Risk sharing properties and labor supply disincentives of pay-as-you-go pension systems

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Publication date
2018

Document Version
Final published version

License
Other

Link to publication

Citation for published version (APA):
Ciurilă, N. (2018). *Risk sharing properties and labor supply disincentives of pay-as-you-go pension systems*. [Thesis, fully internal, Universiteit van Amsterdam].

The thesis consists of three essays discussing the benefits and distortions implied by pay-as-you-go (PAYG) pension systems. The first essay analyses how PAYG pension benefits should be related to a person's lifetime earnings or her pension contributions such as to maximize utility at birth (ex-ante utility). This involves a trade-off between insurance against earnings shocks, labor supply distortions and capital accumulation in the economy. Using a model calibrated for the US economy, I find that a system under which there is no relation between pension benefits and life-time earnings (a flat benefit system) brings a higher utility than a system under which pension benefits reflect life-time earnings (a defined benefit system). The second essay shows how the interaction between unemployment shocks and PAYG pensions leads to earlier retirement of a worker. Using micro data I find that a person's re-employment wage after an unemployment spell is lower the older the person is and the more time she spends outside employment. Using a theoretical model, I show that lower re-employment wages following an unemployment spell lead to earlier retirement among workers but only when a PAYG pension system is in place in the economy. Cross-country data confirms that in countries with higher long-term unemployment or a higher PAYG pension system the labor market participation of older workers is lower. The third essay shows through a theoretical model that PAYG pension systems can be used to share both financial and demographic shocks between young and old agents.

Nicoleta Ciurilă obtained her BSc degree from the Bucharest University of Economic Studies and the MPhil degree Cum Laude from the Tinbergen Institute. In 2014 she started writing her PhD thesis under the supervision of Prof. Roel Beetsma and Dr. Ward Romp at the University of Amsterdam. She is researching topics related to pay-as-you-go pensions, retirement and aging in overlapping generation models with incomplete markets.

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Nicoleta Ciurilă

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Universiteit van Amsterdam

RISK SHARING PROPERTIES AND LABOR SUPPLY DISINCENTIVES OF PAY-AS-YOU-GO PENSION SYSTEMS

ISBN 978 90 3610 525 5

This book is no. 722 of the Tinbergen Institute Research Series, established through cooperation between Rozenberg Publishers and the Tinbergen Institute. A list of books which already appeared in the series can be found in the back.

RISK SHARING PROPERTIES AND LABOR SUPPLY DISINCENTIVES OF PAY-AS-YOU-GO PENSION SYSTEMS

ACADEMISCH PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Universiteit van Amsterdam
op gezag van de Rector Magnificus
prof. dr. ir. K.I.J. Maex
ten overstaan van een door het College voor Promoties ingestelde
commissie, in het openbaar te verdedigen in de Agnietenkapel
op donderdag 20 september 2018, te 10:00 uur

door

Nicoleta Ciurilă

geboren te Braşov, Roemenië

Promotiecommissie:

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Faculteit Economie en Bedrijfskunde

Acknowledgments

I thought that writing the PhD thesis was the toughest thing I have ever done: exciting at times, disheartening at times, deadlocked many times, provoking always. Then I became a mother and realized that the PhD process was in fact a quite manageable event of my life. Still, I would like to thank the people who made my journey towards the PhD title worthwhile.

I thank first of all my promoter and supervisor Roel Beetsma for letting me pursue the research topics that I was interested in, for supporting me, encouraging me and helping me a lot with the intuition and the economics behind my results. Ward, my co-supervisor, for provoking me to think hard about what I was doing and challenging my ideas and my results. My professors at the Tinbergen Institute from whom I learned a lot Eric Bartelsman, Bjoern Brüggeman, Pieter Gautier, Massimo Giuliodori, Franc Klaassen, Kostas Mavromatis, Christian Stoltenberg, Sweder van Wijnbergen: the training I got from TI is a great asset that I take with me from this experience. Jeffrey Campbell and Facundo Piguillem for being truly inspirational as professors, for being honest, modest, fun and showing lots of passion for research. Extra thanks to Facundo for organizing my research visit at the Einaudi Institute for Economics and Finance, one of the highlights of my PhD experience. Felicia Ionescu for being so optimistic, helpful with research and supportive. I am grateful for the opportunity to know and to have very interesting discussions with Casper van Ewijk, John Lorié, Naomi Leefmans, Koen Vermeylen, Siert Jan Vos.

I thank the wonderful staff of the Tinbergen Institute and the University of Amsterdam for being so helpful and friendly, an attitude that always makes me extra happy: Ester van den Bragt, Arianne de Jong, Christina Mansson, Wilma de Kruijf, Robert Helmink, Jolanda Vroons.

I also found a lot of support in the Romanian crew that made me feel at home: Adelina and Andrei Lalu, Oana Furtună and Traian Delcă, Alexandra Rusu, Marius Zoican. The wonderful Wenqian Huang, my MPhil colleague, who is so modest yet so brilliant and nice. My great room mates at UvA Moutaz Altaghibli, Jante Parlevliet and Rui Zhao: I learnt so much about your cultures by talking to you and I really enjoyed your company a lot. My UvA colleagues with which I shared so many days, so many chats and so many coffees: Damiaan Chen, Ron van Maurik, Pim Kastelein, Zina Lekniute, Ioana Neamţu, Swapnil Singh, Rutger Teulings, Jesper

Hansen, Stephanie Chan, Lucyna Gornicka. Egle, a friend in need is a gift from God. This is what you've been to me. We shared laughs, tears, trips, sun, coffee, beer, books, lots of sweets, dinners, movies and many more. Your friendship is a wonderful and valuable thing that I take with me from this PhD.

All my gratitude goes to my parents Mariana and Nicolae Ciurilă who have suffered a lot from my absence from home, but have supported me unconditionally through this process. To my wonderful friends who made my life better by talking to me, visiting me, going on trips with me, sharing my happiness and my pain, supporting me in every way: Alina Bobaşu, Bogdan Muraraşu, Cristian Bulete, Mihai Ţanţaru, Cezar and Cristina Chiţu, Raluca Gheorghiu, Georgiana Obancea, Anca Staicu, Silvia Stănescu, Daniel Ivanuş, Mirela Mardare.

Finally, I thank my beloved husband Iulian. I would not have begun this journey if it was not for you, but I also definitely would not have finished it without your support. Your belief in research, in knowledge, in doing your job the best you can, in learning from every experience whether good or bad have kept me through. We went through some depressing times, but we learned to stick together, to support each other and to have a wonderful time together in spite of the deceptions.

Contents

List of co-authors

1. **Chapter 3**: This chapter is co-authored with Iulian Ciobîcă.
2. **Chapter 4**: This chapter is co-authored with Ward Romp.

Chapter 1

Introduction

Pay-as-you-go pension systems and their reform appeared on policy makers' agenda since the 1990's due to the adverse impact that the large demographic changes had on their sustainability. Future developments in life expectancy and population growth rates are expected to still put pressure on pay-as-you-go pension systems. Hence, discussions regarding the design of these systems are expected in the future as well. This thesis consists of three essays that discuss the trade-offs implied by the design of pay-as-you-go pension systems. The analysis is focused on the major benefits of pay-as-you-go pension systems - the inter- and intra-generational risk sharing they can provide - and the major downsides of the system - labor supply disincentives and crowding out of capital accumulation.

In Chapter 2, I analyse whether the widespread reform of strengthening the link between a person's earnings history and her pension benefits by switching to a Notional Defined Contribution or a points system is welfare improving in the long run. This type of reform was implemented in countries like Italy, Sweden, Poland and Germany. On the one hand, tightening the link between a person's life-time earnings (or a person's life-time pension contributions) and the pension benefit received restores incentives to work more and longer, hence increasing long run welfare. On the other hand, it leaves a person more exposed to earnings risk, hence lowering long run welfare. Specifically, following such a reform, any shock to a person's earnings received throughout the career will be perfectly reflected in post-retirement income. Consequently, insurance is lower after switching to a Notional Defined Contribution or a points system. On the contrary, if pension benefits are constant across individuals and, hence not connected in any way to their earnings history (as it is the case under the so called flat benefit or Beveridge pension system), shocks received to earnings are not carried over to the retirement period. Moreover, I show in the thesis that tightening the link between pension benefits, on the one hand, and the earnings history or pension contributions history, on the other hand, can come at the cost of lower capital accumulation in the economy. This happens because people have higher incentives to work more or longer when they are close to the retirement age and, hence, they need to save less to achieve the same level of old age consumption. A lower level of capital accumulation decreases welfare in dynamically efficient economies. In order to determine the overall implication for welfare of the opposing effects - lower labor supply distortions, on the one hand, and lower insurance and lower capital accumulation, on the other hand - a quantitative evaluation is necessary.

Calibrating a realistic overlapping generations model on the US economy, I find that a tighter link between life-time earnings and pension benefits decreases welfare in the long-run. This result implies that the higher welfare brought about by restoring incentives for working through tightening the link between earnings and pension benefits is dominated by the lower welfare due to less insurance and lower capital accumulation in the economy. Moreover, I find that welfare is the highest if the current pay-as-you-go pension system of the US economy is replaced by a flat benefit pension system.

I also analyze how the result changes in an economy where the contribution to the pension system is higher (33%, the value currently prevailing in Italy, instead of 10.6%, the value currently prevailing in the US economy). Since labor supply distortions increase quadratically in the level of pension contributions, one would expect this effect to dominate at higher levels of pension contributions. I find that the flat benefit system still provides a higher long-run welfare than a Notional Defined Contribution system, but the difference in terms of consumption equivalent compared to the flat benefit system shrinks considerably. Moreover, if one shuts down the impact of capital accumulation on welfare considering the case of a small open economy, the flat benefit and Notional Defined Contribution system become similar in terms of welfare.

Chapter 3 analyses the impact of job loss risk on retirement. It shows empirically and theoretically that in an economy with a higher job loss risk the labor force participation of older workers is lower. The counterfactual experiments performed using the model build in the chapter indicate that the presence of a pay-as-you-go pension system amplifies the impact of job loss risk on the retirement age. This result has important policy implications: if policy makers aim at increasing the retirement age in the economy, they must not only reform the pay-as-you-go pension arrangements but should complement pension reforms with labor markets policies.

Using data from the US Survey of Income and Program Participation (SIPP), this chapter shows that earnings losses following an unemployment spell are higher as people approach the end of their working career and as they spend more time out of employment. Hence, I consider the level of long-term unemployment prevailing in a country as a proxy for the size of earnings losses following an unemployment spell. I construct a cross country regression illustrating that a higher incidence of long term unemployment reduces the labor force participation rate among older workers. In the regression, I control for country specific characteristics such as the size of pension contributions, life expectancy or pension progressivity.

The channels through which a higher job loss risk lowers the average retirement age in an economy are identified in a life-cycle model with endogeneous retirement, incomplete markets and a job loss shock. Higher job loss risk is defined as a higher probability of losing one's job or higher earnings losses following an unemployment spell. All workers, irrespective of whether they are hit by a job loss shock or not, are affected by higher job loss risk through savings. Specifically, anticipating a higher job loss probability or a higher loss in earnings following the unemployment spell, workers make more precautionary savings before the shock can occur and this allows them to retire earlier. Workers that are hit by a job loss shock are furthermore impacted through the fact that: i) the return from working decreases after the shock offering them an incentive to retire earlier (substitution effect); ii) in order to afford the same level of consumption they must work longer (income effect).

Calibrating the model on the US economy, I obtain that workers that are hit by a job loss shock retire earlier than workers that do not suffer a job loss shock. This result implies that, for the parameters of the US economy, the substitution effect of the lower earnings following the job loss shock dominates the income effect. I also obtain that a higher job loss risk lowers the average retirement age in the economy, in line with the empirical findings using cross-country data. I run a counterfactual experiment in which I eliminate the pay-as-you-go pension system in the economy. In this case, workers that are hit by a job loss shock retire much later than workers that do not suffer a job loss shock and the impact of a higher job loss risk on the average retirement age is smaller. Hence, the contribution to the pay-as-you-go pension system amplifies the impact of a higher job loss risk on the average retirement age in the economy. Specifically, the contribution to the pension system reinforces the effect of the lower wage following the jobless spell through the substitution effect, but has a very small income effect because a large part of the contribution to the pension system is returned to the individual as a pension benefit after she reaches the official retirement age.

Chapter 4 analyzes how a politician reforms a pay-as-you-go pension system when the economy is affected by both financial and demographic shocks. The policy function implemented by the politician helps overlapping cohorts share both shocks by appropriately adjusting contributions and benefits each period. Specifically, following an adverse financial shock, the politician increases both contributions and benefits. After an adverse demographic shock, the politician increases contributions but decreases benefits. These results are in line with the major observed developments of pay-as-you-go pension systems.

I also analyze the implications of *permanent* changes in the population growth rate, modeled as a lower mean of the demographic shock. A lower mean of the demographic shock makes the politician's policy function less sensitive to the wealth of the old. Intuitively, when the population growth rate is lower on average, the politician has less room to compensate old agents for their losses from financial shocks. This has two opposing implications. On the one hand, the capacity of the pension system to diversify financial risks decreases. Consequently, young agents must contribute more to the pension system to maintain the same level of protection against financial shocks. On the other hand, the return of the pension system also decreases. This makes young agents less willing to invest in the pension system. In a calibrated version of the model the impact through the lower return dominates so the average contribution to the pension system is smaller if the mean population growth rate is lower. This is consistent with stylized facts: countries in Latin America and Central and Eastern Europe decided to (partially) transform the pay-as-you-go pension system into a fully funded system following a decrease of the average population growth rates.

Chapter 2

How should pension benefits be linked to pension contributions?

2.1 Introduction

In the past 20 years governments reformed the pay-as-you-go (PAYG) pension systems in the sense of strengthening labor supply incentives at both intensive and extensive margin. Many countries such as Italy, Germany, Sweden and Poland opted for a tightening of the link between a person's pension benefits and her history of labor earnings or her history of pension contributions. The present paper asks whether strengthening work incentives in this way increases long run welfare. On the one hand, a tighter link between pension benefits and a person's earnings history reduces labor supply distortions and increases welfare. On the other hand, it reduces the insurance against the earnings shocks received during the working life and, hence, lowers welfare[1]. Moreover, I show that offering people incentives to work more and longer close to their retirement age lowers the savings they make and, hence, the capital accumulated in the economy. This further reduces welfare in the general equilibrium of a dynamically efficient economy[2]. In a model calibrated on the US economy, I find that a pension system that promotes no link between benefits and labor earnings history brings the highest welfare: labor supply distortions are outweighed by the

[1]Such a question is probably of interest for some fully funded pension systems as well. Most of the results of this paper apply to fully funded systems as well, except for the effects coming from the crowding out of capital formation.

[2]An economy is dynamically efficient in the sense of Cass (1972) if capital is lower than its first best level. This implies that the return on capital is higher than the return of the pension system.

insurance offered by such a system and the higher capital accumulation.

In the paper I compare the three most widespread types of pension benefit arrangements: i) the flat benefit (FL) system in which pension benefits are constant across people and do not reflect previous labor earnings; ii) the notional defined contribution system (NDC) in which benefits are based on the contributions accrued throughout the working life and contributions are proportional to labor earnings; iii) the defined benefit system (DB) in which pension benefits are a replacement rate times the average of life-time earnings.

Choosing one of these three pension benefit arrangements involves a trade-off among labor supply distortions, insurance against idiosyncratic earnings shocks and capital accumulation. To illustrate the trade-offs, I build a tractable general equilibrium model with two overlapping generations (young and old), idiosyncratic earnings shocks, incomplete markets, endogeneous labor supply of old individuals and a PAYG pension system. Although the model makes a number of simplifying assumptions, I show that the results obtained with it carry on to a more general setting with many overlapping generations and endogeneous labor supply at both extensive and intensive margin at all ages.

First, I prove in both the small scale and large scale model that old individuals work the least under the FL system. Intuitively, the FL system imposes sizeable distortions on the labor supply of individuals at both the intensive and extensive margin because it features no link between the contributions paid and the benefits received after retirement. People don't have an incentive to pay higher contributions to the pension system by working more or longer because this does not increase the pension benefit they receive after retirement. The DB system restores incentives at the intensive margin by relating benefits to the average of life-time wages. Still, workers have no apparent incentive to work longer once they have passed the peak of their career earnings. The NDC system encourages people to work more and longer by relating pension benefits to the amount of contributions paid throughout the lifetime.

Second, I show that the upside of the FL system is that it offers some insurance against idiosyncratic earnings shocks: any shock that leads to a decrease in earnings at a certain point of an individual's career will not reduce the benefits received after retirement. Hence, consumption inequality is the lowest under the FL system.

Finally, I prove that the impact of the pension benefit arrangement on savings depends on two opposing factors. On the one hand, linking pension benefits to the earnings or pension contributions history as in the DB and NDC systems strengthens work incentives. Individuals close to retirement respond by increasing their labor supply at both intensive and extensive margin to a higher extent than young individuals. A higher labor supply for individuals close to retirement implies that they consume more and, due to smoothing, young individuals also consume more. Since young people increase their labor supply to a lower extent, consuming more implies that they save less. Hence, through this labor supply effect, the capital to labor ratio is lower under the DB and NDC systems than under the FL system. On the other hand, individuals make less precautionary savings under the FL system than under the NDC and DB systems because the FL system offers more insurance against earnings shocks. Whether the effect through labor supply or the one through precautionary savings dominates is a quantitative question.

In order to assess the importance of insurance against idiosyncratic shocks, labor supply distortions and capital accumulation in determining the relative welfare of the three types of pension systems, I build a large scale model calibrated on the US economy. The large scale model considers a realistic demographic structure with 80 overlapping generations, a labor supply and participation decision each period and persistent idiosyncratic earnings shocks. The tax and pension system are modeled consistent with the US economy. I replace the pension benefit arrangement of the US, in turn, with the FL, DB and NDC pension systems. I obtain that the FL system promotes the lowest labor supply because it imposes significant distortions on labor supply at both the intensive and extensive margin, but offers the highest insurance against earnings shocks. Moreover, I show that for the case of the US economy the effect on savings through labor supply dominates the one through precautionary savings and the FL system promotes the highest capital accumulation out of all pension systems. Overall, the FL system provides a higher welfare than the current US pension system as well as the NDC and DB pension systems.

Using the calibrated model, I also analyse whether the welfare ranking of the three pension systems changes if the contribution to the pension system prevailing in the economy is higher. On the one hand, a higher contribution rate increases labor supply distortions and should lower welfare under the FL pension system. On the other hand, a higher contribution rate increases the insurance available in the economy and reduces capital accumulation. From this point of view, the welfare under the FL system should increase compared to the NDC and the DB systems. I conduct a counterfactual experiment in which I increase the contribution to the pension system from 10.6% - the value currently prevailing under the US pension system - to 33% - the contribution rate under the current Italian pension system considering again, in turn, that pension benefits are arranged according to a FL, DB or NDC system. I obtain that the FL system still provides the highest welfare but the welfare difference compared to the NDC and DB systems shrinks considerably. Moreover, when shutting down the impact of capital accumulation on welfare by assuming an open economy framework, the welfare difference between the three pension systems shrinks even further and the FL system brings only a marginally higher ex-ante utility. This shows that the impact of capital accumulation through general equilibrium effects on welfare is quantitatively important.

In the paper I present data from the OECD and the the Cross-National Data Center in Luxembourg (LIS) showing that current arrangements of pension systems around the world largely sustain the welfare results obtained in the paper. Countries with large PAYG pension systems have a tighter link between pension contributions and benefits. Also, countries where the inequality in disposable income increases little during the working life have a tighter link between contributions and benefits.

The paper makes a number of contributions to the literature. First, as far as I am aware, this is the first attempt to compare all three types of pension systems in terms of macroeconomic outcomes. Second, I show that strengthening labor supply incentives by switching to an NDC or a DB system can come at the expense of lower capital accumulation in the economy. In general equilibrium, the lower capital to labor ratio implies a higher return on capital and, hence, a larger difference between the return on capital and the return of the pension system in those economies that have an underaccumulation of capital (dynamically efficient economies in the sense of Cass (1972)). Consequently, DB and NDC pension systems have a more negative impact on welfare from this point of view. Quantitatively, I find the general equilibrium effect to be important.

Finally, in a realistically calibrated model of the US economy, I show that the distortions imposed by the FL system on labor supply at both intensive and extensive margin are outweighed by the higher insurance of idiosyncratic earning shocks. Hence, this type of pension arrangement provides a higher welfare than the current US pension system, the DB and the NDC systems.

The present paper is related to the literature that studies the insurance properties of PAYG pension systems in economies with incomplete markets. Storesletten et al. (2004) show that the US pension system reduces consumption variance with 20% due to the particular, convex shaped link between pension contributions and pension benefits. Imrohoroglu et al. (1995) obtain that, due to its consumption insurance property, a PAYG system with a replacement rate of 30% is welfare improving in the US. However, these papers do not consider the labor supply distortions of PAYG pension contributions. Gruber and Wise (1998), Erosa et al. (2012), Wallenius (2013), Alonso-Ortiz (2014), Bagchi (2015) show how different pension system arrangements impact on the labor supply of agents, but do not consider the insurance properties of the pension systems.

The paper is also related to the literature on the optimal size of a PAYG pension system. Imrohoroglu et al. (1995), Krueger and Kubler (2006), Harenberg and Ludwig (2014), Bagchi (2015) determine the welfare under pension systems with different levels of contributions, but assume a specific link between life-time earnings and benefits. In contrast, I analyse what link between pension benefits and earnings or contributions history maximizes ex-ante welfare by restricting myself to the most widespread types of pension benefit arrangements.

Finally, the paper is the most closely related to the literature that determines the welfare implications of changing the relation between pension benefits and the earnings history. Huggett and Parra (2010) and Golosov et al. (2013) analyse the optimality of the current US pension benefit arrangement and reach the conclusion that a higher progressivity of pension benefits would increase welfare. My analysis departs from these two papers in a number of ways. First, I explicitly analyse the impact of different pension benefit arrangements on labor supply, consumption inequality and capital accumulation and explain how these macroeconomic outcomes impact on welfare. Second, I compare the US, FL, NDC and DB pension system in a model that features idiosyncratic shocks, labor supply responses at both the intensive and extensive margin, in a general equilibrium framework and I show that all these features are important for shaping the relative welfare between these types of pension systems. Third, I also investigate the welfare rankings of the three pension systems in an economy with a higher pension contribution rate. Soares (2012) analyses the welfare difference between the FL and DB systems and their political implementability. However, he does not consider the role of the pension system in providing insurance against idiosyncratic earnings shocks and the existence of labor supply distortions at the extensive margin. Fehr and Habermann (2008) and Fehr et al. (2013) show that replacing the current German pension system based on pension points accumulated throughout a person's working life with a combination between a FL and a points system would improve welfare. Nishiyama and Smetters (2007) conclude that a 50% the privatization of social security can produce efficiency gains if it is accompanied by an increase in the progressivity of pension benefits and is financed by a consumption tax. My analysis departs from these papers by illustrating the role that labor supply distortions, insurance and capital accumulation have in choosing a certain pension benefit arrangement. I also show that in economies with a higher rate of pension contributions the degree of pension benefit progressivity should be lower.

The rest of the paper is structured in the following way. To point out the intuition behind the results, section 2.2 compares the FL, NDC and DB pension systems using a small model with only two overlapping generations. Section 2.3 presents the large scale model with 80 overlapping generations that I will calibrate in Section 2.4 using data for the US economy. I perform the pension policy experiments in Section 2.5. Section 2.6 illustrates the relevance of analysing the link between pension contributions and pension benefits by presenting evidence on the current arrangement of pension systems around the world. Finally, section 2.7 concludes the paper. All the proofs are presented in the Appendix 1.

2.2 The intuition in a small scale model

In order to illustrate the trade-off among insurance, labor supply distortions and capital accumulation implied by different pension benefit arrangements, in this section I analyse the steady state of an infinite horizon economy with only two overlapping generations. The structure of the model is similar to Harenberg and Ludwig (2015), but I abstract from aggregate shocks and introduce a labor supply decision for the old age period of an individual.

2.2.1 The economy

Two overlapping generations are living in the steady state of the economy: young (y) and old (o). Population grows at rate n. Young individuals can be thought of as the group of prime-aged workers (25-55 years) while old individuals can be considered as the group aged 55-85 years. Individuals are homogeneous when young, but in the beginning of their old age they are hit by an idiosyncratic earnings shock z with a cumulative distribution function $F(\bar{z}, \sigma^2)$. Time subscripts are suppressed throughout this subsection because I analyse the economy in its steady state.

2.2.2 Households

The problem that individuals solve is the following:

$$\max_{c^y, c^o(z), l^o(z), a} \eta ln(c^y) + \beta E(\eta ln(c^o(z)) + (1-\eta) ln(1 - l^o(z)))$$
$$c^y + a = w\bar{z}(1-\tau)$$
$$c^o(z) = a(1+r) + wzl^o(z)(1-\tau) + b(l^o(z))$$
$$l^o \in [0, 1)$$

Young individuals supply labor inelastically, pay a contribution to the PAYG pension system proportional to their labor income ($w\bar{z}\tau$), decide how much to consume (c^y) and save (a). The assumption of inelastic labor supply for the young individuals is motivated by evidence from structurally estimated models that the labor supply response of young individuals to changes in taxes is considerably smaller than that of individuals close to their retirement age (French (2005), Domeij and Floden (2006), Imai and Keane (2004), Keane (2015)).

Old individuals decide how much labor to supply ($l^o(z)$) and pay a contribution to the pension system proportional to their labor earnings ($wzl^o(z)\tau$). They consume ($c^o(z)$) the net income obtained by working, the benefit received from the PAYG pension system ($b(l^o(z))$[3]) and their accumulated savings ($a(1+r)$).

For computational convenience, preferences are logarithmic, with $1-\eta$ being the leisure share. I denote by w the wage per unit of efficiency and by r the interest rate. The contribution to the pension system is τ, while the pension benefit is represented by $b(l^o(z))$. The notation $b(l^o(z))$ considers the fact that pension benefits can depend in NDC and DB systems on a person's life-time earnings and, hence, on the labor supplied.

[3]In practice, the pension benefit may depend on more than the labor supplied by workers throughout their lifetime. In asset tested PAYG pension systems for example, benefits also depend on the wealth of a person. In this analysis we abstract from such a type of PAYG pension system.

2.2.3 The pension system

The government runs a pay-as-you-go pension system. The contributions to the pension system come from a proportional tax on labor income τ. I consider the three most widespread types of pension benefits arrangements:

1. A flat benefit (FL) system. Under this system, contributions are proportional to labor earnings but benefits are the same across individuals.

$$b^{FL}(l^o(z)) = b \tag{2.1}$$

2. A Notional Defined Contribution (NDC) system. Under this system, contributions are also proportional to labor earnings and they are accrued each period at the rate of return r^p. All accrued contributions (pension assets) are drawn as benefits by old individuals.

$$b^{NDC}(l^o(z)) = \tau w \bar{z}(1 + r^p) + \tau w z l^o(z) \tag{2.2}$$

3. A Defined Benefit (DB) system. In this case, contributions are proportional to earnings while the benefit is a replacement rate ρ times the average of life-time earnings.

$$b^{DB}(l^o(z)) = \rho(w\bar{z} + wzl^o(z))/2 \tag{2.3}$$

The pay-as-you-go pension system balances in the steady state - total pension benefits equal total pension contributions:

$$\int b(l^o(z))dF(z) = \int (\tau w \bar{z}(1 + n) + \tau w z l^o(z))dF(z) \tag{2.4}$$

The balanced budget condition pins down the value of the benefit b in the FL pension system, the rate of return of the pension system r^p in the NDC system or the replacement rate ρ in the DB system.

2.2.4 Labor supply distortions imposed by the pension system

The intratemporal consumption-leisure choice faced by an old individual in the two overlapping generations model is described by the following first order condition:

$$\frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz(1-\tau) + \frac{\partial b(l^o(z))}{\partial l^o(z)} \tag{2.5}$$

The contribution to the pension system τ imposes distortions on the consumption-leisure decision. However, if pension benefits are positively linked to a person's earnings, a higher labor supply when old increases pension benefits, i.e. $\frac{\partial b(l^o(z))}{\partial l^o(z)} > 0$ and this lowers the labor supply distortion imposed by the pension system: $wz(1-\tau) + \frac{\partial b(l^o(z))}{\partial l^o(z)} > wz(1-\tau)$.

Particularizing relation (2.5) to the FL, DB and NDC pension systems:

$$\text{FL:} \frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz(1-\tau) \tag{2.6}$$

$$\text{NDC:} \ \frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz \tag{2.7}$$

$$\text{DB:} \ \frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz\left(1-\tau+\frac{\rho}{2}\right) \tag{2.8}$$

The NDC pension system leaves the labor supply decision of old workers undistorted, while the FL system imposes labor supply distortions. In the case of the DB system, the participation to the pension system is distortive whenever there is a difference between τ - the marginal contribution to the PAYG system - and $\frac{\rho}{2}$ - the marginal benefit of participating to the PAYG system. Replacing the formula of DB benefits (2.3) in the balanced budget constraint of the pension system (2.4), one can obtain the equilibrium value of the replacement rate:

$$\frac{\rho}{2} = \tau\epsilon, \ \text{where } \epsilon = \frac{\int(\bar{z}(1+n) + zl^o(z))dF(z)}{\int(\bar{z} + zl^o(z))dF(z)} \tag{2.9}$$

With the above expression of the replacement rate, relation (2.8) becomes:

$$\frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = wz\left(1-\tau+\tau\epsilon\right) \tag{2.10}$$

It is straightforward to show that $\epsilon > 1$ if $n > 0$. Hence, the DB system actually introduces a *positive* distortion on the labor supply of old workers, as long as the population growth rate is positive. Intuitively, for old workers, the marginal benefit of contributing to the pension system is higher than the marginal contribution because the pension system balances and the young generation is larger than the old generation. This encourages old workers to supply more labor. Comparing relations (2.7) and (2.10) shows that the NDC and DB pension system are equivalent in terms of labor supply distortions *if and only if* the population growth rate is 0, i.e. $n = 0$. If $n > 0$, the DB system promotes *higher* labor supply than the NDC pension system.

2.2.5 Firms

Firms operate a Cobb-Douglas technology $F(A, K, L) = AK^{\alpha}L^{1-\alpha}$. Profit maximization yields the following conditions:

$$r + \delta = \frac{\partial F(A, K, L)}{\partial K} = A\alpha \left(\frac{L}{K}\right)^{1-\alpha} \tag{2.11}$$

$$w = \frac{\partial F(A, K, L)}{\partial L} = A(1-\alpha) \left(\frac{K}{L}\right)^{\alpha} \tag{2.12}$$

where A is the total factor productivity, K is the capital stock, δ is the depreciation rate of capital. In order to obtain a closed form solution, in the rest of section 2.2, I consider that capital depreciates fully, i.e. $\delta = 1$.

