The spin period of PSR J1455-3330: Evidence for disk instability in wide low-mass X-ray binaries?

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THE SPIN PERIOD OF PSR J1455–3330: EVIDENCE FOR DISK INSTABILITY IN WIDE LOW-MASS X-RAY BINARIES?

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ABSTRACT

We consider the influence on spin evolution, in a neutron star with a low-mass giant companion, of thermal-viscous instability in the accretion disk around the neutron star. Recent investigations on disk instability predicted that most low-mass X-ray binaries (LMXBs) with orbital period \( P_{\text{orb}} \gtrsim 1–2 \) days should be transient. We show that during the LMXB evolution, because of the transient behavior in mass transfer, the neutron star’s spin can deviate from the conventional “equilibrium spin” determined by the stable mass transfer rate. We discuss the application of such a scenario to the prior spin evolution during the LMXB phase in the binary and millisecond radio pulsar PSR J1455–3330. The anomalously high characteristic age of the pulsar indicates that the initial pulse period is close to its current value, 7.99 ms, which is much longer than that attained via stable mass accretion. We argue that such a spin period can be accounted for if only a small fraction of mass transferred from the secondary was accreted by the neutron star during (at least) the late stage of mass transfer, thus presenting positive evidence for disk instability in wide LMXBs.

Subject headings: accretion, accretion disks — binaries: close — stars: neutron — X-rays: stars

1. INTRODUCTION

Disk accretion in cataclysmic variables (CVs) is believed to be thermally and viscosely unstable, leading to outbursts in dwarf novae (DNe), if the rate of mass transfer from the secondary is lower than a critical value at which the temperature at the outer edge of the disk drops below the hydrogen ionization temperature, \( \sim 6500 \) K (cf. Cannizzo 1993 for a review). Such a thermal-viscous instability may also occur in soft X-ray transients (SXTs; Mineshige & Wheeler 1989; Cannizzo, Chen, & Livio 1995), which are low-mass X-ray binaries (LMXBs) characterized by episodic X-ray outbursts during which a soft X-ray spectrum is observed (Tanaka & Shibazaki 1996). The critical mass transfer rate for the disk instability depends on the binary orbital period \( P_{\text{orb}} \), and is strongly influenced by irradiation from the inner region of the accretion disks in LMXBs (van Paradijs 1996). With X-ray heating of the disk included, King, Kolb, & Burderi (1996, hereafter K96) and King et al. (1997, hereafter K97) showed that most neutron star low-mass X-ray binaries (NLMXBs) with \( P_{\text{orb}} > 1–2 \) days will become transient, whereas short-period NLMXBs with a main-sequence donor \( (P_{\text{orb}} < 1 \) day) are transient only if the companion star is significantly nuclear-evolved before mass transfer begins. While most of the known SXTs have orbital periods limited to \( \sim 1 \) day (Tanaka & Shibazaki 1996), observational evidence for accretion instability in wide NLMXBs is rare: among SXTs with a neutron star primary, only one source (Cir X-1) is in a long-period binary \( (P_{\text{orb}} \approx 16.6 \) days; Kaluzienski et al. 1976), whose classification as either a high-mass X-ray binary (HMXB) or an LMXB is still uncertain (cf. White, Nagase, & Parmar 1995). Based on the evolutionary connection between LMXBs and binary and millisecond radio pulsars, here we present indirect evidence for disk instability occurring in wide LMXBs, by exploring the origin of the initial pulse period of the binary and millisecond radio pulsar PSR J1455–3330. We describe the properties of PSR J1455–3330 in § 2. In § 3 we discuss possible scenarios for the period evolution and suggest a new evolutionary track for the neutron star. We summarize in § 4.

