Appendix

The steps below explain how to determine whether testing continues after stage k, and if so, how item selection will proceed alongside the time-limited SPRT stopping rule.

1. After the k–th test item is answered, compute the likelihood ratio

$$\lambda_k = \frac{L(u_1, \dots, u_k|\theta_+)}{L(u_1, \dots, u_k|\theta_-)}, \tag{24}$$

2. Check if $\lambda_k \geq A$ or $\lambda_k \leq B$. If either of the two conditions is satisfied, stop the test and classify the examinee as having mastered the test (if $\lambda_k \geq A$) or having failed the test (if $\lambda_k \leq B$). If neither condition occurs, go to step 3 below.

3. Compute the remaining time that the examinee has, according to $t_k^* = t_{max} - \sum_{i=1}^{k} t_i$. If $t_k^* \leq 0$, stop the test and classify the examinee as having mastered the test if $\lambda_k \geq C$ or having failed the test otherwise. If $t_k^* > 0$, go to step 4 below.

4. For each item $j \in R_k$, evaluate $P\left(T_j > t_k^* | \boldsymbol{t}_k\right)$ using the formula given in Equation 22. If this probability is greater than γ for all items in the set R_k, stop the test and classify the examinee as having mastered the test if $\lambda_k \geq C$ or having failed the test otherwise. If there exists at least one item $j \in R_k$ for which $P\left(T_j > t_k^* | \boldsymbol{t}_k\right) \leq \gamma$, go to step 5 to administer the $(k+1)$–st item.

5. For all remaining items $j \in E_k$ (i.e., the set of items that have not been administered from the pool and have an adequately low probability of exceeding the time limit), calculate

$$\frac{FI_j(\theta_c)}{E\left(T_j | \hat{\tau}_k\right)}, \tag{25}$$

and administer the item with the largest value of (25) as the $(k+1)$–st test item.

Table 3. PCD under M1 and M2 with the time-limited SPRT stopping rule, given at various ability levels when $\rho_{\theta\tau} = 0$ and $\rho_{b\beta} = 0.65$.

Ability	$\theta_c = -1$		$\theta_c = 0$		$\theta_c = 1$	
	M1	M2	M1	M2	M1	M2
-3.00	1.000	1.000	1.000	1.000	1.000	1.000
-2.75	1.000	1.000	1.000	1.000	1.000	1.000
-2.50	1.000	1.000	1.000	1.000	1.000	1.000
-2.25	1.000	1.000	1.000	1.000	1.000	1.000
-2.00	0.998	1.000	1.000	1.000	1.000	1.000
-1.75	0.992	0.998	1.000	1.000	1.000	1.000
-1.50	0.972	0.998	1.000	1.000	1.000	1.000
-1.25	0.850	0.864	0.998	1.000	1.000	1.000
-1.00	0.498	0.496	0.998	1.000	1.000	1.000
-0.75	0.882	0.902	0.996	1.000	1.000	1.000
-0.50	0.978	0.988	0.968	0.980	0.998	1.000
-0.25	0.996	0.998	0.836	0.866	1.000	1.000
0.00	1.000	1.000	0.496	0.556	0.996	0.998
0.25	1.000	1.000	0.846	0.872	0.988	0.996
0.50	1.000	1.000	0.974	0.986	0.946	0.958
0.75	1.000	1.000	0.994	0.998	0.806	0.810
1.00	1.000	1.000	1.000	1.000	0.464	0.498
1.25	1.000	1.000	1.000	1.000	0.812	0.830
1.50	1.000	1.000	1.000	1.000	0.942	0.958
1.75	1.000	1.000	1.000	1.000	0.986	0.996
2.00	1.000	1.000	1.000	1.000	0.998	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000

Table 4. PCD under M1 and M2 with the time-limited SPRT stopping rule, given at various ability levels when $\rho_{\theta\tau} = 0.59$ and $\rho_{b\beta} = 0$.

Ability	$\theta_c = -1$		$\theta_c = 0$		$\theta_c = 1$	
	M1	M2	M1	M2	M1	M2
-3.00	1.000	1.000	1.000	1.000	1.000	1.000
-2.75	1.000	1.000	1.000	1.000	1.000	1.000
-2.50	1.000	1.000	1.000	1.000	1.000	1.000
-2.25	0.998	1.000	1.000	1.000	1.000	1.000
-2.00	1.000	1.000	0.998	1.000	1.000	1.000
-1.75	0.994	0.992	1.000	1.000	1.000	1.000
-1.50	0.932	0.972	1.000	1.000	1.000	1.000
-1.25	0.792	0.846	1.000	1.000	1.000	1.000
-1.00	0.488	0.502	1.000	1.000	1.000	1.000
-0.75	0.818	0.834	0.992	1.000	1.000	1.000
-0.50	0.968	0.976	0.956	0.986	1.000	1.000
-0.25	0.996	0.996	0.786	0.872	1.000	1.000
0.00	0.996	1.000	0.486	0.508	1.000	1.000
0.25	1.000	1.000	0.838	0.884	0.996	0.998
0.50	1.000	1.000	0.980	0.992	0.964	0.994
0.75	1.000	1.000	0.992	1.000	0.816	0.858
1.00	1.000	1.000	1.000	1.000	0.492	0.502
1.25	1.000	1.000	1.000	1.000	0.878	0.888
1.50	1.000	1.000	1.000	1.000	0.974	0.992
1.75	1.000	1.000	1.000	1.000	0.998	0.998
2.00	1.000	1.000	1.000	1.000	0.998	1.000
2.25	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000

Table 5. PCD under M1 and M2 with the time-limited SPRT stopping rule, given at various ability levels when $\rho_{\theta\tau} = 0.59$ and $\rho_{b\beta} = 0.65$.

Ability	$\theta_c = -1$		$\theta_c = 0$		$\theta_c = 1$	
	M1	M2	M1	M2	M1	M2
-3.00	1.000	1.000	1.000	1.000	1.000	1.000
-2.75	1.000	1.000	1.000	1.000	1.000	1.000
-2.50	1.000	1.000	1.000	1.000	1.000	1.000
-2.25	1.000	1.000	1.000	1.000	1.000	1.000
-2.00	1.000	1.000	1.000	1.000	1.000	1.000
-1.75	0.994	0.998	1.000	1.000	1.000	1.000
-1.50	0.970	0.992	1.000	1.000	1.000	1.000
-1.25	0.842	0.864	0.998	1.000	1.000	1.000
-1.00	0.512	0.500	1.000	1.000	1.000	1.000
-0.75	0.886	0.880	0.994	1.000	1.000	1.000
-0.50	0.984	0.992	0.968	0.982	0.998	1.000
-0.25	0.996	1.000	0.834	0.872	0.998	1.000
0.00	1.000	1.000	0.490	0.552	0.998	1.000
0.25	1.000	1.000	0.844	0.874	0.984	0.998
0.50	1.000	1.000	0.974	0.984	0.950	0.964
0.75	1.000	1.000	0.996	0.998	0.802	0.830
1.00	1.000	1.000	1.000	1.000	0.482	0.498
1.25	1.000	1.000	1.000	1.000	0.824	0.846
1.50	1.000	1.000	1.000	1.000	0.946	0.974
1.75	1.000	1.000	1.000	1.000	0.986	0.996
2.00	1.000	1.000	1.000	1.000	0.996	0.998
2.25	1.000	1.000	1.000	1.000	1.000	1.000
2.50	1.000	1.000	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000	1.000	1.000