Assumption 1. *Capital depreciates fully:* $\delta = 1$.

2.2.6 A closed form solution

With assumption 1, the aggregate labor (l) and capital per labor ratio (k) in the steady state of the economy described in subsections 2.2.1-2.2.5 are given by the following relations:

$$l^{FL} = \frac{(1+n+\eta)(1-\alpha)(1-\tau)\bar{z}}{1-\alpha\eta-(1-\alpha)\eta\tau} \tag{2.13}$$

$$l^{NDC} = \frac{(1+n+\eta-\tau(1-\eta)(1+n))(1-\alpha)\bar{z}}{1-\alpha\eta} \tag{2.14}$$

$$l^{DB} = \frac{((1+n+\eta)(1-\tau+\tau\epsilon)-(1-\eta)\tau\epsilon)(1-\alpha)\bar{z}}{1-\alpha\eta+(1-\alpha)\tau(\epsilon-1)} \tag{2.15}$$

$$k^R = \left(\frac{A\bar{z}(1-\alpha)(1-\tau)s^R}{l^R}\right)^{\frac{1}{1-\alpha}} \tag{2.16}$$

where

$$R = FL, NDC, DB \tag{2.17}$$

$$s^R = \frac{\beta\Gamma^R}{\eta+\beta\Gamma^R} \tag{2.18}$$

$$\Gamma^{FL} = \frac{\alpha}{1-\alpha}l^{FL}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{FL}+\tau l^{FL}+z(1-\tau)}\right] \tag{2.19}$$

$$\Gamma^{NDC} = \frac{\alpha}{1-\alpha}l^{NDC}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{NDC}+z+\bar{z}\tau(1+n)}\right] \tag{2.20}$$

$$\Gamma^{DB} = \frac{\alpha}{1-\alpha}l^{DB}E\left[\frac{1}{\frac{\alpha}{1-\alpha}l^{DB}+z(1-\tau+\tau\epsilon)+\bar{z}\tau\epsilon}\right] \tag{2.21}$$

The pension contribution impacts the capital to labor ratio k through three channels: i) the distortion of the intertemporal savings-consumption decision - reflected by the term $1-\tau$ in equation (2.16) -, ii) the savings rate s and iii) aggregate labor l. An increase in the pension contribution lowers the capital to labour ratio through the distortion of intertemporal choices and through the lower savings rate. However, an increase in the pension contribution also lowers labour supply and this triggers an increase in the capital per labor ratio.

Proposition 1 compares the macroeconomic outcomes under the three types of pension systems. The proof of the proposition shows that aggregate labor is decreasing in the pension contribution rate τ under all types of pension systems. Equations (2.6), (2.7) and (2.8) show that the pension contribution distorts the intratemporal choice between leisure and consumption in the FL, but not in the NDC pension system, while under the DB system there is a positive distortion that should incentivise labor supply of old people. However, the pension contribution lowers the accumulation of savings under all types of pension systems. The smaller amount of capital leads, in equilibrium, to a lower demand for labor also in the case of the DB and NDC pension systems.

Proposition 1. *Under Assumption 1:*

1. *Aggregate labour is the highest in the economy with a DB pension system and the lowest in the economy with a FL pension system, i.e.* $l^{DB} > l^{NDC} > l^{FL}$*;*

2. *Consumption inequality is the highest under the DB pension system and the lowest under the FL pension system economy.*

3. *In an economy without idiosyncratic earnings risk, the DB pension system crowds out capital formation the most, while the FL pension system crowds out capital formation the least, i.e.* $k^{DB}_{\sigma=0} < k^{NDC}_{\sigma=0} < k^{FL}_{\sigma=0}$.

The FL pension system promotes lower consumption inequality because contributions are proportional to earnings but benefits are flat. The NDC pension system, however, does not provide direct consumption insurance, because pension benefits are perfectly linked to contributions and, hence, to earnings.

The capital prevailing under the different pension systems is determined by two opposing effects. On the one hand, because the DB and NDC pension system promote higher labor supply, old people work more and, hence, consume more than under the FL system. Due to the intertemporal consumption smoothing effect, young people also consume more. Since their labor supply is inelastic, this implies that they save less. There is substantial microeconometric evidence that the labor supply of young people is substantially less elastic than the labor supply of old people. Hence, the assumption that old people increase their labor supply following changes in the pension benefit system while young agents do not is realistic. Considering this effect, the result that capital per labor ratio is lower under the DB and NDC systems than under the FL system in the absence of shocks, i.e. $k^{DB}_{\sigma=0} < k^{NDC}_{\sigma=0} < k^{FL}_{\sigma=0}$, is intuitive.

On the other hand, when there is idiosyncratic risk in the economy, people make more precautionary savings when young under the DB and NDC system than under the FL system, because the DB and NDC systems offer less insurance. Hence capital accumulation should be higher under the DB and NDC systems from this point of view.

Whether the effect through labor supply or the effect through precautionary savings dominates is a quantitative question and I will answer this using the large scale model presented in Section 2.3. For realistic levels of idiosyncratic risk, I find that the effect through lower labor supply dominates and capital accumulation is higher under the FL pension system.

In conclusion, the analysis in the small scale model implies that the FL system promotes the lowest labor supply, the lowest consumption inequality and the highest level of capital per labor, at least at small levels of idiosyncratic risk. The DB system promotes the highest labor supply, but also the highest consumption inequality and crowds out capital accumulation the most.

The implications for welfare are ambiguous. From the point of view of labor supply incentives, the FL system should provide the lowest welfare since it distorts labor supply the most. However, the FL system also promotes the lowest consumption inequality out of all three systems, hence welfare should be the highest from this point of view. Moreover, as long as the level of idiosyncratic risk is not too high, the FL system promotes the highest capital accumulation out of all three types of pension systems. In a dynamically efficient economy in the sense of Cass (1972) in which there is underaccumulation of capital, a higher level of capital provides a higher welfare. It is not clear whether labor supply incentives, insurance against idiosyncratic shocks or capital accumulation dominates. Hence, in the next section I build a realistically calibrated model of the US economy in order to determine the welfare ranking of the FL, DB and NDC pension systems.

2.3 The large scale model

The economy is comprised of T overlapping generations. An individual enters the labor market at 20 years and lives until at most 20+T years. The probability that an individual survives from age j to $j+1$ is ψ_{j+1}. The population grows at rate n. Time is discrete and each period represents a year.

2.3.1 Households

Households differ with respect to their age $j \in \{20, .., 20+T\}$, idiosyncratic earnings shock $z \in Z$, savings invested in capital $a \in A$ and pension claims $p \in P$ that will be used to compute pension benefits after the official retirement age. We summarize the state of a household of age j in $x = (z, p, a)$. In the next period, the state of the household will be $x' = (z', p', a')$.

Households of age j decide how much to consume (c_j), save (a'_j) and work (l_j) maximizing their expected utility. At the beginning of each period individuals are hit by a persistent idiosyncratic earnings shock z_j. The markets are incomplete, individuals cannot insure against these idiosyncratic earnings shocks in any other way than saving or working more.

$$V_j(x) = \max_{c_j(x), l_j(x), a'_j(x)} u(c_j(x), l_j(x)) + \beta \psi_{j+1} E V_{j+1}(x') \tag{2.22}$$

$$c_j(x)(1+\tau_c) + a'_j(x) = a(x)(1 + r(1-\tau_k)) + y_j(x) + b_j(x) 1_{\{j \geq ORA\}} -$$
$$- T(y_j(x)) \tag{2.23}$$
$$a'_j(x) \geq 0 \tag{2.24}$$
$$l_j(x) \in [0, 1) \tag{2.25}$$

The variable $y(x)$ represents the gross labor earnings of the individual, $b(x)$ is the pension benefit obtained after reaching the official retirement age (ORA), $T(y_j(x))$ are taxes paid on labor earnings, τ_c is the tax on consumption, τ_k is the tax on the return on capital and r is the return on capital.

Relation (2.24) shows that individuals face a tough constraint: they cannot borrow against future earnings.

I denote by $\mu_j(x)$ the measure of households of age j that are in state x. People enter the labor market at age $j = 20$ and in that moment hold no capital and no pension assets. I normalize the mass of individuals entering the labor market to unity:

$$\int_Z d\mu_{20}(z, 0, 0) = 1$$

2.3.2 Preferences

The household's utility is consistent with a balanced growth path:

$$u(c,l)=\begin{cases}\frac{(c^{\eta}(1-l\bar{h}-\theta_P P)^{1-\eta})^{1-\sigma}}{1-\sigma} & \text{, if } \sigma \neq 1\\ (\eta ln(c)+(1-\eta)ln(1-l\bar{h}-\theta_P P)) & \text{, if } \sigma = 1\end{cases} \tag{2.26}$$

Parameters σ and η give the inverse of the intertemporal elasticity of substitution (IES)$= 1+\eta(\sigma-1)$ and the Frisch elasticity of labor supply: $\nu = \frac{1-l\bar{h}-\theta_P}{l\bar{h}}\frac{1-\eta(1-\sigma)}{\sigma}$. The maximum proportion of total time devoted to work in a year is represented by $\bar{h}$.

The parameter θ_P measures the disutility of participating to the labor market. This proxies the fixed costs required to hold a job such as commuting time. The variable P indicates the participation to the labor market: $P = 1$, if $l > 0$ and $P = 0$, if $l = 0$.

2.3.3 Labor income and taxes

If people decide to work ($P = 1$), they receive a gross labor income y. This is given by the average wage $w\bar{h}$, the idiosyncratic earnings shock received by the individual z, a deterministic component that represents age-specific productivity κ_j and the optimally chosen labor supply $l_j(x)$. The government levies a contribution to the PAYG pension system τ and a non-linear tax on labor net of pension contributions $T(y_j(x))$.

$$y_j(x) = w\bar{h}z_j\kappa_j l_j(x) \tag{2.27}$$

$$T(y_j(x)) = \tau_l(y_j(x)/AW)(y_j(x)+TL-\tau y_j(x))-TL \tag{2.28}$$

$$\tau_l(y_j(x)/AW) = a_0 + a_1(y_j(x)/AW) + a_2(y_j(x)/AW)^{\phi} \tag{2.29}$$

where $\tau_l(y_j(x)/AW)$ is the marginal tax of a person earning $y_j(x)$, AW is the average wage in the economy and TL is a lump sum transfer. The lump sum transfer represents government programs such as food stamps and housing subsidies.

The tax paid by workers is computed as the marginal tax rate $\tau_l(y_j(x)/AW)$ times the labor income $y_j(x)$ and the lump sum transfer net of the contribution to the pension system $\tau y_j(x)$. Lump sum transfers reduce the amount of tax paid by the individual. I consider the specification of the marginal tax on labor earnings from Erosa et al. (2012) (relation 2.29). Using OECD tax data, they estimate the marginal tax rate as a quadratic function of labor income net of contributions to the PAYG pension system. For the policy experiments performed in this paper that involve changing the pension benefit arrangement while holding the pension system balanced, this specification of the nonlinear tax on labor is appropriate.

The earnings shock is persistent and lognormally distributed.

$$logz_j = \rho logz_{j-1} + \epsilon_j, \epsilon \sim N(0, \sigma_\epsilon^2) \tag{2.30}$$

2.3.4 The PAYG pension system

The baseline model is calibrated using the current US PAYG pension system. In this system, the contributions come from a proportional tax on labor income τ. People must pay the contribution to the pension system while they work, even after they reach the early retirement age of 62 years and can start collecting pension benefits. Compared to a person that stops working at 62 years, pension benefits of those still in the labor force are not higher so as to reflect the additional contributions paid to the pension system. The simulations performed with the model show that this feature of the US system imposes significant labor supply distortions at the extensive margin and makes people retire earlier. I set the official retirement age (ORA) equal to the early retirement age of 62 years and consider that everyone starts drawing pension benefits at this age[4]. Under the current US pension system, people that draw pension benefits at the early retirement age of 62 years and decide to remain in the labor force are subject to an earnings test until they reach the normal retirement age (66 years). This also imposes labor supply distortions at the extensive margin. In the analysis I ignore this feature of the US pension system. However, just as under the current US pension system, people can choose to work while collecting pension benefits.

Pension claims are based on the average annual earnings in the 35 highest earning years. Since this definition in not tractable in the model, I will compute pension claims based on average annual life-time earnings.

$$p_j(x) = \begin{cases} \frac{jp_{j-1}(x)+y_j(x)}{j+1}, \text{ if } y_j(x) < c \\ \frac{jp_{j-1}(x)+c}{j+1}, \text{ if } y_j(x) \geq c \end{cases} \tag{2.31}$$

[4]In reality, people can defer the collection of pension benefits up to 70 years and they receive a bonus for each year they defer collection. I ignore this feature of the US pension system here as I am not interested in the pension collection behavior of people but in their labor supply decision.

where $p_j(x)$ represents the average life-time income of an individual with age j in state x and c is a cap set on pension claims.

Benefits are determined as a replacement rate times average annual earnings. The replacement rate is decreasing with the size of the average annual earnings (equation (2.32)). This implies that the return of the pension system is higher for low earning individuals who build up smaller pension claims. Hence, the US pension system offers some insurance against idiosyncratic earnings shocks.

$$b_j(x) = \begin{cases} 0.9p \text{ for } p < b_1 \\ 0.9b_1 + 0.32(p - b_1) \text{ for } p \in (b_1, b_2) \\ 0.9b_1 + 0.32(b_2 - b_1) + 0.15(p - b_2) \text{ for } p > b_2 \end{cases} \tag{2.32}$$

Under the US type of benefit arrangement, working more hours pays off because the worker accumulates higher pension claims and, hence, has a higher pension benefit. However, the increase in the pension benefit is not equal to the additional increase in contributions but depends on the replacement rate. This imposes labor supply distortions at the intensive margin. Because the replacement rate decreases with the average life-time earnings, high earning workers benefit less than low income earners from supplying more labor. Hence, the labor supply of higher earners is distorted more than the labor supply of low earners.

2.3.5 Government

The government obtains revenues from taxing consumption at rate τ_c, labor earnings at marginal rate $\tau_l(y(x))$, the return on capital at rate τ_k and from a 100% tax on accidental bequests Beq. These revenues are used to finance government spending G and the lump sum transfers TL. The deficit or surplus of the PAYG pension system (PB) is obtained as a difference between pension contributions and pension benefits and is consolidated in the general government budget. I assume that the government balances the budget in the steady state by adjusting the lump sum transfers TL.

$$\sum_j \int_{Z\times A\times P} (\tau_c c_j(x) + \tau_l(y_j(x))y_j(x) + \tau_k r a_j(x)) d\mu_j(x) + Beq + PB = G + TL \tag{2.33}$$

$$\sum_j \int_{Z\times A\times P} (\tau y_j(x) - b_j(x) 1_{\{j\geq ORA\}}) d\mu_j(x) = PB \tag{2.34}$$

2.3.6 Firms

Firms operate the same Cobb-Douglas technology as in the small scale model of section 2.2: $F(A,K,L) = AK^{\alpha}L^{1-\alpha}$. The same first order conditions of the firms' problem summarized in relations 2.11 and 2.12 apply.

The stationary competitive equilibrium of the economy described in sections 2.3.1-2.3.6 is presented in Definition 1.

Definition 1. *Given the exogeneous survival probabilities $\{\psi_j\}_{j=1,T}$, the population growth rate $\{n\}$ and the government policies $\{G, \tau_c, \tau_l, \tau, b_j\}$, a stationary competitive equilibrium of the model is represented by a set of time-invariant allocations $\{c_j(x), l_j(x), a'_j(x)\}$ for every state x, prices $\{w, r\}$, policies $\{TL\}$, accidental bequests $\{Beq\}$ and a distribution of individuals across states $\mu_j(x)$ such that:*

1. *Individuals take optimal decisions given prices and policies;*

2. *Firms take the optimal decision given prices;*

3. *Bequests are equal to the assets of the deceased:*

$$Beq = \sum_j \int_{Z\times A\times P} (1-\psi_{j+1}) a_j(x) d\mu_j(x)$$

4. *The government budget balances:*

$$\tau_c C + \tau_l L + \tau_k r K + Beq + PB = G + TL$$

5. *Capital market clears:*

$$K = \sum_j \int_{Z\times A\times P} a_j(x) d\mu_j(x)$$

6. Labour market clears:

$$L = \sum_j \int_{Z \times A \times P} l_j(x) z_j \kappa_j d\mu_j(x)$$

7. The law of motion of the population distribution is given by:

$$\mu_{j+1}(x') = \frac{\psi_{j+1}}{1+n} \sum_j \int_{Z \times A \times P} 1_{\{a',p'\}} \pi(z'|z) d\mu_j(x)$$

where $\pi(z'|z)$ is the transition probability density function of productivity from age j to $j+1$ and $1_{\{a',p'\}}$ is an indicator function that takes value 1 if the level of savings invested in capital and pension claims are equal to a' and p', respectively.

2.4 Calibration

The parameters of the model are calibrated to match key variables of the US economy (Table 2.1). The number of overlapping generations is set at T=80. The annual population growth rate is equal to 1.2%, the average for the US over the past 40 years. The survival probabilities ψ_j are taken from Bell and Miller (2005).

The contribution to the US pension system is $\tau = 0.106$. The pension benefit is given by formula (2.32) where the benefit intervals are the ones corresponding to the year 2002: $b_1 = 7104$ USD, $b_2 = 42804$ USD. The annual average wage for this year was 40898 USD (OECD). In the model, the pension system insures a replacement rate of 37%, slightly lower than the value of 39% estimated by OECD (2005).

The government expenditure is equal to 18% of GDP, the average of the variable for the US economy. The depreciation rate is set to $\delta = 0.065$ by targeting the value of 22% for the ratio of investment to GDP. The time preference $\beta = 1.002$ is obtained by targeting a capital to output ratio of 3.1. The capital share in output is $\alpha = 0.35$.

The consumption tax and return on capital tax are taken from the US tax code: $\tau_c = 0.05$ and $\tau_k = 0.36$. For the labor income tax, I use the parameters estimated by Erosa et al. (2012).

The characteristics of the earnings process are taken from Kitao (2014): the autoregressivity $\rho = 0.97$, the variance $\sigma_\epsilon^2 = 0.02$ and the deterministic part of the earnings process that depends on age κ_j. French (2005) calibrates similar values for the autoregressivity and variance of the wage process - $\rho = 0.977$, $\sigma^2_\epsilon = 0.0141$.

Table 2.1: Calibration

Parameter	Description	Value	Source
n	Population growth rate	0.012	
α	Capital share in output	0.35	
ρ	Autoregressivity of earnings process	0.97	
σ^2_ϵ	Variance of earnings process	0.02	
G/Y	Government expenditure to GDP	0.18	
τ_c	Consumption tax	0.05	
τ_k	Return on capital tax	0.36	
τ	Contribution to PAYG pensions	0.106	
a_0		-0.24	Erosa et al. (2012)
a_1		-0.0097	Erosa et al. (2012)
a_2		0.42	Erosa et al. (2012)
ϕ		0.22	Erosa et al. (2012)
η	Consumption share	0.55	French (2005)
Parameter	Description	Value	Target
δ	Depreciation rate	0.065	I/Y=0.22
β	Time preference	1.002	K/Y=3.1
σ	Preference parameter	3	$IES = 0.5$
$\theta_P(j)$	Disutility of labor force participation	$0.16 + 0.14j^2$	Participation rate across the life-cycle
$\bar{h}$	Maximum hours worked	0.45	Drop in hours worked across the life-cycle
TL/Y	Lump sum transfers to GDP	0.06	Balanced budget

To calibrate the preferences of individuals, I set the intertemporal elasticity of substitution to 0.5 and the consumption share η to 0.55, in line with the literature. For calibrating the disutility of labor force participation $\theta_P(j)$, I use as target the labor force participation after the age of 55. To match the data, I consider a specification in which the disutility from labor force participation increases quadratically with the age. I do not aim to match the labor force participation rate before the age of 55, because this is influenced by factors that are not in the model such as education decisions (especially for individuals between 20 and 30 years), disability and discouragement. The number of hours worked $\bar{h}$ is pinned down by the drop in hours worked at the end of the working life. To compute the labor force participation at each age and the annual number of hours worked across the life-cycle, I use data from the 1962-2016 March CPS. I consider the labor supply of men and the cohorts born until 1942.

Figure 2.1 shows the labor force participation rate generated by the model versus the actual data, while figure 2.2 presents the hours worked across the life-cycle.

Figure 2.1: Labor force participation rate: model vs data

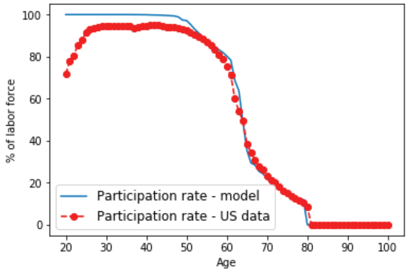
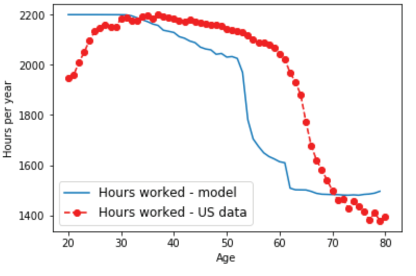


Figure 2.2: Hours worked across life-cycle: model vs data



The model generates a decline in hours worked that starts a bit earlier than in the data. The labor supply smoothing embeded in the model makes individuals adjust the hours they work in order to gradually retire from the labor market. However, the CPS data shows a very abrupt retirement at least for individuals that leave the labor force before the age of 60[5].

In order to validate the calibration of the model, I analyse a number of variables that were not explicitly targeted during the calibration: the elasticity of labor supply, the change in income inequality and in consumption inequality across the life cycle. I compare these with estimated values from the literature.

I compute the elasticity of labor supply by performing the following experiment: I increase the wage of all workers of age j with 20% and I simulate the model in partial equilibrium in order to get the change in labor supply. I find that the model generates elasticities that increase from 0.15 at age 40, to 0.3 at age 50, reaching 1.7 at age 60. Performing the same type of experiment, French (2005) also obtains that the elasticity of labor supply is increasing throughout the individual's life. His

[5]Using CPS data, Kitao (2014) also fails to match the life-cycle profile or hours worked in the data. French (2005) matches the life-cycle profile of hours worked from the PSID, but the PSID data shows a significant decline in hours worked starting from 42 years.

estimates range between 0.19 and 0.37 for individuals aged 40 years and 1.04-1.24 for individuals aged 60 years.

Table 2.2 compares the increase in income inequality and consumption inequality across the life-cycle with values estimated in the literature. The papers included in the comparison use different measures and data sources for earnings and different methods to estimate the change in consumption inequality across the life-cycle. The differences regarding the data sources and the methodology are presented in Appendix 2. One important difference is that Heathcote et al. (2010a) and Heathcote et al. (2005) include only wage earnings, while Storesletten et al. (2004) also include transfers.

Table 2.2: Increase in variance of log earnings and of log non-durable consumption across the life-cycle (log points)

	Age reference	Increase in variance of log earnings	Increase in variance of log consumption
Heathcote et al. (2010a)	25-57 years	0.22*/0.32	0.07*/0.14
Heathcote et al. (2005)	25-60 years	0.3	0.05
Guvenen (2007)	25-65 years	-	0.21
Model	25-60 years	0.27	0.17
	Age reference	Increase in variance of log earnings + transfers	Increase in variance of log consumption
Storesletten et al. (2004)	20-60 years	0.5	0.2
Model	20-60 years	0.9	0.19

* equivalized household income

The increase in the variance of log earnings across the life-cycle seems to be in accordance with Heathcote et al. (2010a) and Heathcote et al. (2005). The increase in the variance of consumption across the life-cycle seems is in accordance with Heathcote et al. (2010a) and Guvenen (2007), but is higher than in Heathcote et al. (2005).

Comparing the model to Storesletten et al. (2004), the increase in log earnings plus transfers is again higher, but consumption inequality is equal. The high increase in variance of log earnings plus transfers is due to the fact that I include in the computation of income inequality individuals that are not participating to the labor market[6]. As individuals approach 60 years and start dropping out of the labor force, the variance in earnings between individuals increases substantially.

[6] This is what Storesletten et al. (2004) also seem to do in their estimations.

2.5 Policy experiments

With the calibrated model, I perform a number of policy experiments. In section 2.5.2, I analyze the macroeconomic and welfare implications of replacing the current US pension system with the three types of pension benefit arrangements analysed in the small model of Section 2.2: the flat benefit (FL) pension system, the Notional Defined Contribution (NDC) and the Defined Benefit (DB) system. When changing the pension benefit arrangement, I consider that the general budget rebalances by changes in the lump sum taxes (TL). In Appendix 3, I present the welfare under the four types of pension systems considering different policy instruments for rebalancing the government budget: the consumption tax τ_c, the return on capital tax τ_k, the parameter that governs the average tax on labor a_2 and the progressivity of the labor tax rate ϕ. The results are similar to the ones obtained by rebalancing the government budget with the lump sum tax.

In section 2.5.3, I analyse the role of contribution size on the welfare of alternative benefit arrangements. I increase the contribution rate from 10.6%, the value prevailing under the current US system to 33%, the current value of contributions in the Italian pension system. Considering this contribution rate, I change the pension benefit arrangement and analyse the impact on welfare.

As mentioned before, under the FL system, the benefit received from the pension system does not depend on the previous contributions and is constant across individuals. Since there is no link between pension contributions and pension benefits, people do not build pension assets during their working life. I assume that, unlike in the US pension system, the payment of pension contributions stops after the official retirement age, so that there are no implicit taxes on working after this age.

$$p_j(x) = 0 \tag{2.35}$$

$$b_j(x) = b \tag{2.36}$$

The NDC system perfectly links life-time contributions with pension benefits. Individuals pay contributions proportional to their labor earnings. These contributions are used to build up a person's claims for the pension (relation (2.37)). Once the individual reaches the official retirement age, he is entitled to receive the pension benefit computed as his pension claims divided by the expected survival period at the official retirement age s_{ret} measured in years (relation (2.38)). The payment of pension contributions stops after the official retirement age.

$$p_j(x) = p_{j-1}(x)(1 + r^p) + \tau y_j(x) \tag{2.37}$$

$$b_j(x) = \frac{p_j(x)}{s_{ret}} \tag{2.38}$$

In line with the way that the NDC system is arranged in practice, the annual accrual rate of a person's pension claims r^p is given by the growth rate of the aggregate wage bill. This ensures that the budget of the pension system is balanced in the stationary competitive equilibrium. The growth rate of the aggregate wage bill is equal to the gross population growth rate in the stationary equilibrium:

$$1 + r^p = 1 + n \tag{2.39}$$

Finally, under the DB pension system, the benefits are computed as a replacement rate ρ times the average of life-time benefits $p_j(x)$:

$$p_j(x) = (p_{j-1}(x) + y_j(x))/j \tag{2.40}$$

$$b_j(x) = \rho p_j(x) \tag{2.41}$$

I determine the parameters of the FL, NDC and DB pension systems (b, s_{ret} and ρ, respectively) by assuming that the PAYG pension system budget balances in the steady state. To make the conditions of the pension systems comparable to the system currently in place in the US, I consider that the official retirement age is 62 years, the same as the early retirement age in the US. Unlike the case of the US system though, I consider that people who continue working after the official retirement age of 62 are exempt from paying the contribution to the PAYG pension system. The lack of implicit taxes after the official retirement age will have significant implications for labor supply at the extensive margin.

2.5.1 Welfare measure

To compare the welfare under the three types of pension benefit arrangements, I use the concept of *ex-ante* utility or utility of individuals at birth:

$$WF = E_0 \sum_{j=1}^{T} \psi_{j+1} \beta^i u(c_j, l_j) \tag{2.42}$$

I compare the different pension arrangements in terms of the consumption compensation that individuals require each year and in each state in order to achieve the same welfare as in the steady of the economy with the policy change (equivalent variation). More specifically, I compute the consumption compensation ω in the following way:

$$E_0 \sum_{j=1}^{T} \psi_{j+1} \beta^i u((1+\omega) c_j^{benchmark}, l_j^{benchmark}) = E_0 \sum_{j=1}^{T} \psi_{j+1} \beta^i u(c_j^R, l_j^R) \tag{2.43}$$

where R is the alternative pension system.

The welfare results should be interpreted in the following way. If $\omega > 0$, individuals living in the steady state of the benchmark economy need a compensation of ω percent out of the consumption they obtain each year and in each state in order to reach the same welfare as in the economy with the alternative pension system. In this case, *welfare is higher in the steady state with the alternative pension system*. If $\omega < 0$, the converse is true: individuals living in the steady state with the alternative pension system are worse off than in the economy with the benchmark pension system.

In order to point out the impact that changing the current US system has on insurance, I use the methodology of Floden (2001) to decompose the equivalent variation ω into i) a component that shows the change in welfare due to the change in consumption levels (ω_{lev}) and ii) a component that shows the change in welfare due to the change in insurance against idiosyncratic shocks (ω_{unc}).

The methodology consists in computing the certainty equivalent consumption and labor bundle $\{\bar{c}, \bar{l}\}$ from:

$$\sum_{j=1}^{T} \psi_{j+1} \beta^i u(\bar{c}, \bar{l}) = E_0 \sum_{j=1}^{T} \psi_{j+1} \beta^i u(c_j, l_j) \tag{2.44}$$

Let C and L be the average consumption and labor and $\bar{C}$, $\bar{L}$ be the average certainty equivalent of consumption and labor, respectively:

$$C = \int c(x)\mu(x); L = \int l(x)\mu(x); \bar{C} = \int \bar{c}\mu(x); \bar{L} = \int \bar{l}\mu(x) \tag{2.45}$$

Also, let the leisure compensated consumption in the economy with the alternative pension system $\hat{C}^R$ be:

$$\sum_{j=1}^{T} \psi_{j+1}\beta^i u(\hat{C}^R, L^{benchmark}) = \sum_{j=1}^{T} \psi_{j+1}\beta^i u(C^R, L^R) \tag{2.46}$$

It is straightforward to compute:

1. The welfare change due to a change in levels:

$$\omega_{lev} = \frac{\hat{C}^R}{C^{benchmark}} - 1 \tag{2.47}$$

2. The welfare change due to a change in uncertainty:

$$\omega_{unc} = \frac{1 - p_{unc}^R}{1 - p_{unc}^{benchmark}} - 1 \tag{2.48}$$

   where p_{unc} is the price of uncertainty computed as:

$$\sum_{j=1}^{T} \psi_{j+1}\beta^i u((1 - p_{unc})C, L) = \sum_{j=1}^{T} \psi_{j+1}\beta^i u(\bar{C}, \bar{L}) \tag{2.49}$$

Floden (2001) shows that:

$$1 + \omega = (1 + \omega_{unc})(1 + \omega_{lev}) \Rightarrow \omega \approx \omega_{unc} + \omega_{lev} \tag{2.50}$$

Since there are infinitely many combinations of certainty equivalent bundles $\{\bar{c}, \bar{l}\}$, I fix labor at the value chosen in the risky world $l_j(x)$.