2. THE INITIAL PULSE PERIOD OF PSR J1455–3330

The millisecond radio pulsar PSR J1455–3330 is among those pulsars with the highest characteristic ages. From its pulse period \( P \approx 7.99 \) ms and period derivative \( \dot{P} \lesssim 6 \times 10^{-21} \) s s\(^{-1} \) (Lorimer et al. 1995b), one can derive its characteristic age \( \tau_c = P/(2 \dot{P}) \gtrsim 2 \times 10^{10} \) yr, exceeding the age \( t_G \sim 9 (\pm 2) \) Gyr of the Galactic disk (Winget et al. 1987). Furthermore, a transverse velocity of 60 \((\pm 20)\) km s\(^{-1}\) of the pulsar was measured with scintillation observation (Nicastro & Johnston 1995), which contributes the Shklovskii term (Shklovskii 1970; Camilo, Thorsett, & Kulkarni 1994) in the apparent period derivative as large as \( \sim 4.3 \times 10^{-21} \) s s\(^{-1}\), and makes the intrinsic value of \( \tau_c \) at least \( 7.5 \times 10^{10} \) yr. Since the pulsar age is \( t = \tau_c [1 - (P_i/P)^2] \) (where \( P_i \) is the initial pulse period) under magnetic dipole radiation (Manchester & Taylor 1977), in order that \( t \leq t_G, P_i \) must be close to 7.99 ms.

A similar case occurs in the millisecond pulsar PSR J1012+5307, which has a characteristic age \( \sim 7 \times 10^9 \) yr, after correction for transverse motion of the pulsar much longer than the cooling age \( \sim 3 \times 10^8 \) yr of its white dwarf companion (Nicastro et al. 1995), suggesting the value of its \( P_i \) is also similar to the present period. Note that the above arguments are based on the assumption of magnetic dipole radiation for a radio pulsar, i.e., the braking index \( n \approx 3 \). The braking index of a millisecond pulsar could be larger than this value (cf. Manchester & Taylor 1977), for example, because of multipolar magnetic field structure (\( n > 3 \)), or angular momentum loss by gravitational radiation (\( n = 5 \)), reducing the value of \( \tau_c \). But, even if \( n \) is as large as 5, the conclusion that \( P_i \) is similar to \( P \) for PSR J1455–3330 does not change because of its very large \( \tau_c \).
Both pulsars are low magnetic field ($B \sim 10^8$ G) binary pulsars ($P_{\text{orb}} \approx 76.2$ days for PSR J1455−3330 and $P_{\text{orb}} \approx 0.6$ day for PSR J1012+5307) with a low-mass (around $0.3 M_\odot$) white dwarf companion (Lorimer et al. 1995a, 1995b, hereafter L95a, L95b; Nicastro et al. 1995). Formation of such low-mass binary and millisecond pulsars can be described by the “recycled” scenario as follows (Verbunt & van den Heuvel 1995). A high-field ($10^{12}$−$10^{13}$ G), rapidly rotating neutron star born in a binary with a low-mass ($\leq 1 M_\odot$) main-sequence companion star spins down under magnetic dipole radiation, and switches off its radio emission within $\sim 10^8$−$10^7$ yr after passing by the so-called death line in the magnetic field–spin period ($B$–$P$) diagram. When the companion evolves to overflow its Roche lobe, mass transfer occurs by way of an accretion disk. Mass accretion onto the neutron star gives rise to X-ray emission, induces magnetic field decay, and spins the star up to short period (the mechanisms for the field decay induced by accretion are, however, not well understood). When mass transfer ceases, the endpoint of the evolution is a circular binary containing a neutron star visible as a low-field, millisecond radio pulsar, and a low-mass ($0.2$−$0.4 M_\odot$) white dwarf, the remaining helium core of the companion. The magnitude of the initial pulse period $P_0$ of the “recycled” radio pulsar depends on the evolution of the mass accretion rate and of the magnetic field of the neutron star. It is generally believed to be close to the so-called equilibrium period at which the accretion torque exerted on the neutron star vanishes (Bhattacharya & van den Heuvel 1991), i.e.,

$$P_{\text{eq}} \approx 2 \omega_0^{-1} \left( \frac{B}{10^8 \text{ G}} \right) \left( \frac{R_s}{10^6 \text{ cm}} \right)^{18/7} \times \left( \frac{M_\chi}{M_\odot} \right)^{-5/7} \left( \frac{M_\odot}{M_\bullet} \right)^{-3/7} \text{ms}, \quad (1)$$

