ have to tend to one of the possible three steady-state values. On the other hand, $\dot{\tilde{c}}_i(t) = 0$ for $0 \leq t \leq T$ and $\dot{\tilde{c}}_i(t)$ is almost 0 for $t > T$: this limits the maximal distance between the initial values c_0 and the limiting values of $\tilde{c}_i(t)$ as $t \to \infty$, leading to a contradiction.

In order to execute the details, note that since the state trajectory tends to the steady state, we have

$$\lim_{t\to\infty} k_i^*(t) = \lim_{t\to\infty} \tilde{k}_i(t) = \frac{\delta}{1+\beta}.$$

We introduce $\Delta_i(t)$ by

$$\tilde{k}_i(t) = \frac{\delta}{1+\beta} + \Delta_i(t).$$

From the theory of differential equations, we can find $C > 0$, $\lambda \in \mathbb{R}$, such that

$$|\Delta_i(t)| \leq Ce^{-\lambda t}.$$

Such an inequality holds clearly for large t and λ close to the largest negative real part of an eigenvalue of the saddle steady state; by taking C sufficiently large, it can be made to hold for all $t \geq 0$.

Substituting these expressions in the state dynamics (2) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\log \tilde{c}_i(t) = \frac{\dot{\tilde{c}}_i}{\tilde{c}_i} = \delta - k_i(t) - \beta k_j(t) = -\Delta_i(t) - \beta\Delta_j(t).$$

From this relation, we derive the estimate

$$|\log \tilde{c}_i(t) - \log \tilde{c}_i(0)| = |\log \tilde{c}_i(t) - \log \tilde{c}_i(T)| \le \int_T^t \left| -\Delta_i - \beta \Delta_j \right| \mathrm{d}s \le \int_T^t (1+\beta) C \mathrm{e}^{-\lambda s} \mathrm{d}s \le 2C\mathrm{e}^{-\lambda T}. \tag{19}$$

Take $\varepsilon > 0$ sufficiently small such that the steady state $(\overline{c}, \overline{c})$ of the differential game does not satisfy $\mathrm{e}^{-\varepsilon} c_0 \le \overline{c} \le \mathrm{e}^{\varepsilon} c_0$; for the rest is ε arbitrary. We now choose T such that $2C\mathrm{e}^{-\lambda T} = \varepsilon$. Then it follows from (19) that

$$\mathrm{e}^{-\varepsilon} c_0 \le \tilde{c}_i(t) \le \mathrm{e}^{\varepsilon} c_0$$

for all t, and hence that $\tilde{c}_i(t)$ tends to a steady-state value in this interval. As $\varepsilon > 0$ was arbitrary, it follows that (c_0, c_0) is a limit point of steady states, and therefore a steady state itself. But (c_0, c_0) has been chosen such that it is not a steady state; hence we have reached a contradiction, and the open-loop Nash equilibrium cannot be subgame perfect.

This shows again that the claim of Lemma 1 is not correct.

4. Concluding remarks

Contrary to what is claimed in CL, the closed-loop solution of the R&D game does not coincide with the open-loop solution. Consequently, the game under consideration is not state-redundant or perfect, and the paper effectively discusses only the open-loop solution.

Some of the main conclusions in CL relate to the private and social desirability of R&D cooperation. This analysis depends on the present value of variables that are affected by the R&D investments of firms over time. As the closed-loop solution differs from the open-loop solution, it is open to question to what an extent (if at all) are conclusions in CL relevant to great many industries in which firms do not commit to the entire investment schedules at the very beginning but can later strategically alter them at will in response to competitors. It is also open to question how realistic the formulation of R&D cartel in CL is. In their derivations for the R&D Cartel (pp. 573–574 in CL), CL implicitly assume that if marginal cost within the R&D cooperative changes, the opponent's quantity does not change. Besides clearly violating the feedback principle underlying the closed-loop solution, this imposition also appears counterintuitive as firms in the R&D cooperative are (usually) supposed to jointly decide on their R&D efforts taking into account that marginal cost in any period affects the ensuing Nash-equilibrium profits in the product market. These specific modelling choices in CL are important to bear in mind also when one potentially considers comparing other possible regimes (i.e., market collusion) to regimes considered in CL. Conclusions in other papers that use the same analytical approach (e.g., Cellini and Lambertini, 2005, 2011) should also be reexamined in a similar way.

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