2.5.2 Changing the US pension benefit arrangement

In this section, I present the results of replacing the pension benefit arrangement of the US system with the FL, NDC and DB arrangement, in turn. Table 2.3 summarizes the changes in macroeconomic variables and welfare determined by the change in the pension benefit arrangement.

Table 2.3: Macroeconomic implications of changing the benefit function

	US system (Benchmark)	FL	NDC	DB
Interest rate	5.2%	5.2%	5.4%	5.4%
Capital per labor	-	-0.3%	-2.7%	-3%
Labor	-	-1.1%	2.5%	3.1%
Consumption equivalent (ω)	-	1.2%	-1.1%	-1.1%
- change due to uncertainty (ω_{unc})	-	2%	-2.6%	-3.1%
- change due to levels (ω_{lev})	-	-0.8%	1.4%	1.8%

The results obtained with the simplified model in Section 2.2 are confirmed in the large scale model. The DB system promotes the highest labor supply while FL the lowest labor supply. Figures 2.3 and 2.4 show that, compared to the FL system, the DB and NDC systems strengthen work incentives at both the intensive and the extensive margin. Under the FL pension system, people leave the labor force earlier and supply less labor starting with the age of 45. A comparison between the current US pension system and the NDC or DB pension system, shows that the difference between the systems is mainly the lower labor force participation after the official retirement age. The difference arises because under the NDC and DB pension systems people are exempt from paying the contribution to the pension system after 62 years, while under the current US system this is payable until people actually leave the work force. This has significant implications on the labor force participation rate of older workers. The labor supply at the intensive margin in the US system is only slightly smaller than the one prevailing under the NDC and DB systems. Hence, the fact that the replacement rate in the US system is decreasing with life-time earnings does not seem to impose large distortions on the number of hours worked.

Figure 2.3: Labor force participation rate under alternative pension systems

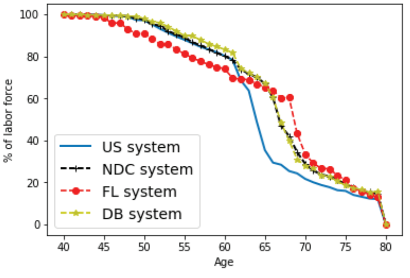
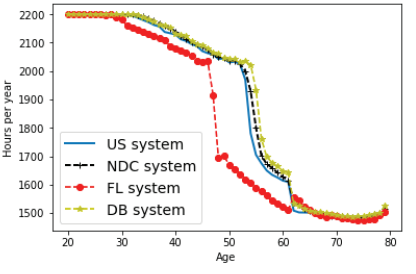


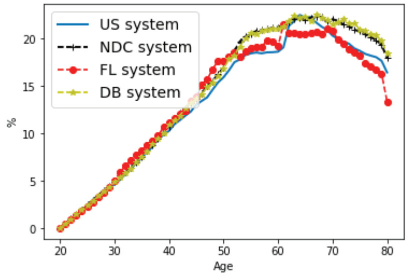
Figure 2.4: Hours worked under alternative pension systems



Compared to the US pension system, the FL system provides strong disincentives to work at both the intensive and extensive margin, but only until workers reach the official retirement age. Since they are not required to pay the contribution after the official retirement age, work incentives are restored even under the FL system.

It is important to notice that the labor force participation of people aged less than 45 years does not respond to changes in the pension benefit arrangement, while the number of hours worked by people aged less than 45 years responds to a very small degree. These results are in line with the literature that structurally estimates labor supply elasticities across the life-cycle and points out that the labor supply of prime-age workers is largely unresponsive to changes in taxation. In Imai and Keane (2004) and Keane (2015) this result is driven by a human capital accumulation mechanism, while in French (2005), Domeij and Floden (2006) and the current paper this is the result of binding borrowing constraints early in the working life. This last result also validates our modeling approach in the two overlapping generations model of section 2.2 where I assumed that the labor supply of young people is fixed.

Figure 2.5: Consumption inequality



The development of consumption inequality across the life-cycle under the three types of pension arrangements (Figure 2.5) is in line with the results obtained in the small model. Consumption inequality is the highest under the DB and NDC pension systems because these systems do not insure against earnings shocks. The FL and the US system promote roughly the same consumption inequality.

Finally, the NDC and DB systems crowd out capital formation more than the FL system. Using the two overlapping generations model of section 2.2, I obtained that the NDC and DB pension system crowd out capital formation more than the FL system in an economy with no idiosyncratic shocks. In a setting with idiosyncratic shocks, the impact of the different pension systems was not clear cut: since the NDC and DB pension systems insure less against idiosyncratic shocks, people make more precautionary savings and this can overturn the impact coming from a higher labor supply of old people. However, in this realistically calibrated model it seems that this is not the case: the NDC and DB pension systems crowd out capital formation more than the FL system even in the presence of a realistic level of idiosyncratic risk.

Overall, individuals are better off in the steady state of the FL economy than under the current US pension system: individuals living in the steady state of the US economy require a compensation of 1.2% of annual consumption in order to achieve the same welfare as under the FL pension system. The higher ex ante utility under the FL system comes entirely from the decrease in uncertainty driven by the pension system (2% of annual consumption), while the lower labor supply promoted by the FL system lowers welfare (-0.8% of annual consumption).

Replacing the current US system with the NDC or the DB system results in a welfare loss: individuals living in the steady state of the US economy can give up 1.1%

of annual consumption and still be as well off as under the alternative pension systems. This is because the current US system provides more insurance against idiosyncratic shocks although it also distorts labor supply more at the extensive margin. The decomposition of the consumption equivalent illustrates this result: when switching from the US to the NDC or DB systems, the change in welfare due to the change in uncertainty is negative while the change in welfare due to higher consumption levels is positive. Finally, this decomposition also shows that the DB pension system provides the least insurance from all the types of pension systems, as predicted using the small model in section 2.2.

2.5.3 The role of the contribution size

In this subsection I analyze the role that the contribution size has on the welfare ranking of FL, NDC and DB pension systems. To this end, starting from the baseline model, I increase the pension contribution from $\tau = 10.6\%$ to $\tau = 33\%$ - the current contribution in the Italian pension system. I simulate the model with a contribution rate of 33% and the arrangement of a DB, NDC and FL pension system, in turn. I let factor prices - interest rate and average wage - adjust with the higher contribution rate, hence considering the case of a closed economy (general equilibrium). In a setting with a higher contribution rate, both aggregate labor and capital will be lower. Hence, a rebalancing of the government budget is necessary. I consider that this is done by an increase the consumption tax rate τ_c.

Table 2.4 summarizes the results considering that the economy with the DB pension system is the benchmark.

Table 2.4: NDC vs FL pension system with $\tau = 33\%$, closed economy

	DB system (benchmark)	FL	NDC
τ_c	12.4%	13%	12.5%
Interest rate	9.5%	8.0%	9.4%
Capital per labor	-	17%	2%
Labor	-	-10%	-1.6%
Consumption equivalent ge. (ω)	-	1.6%	0.3%
- change due to uncertainty (ω_{unc})	-	7.6%	1.1%
- change due to levels (ω_{lev})	-	-5.4%	-0.8%

Qualitatively, the impact of changing the pension benefit arrangement on macroeconomic variables is the same as the one found using the small model in section 2.2. The FL pension system promotes lower labor supply but higher capital accumulation and also offers more insurance against idiosyncratic earnings shocks than the DB and the NDC pension system. The NDC pension system promotes lower labor supply, but higher capital accumulation and insures people better than the DB pension system. Due to the higher size of the pension contribution, the impact of changing the pension benefit arrangement on aggregate macroeconomic variables is quantitatively more important than in the case of the US system.

The FL system provides the highest welfare from all pension systems even at this higher level of pension contributions, but the difference in terms of consumption equivalent is smaller (compare the consumption equivalent in table 2.4 with the one in table 2.3). This is because labor supply distortions are considerably higher when the contribution rate is larger, so the welfare difference between the FL and DB pension systems is lower.

Finally, starting from the benchmark of the US economy, I investigate the implications of having a higher contribution rate, but assuming that the economy is open. Hence, I increase the contribution rate from $\tau = 10.6\%$ to $\tau = 33\%$, but I keep the interest rate and the wage equal to the values prevailing under the US economy. As the results presented in table 2.5 show, increasing the contribution to the pension system under the assumption of an open economy has important implications for the welfare under the three types of pension system arrangements. Specifically, the welfare difference between the three pension systems shrinks even more and the FL system brings only a marginally higher welfare than the NDC and the DB systems. The only difference between the closed and open economy assumptions is that under a closed economy the amount of capital accumulation impacts on the welfare of individuals. The simulations presented in tables 2.4 and 2.5 indicate that this channel is quantitatively important. The FL pension system is substantially better from a welfare point of view even when the contribution rate is as high as 33% under the assumption that the economy is closed. In contrast, in an open economy changes in capital accumulation do not impact on welfare, eliminating one of the advantages of the FL system.

Table 2.5: NDC vs FL pension system with $\tau = 33\%$, open economy

	DB system (benchmark)	FL	NDC
Interest rate	5.2%	5.2%	5.2%
Capital per labor	-	22.1%	2.7%
Labor	-	-6.2%	-0.7%
Consumption equivalent ge. (ω)	-	0.4%	0.2%
- change due to uncertainty (ω_{unc})	-	5.3%	0.7%
- change due to levels (ω_{lev})	-	-4.5%	-0.4%

2.6 Cross country pension systems arrangements

This section presents a cross-country analysis of the link between pension contributions and benefits. Figure 2.6 shows the relationship between the contribution to the pay-as-you-go pension system in a certain country and the link between pension contribution and benefits proxied by the pension progressivity index. Pension systems with a high progressivity index have a weak or even negative link between life-time earnings and pension benefits, while systems with low progressivity index have a tight link between life-time earnings (or contributions) and pension benefits. For example, UK has a FL pension system and a pension progressivity of 60%, while Italy has a NDC system and a pension progressivity of 4%. Figure 2.7 presents the relationship between pension progressivity and the increase in disposable income inequality across the life-time.

To construct the figures, I use data for the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Ireland, Italy, Japan, Germany, Sweden, Spain, New Zealand, UK, US. The pension progressivity index is taken from OECD Pensions at a Glance (2005)[7]. The pension contribution is collected from the OECD Pension Outlook (2012). I compute the increase in disposable income inequality during the working life using data from LIS (Cross-National Data Center in Luxembourg). I determine the coefficient of variation as a ratio between the standard deviation of disposable income and the mean of disposable income. I average the resulting coefficients of variation over the years available for each country. The

[7]In the paper I analyze only the implications of pension benefit progressivity. However, the OECD pension progressivity index also considers the progressivity of pension contributions.

increase in disposable income inequality is computed as the difference between the coefficient of variation of the 45-54 age group and the 25-34 age group.

Figure 2.6 presents the relationship between the progressivity of the pension system and the contribution to the pension system. The figure documents a considerable heterogeneity in the progressivity of pension systems and it indicates a strong negative correlation between pension progressivity and the contribution to the pension system. This agrees with the welfare analysis undertaken in the paper in the following sense. In section 2.5.3, I show that when the contribution to the pension system is higher the welfare difference between the FL system and the NDC system shrinks. A consequence of this result is that the progressivity of the pension system should decrease, moving more towards an NDC system, as the size of pension contribution is higher.

Figure 2.7 shows the relationship between the increase in disposable income inequality across the working life and pension progressivity. The relationship between the two variables is positive, indicating that countries with a larger increase in income inequality across a person's life have more progressive pension arrangements. This also agrees with the welfare analysis undertaken in this paper: countries that have a higher increase in the level of idiosyncratic risk across their life-time should have more progressive pension systems because these type of systems provides individuals with better insurance.

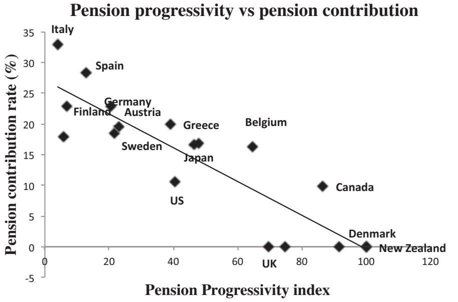


Figure 2.6

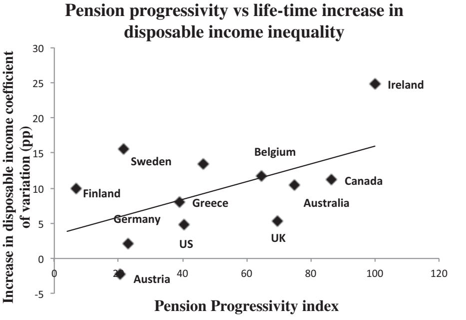


Figure 2.7

Conde-Ruiz and Profeta (2007) explain the negative relationship between the size of the pension system and its progressivity as the outcome of a voting process in economies with different levels of income inequality. In their model, individuals differ in skill and the return on their savings. Under suitable conditions on the return of savings, in countries with high inequality a coalition of low and high income individuals sustain a small and very redistributive PAYG system. In countries with low inequality, the median voter is a middle income individual that sustains a high PAYG pension system with little redistributivity. Koethenbuerger et al. (2008) explain the negative relationship between the size and the progressivity of the pension system also as the outcome of voting on the redistributive character of the pension system when the size of the contributions is fixed. In their model, the trade-off between efficiency and labor supply distortions drives this result. I focus on the insurance properties of the pension system rather than on the redistribution between individuals with different skills. Figure 2.7 indicates that inequality increases during the working life in almost all countries analysed. While the pension system may be an ill-suited mechanism for insuring against shocks that are observable when the individual enters the labor market, it can be a way of insuring against shocks received throughout the working life.

2.7 Conclusion

In this paper I analyze the long-run macroeconomic and welfare implications of the three most widespread types of pension benefits arrangements: the flat benefit (FL), the Notional Defined Contribution (NDC) and the Defined Benefit (DB) pension systems. This paper finds that the FL system leads to lower aggregate labor, but also to lower consumption inequality. As long as the level of idiosyncratic risk is not very high, the FL system also crowds out capital formation less than the DB and the NDC system. The NDC and DB systems are equivalent if there is no population growth. However, with a strictly positive population growth rate, the DB system provides a slightly higher labor supply, less insurance against shocks and crowds out capital formation more than the NDC system.

Comparing the welfare under the three types of pension systems, starting form the benchmark of the US economy, the FL pension system brings the highest welfare. Hence, the higher insurance of idiosyncratic earnings shocks and the lower capital crowding out dominate the higher labor supply distortions that the FL system imposes at both the extensive and intensive margin. Considering a higher contribution rate than under the current US system - 33% instead of 10.6% - I obtain that the FL system still provides the highest welfare but the consumption equivalent compared to the DB and the NDC economies shrinks considerably. Moreover, if one eliminates the welfare impact of the higher capital accumulation under the FL system by considering an open economy framework, the welfare difference between the three pension systems shrinks even further and the FL system provides only a marginally higher welfare.

Appendix 1

Derivations for the equilibrium of the model in Section 2.2

The optimization problem of the household in the model with only 2 overlapping generations is the following:

$$
\begin{aligned}
\max_{c^y, c^o(z), l^o(z), a_t} \quad & u(c^y) + \beta E u(c^o(z), l^o(z)) \\
& c^y + a = w\bar{z}(1-\tau) \\
& c^o(z) = a(1+r) + wzl^o(z)(1-\tau) + b(l^o(z)) \\
& l^o \in [0,1]
\end{aligned}
$$

The first order conditions of the household's problem are:

$$
\frac{-u_l(c^o(z), l^o(z))}{u_c(c^o(z), l^o(z))} = \left(wz(1-\tau) + \frac{\partial b(l^o(z))}{\partial l^o(z)} \right) \tag{2.51}
$$

$$
u_c(c^y) = \beta(1+r)E[u_c(c^o(z), l^o(z))] \tag{2.52}
$$

With the preferences considered in Assumption 1, using (2.51) and the budget constraints of the individuals and government, we find closed form solutions for consumption and labor as a function of savings and prices:

$$
c^y = w\bar{z}(1-\tau) - a
$$

$$
c^o(z) = \begin{cases} FL : \eta(a(1+r) + wz(1-\tau) + w\tau l) \\ NDC : \eta(a(1+r) + wz + w\bar{z}\tau(1+n)) \\ DB : \eta(a(1+r) + wz(1-\tau+\tau\epsilon) + w\bar{z}\tau\epsilon) \end{cases}
$$

$$
l^o(z) = \begin{cases} FL : \eta - (1-\eta)\frac{a(1+r)+w\tau l}{wz(1-\tau)} \\ NDC : \eta - (1-\eta)\frac{a(1+r)+w\bar{z}\tau(1+n)}{wz} \\ DB : \eta - (1-\eta)\frac{a(1+r)+w\bar{z}\tau\epsilon}{wz(1-\tau+\tau\epsilon)} \end{cases}
$$

Dividing the first order conditions of the firm's problem (2.11) and (2.12), we get a relation for the return on capital that we substitute in the formulas for consumption and labor supply:

$$1 + r = \frac{\alpha}{1-\alpha}\frac{l}{a}w$$

We obtain:

$$c^y = w\bar{z}(1-\tau) - a \tag{2.53}$$

$$c^o(z) = \begin{cases} FL : \eta w(\frac{\alpha}{1-\alpha}l + z(1-\tau) + \tau l) \\ NDC : \eta w(\frac{\alpha}{1-\alpha}l + z + \bar{z}\tau(1+n)) \\ DB : \eta w(\frac{\alpha}{1-\alpha}l + z(1-\tau+\tau\epsilon) + \bar{z}\tau\epsilon) \end{cases} \tag{2.54}$$

$$l^o(z) = \begin{cases} FL : \eta - (1-\eta)\frac{\frac{\alpha}{1-\alpha}l+\tau l}{z(1-\tau)} \\ NDC : \eta - (1-\eta)\frac{\frac{\alpha}{1-\alpha}l+\bar{z}\tau(1+n)}{z} \\ DB : \eta - (1-\eta)\frac{\frac{\alpha}{1-\alpha}l+\bar{z}\tau\epsilon}{z(1-\tau+\tau\epsilon)} \end{cases} \tag{2.55}$$

The expressions for consumption are substituted in (2.52). We solve for the level of assets:

$$a = \frac{\beta\Gamma}{\beta\Gamma + \eta}A\bar{z}(1-\tau)(1-\alpha)\left(\frac{a}{l}\right)^{\alpha} \tag{2.56}$$

Using the capital market clearing condition $a = K$ and the notations $s = \frac{\beta\Gamma}{\beta\Gamma+\eta}$, $\Gamma^{FL} = \frac{\alpha}{1-\alpha}lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z(1-\tau)+\tau l}\right]$, $\Gamma^{NDC} = \frac{\alpha}{1-\alpha}lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z+\bar{z}\tau(1+n)}\right]$ and $\Gamma^{DB} = \frac{\alpha}{1-\alpha}lE\left[\frac{1}{\frac{\alpha}{1-\alpha}l+z(1-\tau+\tau\epsilon)+\bar{z}\tau\epsilon}\right]$ we obtain a closed form solution for the capital to labor ratio k:

$$k = \left(\frac{As\bar{z}(1-\tau)(1-\alpha)}{l}\right)^{\frac{1}{1-\alpha}} \tag{2.57}$$

We aggregate individual labor supply expressed by the relations in (2.55) and equate it to aggregate labor supply:

$$l = \bar{z}(1+n) + \int zl^o(z)dF(z)$$

From here we obtain the expressions of aggregate labor supply under all three types of pension systems summarized in (2.13), (2.14), (2.15).

Proof of Proposition 1

1. I compute the derivatives of (2.13), (2.14), (2.15) with respect to τ and obtain:

$$
\begin{aligned}
\frac{\partial l^{FL}}{\partial \tau} &= -\frac{\bar{z}(1-\alpha)(1+n+\eta)(1-\eta)}{(1-\alpha\eta+(1-\alpha)\eta\tau)^2} \\
\frac{\partial l^{NDC}}{\partial \tau} &= -\frac{\bar{z}(1-\alpha)(1-\eta)(1+n)}{1-\alpha\eta} \\
\frac{\partial l^{DB}}{\partial \tau} &= \frac{\bar{z}(1-\alpha)((1+n+\eta)(\epsilon-1)-\epsilon(1-\eta))(\tau(\epsilon-1)(1-\alpha)+1-\alpha\eta)}{(\tau(\epsilon-1)(1-\alpha)+1-\alpha\eta)^2} - \\
&- \frac{\bar{z}(1-\alpha)(\epsilon-1)(1-\alpha)((1+n+\eta)(1+\tau(\epsilon-1))-\tau\epsilon(1-\eta))}{(\tau(\epsilon-1)(1-\alpha)+1-\alpha\eta)^2} = \\
&- \frac{\bar{z}(1-\alpha)(1-\eta)(\epsilon-\alpha\eta(2\epsilon-1)-\alpha(n+1)(\epsilon-1))}{(\tau(\epsilon-1)(1-\alpha)+1-\alpha\eta)^2}
\end{aligned}
$$

The contribution to the pension system distorts labor supply under all three types of pension system: an increase in τ leads to lower aggregate labor supply. I compare the influence of the pension contribution on labor supply at the marginal introduction of the pension system:

$$
\begin{aligned}
\left.\frac{\partial l^{FL}}{\partial \tau}\right|_{\tau=0} &= -\frac{\bar{z}(1-\alpha)(1+n+\eta)(1-\eta)}{(1-\alpha\eta)^2} \\
\left.\frac{\partial l^{NDC}}{\partial \tau}\right|_{\tau=0} &= -\frac{\bar{z}(1-\alpha)(1-\eta)(1+n)}{1-\alpha\eta} \\
\left.\frac{\partial l^{DB}}{\partial \tau}\right|_{\tau=0} &= -\frac{\bar{z}(1-\alpha)(1-\eta)(\epsilon-\alpha\eta(2\epsilon-1)-\alpha(n+1)(\epsilon-1))}{(1-\alpha\eta)^2} \\
\epsilon|_{\tau=0} &= \frac{\bar{z}(1+n+\eta)-\frac{\alpha}{1-\alpha}l^{DB}|_{\tau=0}}{\bar{z}(1+\eta)-\frac{\alpha}{1-\alpha}l^{DB}|_{\tau=0}} = \frac{(1+\eta+n)(1-\alpha)}{(1+\eta)(1-\alpha\eta)-\alpha(1-\eta)(1+n+\eta)}
\end{aligned}
$$

From here, it is straightforward to show that $abs\left(\left.\frac{\partial l^{FL}}{\partial \tau}\right|_{\tau=0}\right) < abs\left(\left.\frac{\partial l^{NDC}}{\partial \tau}\right|_{\tau=0}\right) < abs\left(\left.\frac{\partial l^{DB}}{\partial \tau}\right|_{\tau=0}\right)$.

2. I measure the difference in log consumption of two individuals with shocks $z^H = \bar{z}(1+\sigma)$ and $z^L = \bar{z}(1-\sigma)$ and interpret it as consumption inequality. For simplicity I assume that $\bar{z} = 1$

$$c_{inequality} = log(c^H) - log(c^L)$$

Using the closed for solutions for c^H and c^L obtained in the proof of Proposition 1, we obtain the following relations:

$$c^{FL}_{inequality} - c^{NDC}_{inequality} =$$
$$-\frac{2(1-\alpha)\eta\sigma\tau(\alpha(n+1)(1-\tau)+(n+1)\tau+1)}{(\alpha(\eta\sigma+n+1)+(1-\alpha)(n+1)\tau+1-\sigma)((\alpha(1-\tau)+\tau)(\eta\sigma+n+1)+1-\sigma)} < 0$$
$$c^{DB}_{inequality} - c^{NDC}_{inequality} =$$
$$\frac{2(1-\alpha)\sigma\tau((\epsilon-1)(2\alpha\eta+(1-\tau)(\alpha n-1+\alpha)+n\tau)+n(1-\alpha\eta))}{(\alpha(\eta\sigma+n+1)+(1-\alpha)(n+1)\tau+1-\sigma)(\alpha\eta\sigma+(1-\alpha)\tau(\sigma+(2-\sigma)\epsilon-1)+\alpha(1+n)+1-\sigma)}$$

The sign of the expression $c^{DB}_{inequality} - c^{NDC}_{inequality}$ depends on the sign of:

$$A = (\epsilon-1)(2\alpha\eta+(1-\tau)(\alpha n-1+\alpha)+n\tau)+n(1-\alpha\eta))$$

It is easy to show that A is increasing in τ $\left(\frac{\partial A}{\partial\tau} > 0\right)$, $A(\tau=0)=0$ (using $\epsilon|_{\tau=0} = \frac{(1+\eta+n)(1-\alpha)}{(1+\eta)(1-\alpha\eta)-\alpha(1-\eta)(1+n+\eta)}$) and $A(\tau=1)>0$. Hence A is positive everywhere.

3. The fact that capital accumulation is lower under the NDC pension system than under the FL system in absence of idiosyncratic productivity shocks ($\sigma = 0$) is straightforward to prove for at all levels of pension contributions. Using (2.16):

$$sgn\left(k^{NDC}_{\sigma=0} - k^{FL}_{\sigma=0}\right) = sgn\left(\left(\frac{k^{NDC}_{\sigma=0}}{k^{FL}_{\sigma=0}}\right)^{1-\alpha} - 1\right) =$$
$$= sgn\left(-\frac{\tau\eta(1+\alpha\beta)(1-\eta)(1+\alpha+\tau(1-\alpha)))}{(1-\alpha\eta-\tau\eta(1-\alpha))(\eta(1+\alpha)+\alpha\beta(1+\eta)+\tau(\eta(1-\alpha)-\alpha\beta(1-\eta)))}\right) = -$$

In order to also prove that the DB pension system promotes less capital accumulation than the FL system, I provide an alternative proof that the NDC system crowds out capital accumulation more than the FL at the marginal introduction of the pension system. Using the logarithm version of relation (2.16), I obtain that, in absence of idiosyncratic productivity shocks ($\sigma = 0$), at the marginal introduction of the pension system, an increase in the contribution to the pension system reduces capital under the NDC pension system more than under the FL pension system:

$$\frac{\partial(ln(k^{NDC}_{\sigma=0,\tau=0}) - ln(k^{FL}_{\sigma=0,\tau=0}))}{\partial\tau} = -\frac{(1-\eta)\eta(\alpha\beta+1)(\alpha+\alpha n+1)}{(1-\alpha)(1-\alpha\eta)(\eta+\alpha(\eta+\beta(\eta+n+1)+\eta n))} < 0$$

I use the logarithm version of relation (2.16) in order to compare capital accumulation under the DB and NDC systems:

$$\frac{\partial(ln(k^{DB}_{\sigma=0,\tau=0}) - ln(k^{NDC}_{\sigma=0,\tau=0}))}{\partial\tau} =$$
$$-\frac{(1-\alpha)(1-\eta)\eta n(\alpha\beta+1)(\alpha(1+n)+1)}{(1+\eta-\alpha\eta(1-n)-\alpha(n+1))(\eta(1+\alpha(1+n))+\alpha\beta(\eta+n+1))} < 0$$

Appendix 2 - Insurance in the baseline model of the US economy

This Appendix presents the differences in the methodology for computing the variance of log income and the variance of log consumption in a number of papers used as reference for the calibration of the model in chapter 2.4.

Heathcote et al. (2010a):

- for the variance of log earnings, the paper uses data from CEX that includes annual labor income at household level. Households that report 0 hours worked but positive income in a particular year are eliminate from the sample. Hence, retired individuals or other non-participants are eliminated from the sample. The results presented in table 2.2 are those obtained using cohort fixed effects.

- for the variance of log consumption of households, CEX data is used as well.

Heathcote et al. (2005):

- for the variance of log earnings, the paper uses PSID data including annual labor income at household level. Households that report less than 520 hours worked in a particular year are eliminated from the sample. Hence, retired individuals or other non-participants are not part of the sample. The results presented in table 2.2 are those obtained using cohort fixed effects.

- for the variance of log consumption of households, CEX data is used.

Guvenen (2007) estimates the profile of the variance of log non-durable goods consumption using CEX data.

Storesletten et al. (2004):

– for the variance of log earnings, they use annual wage earnings plus transfers for the house heads and their spouse from PSID. It is not clear whether retired individuals/non-participants are included in the analysis or not. However, since their increase in variance of log earnings is higher than in other papers, I assume that they include households with retired or non-participant individuals.

– for the variance of log consumption, the data is taken from the CEX.

– "cohort effects" are eliminated using dummy variable regressions.

Appendix 3 - Robustness checks

Table 2.6: Using different taxes to re-balance general budget

	US system (Benchmark)	FL	NDC	DB
τ_k	0.36	0.37	0.363	0.363
Consumption equiv. (ω)	-	1.3%	-1.1%	-1%
τ_c	0.05	0.052	0.0504	0.0503
Consumption equiv. (ω)	-	1.2%	-1.2%	-1.1%
a_2	0.42	0.422	0.4202	0.421
Consumption equiv. (ω)	-	1.1%	-1.1%	-1%
ϕ		0.223	0.222	0.221
Consumption equiv. (ω)	-	1.4%	-1.1%	-1.1%

Chapter 3

Job loss risk and retirement

3.1 Introduction

Earnings losses associated with a job loss shock are largely regarded as transitory because unemployment spells are short in the US. However, workers that have been continuously employed for a long time and lose their job due to events that are out of their control (a phenomenon called in the literature *displacement*) suffer significant and long lasting decreases in earnings. Using administrative data for the state of Pensylvania, Jacobson et al. (1993) conclude that the earnings of high tenure workers that were fired during mass layoffs remain 25% lower even 6 years after displacement. Couch and Placzek (2010) use a more recent set of administrative data for the state of Connecticut and reach the conclusion that displacement costs are slightly lower: around 15% of earnings are lost after 6 six years. The literature attributes this persistent decline in earnings to the loss of the match specific human capital (Ljungqvist and Sargent (1998)) or to the fact that job loss implies falling off the job ladder and climbing back up takes time due to frictions (Jung and Kuhn (2012), Krolikowski (2017)).

In this paper we analyse the implications of job loss (or displacement) risk for the retirement decision. We consider there is a *higher job loss risk* in an economy if there is either i) a higher probability of losing one's job or ii) higher earnings losses following the unemployment spell. Displacement is especially relevant for the labor supply decision of workers that are close to their retirement age. These workers usually have higher tenure, so losing their job would imply substantial earnings losses. In a model, we show that a higher job loss risk lowers the retirement age of workers through three channels. First, anticipating the probability of suffering a permanent decrease in their earnings, workers make higher precautionary savings before the job loss shock can occur and this allows them to retire earlier. Workers that go through a jobless spell are furthermore impacted by the fact that: i) the return from working decreases after the shock offering them incentives to retire earlier (substitution effect), ii) in order to afford the same level of consumption, they must work longer since after the shock wages are lower (income effect). Whether the effect through precautionary savings and the substitution effect dominate the income effect is a quantitative question.