where $B$, $R_s$, $M_\chi$, and $M_\odot$ are the surface magnetic field, the radius, and the mass and mass accretion rate of the neutron star, respectively; $M_\bullet$ is the Eddington accretion rate ($\sim 1.5 \times 10^{-9} M_\odot$ yr$^{-1}$ for a $1.4 M_\odot$ neutron star), and $\omega_0$ is the critical value of the fastness parameter $\omega_0 \equiv \Omega_n / \Omega_0(R_0)$ denoting the equilibration spin. Here $\Omega_n = 2\pi/P$ is the spin rate of the neutron star, and $\Omega_0(R_0)$ is the Keplerian angular velocity at the inner edge $R = R_0$ of the accretion disk. Recent improvements in the magnetized accretion disk model initially developed by Ghosh & Lamb (1979) suggest that the value of $\omega_0$ may lie between 0.7 and 0.95 (Campbell 1992; Yi 1995; Wang 1995; Li & Wang 1996). Here we adopt $\omega_0 = 0.7$, to be consistent with the observations of both spin-up and spin-down in most of the binary X-ray pulsars (Nagase 1989; White et al. 1995).

Mass transfer rates in LMXBs can be estimated by evolutionary considerations. The mechanisms that drive mass transfer in LMXBs depend on the initial separations of the binary components (Verbunt & van den Heuvel 1995). In narrow systems with $P_{\text{orb}} \leq 1$−$2$ days, mass transfer is driven by loss of orbital angular momentum via gravitational radiation and/or magnetic braking. The evolution of the binary orbit and the mass transfer rate is not well constrained, however, owing to a poor understanding of the mechanism of magnetic braking. Mass transfer in wide LMXBs with $P_{\text{orb}} \geq 1$−$2$ days is driven by the nuclear expansion of the secondary. Systems of this kind form a quite homogeneous group whose evolutionary seems well understood (Webbink, Rappaport, & Savonije 1983; Taam 1983). The binary orbits widen during the evolution, and there exists a positive correlation between the mass transfer rate $M_{\text{t}}$ from the secondary and $P_{\text{orb}}$ (Webbink et al. 1983). For PSR J1455−3330, the wide, circular orbit and the low mass of the white dwarf companion indicate that it evolved from an LMXB with $P_{\text{orb}} \approx 6$−$7$ days and the mean mass transfer rate $\langle M_{\text{t}} \rangle \approx 4 \times 10^{-9} M_\odot$ yr$^{-1}$. Since $M_{\text{t}} < M_\bullet$, it is natural to assume that all the mass transferred from the secondary was accreted by the neutron star, i.e., $M_\chi = M_{\text{t}}$. Equation (1) then reveals $P_{\text{eq}} \approx 0.7$ ms for PSR J1455−3330 (with $M_\chi = 1.4 M_\odot$), which is $\sim 10$ times shorter than the initial pulse period $P_0$. Note the small power index ($\sim 2$) of $M_\chi$ in equation (1): fluctuations of $M_\chi$ by an order of magnitude can change the value of $P_{\text{eq}}$ by a factor of $\leq 3$.

3. SCENARIOS FOR SPIN EVOLUTION IN PSR J1455−3330

Possible origins of the period discrepancy in PSR J1455−3330 are that the neutron star magnetic field decayed with accretion so rapidly that the neutron star’s spin could not respond accordingly to reach the equilibrium value, or that the neutron star did reach the equilibrium spin, but with a mass accretion rate much lower than the value of $M_\chi$ adopted above.

The former possibility was explored by Burderi, King, & Wynn (1996) to explain the age (or period) discrepancy in PSR J1012+5307. These authors suggested a model for the accretion-induced field decay, assuming that when the blobs of accreting matter fall onto the magnetic cap of a neutron star, the electric currents induced by the blobs give rise to a magnetic field that is opposite to the local dipolar field, and partially neutralize the surface currents that generate the neutron star’s field, causing the field to decay. The field-decay timescale is found to be generally similar to the spin-up timescale, and becomes less than the latter at the late stage of the mass transfer. So most of the accreted matter has been used up to decrease the stellar field before the star reaches the equilibrium spin, resulting in a relatively long $P_i$ close to the current pulse period. The same scenario may also be applied to PSR J1455−3330. However, keeping in mind that the detailed mechanism for accretion-induced field decay is still controversial, we believe that the scenario proposed by Burderi et al. (1996) is unrealistic, because the assumption that the accreting matter induces surface currents to decrease the star’s local field works only when the star is a monopole, while neutron stars are dipoles. Additionally, previous investigations (Taam & van den Heuvel 1986; van den Heuvel & Bitzaraki 1995) suggested that the decay of the fields seems to be correlated with the accreted mass, while in Burderi et al. (1996) it depends essentially on the accretion rate. From equation (2) in Burderi et al. (1996), one can easily derive that the magnetic fields will hardly decay in neutron stars having undergone the HMXB phase, or the LMXB phase with $M_{\text{t}} \lesssim 10^{-10} M_\odot$ yr$^{-1}$, in conflict with the observations of recycled pulsars. We instead adopt a phenomenological form of field decay with accretion, $B = B_0/(1 + \Delta M/\Delta m_0)$, suggested by Shibazaki et al. (1989), which is roughly compatible with both analytic and numerical calculations of period decay (e.g., Zhang, Wu, & Yang 1994; Geppert & Urpin 1994; Urpin & Geppert 1995). Here $B_0$ is the initial magnetic field, $\Delta M$ the accreted mass, and $\Delta m_0$ a constant. The calculated spin evolution in PSR
J1455—3330 is shown in Figure 1. It can be seen that if mass accretion is steady, the neutron star’s spin period, though deviating a bit from the spin-up line, can be accelerated to ~1 ms, still much shorter than $P_t$ (see solid curve in Fig. 1).

We then move to the second possibility. The period discrepancy suggests that at least during the late stage of the mass transfer, the neutron star slowed down to a long period resulting from a lower accretion rate than $M_{\text{cr}}$, which implies unsteady mass accretion during the LMXB evolution.

Observations of SXTs have provided strong evidence for low efficiency of disk accretion with relatively high mass transfer rate in LMXBs (Tanaka & Shibazaki 1996). A most recent example is the black hole candidate A0620—00. Based on the small quiescent X-ray luminosity of $6 \times 10^{30}$ ergs s$^{-1}$ of A0620—00 (for a distance of 0.9 kpc), the mass accretion rate $M_X$ of the black hole was estimated to be $\sim 10^{11}$ g s$^{-1}$ if the accretion disk is optically thick (McClintock, Horne, & Remillard 1995), or $\sim 10^{12}$ g s$^{-1}$ for advective accretion flow (Narayan, McClintock, & Yi 1996). From the optical disk luminosity of A0620—00, the mass transfer rate onto the outer disk $M_*$ can be derived to be $\sim 6 \times 10^{15}$ g s$^{-1}$ (McClintock et al. 1995). The small $M_*$ against $M_X$ is consistent with the idea that a large fraction of the transferred matter is stored in the disk during a quiescent interval, lending support to a thermal-viscous disk instability for outbursts in SXTs.