First, we bring evidence that job loss has more severe consequences in terms of earnings as people progress in life and if people spend more time out of employment. Using data from the US Survey of Income and Program Participation (SIPP), we find that workers aged between 50 and 59 years have lower re-employment wages after a non-employment spell than other age categories of workers (20-29 years, 30-39 years, 40-49 years and over 60 years). Moreover, people that spend more time out of employment have lower re-employment wages and, hence, lose more in terms of earnings. Next, we estimate the impact of job loss risk on the retirement age across countries. Considering the evidence that a longer time spent in unemployment lowers a person's subsequent earnings, we consider the level of long term unemployment in a country as a proxy for the size of earnings losses and, hence, for the job loss risk in the economy. We show that a higher incidence of long term unemployment among prime age workers reduces the retirement age and the labor force participation rate of people aged 55-64 years. The impact of long term unemployment on retirement is statistically significant after controlling for different cross-country institutional characteristics such as the size of the pension system, life expectancy, the progressivity of the pension system.

We identify the channels through which a higher job loss risk reduces the retirement age of workers using a life cycle model with displacement risk, incomplete markets and endogenous retirement. We model displacement as a separation from a job that induces permanent earnings losses. As mentioned before, all workers, irrespective of whether they are hit by a displacement shock or not, are affected by the job loss risk through savings. Because there exists the possibility of losing their job, workers raise their precautionary savings and, hence, retire earlier. Moreover, displaced workers are impacted by the jobless spell through opposing income and substitution effects. We show that for low levels of risk aversion ($\sigma \leq 1$) displaced workers retire earlier than workers that are not displaced as the substitution effect dominates the income effect. However, for higher levels of risk aversion ($\sigma > 1$), displaced workers may end up retiring later than workers that are not displaced. Depending on the parameters of the model, the substitution or the income effect may dominate.

Whether displaced workers retire earlier or later than workers that are not displaced is important for the impact of a higher job loss risk on the average retirement age in the economy. A higher job loss probability lowers the retirement age of both types of workers, but the impact on the average retirement age is ambiguous. This is because a higher job loss probability increases the share of displaced workers in the economy and may increase the average retirement age in the economy if displaced workers retire later than workers that are not displaced. Higher earnings losses following displacement lower the retirement age of workers that are not displaced but the impact on the retirement age of the displaced worker and, hence, on the average retirement age is ambiguous. This is because higher earnings losses decrease a displaced worker's labor supply through the substitution effect (lower return from working) but increase her labor supply through the income effect. The overall result depends on the parameters of the model if risk aversion is moderate to high ($\sigma > 1$).

Using a version of the model calibrated for the US economy, we obtain that displaced workers retire earlier than workers that are not displaced, in line with empirical evidence (Hairault et al. (2015)). Also, we find that a higher job loss risk lowers the average retirement age in the economy, in line with our empirical findings using cross-country data. Using the calibrated model, we run a counterfactual experiment in which we eliminate the pay-as-you-go pension system from the US economy and show that in this case displaced workers retire much later than workers that are not displaced and the impact of a higher job loss risk on the average retirement age is smaller. Hence the contribution to the pay-as-you-go pension system lowers the retirement age of displaced workers more than the retirement age of not displaced workers and it also amplifies the impact of a higher job loss risk on the average retirement age in the economy. Specifically, the contribution to the pension system distorts labor supply reinforcing the effect of the lower wage following displacement through the substitution effect. However, since pension benefits are computed as a replacement rate times average life-time earnings, a large part of the contribution to the pension system is returned to the individual as a pension benefit after she reaches the official retirement age and hence, the income effect of the pension contribution is small.

The cross-country variability in retirement age and labor force participation has been largely attributed to differences in pay-as-you-go pension arrangements (Gruber and Wise (2002), Duval (2003), Blöndal and Scarpetta (1998), Wallenius (2013), Erosa et al. (2012)). In the current paper, we show that cross country differences in job loss risk can explain differences in retirement age even after controlling for the characteristics of the pension system. As far as we know, this is a novel finding. It also has important policy implications. It shows that if policy makers aim at increasing the retirement age in the economy, they must not only reform the pay-as-you-go pension arrangements but must also complement them with labor markets policies.

The impact of job loss on the retirement decision has been analysed in only a few papers. Hairault et al. (2010) present evidence that the distance to retirement affects the employment rate of people close to the official retirement age in the presence of labor market frictions. Hairault et al. (2015) bring empirical and model based evidence that unemployed individuals retire earlier than employed individuals. Lalé (2016) shows that job market frictions and economic turbulence can explain most of the decline in the US and European labor force participation of older workers.

Our paper departs from previous literature that relates job loss to retirement in three respects. First, we consider the role of savings and risk aversion for the retirement decision while the other papers consider a risk neutral framework. Risk aversion introduces a new channel through which job loss impacts on the labor supply decision: workers choose to reduce their exposure to displacement shocks by saving more and retiring earlier. Hence, in our model both individuals that are displaced as well as those that are not displaced are impacted by a higher job loss risk. In contrast, in Hairault et al. (2015) the retirement decision of employed individuals is unaffected by the presence of job loss shocks. Second, we show that for an empirically reasonable level of risk aversion $\sigma > 1$, it may no longer be the case that displaced workers retire earlier than workers that are not displaced as found by Hairault et al. (2015). We actually show that in the absence of the pay-as-you-go system displaced workers would retire later. This happens because the income effect of a lower permanent wage following the unemployment spell dominates the substitution effect. Finally, we prove that the existence of a pay-as-you-go system amplifies the impact of job loss risk on the average retirement age in an economy. This is due to the fact that the contribution to the pension system distorts labor supply amplifying the substitution effect of the permanently lower wage but has a small income effect because life-time pension contributions are partially returned to individuals as pension benefits after the official retirement age.

The rest of the paper is organized as follows. In Section 3.2, we present stylized facts regarding job loss risk and earnings losses associated with job loss across the life-cycle and how these impact on the retirement age across countries. We show the mechanisms through which a higher job loss risk impacts on the effective retirement age in a life-cycle model in Section 3.3. In section 3.4, we extend the model by including a pay-as-you-go pension system and calibrate it on the US economy in order to quantify the impact of job loss risk on the retirement age. Conclusions are comprised in Section 3.5.

3.2 Stylized facts

In this section we present a number of stylized facts regarding job loss risk using both worker level data and cross-country data. In section 3.2.1, using the US Survey of Income and Program Participation (SIPP), we determine the job loss probabilities and the impact of job loss on subsequent earnings for different age categories of workers. We show that re-employment wages following a job loss shock are lower as workers progress in life and, hence, in their career. The highest losses in terms of re-employment wages are registered in the case of workers aged between 50 and 59 years, exactly the years preceding the official retirement age. We also show that job loss has more severe consequences in terms of lower re-employment wages the more time people spend out of employment. In conclusion, job loss shocks have a significant impact on people's earnings if they are older and if re-employment is gained after more than 6 months. We use the estimated job loss probability and the decrease in re-employment wages to calibrate the model in section 3.4.

Taking into consideration the fact that costs associated with job loss are higher for people who spend more time in unemployment, in section 3.2.2 we determine the impact of long term unemployment on the average age at which people retire in an economy. We employ cross-country OECD data, controlling for institutional characteristics of the countries. We reach the conclusion that higher long term unemployment lowers the retirement age in the economy.

3.2.1 Evidence on displacement costs from the SIPP

In this section we use the US Survey of Income and Program Participation (SIPP) to provide evidence that re-employment wages are lower as workers progress in life and, hence, in their career and as they spend more time out of employment. The SIPP is a collection of non-overlapping panels, each of them being named after the first year comprised in the survey. In our analysis, we use the 1996, 2001, 2004 and 2008 panels. The advantage of using the SIPP is that interviewed individuals are followed throughout the whole duration of the panel[1]. Respondents are asked once in four months a set of questions which include the weekly labor market status and the monthly labor earnings since the previous date of the interview. This allows us to identify individuals experiencing non-employment spells, the duration of the spells and the change in wages between the re-employment job and the job before separation.

The sample selection for the spells in our analysis is similar to the one in Fujita and Moscarini (2013). In order to average out the noise in the data coming from the so called "seam" bias[2], we grouped spells with duration lower than 12 months in three months bins and spells with duration higher than 12 months in one bin. We consider individuals that separate from their current employer into unemployment and that we see returning in employment before the end of the panel. We obtain a total of 7838 spells. For these individuals we compute how much the hourly wage changes at re-employment compared to the hourly wage in the month before the separation occurs. To avoid disproportionately higher wage differences on longer spells due to increases in prices, we deflate wages using the Personal Consumption Expenditure deflator.

We regress the change in hourly wage after the non-employment spell on a number of observable characteristics - calendar year, education, gender, race and industry - and a dummy indicating the age group of the worker: less than 20 years, 20-29 years, 30-39 years, 40-49 years, 50-59 years and over 60 years. All wage statistics are computed using longitudinal weights. The result of the estimation is presented

[1]In our sample, the shortest panel is 2001 having a length of 36 months and the longest is the 2008 having a length of 60 months

[2]The SIPP data exhibits sudden changes once in four months coming from the fact that respondents are biased to include important events like wage changes or labor market transitions in the last reference month of the interview.

Table 3.1: Wage change at re-employment by age group

	Coefficient	t-statistic
Constant	0.0765**	(2.53)
Age group 20-29 years	0.00109	(0.06)
Age group 30-39 years	-0.0364**	(-2.09)
Age group 40-49 years	-0.0566***	(-3.21)
Age group 50-59 years	-0.0573***	(-3.05)
Age group over 60 years	-0.0303	(-1.43)
No. of observations	7838	
Adjusted R^2	0.015	

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

in table 3.1[3]. Workers aged between 20-29 years do not suffer any wage losses following a separation. As workers progress in their life and, hence, in their career, the drop in re-employment wages after a separation increases. The maximum loss in terms of re-employment wages is 5.73% for workers in the age group 50-59 years. An economic explanation for the increase in wage losses with age relies on the fact that age is highly correlated with tenure. A higher tenure implies a higher match specific human capital accumulated by a worker at his current employer. Once they become unemployed people lose this match specific human capital (see Jung and Kuhn (2012) and Krolikowski (2017)).

Next, we determine the impact of the time spent out of employment on the size of the wage change at re-employment. To this end, we interact the dummy for each age group with a dummy indicating the time passed until returning to work (d). Table 3.2 presents the results. We obtain that for workers aged less than 30 years, spending more time out of employment does not lead to a higher wage loss. However, as workers progress in their career, wage losses increase with the time spent out of employment. The most affected are workers aged between 50-59 years: their re-employment wages are 12.8% lower if they are out of employment for 6 to 9 months, 22.5% lower if they are out of employment for 9 to 12 months and 26.7% lower if they are out of employment for more than a year.

In conclusion, our analysis indicates that earnings losses after a non-employment

[3]In order to economize space, both in 3.1 and 3.2, we only report the estimated coefficients and t-statistics for our variables of interest: age groups and the interaction between age groups and time spent in unemployment.

Table 3.2: Wage loss at re-employment by age group and time spent in non-employment

	Coefficient	t-statistic
Constant	0.0608^{*}	(1.90)
Under 20 years $\times$ $3 < d \leq 6$	0.0399	(1.14)
Under 20 years $\times$ $6 < d \leq 9$	0.0012	(0.02)
Under 20 years $\times$ $9 < d \leq 12$	0.0614	(0.98)
Under 20 years $\times$ $d > 12$	0.0218	(0.42)
Age group 20-29 years $\times$ $0 < d \leq 3$	0.0248	(1.16)
Age group 20-29 years $\times$ $3 < d \leq 6$	-0.00312	(-0.12)
Age group 20-29 years $\times$ $6 < d \leq 9$	-0.0488	(-1.26)
Age group 20-29 years $\times$ $9 < d \leq 12$	-0.0215	(-0.48)
Age group 20-29 years $\times$ $d > 12$	0.0396	(1.04)
Age group 30-39 years $\times$ $0 < d \leq 3$	-0.00504	(-0.23)
Age group 30-39 years $\times$ $3 < d \leq 6$	-0.032	(-1.19)
Age group 30-39 years $\times$ $6 < d \leq 9$	-0.0724^{**}	(-2.00)
Age group 30-39 years $\times$ $9 < d \leq 12$	-0.0712	(-1.56)
Age group 30-39 years $\times$ $d > 12$	-0.0705^{*}	(-1.95)
Age group 40-49 years $\times$ $0 < d \leq 3$	-0.0145	(-0.66)
Age group 40-49 years $\times$ $3 < d \leq 6$	-0.0369	(-1.33)
Age group 40-49 years $\times$ $6 < d \leq 9$	-0.136^{***}	(-3.71)
Age group 40-49 years $\times$ $9 < d \leq 12$	-0.235^{***}	(-4.81)
Age group 40-49 years $\times$ > 12	-0.122^{***}	(-3.41)
Age group 50-59 years $\times$ $0 < d \leq 3$	-0.00287	(-0.12)
Age group 50-59 years $\times$ $3 < d \leq 6$	-0.0183	(-0.61)
Age group 50-59 years $\times$ $6 < d \leq 9$	-0.128^{***}	(-2.99)
Age group 50-59 years $\times$ $9 < d \leq 12$	-0.225^{***}	(-3.74)
Age group 50-59 years $\times$ $d > 12$	-0.267^{***}	(-6.62)
Age group over 60 years $\times$ $0 < d \leq 3$	0.0131	(0.50)
Age group over 60 years $\times$ $3 < d \leq 6$	-0.0323	(-0.85)
Age group over 60 years $\times$ $6 < d \leq 9$	-0.0432	(-0.65)
Age group over 60 years $\times$ $9 < d \leq 12$	-0.0789	(-0.90)
Age group over 60 years $\times$ $d > 12$	-0.303^{***}	(-4.70)
No. of observations	7838	
Adjusted R^2	0.03	

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

spell can be quite sizable. We determine what is the probability of going through a non-employment spell that entails statistically significant wage losses at re-employment.

Table 3.3: Probability of suffering a significant wage loss by age group

	Prob. of wage loss	Min. wage loss	Max. wage loss
30-39 years	3.18%	7%	7%
40-49 years	2.67%	12.2%	23.5%
50-59 years	2.4%	12.8%	26.7%
Over 60	0.25%	30.3%	30.3%

As the regression summarized in table 3.2 shows, people register significant wage losses if they are older than 30 years and spend more than 6 months out of employment. We divide the number of workers that suffer a non-employment spell that leads to statistically significant wage losses to the number of people in that age category. Table 3.3 reports the probability of suffering statistically significant wage losses following an out of employment spell for each age group. We also summarize for each age group the minimum and the maximum wage loss as determined through the regression presented in table 3.2. The probability of being out of employment for more than 6 months decreases as people progress in their careers. However, the potential wage losses associated with being out of employment increase.

3.2.2 Cross country analysis of retirement decisions

Considering our finding from the previous subsection that people going through longer non-employment spells suffer higher wage losses at re-employment, in this section we analyze whether higher incidence of long term unemployment in an economy and, hence, higher earnings losses associated to a job loss spell impact on the retirement age of workers in that economy. We document the relationship between long term unemployment and retirement age for a number of 43 countries for which job market data is available from the OECD. All the data is averaged for the period 1995-2015.

We analyse two variables that can reveal differences in cross-country retirement decisions. The first variable is the difference between the average effective retirement age (AERA) in the economy and the official retirement age (ORA). As Figure 3.1 shows, there is considerable heterogeneity in the way people retire compared to the age that they are first entitled to draw pension benefits: in some countries individuals retire on average earlier while in others they postpone retirement after the moment that they are entitled to receive their pension benefits. The second variable that we analyse is the labor force participation rate of people aged 55-64 years. Figure 3.2 shows that this variable also exhibits considerable cross-country heterogeneity.

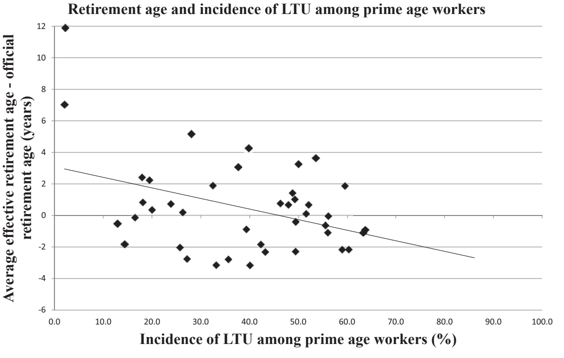


Figure 3.1

The literature starting with Gruber and Wise (2002) explained these differences in the labor supply of old people through the incentives embedded in the pension system arrangements. In this paper we argue that differences in labor market characteristics also have an important role in explaining the cross-country heterogeneity in labor force participation of individuals close to the retirement age. More specifically, we argue that the average effective retirement age is lower in countries where job loss has more severe consequences for individuals in terms of life-time earnings costs.

We take the incidence of long term unemployment (LTU) among prime age workers as a proxy for the cross-country differences in expected earnings losses following an unemployment spell[4]. We consider that a higher incidence of LTU indicates higher earnings losses following an unemployment spell coming from three potential sources: i) a lower job finding rate, ii) more time during which unemployment benefits are the only source of income, iii) lower re-employment wages since these decrease with the time spent in unemployment (following the evidence in Section 3.2.1).

[4]While it would be perhaps better to use the incidence of long term unemployment among old people as a proxy, we believe that this measure is susceptible to reverse causality. Specifically, people that are close to their retirement age and become unemployed have an incentive to remain long term unemployed until they reach the official retirement age.

Figure 3.1 indicates a negative relation between the incidence of LTU and the difference between effective retirement age and official retirement age. Figure 3.2 shows that a higher incidence of LTU is also negatively correlated with the participation rate of people aged 55-64 years.

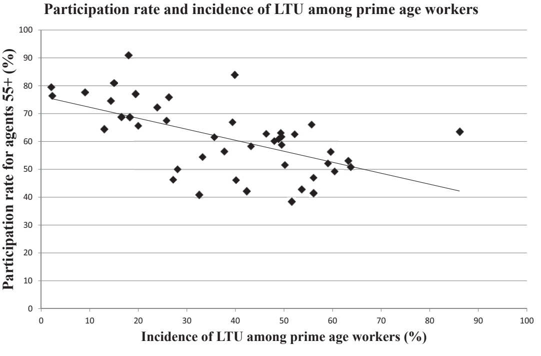


Figure 3.2

We test whether there is a statistically significant relationship between the retirement age or the labor force participation rate of people aged 55-64 years, on the one hand, and job loss risk, on the other hand, by running a cross-country regression. We use data for 43 countries. Not all variables are available for all countries, hence, the number of observations varies from one model to the other.

As explained variable we consider in turn, the difference between the actual average retirement age and the official retirement age (RA) and the participation rate of people aged between 55 and 64 years (PRA).

We proxy the job loss risk by the incidence of long term unemployment of prime age workers (LTU_PAW). We use as control variables the replacement rate of the pension system (RR), the contribution to the pay as you go pension system (τ) and the official retirement age (ORA). Life expectancy and the progressivity of the pension system were not found to be significant and, hence, were omitted from the results.

Tables 3.4 and 3.5 present the results of the estimation. A higher incidence of long-term unemployment lowers the average effective retirement age and the labor force participation of people close to the retirement age and the impact is statistically significant.

The contribution to the pension system and the pension benefit replacement rate have a negative and statistically significant impact on the participation rate. Another variable that helps in explaining the retirement decision is the official retirement age: a higher official retirement age leads to a higher labor force participation among people aged 55-64 years.

Table 3.4: Explained variable: participation rate of people 55-64 years

	(1)	(2)	(3)	(4)
ORA	1.441**	2.410***		
	(0.644)	(0.694)		
τ	-0.382**		-0.450***	
	(0.160)		(0.165)	
ltu_paw	-0.308***	-0.287***	-0.351***	-0.426***
	(0.0915)	(0.0996)	(0.0938)	(0.106)
rr		-0.198***		-0.213**
		(0.0719)		(0.0836)
Constant	-13.36	-69.87	80.94***	89.54***
	(42.30)	(46.23)	(3.449)	(6.341)
Observations	43	34	43	34
Adjusted R^2	0.530	0.572	0.483	0.419

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

ORA represents the official retirement age, τ represents the contribution rate to PAYG pension, ltu_paw is long term unemployment among prime age workers, rr replacement rate of the PAYG pension system.

Table 3.5: Explained variable: difference between effective retirement age and official retirement age

	(1)	(2)
τ	0.0152	
	(0.0498)	
ltu_paw	-0.0714**	-0.0660**
	(0.0298)	(0.0310)
rr		-0.0253
		(0.0244)
Constant	3.021**	4.572**
	(1.114)	(1.848)
Observations	41	34
Adjusted R^2	0.105	0.116

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

τ represents the contribution rate to PAYG pension, *ltu_paw* is long term unemployment among prime age workers, *rr* replacement rate of the PAYG pension system.

In conclusion, the results confirm a statistically significant negative relation between LTU and the retirement age/participation rate of individuals aged 55-64 years. The relationship is significant although we control for the size of the pension system that is considered as the most important explanatory factor for cross country retirement decisions. In section 3.3 we build a model that can rationalize the fact that a higher job loss risk leads to a lower average effective retirement age.

3.3 The model

We analyse the impact of job loss risk on an individual's retirement decision in a life-cycle model with endogeneous labor supply as in Ljungvist and Sargent (2014). We extend the model to incorporate the possibility of a displacement shock in a certain moment of a worker's career in the same way as in Rogerson and Schindler (2002).

3.3.1 The economy

Time is continuous. Each period, a cohort of mass 1 of workers enters the labor market. Everyone dies with certainty in period $t = T$ of their life. We consider that people enter the labor market and die with no assets, i.e. $a_0 = a_T = 0$. We adopt a partial equilibrium framework in which the interest rate r and the wage w are fixed.

We consider that at a certain moment K of their career workers can be hit by a job loss shock with probability p. We will call p the *job loss or displacement probability*. After the shock, the per period earnings of the individual are reduced to $\mu w < w$, where w is the per period wage obtained while working. For simplicity, we normalize wages: $w = 1$. We set $\mu < 1$ in order to capture the reduction in earnings following a job loss shock. Consequently, $1 - \mu$ is the *cost associated with the involuntary job loss*. The fact that $\mu < 1$ is due to the following: i) individuals have to wait for a job opportunity and this depends on the job finding rate of the economy; ii) while unemployed, individuals receive an unemployment benefit that is lower than the wage received before the job loss; iii) re-employment wages are lower than the wages received before the job loss shock. The fact that re-employment wages are lower and the wage loss increases as people progress in life and, hence, in their career, is based on the observations we made using the SIPP in section 3.2.1. An economic interpretation of this stylized fact is that workers lose their match specific human capital when they become unemployed and this match specific human capital is higher the more people advance in their career (see Jung and Kuhn (2012), Krolikowski (2017)).

Markets are incomplete, there is no insurance available against the risk of becoming displaced. Hence people must self insure through savings.

3.3.2 Workers

At each moment t people decide how much to consume c_t and save a_t and whether to work or not $n_t \in \{0, 1\}$. If they work in period t, individuals receive a wage equal to 1 in the case they were not hit by the displacement shock and with μ in the case they were hit by a displacement shock. If people don't work in period t, they finance the consumption from their accumulated savings. In section 3.4, we will consider the possibility of having a pension system in the economy that provides a per period pension benefit after the official retirement age.

The preferences of workers are represented by a utility function that is separable in consumption and labor:

$$U_t = u(c_t) - Bn_t = \frac{c_t^{1-\sigma}}{1-\sigma} - Bn_t \tag{3.1}$$

where B represents the disutility of supplying labor in a certain period.

We write the worker's optimisation problem in terms of the time spent in the labor market (or retirement age) denoted by R. People are hit by a job loss shock with probability p in which case they retire at age R^s. With probability $1-p$, they do not suffer a job loss shock and then they retire at age R^{ns}. Formally, the problem that each individual solves is:

$$\max_{\{c_t, c_t^s, c_t^{ns}, R^s, R^{ns}\}} \int_0^K e^{-\beta t} u(c_t) dt - B \int_0^K e^{-\beta t} dt + p \left(\int_K^T e^{-\beta t} u(c_t^s) dt - B \int_K^{R^s} e^{-\beta t} dt \right)$$

$$+ (1-p) \left(\int_K^T e^{-\beta t} u(c_t^{ns}) dt - B \int_K^{R^{ns}} e^{-\beta t} dt \right) \tag{3.2}$$

$$s.t. \int_0^K e^{-rt} c_t dt + \int_K^T e^{-rt} c_t^{ns} dt = \int_0^{R^{ns}} e^{-rt} dt \tag{3.3}$$

$$\int_0^K e^{-rt} c_t dt + \int_K^T e^{-rt} c_t^s dt = \int_0^K e^{-rt} dt + \int_K^{R^s} \mu e^{-rt} dt \tag{3.4}$$

where β is the per period discount rate, c_t represents consumption in each period before the job loss shock can hit, c_t^s is the consumption of a person affected by a job loss shock at moment K, and c_t^{ns} is the consumption of a person that is not hit by the job loss shock.

Relation (3.3) represents the life-time budget constraint of a person that is not hit by the displacement shock, while (3.4) represents the life-time budget constraint of a person that suffers a displacement shock at time K. In order to obtain the life-time budget constraints, we use the fact that people enter the labor market and die with no assets $a_0 = a_T = 0$.

All the steps necessary for solving the worker's problem are presented in Appendix 1. In order to be obtain a closed form solution to the model, we impose the following assumption:

Assumption 2. *The interest rate and the discount rate are equal, i.e.* $r = \beta$

Denoting by λ^{ns} the Lagrange multiplier for the life-time budget constraint (3.3) and with λ^s the one of the life-time budget constraint (3.4), the first order conditions with respect to consumption c_t, c_t^s, c_t^{ns} and the retirement age R^s, R^{ns} are:

$$u'(c_t) = (\lambda^{ns} + \lambda^s) \tag{3.5}$$

$$(1-p)u'(c_t^{ns}) = \lambda^{ns} \tag{3.6}$$

$$pu'(c_t^s) = \lambda^s \tag{3.7}$$

$$B = \mu u'(c_{R^s}^s) \tag{3.8}$$

$$B = u'(c_{R^{ns}}^{ns}) \tag{3.9}$$

Due to our assumption that the interest rate is equal to the discount rate, consumption is constant in each period before and after the job loss shock hits at time K (relations (3.5), (3.6), (3.7)). Expression (3.5) shows how workers smooth consumption, relating the marginal consumption before the shock occurs with the expected marginal consumption after the shock occurs.

Relations (3.8) and (3.9) are the first order conditions with respect to the retirement age of each type of individual (not displaced and displaced). Workers retire when the marginal benefit from working one more period is equal to the disutility of working measured by B. Following displacement, the wage drops to $\mu < 1$. Hence, the job loss shock lowers the marginal return from working for displaced workers. This substitution effect lowers the labor supply of displaced workers compared to those that are not displaced. However, there is also an income effect. Workers that are displaced have a lower life-time income after the shock hits at time K and this makes them work longer. We will use the closed form solution of the model to determine whether the income or substitution effect dominates and how the displacement shock impacts on the retirement age of displaced workers.

3.3.3 Solution

Replacing the first order conditions (3.5)-(3.9) in the life-time budget constraints of displaced and not displaced workers (3.3) and (3.4), we obtain a closed form solution for the retirement age R^s, R^{ns} and the amount of savings at the moment when the shock occurs a_K.

$$e^{-rR^{ns}} = 1 - \frac{1-e^{-rK}}{\left(B(1-p)+\frac{pB}{\mu}\right)^{\frac{1}{\sigma}}} - \frac{e^{-rK}-e^{-rT}}{B^{\frac{1}{\sigma}}} \tag{3.10}$$

$$e^{-rR^{s}} = \frac{1}{\mu} - \frac{1-\mu}{\mu}e^{-rK} - \frac{1-e^{-rK}}{\mu\left(B(1-p)+\frac{pB}{\mu}\right)^{\frac{1}{\sigma}}} - \frac{e^{-rK}-e^{-rT}}{\mu\left(\frac{B}{\mu}\right)^{\frac{1}{\sigma}}} \tag{3.11}$$

$$a_K = \frac{\left(1-\left(p\frac{B}{\mu}+(1-p)B\right)^{-\frac{1}{\sigma}}\right)(e^{rK}-1)}{r} \tag{3.12}$$

The parameters determining the job loss risk in the economy (p and μ) impact on the retirement age of both displaced R^s and not displaced workers R^{ns}. First, the job loss shock affects the work incentives of both displaced and not displaced workers through savings. Relation (3.12) shows that the job loss shock impacts on the amount of savings people make. More specifically, the presence of the job loss risk makes people save more for precautionary reasons. Because in the presence of displacement shocks workers hold more assets, they need to spend less time in employment to ensure the same old age consumption. Hence, all workers retire earlier in the presence of job loss shocks, including workers that are not displaced.

Second, as we already discussed, in the case of workers that are displaced, the shock impacts the retirement decision through a substitution and an income effect. A permanently lower wage after displacement reduces the return to working and, hence, offers displaced workers an incentive to retire earlier. However, the fact that the income of displaced workers suffers a permanent decline after the shock, offers them incentives to work longer. Hence, the overall impact of the job loss shock on the retirement age of displaced workers is not clear cut.

Using (3.10) and (3.11), we compare the retirement age of workers that are not displaced with the retirement age of displaced workers:

$$\begin{gathered} R^{ns} > R^{s} \text{ iif} \\ (1-e^{-rK})\frac{1-\mu}{\mu}\underbrace{\left(1-B^{-\frac{1}{\sigma}}\left(1-p+\frac{p}{\mu}\right)^{-\frac{1}{\sigma}}\right)}_{>0} + (e^{-rK}-e^{-rT})B^{-\frac{1}{\sigma}}\underbrace{(1-\mu^{\frac{1}{\sigma}-1})}_{\geq 0 \text{ if } \sigma\leq 1} > 0 \end{gathered} \tag{3.13}$$

Workers that are displaced unambiguously retire earlier than those that are displaced only if $\sigma \leq 1$. The job loss shock impacts on the retirement decision of workers that are not displaced only through the fact that they make more precautionary savings and, hence, retire earlier. In the case of workers that are displaced, they are furthermore impacted by the permanently lower wage that they receive after the job loss shock. This has a substitution and an income effect that act in opposite directions: the substitution effect reduces the retirement age while the income effect increases it. When $\sigma \leq 1$ the substitution effect dominates, hence the job loss shock reduces the retirement age.

If $\sigma > 1$, it is no longer clear whether displaced workers retire before those that are not displaced. A higher level of σ implies that people dislike the fluctuation in consumption following the permanent decrease in wages more and prefer adjusting their labor supply to a higher degree. Hence, depending on the rest of the parameters of the model, the income effect can dominate the substitution effect as σ increases above 1 and displaced workers can end up retiring later than workers that are not displaced.