Thermal-viscous instability in accretion disks arises from a steep temperature dependence of the opacity in a partially ionized accretion disk (Cannizzo 1993). When the mass transfer rate is lower than a critical value $M_{\text{cr}}$ (depending on the orbital period), the disk undergoes a thermal limit cycle. In a quiescent state the accretion disk is in the cool state. As matter accumulates in the disk, both the surface density and the temperature increase. When the surface density reaches an upper critical value, the disk jumps to the hot state, which gives rise to a high accretion rate, causing rapid infall of matter onto the central objects. The disk instability model can fairly well explain the observed properties of both DNe and SXTs (Cannizzo 1993; Tanaka & Shibazaki 1996). However, the criterion for instability in LMXB disks may deviate from that in CV disks because in LMXBs the structure of the outer part of the disk is likely to be strongly influenced by irradiation from the central X-ray source or the inner region of the disk (van Paradijs 1996; K96). Recent investigations considering the effects of X-ray heating on disk instability showed that most LMXBs with $P_{\text{orb}} \geq 1$–2 days will undergo thermal-viscous instability regardless of whether the primary is a neutron star or a black hole (K96; K97). Accordingly, the LMXB precursor of PSR J1455—3330 should have been transient, and the neutron star evolution may deviate from the standard scenario as described in, say, Verbunt & van den Heuvel (1995). Figure 1 shows our calculated evolutionary track of PSR J1455—3330 in the B-P diagram, starting from a $P_{\text{orb}} \approx 6.0$ day binary consisting of a 1.4 $M_\odot$ neutron star of magnetic field $B = 10^{12}$ G and spin period $P = 10$ s, and a 1 $M_\odot$ (sub)giant companion with solar chemical composition. We adopt the semianalytical method (Webbink et al. 1983) to calculate the mass transfer rate $M_{\text{in}}$, and use the modified disk stability criterion (van Paradijs 1996; K96; K97) to determine whether the mass transfer through the disk is stable. Disk accretion was steady during the initial phase of the mass transfer lasting $\sim 4 \times 10^5$ yr. When about 0.2 $M_\odot$ mass was transferred, the star’s field decayed to $\sim 10^8$ G ($M_\odot$ is assumed to be $2 \times 10^{-5} M_\odot$) and the spin period was accelerated to $\sim 1$ ms (solid curve). Then $M_{\text{in}}$ became lower than the critical value $M_{\text{cr}}$ for disk instability, and the disk underwent limit cycles (the reason is that when the orbit widens, $M_{\text{in}}$ increases faster with $P_{\text{orb}}$ than $M_{\text{cr}}$). The field and the spin of the neutron star then followed a new evolutionary track shown with the dashed curve: the magnetic field remained nearly constant and the spin period increased. To explain this behavior in detail, we first define $M_{\text{out}}$ and $M_{\text{in}}$ to represent the mass transfer rate through the inner edge of the disk during outbursts and quiescence, respectively. The former can be estimated as $M_{\text{out}} \sim (t_{\text{rec}}/\tau)M_{\text{in}}$, where $\tau$ and $t_{\text{rec}}$ are the typical duration and the recurrence time of the outbursts, respectively. Observed SXTs showed transient duty cycles $t/t_{\text{rec}} \approx 10^{-2}$ (Tanaka & Shibazaki 1996), giving $M_{\text{out}} \approx 10^{-9} M_\odot$ yr$^{-1}$ if $M_{\text{in}} \approx 10^{-9} M_\odot$ yr$^{-1}$. Since the binary considered here is wider than most SXTs, we would expect larger disk size, longer recurrence time, and hence higher $M_{\text{out}}$ (Cannizzo, Shafer, & Wheeler 1988). Unlike black holes, neutron stars can accrete at a rate limited to $M_{\text{in}}$, so only a very small fraction of the mass transferred during outbursts could fall onto the neutron star, the stellar field remaining hardly changed. The spin of the neutron star was alternately accelerated during outbursts or decelerated due to the “propeller” effect during quiescence. Because $t_{\text{rec}} \gg \tau$, the spin of the neutron star was dominated by slowdown during quiescence, until a new equilibrium spin was reached, which is determined by equation (1) with $M_{\text{cr}} = M_{\text{in}}$.

To verify the above arguments, we estimate the spin changes in the neutron star during outburst and quiescent states, respectively. First, since $M_{\text{out}}$ is generally super-
Eddington and $B$ is low, the magnetosphere of the neutron star is compressed to close the stellar surface. The spin-up timescale is

$$t_{su} = \frac{I}{T_{su}} \approx 1.8 \times 10^7 \left( \frac{P}{1\text{ ms}} \right)^{-1} \left( \frac{I}{10^{45}\text{ g cm}^2} \right) \left( \frac{M_X}{M_\odot} \right)^{-1/2} \left( \frac{R_s}{10^6\text{ cm}} \right)^{-3/2} \text{ yr}, \quad (2)$$

where the spin-up torque is

$$T_{su} \approx M_{\text{eq}}(GM_X R_s)^{1/2}. \quad (3)$$

Here $G$ is the gravitational constant, and $I$ is the moment of inertia of the neutron star. Compared with the typical duration (days to months) of the outbursts observed in SXTs, $t_{su}$ is much longer; the neutron star’s spin hardly changes during outbursts, because of the small specific angular momentum carried by the accreting matter.

Can the neutron star spin down from $\sim 1$ ms to $\sim 7$ ms within the mass transfer lifetime? During quiescence, since $M_{\text{eq}} \ll M_r$ and $M_{\text{out}}$, the inner edge of the disk exceeds the corotation radius of the neutron star, and the neutron star is in “propeller” regime (Illarionov & Sunyaev 1975). The accretion rate in the disk is negligibly small, and angular momentum extracted from the neutron star flows outward over the disk from the inner edge. The timescale in which the neutron star is slowed down by such a (dead) disk can be estimated as (cf. Lipunov 1992)

$$t_{sd} = \frac{I}{T_{sd}} \approx 3 \times 10^7 \left( \frac{I}{10^{45}\text{ g cm}^2} \right) \left( \frac{P}{1\text{ ms}} \right)^{-1} \times \left( \frac{M_{\text{eq}}}{10^{-11} M_\odot \text{ yr}^{-1}} \right)^{-6/7} \left( \frac{B}{10^8\text{ G}} \right)^{-2/7} \times \left( \frac{R_s}{10^6\text{ cm}} \right)^{-6/7} \left( \frac{M_X}{M_\odot} \right)^{-3/7} \left( \frac{R_\odot/R_s}{10^5} \right)^{-1/2} \text{ yr}, \quad (4)$$

where the spin-down torque

$$T_{sd} = M_{\text{eq}}(GM_\odot R_\odot)^{1/2} \left[ (R_\odot/R_s)^{1/2} - 1 \right], \quad (5)$$

where $R_\odot$ and $R_s$ are the inner and outer radii of the accretion disk, respectively (note that $T_{sd}$ is independent of the detailed structure of the disk). Equation (4) demonstrates that $t_{sd}$ is significantly less than the nuclear evolution timescale (a few times $10^8$ yr) of the secondary. Thus there is sufficient time for the neutron star to spin down to a long period. When the corotation radius of the neutron star becomes equal to the inner edge of the disk, accretion occurs and the neutron star’s spin reaches the equilibrium spin, the magnitude of which depends on $M_{\text{eq}}$. If $M_{\text{eq}} \approx 10^{-11} M_\odot \text{ yr}^{-1}$, then $P_I = P_{\text{eq}} \approx 7$ ms.

A related question is how the system appears during the propeller phase. It is of course transient during this phase, and its outbursts probably appear quite normal given that the magnetosphere is squashed down to the neutron star surface during them. However, we apparently do not observe radio pulsars in quiescent transients. This presumably means that even in quiescence there is enough interaction between the disk and the stellar magnetosphere to suppress the pulses. This of course supports the idea of a reasonably short spin-down time, even for a nonsteady disk.

4. DISCUSSION AND CONCLUSION

Our calculations imply that unsteady mass transfer in LMXB evolution can interpret the period discrepancy in PSR J1455−3330 self-consistently. We regard such an example as evidence for disk instability in wide LMXBs, which has not been mentioned in observations though theoretically predicted. We briefly discuss some implications of the results for the evolution of LMXBs and low-mass binary pulsars. First, the transient nature of wide LMXBs makes them difficult to observe as persistent X-ray sources, influencing the evaluation of the number and the birthrate of LMXBs. Second, for neutron stars recycled from LMXBs that had experienced disk instability, the initial pulse periods may be significantly longer than the standard equilibrium periods. This can explain the fact that several recycled pulsars have unusually high characteristic ages that exceed the age of the Galaxy (Camillo et al. 1994). The true pulsar ages would be much shorter than the characteristic ages, and the birthrate of low-mass binary pulsars required to sustain the observed Galactic population would be much greater than previous calculations have suggested (L95b). One can also speculate that there exist a large number of “sleeping” neutron stars located beyond the death line in the $B$-$P$ diagram, not shining as radio pulsars. Third, the amount of mass accreted by a neutron star during the LMXB phase is likely to be significantly lower than previously considered (Taam & van den Heuvel 1986; van den Heuvel & Bittzaraki 1995). Most of the mass stored in the quiescent state is probably ejected during outbursts, given that the rate of mass accretion is super-Eddington. To some extent this will change the correlation between the field decay and the accreted mass obtained by van den Heuvel & Bittzaraki (1995), which may provide an empirical test for the theoretical models of accretion-induced field decay.

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