The following parameters of the model are crucial in determining the overall result: i) the timing of the shock K, ii) the disutility of labor supply B and iii) the size of earnings losses following the job loss shock μ. The later in the career the shock can hit (higher K), the less important is the income effect because the lower wage affects displaced workers for a smaller part of their career. Hence, if the job loss shock is expected to impact on workers late in their career, it is more likely that displaced workers retire earlier than not displaced workers. A higher disutility of labor supply B also makes it more likely for displaced workers to retire earlier than not displaced workers. If the disutility of labor supply is high, the fact that the return from working is lower following the job loss shock (the substitution effect) is more important. Finally, the more workers lose in terms of earnings following the job loss shock (the lower is μ), the more important is the substitution effect. In section 3.4 we analyze a calibrated version of the model in which we determine whether the income or substitution effect dominates.

3.3.4 Analysis

In this subsection we analyse the way in which the optimal career length varies with the parameters of the model: the job loss probability p, the cost of undergoing a job loss spell $1-\mu$ and the moment that the job loss shock can occur K. Proposition 2 summarizes the impact of p and μ on the retirement decision of displaced R^s and not displaced workers R^{ns}.

Proposition 2. *A higher job loss probability p:*

1. *decreases the retirement age of all types of workers (lower R^s and R^{ns});*
2. *increases the amount of savings made before the moment when the job loss shock can occur a_K.*

A higher displacement cost (lower μ):

1. *decreases the retirement age of workers that are not displaced R^{ns};*
2. *increases the amount of savings made before the moment when the job loss shock can occur a_K;*
3. *the impact on the retirement age of displaced workers R^s is ambiguous.*

Proof. Using (3.10), (3.11) and (3.12):

$$\frac{\partial R^{ns}}{\partial p} = -\frac{e^{rR^{ns}}}{r} B^{-\frac{1}{\sigma}} \frac{1-e^{-rK}}{\sigma} \left(1-p+\frac{p}{\mu}\right)^{-1-\frac{1}{\sigma}} \frac{1-\mu}{\mu} < 0 \tag{3.14}$$

$$\frac{\partial R^{s}}{\partial p} = -\frac{e^{rR^{s}}}{r} B^{-\frac{1}{\sigma}} \frac{1-e^{-rK}}{\mu\sigma} \left(1-p+\frac{p}{\mu}\right)^{-1-\frac{1}{\sigma}} \frac{1-\mu}{\mu} < 0 \tag{3.15}$$

$$\frac{\partial a_K}{\partial p} = \frac{e^{rK}-1}{r\sigma} \left(\frac{pB}{\mu} + (1-p)B\right)^{-\frac{1}{\sigma}-1} \frac{B(1-\mu)}{\mu} > 0 \tag{3.16}$$

$$\frac{\partial R^{ns}}{\partial \mu} = \frac{e^{rR^{ns}}}{r} B^{-\frac{1}{\sigma}} \frac{1-e^{-rK}}{\sigma} \left(1-p+\frac{p}{\mu}\right)^{-1-\frac{1}{\sigma}} \frac{p}{\mu^2} > 0 \tag{3.17}$$

$$\frac{\partial R^{s}}{\partial \mu} = \frac{e^{rR^{s}}}{r} \left[\underbrace{\frac{1-e^{-rK}}{\mu^2} \left(1 - B^{-\frac{1}{\sigma}} \frac{1-p+\frac{p(\sigma-1)}{\mu\sigma}}{\left(1-p+\frac{p}{\mu}\right)^{\frac{1}{\sigma}+1}} \right)}_{\omega_1>0} + \underbrace{\mu^{\frac{1}{\sigma}-1}(e^{-rK}-e^{-rT})\frac{1-\sigma}{\sigma}}_{\omega_2\leq 0 \text{ iif } \sigma\geq 1} \right] \tag{3.18}$$

$$\frac{\partial a_K}{\partial \mu} = -\frac{e^{rK}-1}{r\sigma} \left(\frac{pB}{\mu} + (1-p)B\right)^{-\frac{1}{\sigma}-1} \frac{pB}{\mu^2} < 0 \tag{3.19}$$

□

If the probability of a job loss shock is higher (higher p), both displaced and not displaced workers retire earlier. The intuition for this effect is as follows. Faced with a higher probability of losing their jobs, in an incomplete markets economy, individuals self insure by saving more: a increases at each point in time before K. This implies that people consume less before the shock can occur at K. Because people smooth consumption across their life-time (relation (3.5)), this means that they also consume less after K. This is consistent with workers retiring earlier. In conclusion, people reduce their exposure to the job loss loss shock by both saving more and retiring earlier.

If the cost associated with the job loss increases (lower μ), workers that are not displaced retire earlier because they make higher precautionary savings. For displaced workers, the impact is ambiguous because of two offsetting effects. Permanently lower earnings after the job loss shock make people retire earlier through a substitution effect and later through the income effect. We discuss which of the two effects is expected to dominate depending on the parameters of the model (σ, K, B, μ) by analyzing equation (3.18) from the proof of Proposition 2. The way that the result depends on the parameters of the model is similar to how the difference in retirement age between the displaced and not displaced workers depends on the parameters of the model (relation (3.13)).

The overall impact of a lower μ on the retirement age of displaced agents R^s is determined by the terms ω_1 and ω_2 of relation (3.18). When $\sigma \leq 1$, both terms are positive. Hence, the derivative of R^s with respect to μ (relation (3.18)) is positive, implying that a higher cost of displacement (lower μ) decreases the retirement age of displaced workers. However, one can argue that this is not a plausible case because most of the literature uses values for risk aversion σ greater than 2.

For $\sigma > 1$, the term ω_1 is positive, while ω_2 is negative, so the overall impact of μ on R^s is not clear and depends on the other parameters of the model, with the most important being K, μ, B. If the shock hits late in a person's career (high K), the positive term ω_1 is expected to dominate the negative term ω_2 and, hence, the derivative from relation (3.18) will be positive. Intuitively, when the shock hits late in a person's career, the income effect is less important because it affects the workers for a shorter part of their career. Consequently, if the job loss shock hits late in a person's career, a higher displacement cost (lower μ) leads to a lower retirement age for the displaced worker.

If displacement costs are high (μ is low), the term $\frac{1}{\mu^2}$ of ω_1 dominates the term $\mu^{\frac{1}{\sigma}-1}$ of ω_2 and the derivative from relation (3.18) is positive. Intuitively, when earnings costs following displacement are high, the substitution effect dominates and a decrease in μ lowers the retirement age of displaced workers.

Finally, the disutility from supplying labor (B) also matters. A higher B increases the positive term ω_1 and leaves the negative term ω_2 unchanged. Hence, the derivative from (3.18) is expected to be positive. Intuitively, a high disutility from labor gives more importance to the lower return from working after the job loss shock. Consequently, with a high disutility of labor, the substitution effect dominates and an increase in the costs associated with job loss (lower μ) leads to a lower retirement age in the case of displaced workers.

The implication of a higher p or a lower μ for the overall retirement age in the economy is not clear. We define the average effective retirement age ($AERA$) and determine how it changes with p and μ:

$$AERA = (1-p)R^{ns} + pR^{s} \tag{3.20}$$

$$\frac{\partial AERA}{\partial p} = \underbrace{(1-p)\frac{\partial R^{ns}}{\partial p} + p\frac{\partial R^{s}}{\partial p}}_{<0} + \underbrace{R^{s} - R^{ns}}_{>0 \text{ if } \sigma \geq 1} \tag{3.21}$$

$$\frac{\partial AERA}{\partial \mu} = (1-p)\underbrace{\frac{\partial R^{ns}}{\partial \mu}}_{>0} + p\underbrace{\frac{\partial R^{s}}{\partial \mu}}_{<0 \text{ if } \sigma \geq 1} \tag{3.22}$$

An increase in p has two effects on aggregate retirement age: i) both displaced and not displaced workers retire earlier (first part of relation (3.21)) and ii) the proportion of displaced workers to not displaced workers in the economy increases (second part of relation (3.21)). As long as displaced workers retire earlier than not displaced workers, both effects have the same impact of lowering the aggregate retirement age. However, if displaced workers retire later than not displaced workers, a higher job loss probability p may increase the aggregate retirement age in the economy because displaced workers become a higher share of the population.

As Proposition 2 shows, if the costs associated with displacement increase (μ decreases), workers that are not displaced retire earlier. However, the impact on the retirement age of displaced workers is unclear. Consequently, the impact on the aggregate retirement age is also ambiguous, depending on the relative impact of μ on the retirement age of displaced and not displaced workers.

In order to clarify the impact of the higher job loss risk on the retirement age of each type of worker and on the aggregate, in section 3.4 we analyze a version of the model calibrated on the US economy.

Finally, we investigate how the timing of the shock impacts on the optimal career length. As we saw in proposition 2, the possibility of a job loss shock at a certain time period K makes people accumulate more savings. Due to the fact that they accumulate more savings than in the absence of job loss shocks, people can retire earlier. If an individual anticipates that the job loss shock can hit him very late in the career, then the impact through the amount of precautionary savings is even more important because she has more time to accumulate higher savings. Hence, the latter in the career the job loss shock can occur, the earlier people retire.

Proposition 3. *The optimal career length (R^{s}, R^{ns}) is inversely related to the mo-*

ment the job loss shock can hit the worker (K).

Proof.

$$\frac{\partial R^{ns}}{\partial K} = -\frac{e^{r(R^{ns}-K)}K}{r}\left[\frac{1}{B^{\frac{1}{\sigma}}} - \frac{1}{(B(1-p)+\frac{pB}{\mu})^{\frac{1}{\sigma}}}\right] < 0$$

$$\frac{\partial R^{s}}{\partial K} = -\frac{e^{r(R^{s}-K)}K}{\mu r}\left[1-\mu+\frac{1}{\left(\frac{B}{\mu}\right)^{\frac{1}{\sigma}}} - \frac{1}{(B(1-p)+\frac{pB}{\mu})^{\frac{1}{\sigma}}}\right] < 0$$

□

3.4 The impact of a pay-as-you-go pension system on the retirement age

In this section we extend our model by including a pay-as-you-go pension system. The scope of this section is twofold. First, since the literature has shown that the existence of a pay-as-you-go pension system is an important driver of retirement decisions, we analyze how it interacts with the job loss shock in determining the retirement of workers. Second, we showed in section 3.3 that the impact of a higher job loss risk on the average retirement age in the economy is ambiguous if $\sigma > 1$. Hence, we calibrate the model on the US economy and investigate whether the substitution or income effect dominates. Since the US economy features a pay-as-you-go pension system, we need to include this feature in the model.

Workers pay a contribution equal to τ each period they are working until they withdraw from the labor force. The pension system pays a benefit b^{ns} or b^{s}, depending on the type of the worker, each period after the official retirement age R^{of}.

The budget constraints of workers from section 3.3 (relations (3.3),(3.4)) become:

$$\int_0^K e^{-rt}c_t dt + \int_K^T e^{-rt}c_t^{ns}dt = \int_0^{R^{ns}} e^{-rt}(1-\tau)dt + \int_{R^{of}}^T e^{-rt}b^{ns}dt \tag{3.23}$$

$$\int_0^K e^{-rt}c_t dt + \int_K^T e^{-rt}c_t^{s}dt = \int_0^{K} (1-\tau)e^{-rt}dt + \int_K^{R^s} (1-\tau)e^{-rt}\mu dt + \int_{R^{of}}^T e^{-rt}b^{s}dt \tag{3.24}$$

We consider the case of an earnings related pension system where the pension benefit is computed as a replacement rate ρ times the average of a number of years of earnings. Because in practice the number of years of earnings comprised in the average varies across countries, we will consider two cases. In subsection 3.4.1, the pension benefit is based on the average of the K highest earnings years. In this case, given the assumptions of our model, the displacement shock does not influence the average of lifetime earnings because the K years with the highest earnings are the first K years for both displaced and not displaced individuals. Hence, the pension benefit is the same for both displaced and not displaced workers.

In subsection 3.4.2, we consider the case of an earnings related pension system in which the pension benefit is computed based on the average of earnings received throughout a person's career. In this case displacement impacts on the computation of the pension benefit. Displaced and not displaced workers have different pension benefits and this affects their retirement decision.

3.4.1 An earnings related system based on the K highest earning years

If the pension benefit is computed based on K years with the highest earnings, then the reduction in wages following displacement does not impact on the pension benefit of displaced workers. This is computed as:

$$b^{ns} = b^s = \rho \frac{\int_0^K 1dt}{K} = \rho \tag{3.25}$$

The first order conditions with respect to consumption and assets are the same as in section 3.3 relations (3.5)-(3.7). The first order conditions with respect to the time spent in employment (R^s, R^{ns}) become:

$$\mu(1-\tau)u'(c^s) = B \tag{3.26}$$

$$(1-\tau)u'(c^{ns}) = B \tag{3.27}$$

The contribution to the pension system τ reduces the marginal benefit of working (left-hand side of equations (3.26), (3.27)). This induces workers to retire earlier than they would in the absence of the pension system.

3.4.2 An earnings related system based on career average earnings

In this case pension benefits are equal to:

$$b^s = \rho \frac{K + (R^s - K)\mu}{R^s} \tag{3.28}$$

$$b^{ns} = \rho \frac{(K + (R^{of} - K))}{R^{of}} = \rho \tag{3.29}$$

The displacement shock reduces the pension benefits of displaced workers b^s.

The first order conditions with respect to the time spent in employment R^s and R^{ns} are:

$$\left((1-\tau)\mu - \frac{K(1-\mu)\rho(e^{-r(R^{of}-R^s)} - e^{-r(T-R^s)})}{(R^s)^2}\right) u'(c^s) = B \tag{3.30}$$

$$(1-\tau)u'(c^{ns}) = B \tag{3.31}$$

Once again, the contribution to the pension system lowers the marginal benefit of working (left-hand side of equations (3.30), (3.31)). Comparing relation (3.30) with relation (3.31), we notice that the pension system based on career average earnings is more distortive for displaced workers that the pension system based on K years with the highest earnings. The reason for this effect is that following displacement, every period spent in employment reduces the career average earnings.

3.4.3 Balanced budget constraint

The size of the replacement rate ρ is determined from the balanced budget constraint of the pension system: pension benefits paid to retired individuals are equal to contribution paid by both displaced and not displaced workers:

$$(pb^s + (1-p)b^{ns}) \int_{R^{of}}^{T} e^{-rt} dt = \tau \int_0^K e^{-rt} dt + p \int_K^{R^s} e^{-rt}\tau\mu dt + (1-p) \int_K^{R^{ns}} e^{-rt}\tau dt \tag{3.32}$$

In the ***recursive competitive equilibrium*** of the economy, individuals make optimal consumption, savings and labor supply decisions taking the pension contribution and pension benefits as given and the budget of the pay-as-you-go pension system balances.

3.4.4 Calibration

In order to solve the model numerically, we consider that a period of the model represents a month. People live for 80 years, but do not supply labor in the first 20 years, implying $T = (80 - 20) \cdot 12$ months. They enter the labor market and die with no assets. The official retirement age is $R^{of} = (65 - 20) \cdot 12$ months. We calibrate the disutility of labor supply to $B = 1.045$ by targeting the average effective retirement age in the US economy of 64.9 years (OECD data). The contribution to the pension system is the one currently prevailing in the US system $\tau = 0.106$. We set $\sigma = 2$ in line with the literature and $r = \beta = 0.0033$ such as to obtain an annualized interest rate of 4%.

We calibrate the characteristics of the job loss shock based on the evidence we obtained with SIPP data in section 3.2.1. We focus on the workers aged between 50 and 59 because they suffer the highest losses in terms of re-employment wages and they also are the age group that is the closest to the official retirement age in the US economy. Hence, we set the job loss shock at age 55, i.e. $K = (55 - 20) \cdot 12$. Using the findings summarized in table 3.3, we set p equal to the probability of suffering a statistically significant wage loss (spending more than 6 months outside employment) if one belongs to the group of workers aged between 50 and 59 years. The probability of such an event happening is 0.024 in 24 months (see table 3.3). Hence, between 50 and 59 years a person has a probability of $p = 0.024 \cdot 5 = 0.12$ to suffer a significant wage loss following a non-employment spell. For the job loss cost, we set $1 - \mu = 0.13$ equal with the wage loss encountered following a non-employment spell of 6-9 months for the workers aged between 50 and 59 (see table 3.3). In comparison, Rogerson and Schindler (2002) use $p = 0.25$, $1 - \mu = 0.3$ using the data from Jacobson et al. (1993). The displacement probability and the displacement costs found by Jacobson et al. (1993) are biased upwards due to the fact that the data corresponds to a recession. Using newer data and outside a recession, Couch and Placzek (2010) find figures consistent with $p = 0.17$ and $\mu = 1 - 0.15$, closer to our calibration. Krebs (2007) uses $p = 0.04$ (annual probability of displacement) and $\mu = 1 - 0.15$. Both figures are higher than the ones we employ. In conclusion, our calibration is close to the literature, but both the displacement probability and the displacement costs are slightly lower.

3.4.5 Results

Table 3.6 presents the effective retirement age of each type of worker and as an average in the economy under the pension system described in section 3.4.2 where benefits are computed based on career average earnings. We determine the impact on the retirement age of a higher job loss probability p, a higher displacement cost $1-\mu$ and a higher contribution to the PAYG pension system τ.

Table 3.6: Pension benefits based on career average earnings

	Baseline	p=0.024 $\cdot$ 5	$1-\mu$=0.14	τ=0.116
R^{ns}	64.9	-5.8	-1.3	-8.6
R^{s}	64.8	-7.0	-1.8	-10.4
AERA	64.9	-6.1	-1.4	-8.9

Note: changes compared to the baseline are expressed in months. R^{ns} is the retirement age of individuals who do not suffer a job loss shock, R^{s} is the retirement age of individuals who suffer a job loss shock, AERA is average effective retirement age.

As we showed using the model with a closed form solution in section 3.3, for $\sigma > 1$ it is not clear whether displaced workers retire earlier than not displaced agents and how the characteristics of the job loss shock (p, μ) impact on the average retirement age in the economy. The results depended on the parameters of the model. Using US data for the calibration, we obtain that displaced workers retire earlier than workers that are not displaced $R^s < R^{ns}$. Hence, the substitution effect of the displacement shock and the effect through higher precautionary savings accumulated before the moment the shock hits dominate the income effect. The fact that displaced workers retire earlier than workers that are not displaced is in accordance with US micro-data evidence. Employing the US Health and Retirement Survey (HRS), Hairault et al. (2015) show that people aged over 50 years that are looking for a job have a probability to retire next period with 17% higher than people who work.

If the job loss probability is higher (3.4% in 24 months instead of 2.4%), the retirement age of displaced workers decreases by 7 months while the retirement age of workers that are not displaced decreases by 6 months. The fact that a higher job loss probability impacts more on the retirement age of the displaced worker confirms the fact that the substitution effect of a permanently lower wage dominates the income effect.

If we consider a higher cost associated with displacement ($1-\mu = 0.14$ instead of 0.13) the retirement age of all types of workers and as an average on the economy decreases. The effect is less pronounced for workers that are not displaced. This is because the higher displacement cost only impacts them through the fact that they make higher precautionary savings and, hence, retire earlier. Displaced workers also make higher precautionary savings and, hence, retire earlier but they are also impacted by the fact that their return from working decreases after the job loss spell. This makes them withdraw from the labor market earlier. For the calibration employed in this section, the effect through savings and the substitution effect dominate the income effect of a lower wage after displacement.

A higher contribution to the pension system lowers the retirement age of both types of workers and has a higher impact on displaced workers. This suggests that the size of the pension system is also a determinant of the relative importance of the substitution and income effect and their final impact on the retirement age. As relations (3.26) and (3.30) show, the contribution to the pension system τ reinforces the substitution effect of a permanently lower wage μ on the retirement age. However, the income effect of the contribution to the pension system is substantially lower than the one of the permanently lower wage μ, because the contribution to the pension system entitles the worker to receive a benefit after the official retirement age and the benefit is proportional to the contributions made due to the balanced budget constraint.

Table 3.7: Pension benefits based on K highest earning years

		p=$0.024 \cdot 5$	$1-\mu$=0.14	τ=0.116
R^{ns}	65.2	-4.4	-1.0	-8.3
R^{s}	65.2	-5.1	-1.2	-9.7
AERA	65.2	-4.5	-1.0	-8.6

Note: changes compared to the baseline are expressed in months. R^{ns} is the retirement age of individuals who do not suffer a job loss shock, R^{s} is the retirement age of individuals who suffer a job loss shock, AERA is average effective retirement age.

Table 3.7 shows the implications of computing the pension benefits based on the

K highest earning years and not on the entire history of earnings. The retirement age of both types of workers is higher in this case: 65.2 years for both types of workers instead of 64.9 years for workers that are not displaced and 64.8 years for displaced workers. Hence, not including the post-displacement wages in the base for computing pension benefits encourages people to spend more time in employment. The reason for this is twofold. First, for people that are displaced, spending more time in employment after the job loss spell occurs does not reduce their pension benefits if only the first K years of earnings are taken into account. Hence, the pension system does not offer them supplementary incentive to retire earlier compared to people that are not affected by a job loss shock. Second, because the job loss shock does not reduce the pension benefits of displaced workers, the pension system offers people some insurance against the job loss shock. Hence, both displaced and not displaced workers make lower precautionary savings before the shock can hit and they retire later.

A higher job loss probability p or a higher job loss cost $1-\mu$ lowers the retirement age of each type of workers and as an average on the economy. However, the impact of a higher job loss risk on the retirement age is smaller than in the case of the pension system based on career average earnings (table 3.6). As explained above, this is because the pension system offers workers some insurance against the job loss shock. Hence, changes in the characteristics of the job loss risk impact on the retirement age to a lesser degree.

Finally, we analyse the interaction between job loss risk and the existence of a pay-as-you-go pension system. We make a counterfactual experiment and eliminate the pay-as-you-go pension system in the US economy by setting $\tau = 0$. We analyse the impact on the retirement age. The results are presented in table 3.8.

Table 3.8: Retirement in the absence of the pension system $\tau = 0$

	Baseline	p=0.024 $\cdot$ 5	$1-\mu$=0.14
R^{ns}	73.7	-6.7	-1.5
R^{s}	75.4	-8.2	0.0
AERA	73.9	-5.9	-1.3

Note: changes compared to the baseline are expressed in months. R^{ns} is the retirement age of individuals who do not suffer a job loss shock, R^{s} is the retirement age of individuals who suffer a job loss shock, AERA is average effective retirement age.

There are four major differences between the model calibrated on the US economy and the economy without a pension system (table 3.6 and table 3.8, respectively). First, all workers retire earlier in the US economy. The fact that people must pay a contribution to the pension system until they leave the work force distorts their labor supply decision and makes them retire earlier. Second, under the US pension system, workers that are not displaced retire later than those that are displaced. This result is also confirmed by micro-data evidence. However, in the absence of the pension system this result would not hold: displaced workers retire later than workers that are not displaced. Hence, the distortion of the pension system is crucial for obtaining the result that displaced workers retire earlier than workers that are not displaced. The distortion of the contribution to the pension system reinforces the substitution effect of the lower wage after the job loss shock. However, the income effect of the pension contribution is very small because people obtain pension benefits in return for their contributions and their size is similar due to the balanced budget constraint of the pension system. Overall, the substitution effect dominates the income effect in the presence of a pay-as-you-go pension system with a contribution of $\tau = 10.6\%$.

Third, a higher job loss probability p leads to a more important reduction in the retirement age of each type of worker when the pension system is absent. However, the effect on the retirement age on the entire economy is less pronounced than under the US system. As we explained using the analytical solution of the model in section 3.3, this happens because a higher p increases the share of displaced workers in the economy and this category of workers retires later in the absence of the pension system. Finally, a higher job loss cost $1 - \mu$ does not impact on the retirement age of displaced workers in the absence of the pay-as-you-go pension system. On aggregate, a higher job loss cost reduces the retirement age but the impact is smaller than in the presence of the pay-as-you-go pension system.

In conclusion, we obtain the fact that the interaction between job loss shocks and the pay-as-you-go pension system increases the impact of job loss risk on the average retirement age in the economy.

3.5 Conclusions

In this paper we analyse how job loss risk impacts on the retirement decisions of workers. We build a life cycle model with endogenous retirement, displacement risk and incomplete markets and show that a higher probability of displacement as well as higher earnings losses following displacement lower the average effective retirement age. We also show that the impact of job loss risk on the average retirement age in the economy is amplified by the presence of a pay-as-you-go pension system.

Cross country data confirms the negative impact of job loss risk on the average retirement age. We document a negative relationship between the incidence of long term unemployment, on the one hand, and the actual retirement age on the other hand. We show that this relationship is statistically significant even when we control for other factors such as the size of the pension systems prevailing in each country.

Appendix 1. Solution of the model in Section 3.3

The worker's optimization problem is:

$$\max_{\{c_t,c_t^s,c_t^{ns},R^s,R^{ns}\}} \int_0^K e^{-\beta t}u(c_t)dt - B\int_0^K e^{-\beta t}dt + p\left(\int_K^T e^{-\beta t}u(c_t^s)dt - B\int_K^{R^s} e^{-\beta t}dt\right)$$
$$+ (1-p)\left(\int_K^T e^{-\beta t}u(c_t^{ns})dt - B\int_K^{R^{ns}} e^{-\beta t}dt\right) +$$
$$\lambda^{ns}\left(\int_0^{R^s} e^{-rt}dt - \int_0^K e^{-rt}c_t dt - \int_K^T e^{-rt}c_t^{ns}dt\right) +$$
$$\lambda^s\left(\int_0^K e^{-rt}dt + \int_K^{R^s} e^{-rt}\mu dt - \int_0^K e^{-rt}c_t dt - \int_K^T e^{-rt}c_t^s dt\right)$$

The first order conditions with respect to c_t, c_t^s, c_t^{ns}, R^s and R^{ns} are:

$$e^{-\beta t}u'(c_t) = (\lambda^{ns}+\lambda^s)e^{-rt} \tag{3.33}$$
$$(1-p)e^{-\beta t}u'(c_t^{ns}) = \lambda^{ns}e^{-rt} \tag{3.34}$$
$$pe^{-\beta t}u'(c_t^s) = \lambda^s e^{-rt} \tag{3.35}$$
$$B = \mu u'(c_{R^s}^s) \tag{3.36}$$
$$B = u'(c_{R^{ns}}^{ns}) \tag{3.37}$$

Replacing $u'(c) = c^{-\sigma}$, we obtain the expressions of c_t, c_t^{ns}, c_t^s as a function of

λ^s, λ^{ns}:

$$c_t = \left(e^{(\beta-r)t}(\lambda^{ns}+\lambda^s)\right)^{-\frac{1}{\sigma}} \tag{3.38}$$

$$c_t^s = \left(e^{(\beta-r)t}\frac{\lambda^s}{p}\right)^{-\frac{1}{\sigma}} \tag{3.39}$$

$$c_t^{ns} = \left(e^{(\beta-r)t}\frac{\lambda^{ns}}{1-p}\right)^{-\frac{1}{\sigma}} \tag{3.40}$$

These are replaced in the budget constraints of the individuals (3.3) and (3.4). Together with the first order conditions with respect to time spent in employment (3.36) and (3.37), we obtain the following system of equations with unknowns R^s, R^{ns}, λ^s, λ^{ns}:

$$B = \mu\frac{\lambda^s}{p}e^{(\beta-r)R^s} \tag{3.41}$$

$$B = \frac{\lambda^{ns}}{1-p}e^{(\beta-r)R^{ns}} \tag{3.42}$$

$$(\lambda^{ns}+\lambda^s)^{-\frac{1}{\sigma}}\left(1-e^{-rK-\frac{(\beta-r)K}{\sigma}}\right)+\left(\frac{\lambda^{ns}}{1-p}\right)^{-\frac{1}{\sigma}}\left(e^{-rK-\frac{(\beta-r)K}{\sigma}-e^{-rT}}\right) = (1-e^{-rR^{ns}}) \tag{3.43}$$

$$\begin{aligned}&(\lambda^{ns}+\lambda^s)^{-\frac{1}{\sigma}}\left(1-e^{-rK-\frac{(\beta-r)K}{\sigma}}\right)+\left(\frac{\lambda^s}{p}\right)^{-\frac{1}{\sigma}}\left(e^{-rK-\frac{(\beta-r)K}{\sigma}-e^{-rT}}\right) = \\ &(1-e^{-rK})+\mu(e^{-rK}-e^{-rR^s})\end{aligned} \tag{3.44}$$

To get an expression for savings at time of displacement (a_K), we integrate the per period budget constraints over [0,K]:

$$\int_0^K (c+\dot{a}_t)e^{-rt}dt = \int_0^K (1+ra_t)e^{-rt}dt \Rightarrow c\frac{1-e^{-rK}}{r} + a_K e^{-rK} = \frac{1-e^{-rK}}{r}$$

$$a_K = \frac{(1-c)(e^{rK}-1)}{r}$$

Using Assumption 2, the solution of the model becomes:

$$
\begin{aligned}
\lambda^{ns} &= (1-p)B \\
\lambda^{s} &= \frac{pB}{\mu} \\
c^{s} &= \left(\frac{B}{\mu}\right)^{-\frac{1}{\sigma}} \\
c^{ns} &= B^{-\frac{1}{\sigma}} \\
c &= \left(p\frac{B}{\mu} + (1-p)B\right)^{-\frac{1}{\sigma}} \\
a_K &= \frac{\left(1 - \left(p\frac{B}{\mu} + (1-p)B\right)^{-\frac{1}{\sigma}}\right)(e^{rK}-1)}{r}
\end{aligned}
$$

Replacing the above solutions in the life-time budget equations (3.43) and (3.44), we obtain the solutions for the retirement age (3.10) and (3.11).

Appendix 2. Solution of the model in Section 3.4.2

The worker's optimization problem is:

$$
\begin{aligned}
&\max_{\{c_t, c_t^s, c_t^{ns}, R^s, R^{ns}\}} \int_0^K e^{-\beta t} u(c_t)dt - B\int_0^K e^{-\beta t}dt + p\left(\int_K^T e^{-\beta t}u(c_t^s)dt - B\int_K^{R^s} e^{-\beta t}dt\right) \\
&+ (1-p)\left(\int_K^T e^{-\beta t}u(c_t^{ns})dt - B\int_K^{R^{ns}} e^{-\beta t}dt\right) + \\
&\lambda^{ns}\left(\int_0^{R^{ns}} (1-\tau)e^{-rt}dt + b^{ns}\int_{R^{of}}^T e^{-rt}dt - \int_0^K e^{-rt}c_t dt - \int_K^T e^{-rt}c_t^{ns}dt\right) + \\
&\lambda^{s}\left(\int_0^K (1-\tau)e^{-rt}dt + \int_K^{R^s} e^{-rt}\mu(1-\tau)dt + b^s\int_{R^{of}}^T e^{-rt}dt - \int_0^K e^{-rt}c_t dt - \int_K^T e^{-rt}c_t^s dt\right)
\end{aligned}
$$

The first order conditions are:

$$e^{\beta t}u'(c_t) = \lambda^{ns}e^{-rt} + \lambda^{s}e^{-rt} \tag{3.45}$$

$$(1-p)e^{-\beta t}u'(c_t^{ns}) = \lambda^{ns}e^{-rt} \tag{3.46}$$

$$pe^{-\beta t}u'(c_t^{s}) = \lambda^{s}e^{-rt} \tag{3.47}$$

$$pBe^{-\beta t} = \lambda^{s}\left((1-\tau)\mu e^{-rR^s} - \frac{K(1-\mu)\rho(e^{-rR^{of}} - e^{-rT})}{rR^{s2}}\right) \tag{3.48}$$

$$(1-p)Be^{-\beta R^{ns}} = (1-\tau)\lambda^{ns}e^{-rR^{ns}} \tag{3.49}$$

Replacing $u'(c) = c^{-\sigma}$, we obtain the expressions of c_t, c_t^{ns}, c_t^{s} as a function of λ^s, λ^{ns}:

$$c_t = \left(e^{(\beta-r)t}(\lambda^{ns} + \lambda^{s})\right)^{-\frac{1}{\sigma}}$$

$$c_t^{s} = \left(e^{(\beta-r)t}\frac{\lambda^{s}}{p}\right)^{-\frac{1}{\sigma}}$$

$$c_t^{ns} = \left(e^{(\beta-r)t}\frac{\lambda^{ns}}{1-p}\right)^{-\frac{1}{\sigma}}$$

These are replaced in the budget constraints of the individuals. Together with the balanced budget constraint of the pay-as-you-go system, we obtain the following system of equations with unknowns R^s, R^{ns}, λ^s, λ^{ns}, ρ:

$$(3.49), (3.48)$$

$$(\lambda^{ns} + \lambda^{s})^{-\frac{1}{\sigma}}\left(1 - e^{-rK - \frac{(\beta-r)K}{\sigma}}\right) + \left(\frac{\lambda^{ns}}{1-p}\right)^{-\frac{1}{\sigma}}\left(e^{-rK - \frac{(\beta-r)K}{\sigma}} - e^{-rT}\right) =$$

$$(1-\tau)(1 - e^{-rR^{ns}}) + b^{ns}(e^{-rRof} - e^{-rT})$$

$$(\lambda^{ns} + \lambda^{s})^{-\frac{1}{\sigma}}\left(1 - e^{-rK - \frac{(\beta-r)K}{\sigma}}\right) + \left(\frac{\lambda^{s}}{p}\right)^{-\frac{1}{\sigma}}\left(e^{-rK - \frac{(\beta-r)K}{\sigma}} - e^{-rT}\right) =$$

$$(1-\tau)(1 - e^{-rK}) + \mu(1-\tau)(e^{-rK} - e^{-rR^s}) + b^{s}(e^{-rRof} - e^{-rT})$$

$$(pb^{s} + (1-p)b^{ns})(e^{-rRof} - e^{-rT}) = \tau(1 - e^{-rK}) + p\tau\mu(e^{-rK} - e^{-rR^s}) + (1-p)\tau(e^{-rK} - e^{-rR^{ns}})$$

$$b^{ns} = \rho$$

$$b^{s} = \rho\frac{K + (R^s - K)\mu}{R^s}$$

This system of equations is solved numerically.

Chapter 4

The political arrangement of pay-as-you-go pension systems in the presence of financial and demographic shocks

4.1 Introduction

Developments of pay-as-you-go (PAYG) pension systems are tightly connected with shocks to the real return on capital and demographic shocks. Narrative evidence summarized in Section 2 suggests that the introduction of PAYG pension systems in countries such as Germany, Italy, Japan, the United States and the United Kingdom was triggered by shocks that reduced accumulated savings of old agents and agents close to retirement (see Flora (1987), Gordon and Varian (1988), Perotti and Schwienbacher (2009)). In many countries, PAYG pension systems were expanded following other major economic downturns through early retirement schemes and decreases in retirement age.

Beginning with the 1980s, demographic projections worsened the outlook for the financial sustainability of PAYG pension systems, leading many governments to enact substantial reforms. In some countries, these consisted of increases in contributions and cuts in benefits or a tightening of the eligibility conditions for a pension. In other countries, mainly in Central and Eastern Europe and Latin America, the reforms entailed a partial or complete reduction of the public PAYG pension system in favor of private fully funded pension funds (see OECD (2012) for a discussion on pension reforms implemented since mid 1980's).

This paper explains these reforms using a political economy model where consensus between young and old agents is achieved through voting. We focus on the role of PAYG pension systems in sharing financial shocks among different cohorts in an economy with incomplete asset markets. Young agents are prepared to contribute to the pension system because this offers them partial protection against shocks to the real return on capital.

The contributions of this paper are the following. First, we study the interaction between demographic shocks and the role of PAYG pension systems in sharing financial shocks among cohorts in a setting with voting. Second, through the combination of financial and demographic shocks we can account for the most important developments of the PAYG pension systems around the world. Finally, we compare our results to the normative benchmark of the Ramsey planner.

We consider a small open economy in which the real rate of return[1] on capital and the population growth rate are stochastic. We call these variables *financial shock* and *demographic shock*, respectively. Young and old agents vote each period over the size of the PAYG contributions. The electoral competition is modeled as a probabilistic voting game as in Persson and Tabellini (2000). We consider differentiable Markov policies, i.e. differentiable policies that depend only on current state variables. Since the contribution to the pension system depends negatively on the amount of savings, young agents incorporate in their voting decision the fact that larger pension contributions imply lower savings, thereby causing a larger PAYG scheme in the next period as well. Hence, this "strategic effect"[2] in the political process weakens the opposition of the young to a social security scheme.

We show that the policy function resulting from the voting process helps overlapping cohorts share both financial and demographic shocks by appropriately adjusting

[1]We will henceforth refer to shocks to the return on capital by the term "financial shocks", but we bear in mind that these may also represent inflation or capital depreciation shocks.

[2]The term "strategic effect" was first used by Song (2011) and denotes the strategic behavior of agents who internalize the fact that the next period's politician will base his policy decisions on the policy implemented in the current period. Although using a different setting, the strategic effect in Song (2011) is positive and has the same role as in our paper: current voters have the incentive to vote for a higher contribution to the PAYG system in the current period in order to enjoy higher benefits next period.

contributions each period. More specifically, following a decrease in the return on capital, the politician increases both contributions and benefits. After a decrease in the population growth rate however, the politician increases contributions but decreases benefits. These results are in line with the major observed developments of the PAYG pension systems.

We also analyze the implications for pension contributions and benefits of *permanent* changes in the population growth rate, modeled as a lower mean of the demographic shock. We find that a lower mean of the population growth rate makes the politician's policy function less sensitive to the wealth of the old. Intuitively, when the population growth rate is lower on average, the politician has less room to compensate old agents for their losses from financial shocks. This is what we observed during the Great Recession; governments did not expand PAYG pension systems to compensate old agents or agents close to retirement for financial losses as they did in previous major recessions[3].

The fact that a lower mean of the population growth rate makes the politician's policy function less sensitive to wealth losses has two opposing implications. On the one hand, the capacity of the PAYG pension system to diversify the financial risks decreases. Consequently, young agents must contribute more to the pension system to maintain the same level of protection against financial shocks. On the other hand, the return of the pension system also decreases due to a weaker strategic effect. This makes young agents less willing to invest in the pension system. In a calibrated version of the model we show that the impact through the strategic effect dominates. This implies that the average contribution to the PAYG pension system is smaller if the mean population growth rate is lower. This is consistent with events described in section 4.2. Countries in Latin America and Central and Eastern Europe decided to (partially) transform the PAYG pension system into a fully funded after a decrease of the average population growth rates in these countries.

Analyzing the Ramsey planner's problem, we find that, following financial and demographic shocks, contributions and benefits change in the same direction as in the politician's case, but the adjustments are of a smaller magnitude. If the mean of the population growth rate is smaller, the Ramsey planner implements a larger average contribution. This result is contrary to the one obtained in the politician's

[3]However, many governments took measures to alleviate the conditions of the poorest pensioners, for example by expanding means tested benefits. See OECD (2012).

case. Because in the Ramsey planner's problem the strategic effect plays no role, a lower mean of the demographic shock weakens the pension system's ability of sharing the financial risk without making the investment in capital more attractive. Hence, the Ramsey planner expands the size of the pension system in order to better protect young agents against financial shocks.

Previous literature has focused on how either financial or demographic shocks affect PAYG pension systems. In an economy featuring only financial shocks, D'Amato and Galasso (2010) show that both a politician and a Ramsey planner would organize transfers that feature a negative relationship with the wealth of the old. However, the policy rule established by the politician exhibits more persistence and entails higher benefits for the old than the Ramsey planner's. We extend their model by incorporating demographic shocks since these are empirically relevant for the development of PAYG pension systems. We also analyze whether our results hold in a model extended with a realistic utility function, savings and labor supply decisions of young agents. In a model featuring only demographic shocks, Gonzalez-Eiras and Niepelt (2008) find that a decrease in the population growth rate leads to an increase in the contributions and a decrease in the pension benefits established by a politician. In their model however, the introduction of PAYG pension systems is explained by a positive demographic shock. This is not consistent with the narrative evidence. Moreover, we show that the interaction between financial and demographic shocks plays an important role in determining the equilibrium size of the PAYG pension system. Bohn (2001) considers both financial and fertility shocks but does not analyze the role of the financial shock in determining the contributions to and benefits from the PAYG pension system. Also, he only addresses the policy implemented by a Ramsey planner, while we focus mainly on the politician's policy function. Bonenkamp et al. (2017) show that current pension reforms can be rationalized as the welfare-best response of a social planner confronted with population aging. However, this framework cannot explain the introduction and subsequent up-scaling of PAYG pension systems. We also show that the same type of pension reforms would arise in a setting with voting.

The remainder of the paper is organized as follows. In Section 4.2 we present the stylized facts motivating the analysis in this paper. Section 4.3 presents a simple version of the political equilibrium model and derives the voting outcome for pension contributions and benefits in an economy affected by both financial and demographic shocks. Section 4.4 presents, for comparison, the policy implemented by a Ramsey planner. We analyze an extension of the model that includes savings, labor supply decisions and a realistic utility function in Section 4.5. Finally, Section 4.6 concludes the main body of this paper. The proofs of the Propositions are included in Appendix 1.

4.2 Motivating facts

In this section we present narrative evidence on the impact that financial and demographic shocks had on pension systems throughout the years. Ideally, one should provide econometric evidence on this relationship. Unfortunately, data about the size of contributions in a large number of countries is difficult to find for pre-1980, while we would also need data for the period 1940-1980 when major financial shocks occurred. Moreover, the size of the contributions does not always give all the information about the size of the pension system. For example, the changes in the retirement age, the way pension benefits are computed, the coverage of the pension system are also relevant. For these reasons, we have to rely on narrative evidence only.

The information available about the early days of PAYG pension systems shows that, in many countries, these were introduced after events that had large negative effects on existing retirement savings (Flora (1987), Gordon and Varian (1988), Perotti and Schwienbacher (2009)). Table 4.1 summarizes the key moments in the development of PAYG pension systems in a number of selected countries. In the US, the PAYG pension system emerged after the Great Depression[4]. Old people were affected the most by the economic downturn since they were left with few savings and they also lacked the capacity to return to work after the recession ended. In Germany, the old age pension system that was set up by Otto von Bismarck as a fully funded system was transformed into a PAYG system since the accumulated capital was depleted during World War II (Rürüp (2002)). Fully funded pensions systems were transformed to PAYG system in Japan and Italy for the same reason. The UK introduced the basic state pension in 1946 on a PAYG basis and not as a fully funded system like Beveridge envisaged it. The reason was to cover also those who were affected by the Great Depression and contributed to the World War II effort (Bozio et al. (2010)).

[4]Historical Background and Development of Social Security, Social Security Administration, https://www.ssa.gov/history/briefhistory3.html, visited on February 1st 2014.

The reforms summarized in Table 4.1 also indicate that PAYG pension systems have been expanded following the economic downturns of the 1970s and the 1980s. In France, the Netherlands, Spain, US and Poland, during recessions, early retirement schemes were introduced and the retirement age was reduced.

Beginning with the 1990s, on the backdrop of important decreases in the population growth rates (see Figure 4.1), substantial reforms aimed at restoring the viability of the PAYG pension systems were enacted. These pension reforms involved increases in contributions and cuts in benefits. The reductions in pension benefits were achieved through a variety of means: decreases in accrual rates, indexation or valorization, increases in the number of reference years or the retirement age or strengthening of work incentives, increases in the number of years of contribution. A few countries (Germany, Spain and countries that shifted to a Notional Defined Contribution system) established a direct link between benefits paid and demographic indicators.

The reforms aimed at restoring the viability of pension systems persisted even during the Great Recession: most of the adjustments to PAYG pension systems entailed increases in contributions or in the retirement age. Although agents over 60 years suffered higher wealth losses than young and middle-aged agents (Glover et al. (2011)), there was little compensation for wealth losses through the PAYG pension system.

While many countries retained their public PAYG pension systems but tried to restore their financial sustainability, a number of countries decided to partially or completely transform them in private fully funded ones in the face of demographic changes. Many Central and Eastern European and Latin American countries downsized their PAYG pension systems in this way. Figure 4.1 shows that, compared to other regions of the world, Eastern European and Latin American countries experienced more sizable reductions in population growth rate that seem to be *permanent*. In this paper we model permanent reductions in the population growth rate as a decrease *in the mean of the population growth rate*. In contrast, Western European and Northern American countries experienced less severe reductions in the population growth rates and, starting with the 1960s, these can be qualified more as *shocks*.

Another puzzling fact about pension provision is the considerable cross-country heterogeneity in the size of PAYG pension contributions and benefits (see table 4.2). Tabellini (2000) explains this by the proportion of the elderly in the economy and the pre-tax income inequality. In his paper, higher income inequality of young people

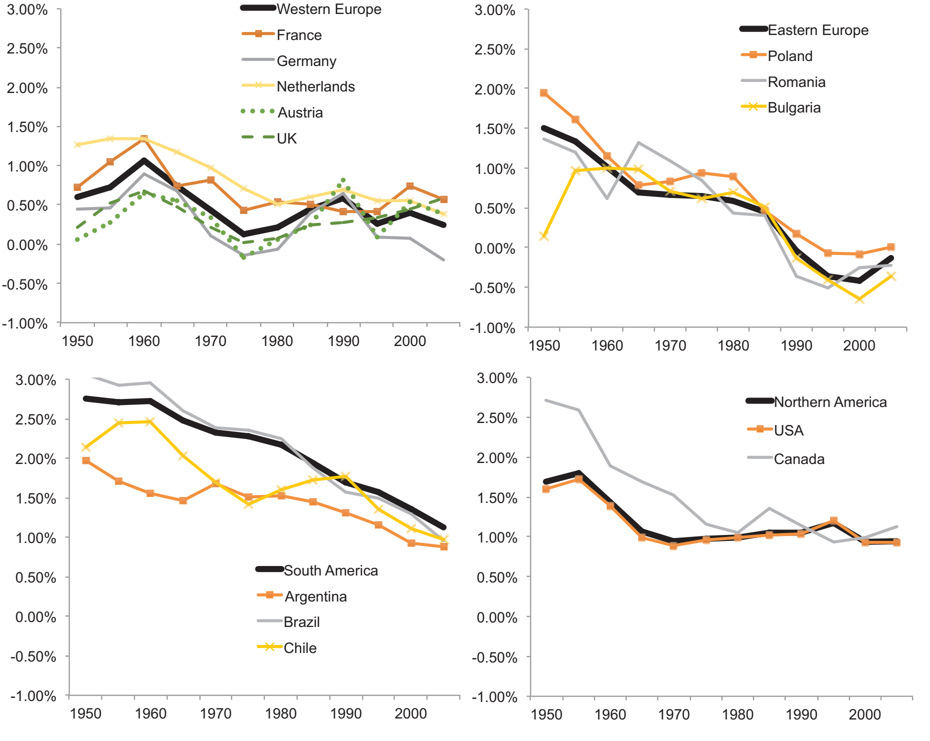
Table 4.1: PAYG pension system reforms in selected countries

Country	Year of introduction and context	1960-1985	1985-2008	Great Recession
Austria	1906 - Pension system for white collar workers 1926;1935;1939 - Pension system for blue collar workers	1958;1961-1966 - Introduction of early retirement schemes 1984 – Number of reference years extended from 5 to 10	1993 - Increase in contributions of civil servants; net wage instead of gross wage indexation 1997;2000;2003 - Strengthening work incentives; reforms aimed at cutting benefits (including a reduction in accrual rate) 2004 - Switch to individual accounts	
France	1945 - Aftermath of WWII	1982 - Retirement age decreased to 60 years	1993 - Pension indexation with inflation; reference years extended from 10 to 25 years; increase in required contribution period 2003 - Increase in contribution years to 42; incentives to work after retirement age; indexation according to cost of living in the public sector.	Increase in contributions
Germany	1957-1969 - Funded system transformed into PAYG after massive destruction of capital stock in WWII	1972 - Expansion of the statutory pension scheme	1989 - Increase in retirement age; indexation linked to net wage growth (instead of gross) 2004 - Sustainability factor links pension benefits to demographic factors; taxation of pension benefits 2007 - Increase in retirement age	
Italy	1952 - Funded system transformed in PAYG - the previously accumulated capital was used for war spending or eroded by inflation	1956;1965 - Introduction of "seniority pensions"	1992 - Increase in retirement age and the number of reference years, change of indexation from wages to prices, increase in years of contribution 1995 - Gradual switch to NDC system; sustainability factor that links pension benefits to the age of retirement, economic trends and demographic dynamics; increase in qualifying period for seniority pensions, increase in contribution rates 1997 - Tightening conditions for seniority pensions, partial compatibility between pensions and income from work 2004 - Fixed higher retirement age, bonus for deferred retirement, extra tax on very high pensions	Increase in retirement age
Japan	1954 - Rebuilding of the Kosei Nenkin Hoken (KNH) pension system whose assets were eroded by post war inflation	1961 - Kokumin Nenkin (KN) pension system introduced - covers all categories not included in KNH 1965-1973 - Increase in contributions and benefits of KNH 1973 - Indexation of pension entitlements and benefits	1985 - Decrease in the accrual rate of pension benefit (KNH); Increase in the number of contribution years (KN) 1994 - Gradual phasing out of basic pensions; indexation with net wages 1999 - Decrease in accrual rate, shift to CPI indexation, new earnings test introduced, pensionable age increased	Eligibility period for KN shortened Insurance extended to more part time workers

Country	Year of introduction and context	1960-1985	1985-2008	Great Recession
Netherlands	1947-1957 - Gradual introduction of basic pension following WWII	1976-1985 - Introduction of early retirement schemes on a PAYG basis	1997;2006 - Early retirement schemes integrated in the fully funded occupational schemes. Tax advantages for early retirement abolished	Increase in retirement age
Poland	1950 - Funded pension system becomes PAYG during communist regime	1980 - Coverage extended to rural population	1990-1996 - Early retirement facilitated to fight unemployment during the restructuring of the economy 1999 - Three pillar pension system with a mandatory private defined contribution (DC) pillar	2009 - Early retirement eliminated for certain categories of agents 2011 - Reduction in contribution diverted to the DC pillar 2012 - Retirement age raised
Spain	1963;1972 - Introduction of PAYG pension	1978-1982 - Increase in coverage of PAYG system 1985 - Increase in contribution years and reference period, indexation with estimated CPI increase	1990 - Introduction of mean-tested non-contributory pensions 1997 - Switch back to past CPI indexation; reduction in accrual rates, increase in reference period for benefit calculation 2002 - Incentives to work longer	Increase in normal pension age, incentives to work after retirement Sustainability factor linked to life expectancy
UK	1908 - Means tested non-contributory benefits 1948 - Basic State Pension - flat pension paid on a PAYG basis	1959 - Graduated retirement benefit (GRB) pensions that depend on earnings introduced 1975 - State Earnings Related Pension Scheme (SERPS) replaces and expands GRB; Basic pension indexed with inflation or wages whichever is higher	1986 - Earnings-related benefit calculation includes lifelong earnings; Reduction in SERPS replacement rate and survivors' pensions Extension of means tested supplements 1996 - Second State Pension replaces SERPS 2004 - Rewards for late retirement	Increase in retirement age Increase in contributions
US	1935 - During the Great Depression, due to the increase in the poverty of old agents 1939 - Social security extended to the survivors of the contributors	1950 - Benefits increased with cost of living at certain intervals 1961- Eligibility for early retirement at 62 years 1972 - Automatic Cost of Living Adjustment of benefits 1977- Increase in payroll tax, small decrease in benefits 1983 - Taxation of benefits, increase in retirement age		Changes in early and late retirement

Sources: Immergut et al. (2006), Tálos (2006), Rürüp (2002), Bonin (2009), Franco (2002), Conrad (2001), Takayama (2003), Euwals et al. (2011), Bozio et al. (2010), the site of the Social Security Administration, OECD (2007), OECD (2009), OECD (2011), OECD (2012), OECD (2013).

Figure 4.1: Annual average population growth rate in selected countries and regions



Source: United Nations, Department of Economic and Social Affairs, Population Division (2012)

Table 4.2: Size of the PAYG pension system in selected countries in 2012

Country	Contribution Rate (%)	Gross replacement rate (%)
Austria	22.8	77
France	16.7	59
Germany	19.6	42
Italy	33.0	71
Japan	16.8	36
Netherlands	17.9	30
Poland	19.5	25
Spain	28.3	74
UK	–	33
US	10.4	38

Source: OECD (2013). The contribution rates reported are for the year 2012. For Poland the figure includes the contribution diverted to the DC pillar. The replacement rate corresponds to an agent which earns an average wage.

leads to a larger social security system. Chen and Song (2014) also single out income inequality but they argue in favor of a negative relation with the size of the PAYG pension system. Song (2011) explains the development of social security in the US through the wealth inequality among old agents. However, our narrative evidence shows that the introduction and subsequent changes in the size of PAYG pension systems are more related to the financial and demographic shocks that affected the economies.

Perotti and Schwienbacher (2009) provide another explanation for the cross country differences in the size of PAYG pension systems. They explain this heterogeneity by the fact that different countries experienced inflation shocks of different magnitudes prior to the introduction of the PAYG pension systems. While this may be a good explanation for the introduction of PAYG pension systems, it cannot explain the subsequent expansion of the programs or the reforms enacted after the 1980s. Our model brings an important addition to this strand of literature by exploring other factors that can explain this cross country heterogeneity: the size of the financial and demographic shocks hitting the economy, the mean of these shocks and the political power of the young agents relative to the old.

4.3 A stylized political equilibrium model

We illustrate the mechanisms that determine the pension contributions and benefits and explain the intuition of our results in a simple model that admits an analytic solution. We will relax the assumptions of this simple model in Section 4.5, where we show that our analytic results carry over to a more general model with savings and labor supply decisions and a more realistic class of utility functions. Due to its complexity, the extended model can be solved only numerically.

The setup of the simplified model is closely related to D'Amato and Galasso (2010). We consider an infinite horizon economy inhabited by cohorts of overlapping generations, each living for two periods. The size of the cohort born at time t is N_t. Agents work when young, consume when old and derive utility only from the consumption of goods, an assumption we will relax in section 5. When young, they work their entire time endowment which is standardized to one unit. Aggregate labor supply L_t is equal to the size of the young population N_t.

In this section, we assume a quadratic utility function:

$$u(c_{t+1}) = -\frac{(c_{t+1} - \gamma)^2}{2}$$

where γ is the "bliss" level of consumption[5] and c_{t+1} is the per capita consumption when old of agents born at time t.

The economy has a constant returns to scale technology which gives each period the output $Y_t = wL_t + R_t S_{t-1}$, where S_{t-1} is the stock of capital at time t given by the amount of savings made in the economy at time $t-1$. Capital depreciates fully every period. We consider the case of a small open economy in which the gross return on capital is exogenous but stochastic, independent in time and distributed according to a cumulative distribution function with mean $\bar{R}$ and variance σ_R^2. The gross population growth rate $n_t = \frac{N_t}{N_{t-1}}$ is also stochastic, independent in time[6] and distributed according to a cumulative distribution function with mean $\bar{n}$ and variance σ_n^2. We assume that $\bar{R} > \bar{n}$. Wages w are exogenous, deterministic and normalized to 1.

[5]Expected utility can be equivalently written: $E_t c_{t+1} - \frac{1}{2\gamma} E_t c_{t+1}^2$. In this form it becomes apparent that the parameter γ is inversely related to the degree of risk aversion.

[6]We do not consider any persistence in the demographic process because we analyze very low frequency changes. However, the stylized model presented in Section 4.3 maintains its analytical tractability even the case of an autocorrelated demographic process.

We consider a PAYG pension system that runs a balanced budget each period[7]. Denoting the young's per capita contribution to the PAYG pension system by τ_t and the old's per capita benefit at time t by b_t, the balanced budget condition becomes:

$$b_t N_{t-1} = \tau_t N_t \Rightarrow b_t = \tau_t n_t$$

In the economy described above, the budget constraints of the agents are:

$$s_t + \tau_t = 1 \tag{4.1}$$

$$c_{t+1} = R_{t+1} s_t + \tau_{t+1} n_{t+1} \tag{4.2}$$

where $s_t = \frac{S_t}{N_t}$ is per capita savings of the young.

4.3.1 The politician's problem

The contribution to the pension system is determined every period by a voting process that takes place after the shocks R_t and n_t have materialized. Due to the simplifying assumptions made in this section, agents consume only when they are old, so they make no economic choices when young. They pay the contribution to the pension system and save the rest of their income. This allows us to directly define the political equilibrium of the model.

We consider the setting of a probabilistic voting game[8] in which two candidates compete each period in an election. The candidates are office seeking so they only care about maximizing the probability of being elected. They are also non-partisan. The fact that a voting process is organized in the beginning of each period makes the elected politician unable to commit to implement the policy chosen in his program in the next period as well. We assume however that he can commit to implement the policy outlined in his electoral platform after he is elected. In the voting game, the two candidates take into account the preferences of agents with respect to the policies but also the ideological views of the agents which can make them more inclined to vote for one candidate or another.

[7]One could also consider public debt alongside PAYG pensions for spreading shocks across generations. However, empirically, government debt is used to smooth the impact of business cycle fluctuations on the public budget and not to facilitate intergenerational risk sharing.

[8]The set up of the probabilistic voting game used in this paper is the one presented in Persson and Tabellini (2000).

We assume that ideology is uniformly distributed in the group of young and old agents with density ϕ_1 and ϕ_0, respectively. In the equilibrium of this probabilistic voting game, both candidates choose the same contribution to the pension system. They do so by maximizing the joint welfare of young and old individuals weighed with the density distribution of their ideologies and their relative size in the population, subject to the budget constraints of the agents (4.1) and (4.2):

$$\max_{\tau_t} \phi_0 u(c_t) + \phi_1 n_t E_t u(c_{t+1}) \tag{4.3}$$

$$c_t = R_t s_{t-1} + \tau_t n_t \tag{4.4}$$

$$s_t + \tau_t = 1 \tag{4.5}$$

$$c_{t+1} = R_{t+1} s_t + \tau_{t+1} n_{t+1} \tag{4.6}$$

Because the politician elected at time t cannot commit to implementing the same policy at τ_{t+1}, young agents must have a perception regarding the policy's law of motion. We restrict our attention to differentiable Markov policy functions, i.e. to policies that depend on the current state of the economy defined in our model by the savings of the old, the financial shock and the demographic shock. Hence, we impose the following general form of the policy function:

$$\tau_t = f(s_{t-1}, R_t, n_t) \tag{4.7}$$

with $f(\cdot, \cdot, \cdot)$ continuous and differentiable.

4.3.2 The intergenerational conflict settled by the voting process

It is worth discussing at this point the intergenerational conflict that arises in an economy where a policy τ_t must be implemented at time t. The marginal utility of an old agent with respect to the pension contribution is:

$$\frac{\partial U^o}{\partial \tau_t} = n_t u'(c_t) > 0 \tag{4.8}$$

where the relation comes from the fact that the utility function is strictly increasing[9]. The old strictly prefer higher contributions irrespective of the state of the economy. Consequently, the higher the weight of the old relative to the young in the political process, the higher will be the contribution implemented.

In the case of a young agent, his preferences over the policy that must be implemented at time t are less clear cut. The marginal utility of an increase in contribution at time t is:

$$\frac{\partial U^y}{\partial \tau_t} = E_t \left[u'(c_{t+1}) \left(n_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t} - R_{t+1} \right) \right] \tag{4.9}$$

The term $n_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t}$ captures the "strategic effect" that arises from the politician's lack of committment. This effect works through the savings of the young generation. An increase in τ_t leaves the current young generation with a smaller amount of savings s_t. Since the contribution set at period $t+1$ depends on the amount of savings prevailing at the beginning of that period, the next period's politician will set a higher contribution τ_{t+1}[10].

The support of the young agents towards a higher contribution to the PAYG pension system depends on the strength of the strategic effect relative to the return on capital. The stronger is the strategic effect, the higher is the return of the PAYG pension system compared to the return on capital. This makes young agents willing to expand the pension system.

With quadratic utility, relation (4.9) becomes:

$$\begin{aligned} \frac{\partial U^y}{\partial \tau_t} = {} & cov\left[c_{t+1}, R_{t+1}\right] - cov\left(c_{t+1}, n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right) \\ & + E_t c_{t+1} E_t \left(R_{t+1} - n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right) - -\gamma E_t \left[R_{t+1} - n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right] \end{aligned} \tag{4.10}$$

The first and last term of relation (4.10) dominate the preferences of young agents

[9]We assume that the condition $c_{t+1} < \gamma$ holds in every state. There is no formal restriction that we can impose on the parameters of the model to insure that this condition holds. However, using numerical simulations, we found that for low values of ϕ the condition is violated in less that 1% of all cases. The utility specification in our extended model ensures that this condition always holds.

[10]Forni (2005) proves that the policy function is a decreasing function of the savings, meaning that the strategic effect must be positive in the equilibrium of the game between subsequent politicians. The argument is as follows: if the contribution is an increasing function of savings, then the young generation has an incentive to save more in order to get a higher benefit from the pension system when old. But this cannot be an equilibrium, because a higher benefit of the old reduces the savings of the young.

towards higher or lower contributions to the pension system.

The first term is the covariance between consumption and the financial shock. We will denote this term by "the risk sharing component" because it captures the property of the PAYG pension system of sharing shocks between young and old agents. To see this, we substitute consumption and savings using the budget constraints (4.1) and (4.2):

$$cov\left[c_{t+1}, R_{t+1}\right] = (\bar{R}^2 + \sigma_R^2)(1-\tau_t) + cov(b_{t+1}, R_{t+1}) \tag{4.11}$$

A higher contribution to the pension system lowers the covariance between consumption and return on capital. The pension system has an additional risk sharing property if the covariance between benefits and the financial shock is negative. The more negative this covariance, the less young agents must invest in social security in order to achieve the same level of risk sharing.

The last term of relation (4.10) shows how the preferences of young agents for the pension system are determined by the return on capital R_{t+1} in excess of the return of the pension system measured by the strategic effect $n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}$. We will henceforth denote this term "the return component". A lower strategic effect makes the investment in capital more attractive and weakens the support of young agents for the pension system. The return component will dominate the incentives of young agents for high values of γ (low risk aversion).

4.3.3 The political equilibrium

At the optimum, the politicians equate the welfare gain of a marginal transfer through the pension system coming from the increase in the utility of the old with the welfare loss coming from the decrease in the utility of the young:

$$u'(c_t) = \phi E_t\left[\left(R_{t+1} - n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\right)u'(c_{t+1})\right] \tag{4.12}$$

where we denote with $\phi = \frac{\phi_1}{\phi_0}$ the political weight of the young generation relative to the old generation.

The resulting contribution to the pension system depends on the weight that the two groups of agents have in the voting process. A higher dispersion of ideology among the young (relative to the old) gives them a lower political weight. In this case, the politician implements a higher contribution that implies lower consumption for the young agents.

Given a perceived policy function $\tau_{t+1} = f(s_t, R_{t+1}, n_{t+1})$, the current contribution to the pension system is set by solving the politician's problem formulated in relations (4.3)-(4.6). The solution will yield the actual policy function $\tau_t = \bar{f}(s_{t-1}, R_t, n_t)$.

Definition 2. *A Markov perfect equilibrium of the game between subsequent political candidates is obtained if the perceived and the actual policy function are equal, i.e.* $f = \bar{f}$.

In order to have a meaningful model for analyzing PAYG pension systems we make two assumptions[11]. The first one rules out cases in which young agents are so risk adverse that they are willing to participate in the PAYG pension system even if they expect no benefits from the pension system next period.

Assumption 3. *Young agents will not sustain positive contributions to the pension system if these do not bring them benefits when old. This is equivalent to*[12]*:* $\gamma > \gamma_{min} = \Gamma/\bar{R}$, *where* $\Gamma = \bar{R}^2 + \sigma_R^2$.

The second assumption comes from the fact that our main aim is to study pension systems. While transfers from old to young are also possible in reality - for example through taxes on wealth -, we analyze transfers from young to old. Hence, we impose the restriction that, on average, the contribution to the PAYG pension system is positive. We also rule out the possibility that old agents expropriate the young through the pension system by imposing that the contribution to the PAYG pension system is, on average, less than 1.

Assumption 4. *The average contribution to the pension system is strictly positive and non-expropriating:* $E(\tau_t) \in (0, 1)$.

The following proposition (for proof see Appendix 1) lays down a closed form solution to the politician's problem.

Proposition 4. *The politician's optimization problem is solved by the following policy function:*

$$\tau_t^P = \frac{A^P - R_t s_{t-1}}{B^P + n_t} \tag{4.13}$$

[11]In the extended model presented in section 4.5, assumptions 3 and 4 are not necessary.

[12]We obtain this relation by imposing the condition $\frac{\partial U^y}{\partial \tau_t}|_{\tau_t=0, b_{t+1}=0} < 0$.

where the coefficients A^P and B^P satisfy[13]*:*

$$1 = \phi B^P E_t \frac{R_{t+1}^2}{(B^P + n_{t+1})^2} \tag{4.14}$$

$$A^P = \frac{\gamma - \phi\gamma B^P E_t \frac{R_{t+1}}{B^P + n_{t+1}} + B^P}{1 - \phi B^P E_t \frac{R_{t+1} n_{t+1}}{(B^P + n_{t+1})^2}} \tag{4.15}$$

Equation (4.14) can have no solution, one solution or multiple solutions in the domain of real numbers. However, we can conclude that, if a solution exists, it must be strictly positive ($B^P > 0$). With this result, we can already analyze how politicians share financial and demographic risks between agents.

Proposition 5. *An adverse financial shock (a decrease in R_t) leads to an increase in both pension contributions and pension benefits. Under Assumption 4, an adverse demographic shock (a decrease in n_t) leads to an increase in contributions but to a decrease in benefits.*

Since $B^P + n_t > 0$, the contribution is inversely related to the wealth of the old $R_t s_{t-1}$. This means that, at the same level of savings, following a decrease in the return on capital, the politician establishes a higher contribution for the young and hence a higher level of benefits for the old. Consequently, the benefits of the PAYG pension system are negatively correlated with the return on capital. This helps agents to partially protect against financial shocks. Also, a smaller amount of savings of the old leads to a higher contribution required by the politician and, hence, to a higher benefit paid to the old.

The decrease in the population growth rate has an adverse impact on both young and old agents: contributions are increased and benefits are decreased. This impact of the demographic shock on the political arrangement of the PAYG pension system is the same as the one obtained by Gonzalez-Eiras and Niepelt (2008). In our paper, this result holds in an economy hit by both financial and demographic shocks. Hence, we establish the result that a PAYG pension system can share both types of shocks, if the contribution is adjusted correspondingly each period.

[13] A trivial solution to the policy function is given by $A^P = \gamma, B^P = 0$. However, this solution is explosive, so we will not consider it further.

4.3.4 An analytical solution of the model

In this subsection we investigate an analytic solution of the model. In order to obtain such a solution we need only two additional assumptions:

Assumption 5. *i) The financial and demographic shock are independent and ii) the demographic shock has a uniform distribution on the interval* $[\bar{n}-a, \bar{n}+a]$ *with* $\bar{n}-a>0$.

In the setup we considered, of a small open economy in which the return on capital is determined at global level, the assumption that the financial and demographic shock are independent is not implausible.

Proposition 6. *Under Assumption 5, for* $\phi\Gamma > \phi_{min}\Gamma = \frac{4a^2}{\left(1-e^{-2a/\bar{R}}\right)\left(\bar{n}+a-(\bar{n}-a)e^{2a/\bar{R}}\right)}$, *there exists a unique political equilibrium that has a finite average contribution. The coefficients of the policy function are:*

$$B^P = \frac{\phi\Gamma - 2\bar{n} + \sqrt{(2\bar{n}-\phi\Gamma)^2 - 4(\bar{n}^2-a^2)}}{2} \tag{4.16}$$

$$A^P = \frac{\gamma - \phi\gamma\frac{B^P\bar{R}}{2a}\ln\frac{B^P+\bar{n}+a}{B^P+\bar{n}-a} + B^P}{1 - \phi\frac{B^P\bar{R}}{2a}\ln\frac{B^P+\bar{n}+a}{B^P+\bar{n}-a} + \frac{B^P\bar{R}}{\Gamma}} \tag{4.17}$$

With this analytic solution we can investigate the impact of a lower mean of the population growth rate on the politician's policy function. We interpret the decline in the mean of the demographic shock as a *permanent* demographic shift and we analyze its implications on the risk sharing properties and the return of the pension system.

Proposition 7. *Under the assumptions of Proposition 6, in an economy with a lower mean of the demographic shock the politician implements a policy function that is less sensitive to the wealth of the old and for which the average strategic effect is lower, i.e.* $\frac{\partial B^P}{\partial \bar{n}} < 0, \frac{\partial E\frac{n_t}{B^P+n_t}}{\partial \bar{n}} > 0$. *The impact on the average contribution to the pension system is ambiguous.*

The fact that the policy function is less sensitive to the wealth of the old when the mean of the demographic shock is lower has two opposing implications. On the one hand, this means that pension benefits have a weaker negative correlation with the financial shock. Hence young agents should invest more in the pension system in order to achieve the same level of risk sharing.

On the other hand, this lower sensitivity of the policy function with respect to the wealth of the old also implies a lower strategic effect and, consequently a lower return of the pension system. From this point of view, young agents stronger oppose social security.

Whether the impact of the lower mean of the demographic shock on the risk sharing property of the pension system or the impact on the return of the pension system dominates in equilibrium is a quantitative question. In Section 4.5, we analyze a calibrated version of a more realistic model in order to determine how the average contribution to the PAYG pension system depends on the mean of the demographic shock.

4.4 The Ramsey planner's problem

In this section we analyze how a Ramsey planner sets pension contributions and benefits in the economy. The Ramsey planner's problem differs from the politician's in three ways. First, the Ramsey planner can commit to implement the selected policy now and in the future, so there is no strategic effect. Second, the Ramsey planner maximizes the welfare of both living and unborn agents. Finally, the cohort specific welfare weights used by the Ramsey planner may differ from the politician's.

The Ramsey planner maximizes the social welfare function, subject to the budget constraints of the agents.

$$\max_{\{\tau_t\}_{t=0}^{\infty}} \sum_{t=-1}^{\infty} \rho^{t+1} E_0[N_t u(c_{t+1})]$$

$$\text{s.t.}\ \ s_t + \tau_t = 1 \tag{4.18}$$

$$c_{t+1} = R_{t+1} s_t + \tau_{t+1} n_{t+1} \tag{4.19}$$

given N_{-1}, s_{-1}, R_0 and N_0. The welfare weight of the generation born at time t is ρ^{t+1}. We impose $\rho < \frac{1}{\bar{n}}$ to ensure that the social welfare function is bounded.

The first order condition of the Ramsey planner's problem is:

$$u'(c_t) = \rho E_t[R_{t+1} u'(c_{t+1})] \tag{4.20}$$

The form of the policy function implemented by the Ramsey planner is identical to that of the politician, but the coefficients of the policy function will be different.

Proposition 8. *The Ramsey planner's optimization problem is solved by the following policy function:*

$$\tau_t^R = \frac{A^R - R_t s_{t-1}}{B^R + n_t} \tag{4.21}$$

where the coefficients A^R and B^R satisfy:

$$1 = \rho E_t \frac{R_{t+1}^2}{B^R + n_{t+1}} \tag{4.22}$$

$$A^R = \frac{\gamma - \rho\gamma\bar{R} + B^R}{1 - \rho E_t \frac{R_{t+1} n_{t+1}}{B^R + n_{t+1}}} \tag{4.23}$$

The right hand side of equation (4.22) is decreasing in B^R while the left hand side is constant[14]. Hence, there always exists a solution with $B^R + n_t > 0$ and this is unique.

Without making any assumptions regarding the demographic and financial shocks, we can already compare the politician and the Ramsey planner's policy functions. The two policy functions are different due to the absence of the strategic effect in the Ramsey planner's problem but also because the weights they use may be distinct. In order to study only the implications of the strategic effect we consider $\rho = \phi$[15].

Proposition 9. *If the Ramsey planner and the politician use the same weights for the young generation ($\rho = \phi$), the following relations between the policy functions of the politician and the Ramsey planner hold:*

1. *The contribution set by the politician is higher than the contribution set by the Ramsey planner, i.e. $\tau^P > \tau^R$;*

2. *Following a financial or a demographic shock, the politician and the Ramsey planner change the contributions and benefits in the same direction;*

3. *Following a financial or a demographic shock, the politician adjusts the contributions more than the Ramsey planner, i.e. $|\frac{\partial \tau_t^P}{\partial R_t s_{t-1}}| > |\frac{\partial \tau_t^R}{\partial R_t s_{t-1}}|$ and $|\frac{\partial \tau_t^P}{\partial n_t}| > |\frac{\partial \tau_t^R}{\partial n_t}|$.*

[14] As in the case of the political equilibrium, we will not consider further the solution $B^R = 0$ which features an explosive dynamics.

[15] We implicitly assume that $\rho = \phi \geq \underline{\phi}$, where $\underline{\phi}$ is the lowest political weight for which at least one solution for the political equilibrium exists. However, the Ramsey planner's problem admits solutions also for $\rho < \underline{\phi}$.

The above proposition implies that the response of the politician and the Ramsey planner is qualitatively the same following a financial or a demographic shock. Quantitatively, however, their reaction is different. The politician changes the contributions more than the Ramsey planner because of the existence of the strategic effect.

In order to be able to study further the Ramsey planner's policy function, we impose the assumptions summarized in 5.

Proposition 10. *Under Assumption 5, the solution to the Ramsey planner's problem has the following coefficients:*

$$B^R = \frac{\bar{n} + a - (\bar{n} - a)e^{\frac{2a}{\rho\Gamma}}}{e^{\frac{2a}{\rho\Gamma}} - 1} \tag{4.24}$$

$$A^R = \frac{\gamma - \rho\gamma\bar{R} + B^R}{1 - \rho\bar{R} + \frac{B^R\bar{R}}{\Gamma}} \tag{4.25}$$

The average contribution to the pension system is finite for $\rho > \frac{\bar{R}}{\Gamma}$.

The following proposition shows how the mean of the demographic shock affects the policy function of the Ramsey planner.

Proposition 11. *Under Assumptions 3-5, in an economy with a lower mean of the demographic process, the Ramsey planner will impose a higher average contribution, i.e.* $\frac{\partial E(\tau_t^R)}{\partial \bar{n}} < 0$.

Unlike in the politico-economic equilibrium, the result is unambiguous. Intuitively, in the absence of the strategic effect, when the mean of the population growth rate is lower, the incentives of the old and young agents change in the same direction of sustaining a higher size of the pension system. For the young agents, the return on capital in excess of the pension system does not change when the characteristics of the demographic shock change. But the covariance between the benefits and the financial shock becomes less negative and this makes a higher size of the pension system optimal.

4.5 An extended model

In this section, we determine whether the results obtained with the model presented in Section 4.3 hold in a more general setting. First, we introduce the savings and labor supply decision of young agents. Both these extensions are important for shaping the preferences of young agents for the PAYG pension contribution. Since they are able to adjust their level of savings, agents can choose a lower exposure towards financial shocks and, hence, vote for smaller contributions to the pension system. Endogenising labor supply also has an important implication: the contribution to the PAYG pension system becomes distortionary so young agents dislike it more than in the setting with exogenous labor supply. Second, we relax the assumption of the analytically convenient quadratic utility and consider a utility function of Greenwood, Hercowitz and Huffman (GHH) type that is typically used in open economy Real Business Cycle models (Mendoza (1991)). Apart from checking whether the results obtained in the small model hold in a more realistic setting, we also use the extended model to establish the implications of a lower mean population growth rate for the average contribution to the pension system in the political equilibrium. That result was ambiguous in the analytic solution of Section 4.3.

4.5.1 The economic equilibrium

Households maximize expected utility given the budget constraints faced in both periods of life:

$$\max_{c_t^y, c_t^o, s_t, l_t} u\left(c_t^y, l_t\right) + \beta E_t u(c_{t+1}^o) \tag{4.26}$$

$$c_t^y + s_t = l_t(1 - \tau_t) \tag{4.27}$$

$$c_{t+1}^o = R_{t+1} s_t + b_{t+1} \tag{4.28}$$

where c_t^y is the consumption of young agents at time t, c_{t+1}^o is the consumption of old agents at time $t+1$, s_t are the savings, l_t is the amount of labor supplied by young agents. As before, the contribution to the pension system is τ_t, while the benefits from the pension system are represented by b_t. Wages are again normalized to 1.

We consider a utility function of Greenwood, Hercowitz and Huffman (GHH) type:

$$u\left(c_t^y, l_t\right) = \frac{\left(c_t^y - \psi \frac{l_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\eta}}{1-\eta} \tag{4.29}$$

where η is the inverse of the intertemporal elasticity of substitution, ν is the Frisch elasticity of labor supply and ψ quantifies the disutility of labor.

The first order conditions of the household with respect to s_t and l_t give us the Euler equation of the young agent (relation (4.30)) and a closed form solution for the amount of labor supplied by the household (relation (4.31)), respectively:

$$u_c\left(c_t^y, l_t\right) = \beta E_t\left[u_c(c_{t+1}^o)R_{t+1}\right] \tag{4.30}$$

$$\psi l_t^{\frac{1}{\nu}} = 1 - \tau_t \Rightarrow l_t = \left(\frac{1-\tau_t}{\psi}\right)^{\nu} \tag{4.31}$$

The government runs the PAYG pension system and ensures a balanced budget each period.

$$N_t^y l_t \tau_t = N_t^o b_t \Rightarrow b_t = n_t l_t \tau_t \tag{4.32}$$

Households make their decisions taking pension benefits as given and the government balances the budget of the pension system.

4.5.2 Political equilibrium

In the political equilibrium, agents vote for the politician that will implement their desired level of contributions to the pension system. As in the small model presented in Section 4.3, the politicians maximize the present value of the utility of the two living cohorts weighed with the size of the cohorts and their ideological dispersion, subject to the agents' budget constraints, their optimality conditions and the balanced budget constraint of the government.

$$\max_{\tau_t} \phi_0 u(c_t^o) + \phi_1 n_t\left[u\left(c_t^y, l_t\right) + \beta E_t u(c_{t+1}^o)\right]$$
$$\text{s.t. } (4.27), (4.28), (4.30), (4.31), (4.32)$$

We restrict our attention to the same family of differentiable Markov policy functions as in the stylized model presented in Section 4.3. Hence, the contributions to the pension system are set according to the policy function from relation (4.7).

In the Markov perfect equilibrium, the current contribution to the pension system influences the savings made by young agents. In turn, the amount of savings young agents make influence the contribution to the pension system set next period through the policy function of the next period's politician. To show the impact of current and future contributions on the savings of young agents, we use the Euler equation (4.30), together with the solution for labor supply (4.31), the balanced budget condition (4.32) and the policy function for the contribution to the pension system (4.7). We obtain an equation that implicitly defines the amount of savings as a function of the current (τ_t) and future (f) contributions to the pension system:

$$u_c\left(\left(\frac{1-\tau_t}{\psi}\right)^{\nu}(1-\tau_t)-s_t-\psi\frac{\left(\frac{1-\tau_t}{\psi}\right)^{1+\nu}}{1+\frac{1}{\nu}}\right)- \\ -\beta E_t\left[u_c\left(R_{t+1}s_t+n_{t+1}\left(\frac{1-f(s_t,R_{t+1},n_{t+1})}{\psi}\right)^{\nu}f(s_t,R_{t+1},n_{t+1})\right)R_{t+1}\right]=0 \tag{4.33}$$

The first order condition of the politician's problem is:

$$u_c(c_t^o)\frac{\partial Q_t}{\partial \tau_t}=\phi\left[l_t u_c\left(c_t^y,l_t\right)-\beta E_t\left[u_c(c_{t+1}^o)n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}\frac{\partial Q_{t+1}}{\partial \tau_{t+1}}\right]\right] \tag{4.34}$$

where $n_{t+1}\frac{\partial \tau_{t+1}}{\partial \tau_t}$ is the strategic effect and $Q_t=\tau_t l_t$ are government revenues.

We are now ready to define the political equilibrium of the model.

Definition 3. *A Markov perfect political equilibrium is defined by a function f for which:*

1. *Given the policy rule f, s_t is obtained by solving equation* (4.33)*;*
2. *Given f and s_t, $\bar{f}$ solves the politician's problem defined by the first order condition* (4.34)*;*
3. *The perceived and the actual policy function are the same: $f=\bar{f}$.*

4.5.3 The Ramsey planner's problem

The Ramsey planner sets the contributions to the pension system by maximizing the welfare of all living and unborn agents subject to their budget constraints, their

Table 4.3: Calibration

Parameter	Description	Value	Source
$\bar{R}$	Mean of financial shock	1.82	US Data
σ_R	Std. deviation of financial shock	0.70	US Data
$\bar{n}$	Mean of demographic shock	1.59	US Data
$\sigma_n = {}^{a}/\sqrt{3}$	Std. deviation of demographic shock	0.09	US Data
η	Inverse intertemporal elasticity of substitution	2	-
β	Time preference	${}^{1}/\bar{R}$	-
ν	Frisch elasticity	0.7	-
Parameter	Description	Value	Target
ψ	Labor disutility	4	Average labor supply l: 0.35
ϕ	Political weight of young agents	1.7	Average contribution τ: 0.106

optimality conditions and the balanced budget constraint of the government.

$$\max_{\{\tau_t\}_{t=0}^{\infty}} \beta N_{-1} u(c_0^o) + E_t \left[\sum_{t=0}^{\infty} N_t \rho^t \left(u\left(c_t^y, l_t\right) + \beta u(c_{t+1}^o) \right) \right]$$
$$\text{s.t. } (4.27), (4.28), (4.30), (4.31), (4.32)$$

where ρ is the weight that the Ramsey planner places on future generations. N_{-1} and s_{-1} are given.

The first order condition of the Ramsey planner's problem in this extended model is:

$$\beta u_c(c_t^o) \frac{\partial(\tau_t l_t)}{\partial \tau_t} = \rho l_t u_c\left(c_t^y, l_t\right) \tag{4.35}$$

4.5.4 Calibration and qualitative explanatory power

We calibrate the model using a combination of empirical targets, data based parameters and values usually employed in the literature. These are presented in Table 4.3.

Since we consider only two overlapping generations, the frequency of our model's data is very low: we consider it to be 40 years. Hence, we require a very long data set for the calibration of the financial and demographic shock. We can only obtain such a long data set (comprising the years 1900-2014) for the US. We take the data for the S&P500 annual returns and long term interest rates from the updated database of Robert Shiller[16] and the data for the nominal wage and the population growth rate

[16] *Irrational Exuberance* [Princeton University Press 2000, Broadway Books 2001, 2nd ed., 2005], http://www.econ.yale.edu/ shiller/data.htm

from the Federal Reserve of St. Louis (FRED) database.

For the calibration of the financial shock, we compute the gross return of a portfolio comprised 50% of stocks and 50% of long term government bonds on a 40 years horizon between 1940-2014. The gross returns of the portfolio are deflated with the nominal wage growth rate. For the calibration of the demographic shock, we compute the gross annual population growth rates on a 40 years horizon. In both cases, we take the minimum and the maximum value and, assuming a uniform distribution for the shocks, we compute the mean and standard deviation of the variables as presented in table 2.3.

The Frisch elasticity of labor supply is set equal to 0.7 following Song et al. (2012), Heathcote et al. (2010b), Pistaferri (2003). We take $\eta = 2$, a value usually employed in the literature (see Song (2011)). The parameter that determines the disutility of labor is set by targeting an average labor supply of 0.35 (Gonzalez-Eiras and Niepelt (2008)). The political weight of young agents is calibrated in order to obtain an average contribution equal with 10.6%, the value currently in place in the US.

We solve the model using the projection method. The numerical procedure is detailed in Appendix 2. We use the calibrated version of the model to perform two analyses. First, we show that the impact of demographic and financial shocks on pension contributions and benefits is the same as in the stylized model of Section 4.3 even if we allow for savings and endogenous labor supply. Second, we show that for our calibration a lower mean of the demographic shock leads to a smaller average contribution set by the politician. This is contrary to the result obtained in the Ramsey planner's case.

The impact of financial and demographic shocks on the politician's policy function

We first analyze the impact of demographic and financial shocks on pension contributions and benefits. Figures 4.2 and 4.3 present the impact of demographic shocks on the arrangement of the pension system. Pension contributions and benefits are presented for different levels of the wealth of the old and for two extreme realizations of the population growth rates: $n - \bar{a} = 1.44$ - the lowest value that the population growth rate can take in our calibration and $\bar{n} + a = 1.73$ - the highest value that the population growth rate can take. At the same level of the wealth of the old, a lower population growth rate implies a higher level of contributions but a lower level of benefits. As we showed in the narrative evidence of Section 4.2, this result is consistent with the pension reforms enacted in many countries starting with the 1990s. Confronted with a decrease in the population growth rate, Western European countries implemented reforms that increased the contribution to the pension system and decreased the life-time pension benefits of agents.

Figure 4.2: Contributions

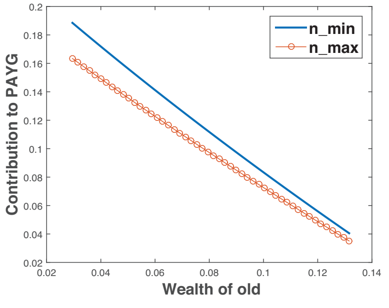


Figure 4.3: Benefits

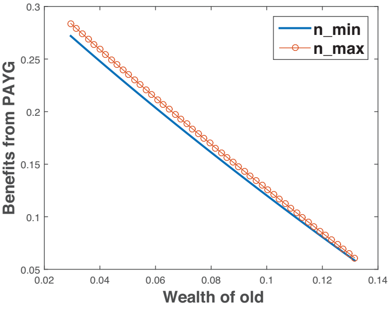


Figure 4.2 shows that the sensitivity to demographic shocks of the contribution rate set by the politician depends on how wealthy old agents are. When old agents are poor, i.e. they are hit by an adverse financial shock in the current period - the contribution rate adjusts substantially in order to accommodate changes in the population growth rate. Hence, demographic risk is borne more by the young generation. The converse is true when old agents are wealthy: the contribution rate is not very sensitive to changes in the population growth rate and the young generation is less affected by demographic shocks.

Figures 4.4 and 4.5 show the pension contribution and benefits across different population growth rates and for two levels of the financial shock $R_{min} = 0.6$ and $R_{max} = 3.03$. Financial shocks change contributions and benefits in the same direction: a lower return on financial investment increases both contributions and benefits. We obtained the same result with the stylized model of Section 4.3.

The impact of financial shocks on pension contributions and benefits is also consistent with the stylized facts regarding changes in PAYG pension systems after major economic downturns. As we pointed out in our narrative evidence of Section 4.2, governments increased both contributions and benefits after the substantial savings losses caused by the Great Depression and World War II.

Figure 4.4: Contributions

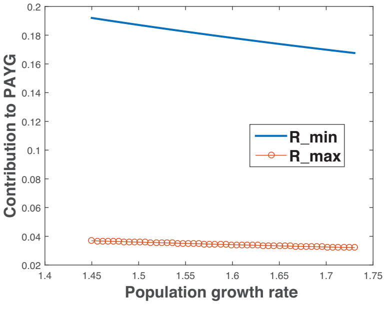
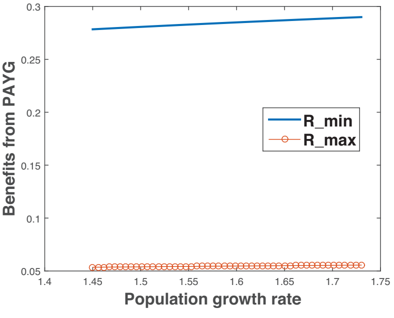


Figure 4.5: Benefits



The difference between the political equilibrium and the Ramsey planner's model

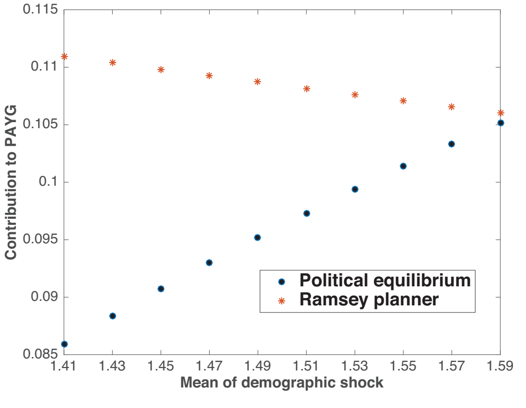
For the Ramsey planner we were able to determine in Section 4.4 that a lower mean of the population growth rate leads to a higher average contribution to the pension system. The impact of a lower mean of the population growth rate in the political equilibrium was ambiguous. Here, we resort to a numerical simulation to answer this question.

Figure 4.6 presents the average contribution to the pension system set by the politician and the Ramsey planner for different means of the population growth rate. For the Ramsey planner's problem we set $\rho = 0.3525$, a value for which the average contribution set by the Ramsey planner and the politician is the same. As in the stylized model presented in Section 4.4, the Ramsey planner sets, on average, a higher contribution when the demographic shock has a lower mean. At the the same level of pension contribution, the pension system offers a lower insurance against financial shocks when the demographic shock has a lower mean. The Ramsey planner increases the average contribution to the pension system to restore the pension system's property of insuring against financial shocks. In contrast, the politician sets a lower average contribution. This is because in the political equilibrium a lower mean of the population growth rate implies that the pension system offers less insurance against financial risks but also a lower return due to a smaller strategic effect. Numerical simulations indicate that the return component dominates the risk sharing component and the politician sets a lower average contribution.

These findings are qualitatively consistent with two stylized facts presented in Section 4.2. First, although individuals over 60 years suffered the highest wealth losses during the Great Recession, governments did very little to compensate them through the pension system. This is unlike the response of governments after the Great Depression and World War II. We explain this through the fact that the decline in the mean population growth rate in some countries weakened the response of governments to financial shocks.

Second, countries that experienced permanent reductions in their population growth rate (Central and Eastern European and Latin American countries) decided to partially or completely downsize their PAYG pension systems. We interpret this as evidence that a lower mean of the population growth rate leads to a lower average contribution to the pension system in the political equilibrium.

Figure 4.6: Impact of the mean of the demographic shock on the contribution rate



4.6 Conclusions

The present paper shows how a PAYG pension system responds to financial and demographic shocks in a probabilistic voting model with overlapping generations. We find that, after a decrease in the return on capital, the politician increases contributions and benefits. In response to a decrease in the population growth rate, contributions are raised, but benefits are reduced. These responses correspond with major developments of PAYG pension systems observed in reality.

The mean of the demographic shock impacts on the politician's policy function: if the mean of the demographic shock is lower, the compensation for financial losses offered to old agents through the pension system is smaller. This result can account for the stylized fact discussed in Section 2 that during the Great Recession, unlike in previous major downturns, governments did not increase the pension benefits. In the political equilibrium, a weaker sensitivity of the policy function with respect to the wealth of the old leads to a lower average contribution to the pension system. This effect can explain reforms that involve the (partial) transition from PAYG pension systems to fully funded ones.

Following financial and demographic shocks, the Ramsey planner changes contributions and benefits in the same direction as the politician. However, the adjustments made are of a smaller magnitude. Unlike the politician, if the mean of the demographic shock is smaller, the Ramsey planner increases the average contribution to the pension system.

The analysis in this paper can be extended into several interesting directions. First, it would be important to introduce heterogeneity in young agents' productivity. With heterogeneity in productivity, the choice over pension contributions would reflect the diverging interests of young and old agents as well as those of rich and poor agents. We do not expect the heterogeneity in productivity to change the qualitative results of the present paper. However, we expect it to impact on the size of changes in contributions and benefits following financial shocks. This is because financial shocks impact rich and poor agents to different degrees. Also, in such a framework we can study the provision of means and asset tested old-age benefits alongside earnings related pensions. We can analyze how the relative size of these two pension pillars is influenced by the business cycle. This is a relevant policy question since we observe that during the Great Recession many governments expanded means and asset tested benefits but kept earnings related pension benefits unchanged or even downsized them[17].

Second, the model can be used to study the political arrangement of pension systems that have a mandatory fully funded component. The downsizing of this component that took place in most of the Central and Eastern European countries

[17]Through increases in the retirement age for example. See OECD (2012).

during the Great Recession shows that financial shocks may influence its size.

Appendix 1

Proof of Proposition 4

Based on the solution we guessed in (4.13), the strategic effect is:

$$\frac{\partial \tau_{t+1}^P}{\partial \tau_t^P} = \frac{\partial \tau_{t+1}^P}{\partial s_t}\frac{\partial s_t}{\partial \tau_t^P} = -\frac{R_{t+1}}{B^P + n_{t+1}}\frac{\partial(1-\tau_t^P)}{\partial \tau_t^P} = \frac{R_{t+1}}{B^P + n_{t+1}} \tag{36}$$

We substitute (36), the guess for the policy function (4.13) and the budget constraints (4.1) and (4.2) in relation (4.12). We obtain the following expression for the policy function:

$$\tau_t^P = \frac{\gamma - \phi\gamma B^P E_t \frac{R_{t+1}}{B^P + n_{t+1}} + \phi B^P A^P E_t \frac{R_{t+1} n_{t+1}}{(B^P + n_{t+1})^2} + \phi (B^P)^2 E_t \frac{R_{t+1}^2}{(B^P+n_{t+1})^2} - R_t s_{t-1}}{\phi (B^P)^2 E_t \frac{R_{t+1}^2}{(B^P+n_{t+1})^2} + n_t}$$

Comparing the above with the solution we proposed in (4.13), we obtain the system of equations for the coefficients A^P and B^P.

Proof of Proposition 5

Differentiation of equation (4.13) gives:

$$\begin{aligned}\frac{\partial \tau_t^P}{\partial n_t} &= -\frac{\tau_t^P}{B^P + n_t} < 0 \text{ if } \tau_t^P > 0\\ \frac{\partial b_t^P}{\partial n_t} &= \frac{B^P \tau_t^P}{B^P + n_t} > 0 \text{ if } \tau_t^P > 0\end{aligned}$$

In the last inequality we used the fact that $B^P > 0$.

Proof of Proposition 6

We provide the proof in steps. First, under Assumption 5, equation (4.14) becomes:

$$1 = \phi B^P \Gamma \int_{\bar{n}-a}^{\bar{n}+a} \frac{1}{(B^P + n_{t+1})^2}\frac{1}{2a} dn_{t+1} = \frac{\phi \Gamma B^P}{(B^P + \bar{n} + a)(B^P + \bar{n} - a)}$$

We obtain two solutions for B^P:

$$B_1^P = \frac{\phi\Gamma - 2\bar{n} + \sqrt{\Delta}}{2}; B_2^P = \frac{\phi\Gamma - 2\bar{n} - \sqrt{\Delta}}{2} \tag{37}$$

where $\Delta = (\phi\Gamma - 2\bar{n})^2 - 4(\bar{n}+a)(\bar{n}-a)$. From (4.15) we obtain the solution for A^P.

Second, we impose the condition that the policy function has a finite mean. We substitute (4.1) in (4.13) and obtain a recursive expression for τ_t^P:

$$\tau_t^P = \frac{A^P - R_t}{B^P + n_t} + \frac{R_t}{B + n_t}\tau_{t-1}^P$$

We denote $\frac{A^P - R_t}{B^P + n_t} \equiv Q_t$ and $\frac{R_t}{B+n_t} \equiv M_t$. Iterating backwards to period $t = 0$, we obtain:

$$\tau_t^P = \sum_{i=1}^{t} Q_i \prod_{j=i+1}^{t} M_j + \tau_0^P \prod_{j=1}^{t} M_j$$

We compute the unconditional mean of τ_t^P allowing $t \to \infty$ and using the time independence of shocks.

$$\tau_\infty^P = \sum_{t=1}^{\infty} Q_t \prod_{j=1}^{t} M_j + \tau_0^P \prod_{j=1}^{\infty} M_j \Rightarrow$$
$$E(\tau_\infty^P) = EQ \sum_{t=0}^{\infty} (EM)^t + E(\tau_0^P) \prod_{j=1}^{\infty} EM$$

The unconditional mean of τ_t exists if and only if $-1 < EM < 1$. This is equivalent to $-1 < E\frac{R_t}{B^P+n_t} < 1$. Since $E\frac{R_t}{B^P+n_t} > 0$, we only need to determine under what conditions $E\frac{R_t}{B^P+n_t} < 1$.

Under Assumption 5:

$$E\left[\frac{R_t}{B^P + n_t}\right] = \bar{R}\int_{\bar{n}-a}^{\bar{n}+a} \frac{1}{B^P + n_{t+1}}\frac{1}{2a} dn_t = \frac{\bar{R}}{2a} ln \frac{B^P + \bar{n} + a}{B^P + \bar{n} - a}$$

Since $ln\frac{B^P+\bar{n}+a}{B^P+\bar{n}-a}$ is decreasing in B^P:

$$\frac{\bar{R}}{2a} ln \frac{B_1^P + \bar{n} + a}{B_1^P + \bar{n} - a} < \frac{\bar{R}}{2a} ln \frac{B_2^P + \bar{n} + a}{B_2^P + \bar{n} - a}$$

We determine the value of $\phi\Gamma$ for which only solution B_1^P achieves a finite mean of the contribution to the pension system:

$$\frac{\bar{R}}{2a} ln \frac{B_1^P + \bar{n} + a}{B_1^P + \bar{n} - a} < 1 < \frac{\bar{R}}{2a} ln \frac{B_2^P + \bar{n} + a}{B_2^P + \bar{n} - a} \iff$$
$$\phi\Gamma > \frac{4a^2}{\left(1 - e^{-2a/\bar{R}}\right)\left(\bar{n} + a - (\bar{n} - a)e^{2a/\bar{R}}\right)}$$

where we used (37).

A finite mean of τ_t also implies absolute convergence of τ_t. Following Goldie and Maller (2000), a necessary and sufficient condition for the series τ_t^P to be absolutely convergent is $E\left[ln\frac{R_t}{B^P+n_t}\right] < 0$. Jensen's inequality gives $E\left[ln\frac{R_t}{B^P+n_t}\right] < lnE\frac{R_t}{B^P+n_t} < 0$ where the last upper bound follows from $E\frac{R_t}{B^P+n_t} < 1$.

Proof of Proposition 7

The sensitivity of the politician's policy function with respect to the wealth of the old is given by $\frac{1}{B^P+n_t}$, while the average strategic effect is equal to $\bar{R}E\frac{n_t}{B^P+n_t}$.

We can immediately obtain the following derivatives:

$$\frac{\partial B^P}{\partial \bar{n}} = -1 - \frac{\phi\Gamma}{\sqrt{\Delta}} < 0 \tag{38}$$

$$\frac{\partial E\frac{1}{B^P+n_t}}{\partial \bar{n}} = -\left(\frac{\partial B^P}{\partial \bar{n}} + 1\right)\frac{1}{(B^P + \bar{n} + a)(B^P + \bar{n} - a)} =$$
$$= \frac{\phi\Gamma}{\sqrt{\Delta}(B^P + \bar{n} + a)(B^P + \bar{n} - a)} > 0 \tag{39}$$

$$\frac{\partial E\frac{n_t}{B^P+n_t}}{\partial \bar{n}} = -\frac{\partial B^P}{\partial \bar{n}}\left(E\frac{1}{B^P + n_t} - \frac{1}{\phi\Gamma}\right) + \frac{1}{\phi\Gamma} > 0 \tag{40}$$

To sign the derivatives we used the result $E\frac{1}{B^P+n_t} - \frac{1}{\phi\Gamma} > 0$, obtained from rewriting (4.12):

$$E_t\frac{n_{t+1}}{(B^P + n_{t+1})^2} = E_t\frac{1}{B^P + n_{t+1}} - \frac{1}{\phi\Gamma} > 0$$

The sensitivity of the contribution to the pension system with respect to the wealth of old is inversely related to B^P, hence we get the first result of the proposition.

Proof of Proposition 8

We substitute (4.18), (4.19) and (4.21) in (4.20) and obtain:

$$\tau_t^R = \frac{\gamma - \rho\gamma\bar{R} + \rho A^R E_t \frac{R_{t+1}n_{t+1}}{B^R + n_{t+1}} + \rho B^R E_t \frac{R_{t+1}^2}{B^R + n_{t+1}}}{B^R \rho E_t \frac{R_{t+1}^2}{B^R + n_{t+1}} + n_t} \tag{41}$$

Comparing this solution with the initial guess in (4.21), we obtain the system of equations defining the solutions for B^R and A^R.

Proof of Proposition 9

1. We replace in (4.12) the formula of the strategic effect (36). The first order condition of the politician's problem becomes:

$$u'(c_t^P) = \phi E_t \left[\frac{B_t^P}{B_t^P + n_{t+1}} R_{t+1} u'(c_{t+1}^P) \right] \tag{42}$$

Consider a level of ϕ denoted by ϕ' such that $\phi' E_t \left[\frac{B^P}{B^P + n_{t+1}} R_{t+1} u'(c_{t+1}^P) \right] = \rho E_t \left[R_{t+1} u'(c_{t+1}^P) \right]$. Relation (42) becomes:

$$u'(c_t^P) = \rho E_t \left[R_{t+1} u'(c_{t+1}^P) \right] \tag{43}$$

which is the first order condition of the Ramsey planner (4.20). Consequently, for $\phi = \phi'$ the policy functions of the politician and the Ramsey planner are the same $\tau^P = \tau^R$.

Since $B^P, n_{t+1} > 0$, we obtain:

$$\rho E_t \left[R_{t+1} u'(c_{t+1}^P) \right] = \phi' E_t \left[\frac{B^P}{B^P + n_{t+1}} R_{t+1} u'(c_{t+1}^P) \right] < \phi' E_t \left[R_{t+1} u'(c_{t+1}^P) \right] \Rightarrow \phi' > \rho \tag{44}$$

Consequently, we prove that for $\phi = \phi' > \rho$ the policy functions are the same $\tau_{\phi'}^P = \tau_\rho^R$. Then for a lower political weight of the young $\phi = \rho < \phi'$ the contribution to the pension system must be higher, so $\tau_\rho^P > \tau_{\phi'}^P = \tau_\rho^R$.

2. We show that $B^R > B^P$. We compare equations (4.12) and (4.20). For $\rho = \phi$, we obtain:

$$E_t \frac{B^P}{(B^P + n_{t+1})^2} = E_t \frac{1}{B^R + n_{t+1}} \Leftrightarrow E_t \frac{1}{B^P + n_{t+1}} - E_t \frac{n_{t+1}}{(B^P + n_{t+1})^2} = E_t \frac{1}{B^R + n_{t+1}}$$

Since $E_t \frac{n_{t+1}}{(B^P+n_{t+1})^2} > 0$, we have:

$$E_t \frac{1}{B^P + n_{t+1}} > E_t \frac{1}{B^R + n_{t+1}} \Leftrightarrow (B^R - B^P) E_t \frac{1}{(B^P + n_{t+1})(B^R + n_{t+1})} > 0 \Leftrightarrow B^R > B^P$$

We already know that $B^P > 0$ always and from the proof above $B^R > B^P$. Then also $B^R > 0$. Since $B^R + n_t > 0$ and $B^P + n_t > 0$, both the politician and the Ramsey planner decrease the pension contributions and benefits when the return on capital increases.

Focusing only on positive equilibrium contributions to the pension system, as shown in the proof of Proposition 5, the fact that $B^P, B^R > 0$ also insures that both the politician and the Ramsey planner decrease the pension contribution and increase the benefits when the population growth rate increases.

3. See the proof at point 2 for $B^R > B^P$. This shows that the politician decreases contributions to a higher extent than the Ramsey planner when there is an increase in the return on capital or in the population growth rate.

Proof of Proposition 10

We solve for B^R and A^R from (4.22) and (4.23) using the assumption of independence between R_t and n_t and the assumption of a uniform distribution for n_t. The condition for a finite mean is the same as in the politician's case $E \frac{1}{B^R+n_t} \in (-1/\bar{R}, 1/\bar{R})$. But now, $E \frac{1}{B^R+n_t} = \frac{1}{\rho\Gamma}$. Hence we must have $\rho > \frac{\bar{R}}{\Gamma}$.

Proof of Proposition 11

The derivative of B^R with respect to $\bar{n}$ is:

$$\frac{\partial B^R}{\partial \bar{n}} = -1 < 0$$

The average contribution to the pension system has the form:

$$E(\tau_t^R) = \frac{(A^R - \bar{R}) E \frac{1}{B^R+n_t}}{1 - \bar{R} E \frac{1}{B^R+n_t}} = \frac{A^R - \bar{R}}{\rho\Gamma - \bar{R}}$$

The derivative of $E(\tau_t^R)$ with respect to $\bar{n}$ is:

$$\frac{\partial E(\tau_t^R)}{\partial \bar{n}} = \underbrace{\frac{\partial E(\tau_t^R)}{\partial A^R}}_{>0} \underbrace{\frac{\partial A^R}{\partial B^R}}_{>0} \underbrace{\frac{\partial B^R}{\partial \bar{n}}}_{<0} < 0$$

$$\frac{\partial E(\tau_t^R)}{\partial A^R} = \frac{1}{\rho\Gamma - \bar{R}} > 0$$

$$\frac{\partial A^R}{\partial B^R} = \frac{\left(\bar{R}\rho - 1\right)\left(\frac{\gamma\bar{R}}{\Gamma} - 1\right)}{\left(1 - \rho\bar{R} + \frac{\bar{R}B^R}{\Gamma}\right)^2} > 0$$

To sign the above derivatives we use:

- Assumption 3 ($\gamma > \Gamma/\bar{R}$);
- the result of Proposition 10 ($\rho > \frac{\bar{R}}{\Gamma}$);
- an additional restriction on the welfare weight that insures Assumption 4 is satisfied ($\rho > 1/\bar{R}$). The proof for this last restriction entails the following steps. First, we first establish that $E(\tau_t^R) < 1 \Leftrightarrow \frac{(\gamma + B^R - \rho\Gamma)(1-\rho\bar{R})}{1-\rho\bar{R}+\frac{\bar{R}B^R}{\Gamma}} < 0$.

  Second, using the properties of $\frac{\partial B^R}{\partial \sigma_n}$ derived above we can establish that $B^R > \lim_{\sigma_n \to 0} B^R = \rho\Gamma - \bar{n}$. Based on this inequality we can prove the relations:

$$1 - \rho\bar{R} + \frac{\bar{R}B^R}{\Gamma} > 1 - \rho\bar{R} + \frac{\bar{R}(\rho\Gamma - \bar{n})}{\Gamma} = \frac{\Gamma - \bar{R}\bar{n}}{\Gamma} > 0$$

$$\gamma + B^R - \rho\Gamma > \gamma - \bar{n} > \frac{\Gamma}{\bar{R}} - \bar{n} > 0$$

Consequently, for Assumption 4 to hold it must be the case that $1 - \rho\bar{R} < 0$.

Appendix 2

We solve the model using the projection method. We approximate the policy functions for the contribution to the pension system $\tau_t = f(s_{t-1}, R_t, n_t)$ and the savings made by young agents $s_t = g(s_{t-1}, R_t, n_t)$ by 5^{th} order Cebyshev polynomials.

The numerical solution to the politician's problem is given by the policy functions for the contribution rate $\tau_t = f(s_{t-1}, R_t, n_t)$ and savings $s_t = g(s_{t-1}, R_t, n_t)$ obtained by fitting the Euler equation (4.30) and the first order condition of the politician

(4.34). When we solve the Ramsey planner's problem we replace the first order condition of the politician we the one of the Ramsey planner's (4.35).

To implement the numerical procedure, we need a good guess for the initial policy function to ensure that convergence is achieved quickly. The procedure of finding a good starting point entails the following steps.

1. We start from the solution of the Ramsey planner's problem with exogeneous labor supply, savings, no demographic shocks and quadratic utility. We prefer to start from the Ramsey planner's problem and not from the politician's problem because in the case of quadratic utility the latter can have no solution in the domain of real numbers for some parametrizations.

2. We take $s_{min} = 0.001$ and $s_{max} = 0.3$ and we solve for the policy functions using the calibration $\eta = 1$, $\psi = 2$, $\sigma_n = 0.001$ and $\nu = 0.25$. A low level of ν (low elasticity of labor supply) is key to achieve a quick convergence.

3. The policy functions obtained with this calibration are used as an initial guess for subsequent calibrations. Specifically, we increase in steps the Frisch elasticity of labor supply ν from 0.25 to the calibrated value of 0.7. At each step, the initial guess is the policy function obtained in the previous step.

With this guess for the initial policy function the model is solved very quickly and the results are quite accurate. Simulations performed with the model show that the contribution to the pension system is always contained between (0,1).

Summary

This thesis consists of three essays that analyse the risk sharing and labor supply distortions embedded in pay-as-you-go pension systems.

In Chapter 2, I compare the three most widespread types of pay-as-you-go pension systems - Defined Benefit (DB), Notional Defined Contribution (NDC) and flat benefit (FL) - in terms of long run macroeconomic implications and welfare. On the one hand, having a tight link between a person's life-time earnings (or a person's life-time pension contributions) and the pension benefit as in the NDC and DB systems restores incentives to work more and longer, hence increasing long run welfare. On the other hand, it leaves a person more exposed to earnings risk, hence lowering long run welfare. Specifically, any shock to a person's earnings received throughout the career will be perfectly reflected in post-retirement income. Consequently, insurance is lower under a NDC or DB system. Moreover, I show that tightening the link between pension benefits and the earnings history can come at the cost of lower capital accumulation in the economy. This happens because people have higher incentives to work more or longer when they are close to the retirement age and, hence, they need to save less to achieve the same level of old age consumption. A lower level of capital accumulation decreases welfare in dynamically efficient economies.

Calibrating a realistic overlapping generations model on the US economy, I find that switching to an NDC or DB pension system decreases welfare in the long-run. Moreover, welfare is the highest if the current pay-as-you-go pension system of the US economy is replaced by a FL pension system. This result implies that the higher welfare brought about by restoring incentives for working through tightening the link between earnings and pension benefits is dominated by the lower welfare due to less insurance and lower capital accumulation in the economy.

I also analyze how the result changes in an economy where the contribution to the pension system is higher (33%, the value currently prevailing in Italy, instead

of 10.6%, the value currently prevailing in the US economy). Since labor supply distortions increase quadratically in the level of pension contributions, one would expect this effect to dominate at higher levels of pension contributions. I find that the FL system still provides a higher long-run welfare than a NDC system, but the difference in terms of consumption equivalent compared to the flat benefit system shrinks considerably. Moreover, if one shuts down the impact of capital accumulation on welfare considering the case of a small open economy, the FL and NDC system become similar in terms of welfare.

Chapter 3 analyses the impact of a higher job loss risk on retirement. It shows empirically and theoretically that in an economy with a higher job loss risk the labor force participation of older workers is lower. The counterfactual experiments performed using the model build in the chapter indicate that the presence of a pay-as-you-go pension system amplifies the impact of job loss risk on the retirement age.

Using data from the US Survey of Income and Program Participation (SIPP), this chapter shows that the risk of suffering a substantial earnings loss following an unemployment spell is higher as people approach the end of their working career and as they spend more time out of employment. Hence, I consider as a proxy for the size of earnings losses following an unemployment spell the level of long-term unemployment prevailing in a country. I construct a cross country regression illustrating that higher earnings losses following an unemployment spell reduce the labor force participation rate among older workers. In the regression, I control for country specific characteristics such as the size of pension contributions, life expectancy or pension progressivity.

The channels through which a higher job loss risk lowers the average retirement age in an economy are identified in a life-cycle model with endogeneous retirement, incomplete markets and a job loss shock. All workers, irrespective of whether they are hit by a job loss shock or not, are affected by job loss risk through savings. Specifically, anticipating the probability of suffering a substantial decrease in their earnings after the job loss shock, workers make higher precautionary savings before the shock can occur and this allows them to retire earlier. Workers that are hit by a job loss shock are furthermore impacted through the fact that: i) the return from working decreases after the shock offering them an incentive to retire earlier (substitution effect); ii) in order to afford the same level of consumption they must work longer (income effect).

Calibrating the model on the US economy, I obtain that workers that are hit by a job loss shock retire earlier than workers that do not suffer a job loss shock. Also, a higher job loss risk lowers the average retirement age in the economy, in line with the empirical findings using cross-country data. I run a counterfactual experiment in which I eliminate the pay-as-you-go pension system in the economy. In this case, workers that are hit by a job loss shock retire much later than workers that do not suffer a job loss shock and the impact of a higher job loss risk on the average retirement age is smaller. Hence the contribution to the pay-as-you-go pension system amplifies the impact of a higher job loss risk on the average retirement age in the economy. Specifically, the contribution to the pension system reinforces the effect of the lower wage following the jobless spell through the substitution effect, but has a very small income effect because a large part of the contribution to the pension system is returned to the individual as a pension benefit after she reaches the official retirement age.

Chapter 4 analyzes how a politician reforms a pay-as-you-go pension system when the economy is affected by both financial and demographic shocks. The policy function implemented by the politician helps overlapping cohorts share both shocks by appropriately adjusting contributions and benefits each period. Specifically, following an adverse financial shock, the politician increases both contributions and benefits. After an adverse demographic shock, the politician increases contributions but decreases benefits. These results are in line with the major observed developments of pay-as-you-go pension systems.

I also analyze the implications of *permanent* changes in the population growth rate, modeled as a lower mean of the demographic shock. A lower mean of the demographic shock makes the politician's policy function less sensitive to the wealth of the old. Intuitively, when the population growth rate is lower on average, the politician has less room to compensate old agents for their losses from financial shocks. This has two opposing implications. On the one hand, the capacity of the pension system to diversify financial risks decreases. Consequently, young agents must contribute more to the pension system to maintain the same level of protection against financial shocks. On the other hand, the return of the pension system also decreases. This makes young agents less willing to invest in the pension system. In a calibrated version of the model the impact through the lower return dominates so the average contribution to the pension system is smaller if the mean population growth rate is lower. This is consistent with stylized facts: countries in Latin America and Central and Eastern Europe decided to (partially) transform the pay-as-you-go pension system into a fully funded system following a decrease of the average population growth rates.

Samenvatting

Dit proefschrift bestaat uit drie essays die de risicodeling en arbeidsaanbodverstoringen analyseert die ingebed zijn in omslagstelsels.

In het eerste hoofdstuk vergelijk ik de drie meest voorkomende typen omslagstelsels - beschikbare-uitkeringssysteem (engels, Defined Benefit - DB), theoretische beschikbaar premiesysteem (engels, Notional Defined Contribution - NDC) en vast bedrag pensioen (engels, flat benefit - FL) - op het gebied van de macroeconomische- en welvaartsimplicaties op de lange termijn. Er zijn hierbij twee argumenten van belang. Aan de ene kant zorgt een nauw verband tussen iemands arbeidsinkomen over de levensloop (of iemands betaalde pensioencontributies over de levensloop) en de pensioenopbrengsten, zoals in NDC en DB systemen, voor een prikkel om meer en langer te werken, resulterende in meer welvaart op de lange termijn. Aan de andere kant zijn mensen in dergelijke systemen vatbaarder voor inkomensrisico, resulterende in minder welvaart op de lange termijn. Meer specifiek, een schok op iemands arbeidsinkomen tijdens de werkfase zal perfect gereflecteerd worden in het inkomen na pensionering. Als gevolg hiervan is er minder verzekering in NDC of DB omslagstelsels. Daarnaast laat ik zien dat een nauwer verband tussen pensioenopbrengsten en iemands arbeidsinkomen over de levensloop kan zorgen voor minder opgebouwd kapitaal in de economie. In dergelijke systemen worden mensen namelijk geprikkeld om meer of langer te werken wanneer ze dicht bij pensionering zijn en dit zorgt ervoor dat zij minder hoeven te sparen voor een gegeven hoeveelheid consumptie na pensionering. Een lagere hoeveelheid opgebouwd kapitaal zorgt voor een lager welvaartsniveau in dynamisch-efficinte economin.

Na een realistisch overlappende-generaties model gecalibreerd te hebben voor de Amerikaanse economie vind ik dat de overgang naar een NDC of DB omslagstelsel zorgt voor een lager welvaartsniveau op de lange termijn. Bovendien, het welvaartsniveau is het hoogst indien het huidige Amerikaanse omslagstelsel ingeruild wordt voor een FL systeem. Dit resultaat impliceert dat de welvaartsverhoging als gevolg van een nauwer verband tussen arbeidsinkomen en pensioenopbrengsten kleiner is dan het welvaartsverlies dat voortvloeit uit de mindere verzekering en verminderde opbouw van kapitaal in de economie.

Ik analyseer ook hoe dit resultaat verandert wanneer de contributievoet van het omslagstelsel hoger is (33% van arbeidsinkomen, zoals in Itali op het moment, in plaats van de 10.6% die nu in Amerika gehandhaafd wordt). Aangezien de arbeidsvertoringen quadratisch stijgen in de pensioencontributies zou men verwachten dat deze verstoringen zwaarder wegen bij hogere waarden van de pensioencontributie. Ik vind dat in dit geval het FL systeem nog steeds een hoger welvaartsniveau op de lange termijn oplevert dan een NDC systeem, maar het verschil tussen de twee omslagstelsels, uitgedrukt in consumptie equivalenten, daalt aanzienlijk. Bovendien, wanneer men het effect van de opbouw van kapitaal op welvaart uitschakelt door een kleine, open economie te bestuderen, leveren het FL en NDC systeem vergelijkbare welvaartsniveaus op.

Het tweede hoofdstuk bestudeert het effect van een hoger risico op baanverlies op pensionering. Ik laat empirisch en theoretisch zien dat in een economie met een hoger risico op baanverlies de arbeidsparticipatie van oudere werkers kleiner is. De hypothetische experimenten uitgevoerd in dit hoofdstuk met behulp van het geconstrueerde model wijzen erop dat de aanwezigheid van omslagstelsels zorgt voor een sterker effect van een hoger risico op baanverlies op de pensioneringsleeftijd.

Met gebruik van data van de US Survey of Income and Program Participation (SIPP) laat dit hoofdstuk zien dat het risico van een substantieel verlies van arbeidsinkomen als gevolg van een werkloze periode hoger is voor mensen die aan het einde van hun carrire zitten aangezien zij meer tijd buiten hun werk spenderen. Ik besluit hierdoor de langdurige werkloosheid in een land te gebruiken als proxy voor het verlies van arbeidsinkomen als gevolg van een werkloze periode. Ik schat een regressiemodel met behulp van data van meerdere landen om te illustreren dat grotere verliezen in arbeidsinkomen als gevolg van een werkloze periode de arbeidsparticipatie van oudere werknemers doet afnemen. In dit geschatte regressiemodel controleer ik voor landspecifieke karakteristieken zoals de grootte van pensioencontributies, levensverwachting en de progressiviteit van het pensioenstelsel.

De kanalen die ervoor zorgen dat een hoger risico op baanverlies leidt tot een verlaging in de gemiddelde pensioneringsleeftijd worden gedentificeerd in een levenscyclus model met endogene pensionering, incomplete markten en een risico op baanverlies. Alle werkers, onafhankelijk van of ze nu door een schok van baanverlies geraakt zijn of niet, worden benvloed door het risico op baanverlies via hun spaargedrag. Meer specifiek, werkers voorzien de kans op een substantile verlaging in arbeidsinkomen als gevolg van plotseling baanverlies en sparen daarom uit voorzorg meer voordat de schok op baanverlies hen kan raken. Dit stelt hen in staat om vroeger met pensioen te gaan. Werkers die daadwerkelijk door de schok op baanverlies geraakt worden zijn daarnaast benvloed vanwege het feit dat i) het lagere arbeidsinkomen na plotseling baanverlies een prikkel geeft tot vervroegde pensionering (een substitutie effect) en het feit dat ii) zij langer zullen moeten werken om hetzelfde consumptieniveau te behalen (een inkomenseffect).

Na het model gecalibreerd te hebben voor de Amerikaanse economie vind ik dat werkers die plotseling hun baan verliezen vroeger met pensioen gaan dan werkers die niet plotseling hun baan verloren hebben. Daarnaast zorgt een hoger risico op baanverlies in het model voor een lagere gemiddelde pensioneringsleeftijd, hetgeen in lijn is met het hierboven beschreven empirische onderzoek. Ik voer ook een hypothetisch experiment uit waarbij ik het omslagstelsel in de economie elimineer. In dit geval gaan de werkers die plotseling hun baan verliezen veel later met pensioen dan de werkers die niet plotseling hun baan verloren hebben. Het effect van een hoger risico op baanverlies op de gemiddelde pensioneringsleeftijd is in dit geval ook kleiner. De contributies aan het omslagstelsel versterken het effect van een hoger risico op baanverlies op de gemiddelde pensioneringsleeftijd dus. De contributie aan het pensioensysteem versterkt namelijk het effect van het lagere arbeidsinkomen als gevolg van de plotselinge werkloosheid via het substitutie effect, terwijl het slechts een klein inkomenseffect heeft omdat een groot deel van de pensioencontributies naar de werker terugvloeit in de vorm van pensioeninkomen na pensionering.

Het derde hoofdstuk analyseert hoe een politicus een omslagstelsel hervormt wanneer de economie geraakt wordt door financile en demografische schokken. De beleidsfunctie die de politicus implementeert stelt overlappende cohorten in staat om beide typen schokken te delen door in elke periode de contributies en uitbetalingen op de juiste manier aan te passen. Wanneer de economie geraakt wordt door een nadelige financile schok verhoogt de politicus de contributies en de uitbetalingen. Wanneer de economie geraakt wordt door een nadelige demografische schok verhoogt de politicus de contributies maar verlaagt hij de uitbetalingen. Deze resultaten zijn in lijn met recente aanpassingen aan bestaande omslagstelsels.

Ik analyseer tevens de implicaties van permanente veranderingen in de groeivoet van de populatie, gemodeleerd als een lager gemiddelde van de demografische schok. Dit zorgt ervoor dat de beleidsfunctie van de politicus minder gevoelig is voor het vermogen van de ouderen. Intuitief gezien heeft de politicus dan namelijk minder ruimte om de vermogensverliezen van de ouderen te compenseren in het geval van een nadelige financile schok. Dit heeft twee tegenstrijdige gevolgen. Aan de ene kant is het pensioensysteem minder capabel in het diversificeren van financiel risico. Hierdoor moeten de jongeren meer bijdragen aan het pensioensysteem om dezelfde hoeveelheid bescherming tegen financile schokken te krijgen. Aan de andere kant daalt de opbrengst van het pensioensysteem, hetgeen er voor zorgt dat de jongeren minder graag investeren in het pensioensysteem. In een gecalibreerde versie van het model domineert het effect van de lagere pensioensysteemopbrengsten. Wanneer de gemiddelde groeivoet van de populatie lager is, zal de gemiddelde contributie aan het pensioensysteem dus lager zijn. Dit is consistent met bestaande pensioenhervormingen: landen in Latijns-Amerika en Oost-Europa besloten omslagstelsels te hervormen tot kapitaalgedekte pensioensystemen toen de gemiddelde groeivoet van de populatie daalde.

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