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Sphaleron transition rate at high temperature in the 1+1 D abelian Higgs model

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New results for the rate are presented using the canonical ensemble in the classical approximation on a spatial lattice. We find that the rate at high temperatures is proportional to T^2 , and strongly dependent on the lattice spacing a . We conclude that a better effective action is needed for the classical approximation.

1. Introduction

The rate Γ of baryon number violation in the Standard Model is difficult to calculate at finite temperature T . Therefore, the classical approximation was proposed which allows for making a numerical estimate. It was tested in the 1+1 dimensional abelian Higgs model [1–4], for which a semiclassical calculation of the rate is available,

$$F\left(\frac{T}{m_\phi}, \xi, v\right) \equiv \frac{\Gamma}{m_\phi^2 L} = f(\xi) \sqrt{\frac{E_s}{T}} \exp\left[-\frac{E_s}{T}\right]. \quad (1)$$

Here L is the one dimensional volume, m_ϕ is the Higgs mass, $v^{-2} = 2\lambda/m_\phi^2$ is the effective dimensionless coupling, $\xi = g^2/\lambda$, g is the gauge coupling, λ is the quartic Higgs self coupling, $E_s = (2/3)v^2 m_\phi$ is the classical sphaleron energy, and $f(\xi)$ is calculated in [5]. This formula is expected to be valid at temperatures $m_\phi \ll T \ll E_s$ which is confirmed by numerical simulations [4,3].

At high temperatures the rate is still unknown even for this rather simple model. From dimensional arguments one expects that the rate behaves as T^2 ,

$$F = \kappa(\xi, v) \frac{(v^{-2}T)^2}{m_\phi^2} \quad \text{for } T \gg E_s. \quad (2)$$

2. Classical approximation

At high temperature one expects high occupation numbers for quanta with energies much less than T and for their contribution a classical approximation seems reasonable. Suppose we

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Figure 1. Results for $\ln F = \ln(\Gamma/m_\phi^2 L)$ for $\xi = 4$. The diagonal line represents the analytic formula (1). The other two lines are fits to the form $c_0 + c_2 T^2$. The upper data are for $a' = 0.32$, the lower data for $a' = 0.16$.

derive an effective action in which the spatial momenta are limited to $|p| < \Lambda$. Then the effective energies will be restricted by Λ which suggests a classical approximation based on this effective action for $\Lambda \ll T$. Such an effective action $\bar{S}$ might be defined (in a generic field theory with generic fields φ) as follows,

$$\exp[\bar{S}(\bar{\varphi}, \Lambda)] = \int D\varphi B_\Lambda(\bar{\varphi}, \varphi) \exp[S(\varphi)]. \quad (3)$$

Here B_Λ is a blocking function such that $\int D\bar{\varphi} B_\Lambda(\bar{\varphi}, \varphi) = 1$, e.g.

Figure 2. $\langle |\phi'|^2 \rangle$ versus inverse temperature β' .

$$B_\Lambda(\bar{\varphi}, \varphi) = \prod_{(x,t)} \delta[\bar{\varphi}(x,t) - \int_{|p|<\Lambda} dp \bar{\varphi}(p,t) e^{ipx}]. \quad (4)$$

In this spirit we interpret the effective action for the abelian Higgs model on a lattice with lattice spacing a ,

$$\begin{aligned} \bar{S} = & \int dt a \sum_n \left[\frac{1}{4g^2} \left(\frac{A_{0n+1} - A_{0n}}{a} - \partial_t A_{1n} \right)^2 \right. \\ & - \frac{1}{a^2} |\exp(-iaA_{1n})\phi_{n+1} - \phi_n|^2 \\ & \left. + |(\partial_t - iA_{0n})\phi_n|^2 - \lambda \left(|\phi_n|^2 - \frac{v^2}{2} \right) \right], \end{aligned} \quad (5)$$

as an approximation to such an effective action (we omitted the ‘bar’ on the fields, $\Lambda = \pi/a$). For convenience we make a scale transformation to dimensionless variables [2,3], $a = a'/v\sqrt{\lambda}$, $t = t'/v\sqrt{\lambda}$, $\phi = v\phi'$, $A_\mu = v\sqrt{\lambda}A'_\mu$, $m_\phi = \sqrt{\lambda}vm'_\phi$, $H = \sqrt{\lambda}v^3 H'$, $T = \sqrt{\lambda}v^3 T'$, $\beta' = 1/T'$, where H is the hamiltonian.

3. Results

Fig. 1 gives an overview of the rate obtained in our simulation, which was carried out in the same

Figure 3. F versus T'^2 for $a' = 0.16$. The line represents the fit $F = 0.00335 + 0.00512 T'^2$.

way as in [3]. The classical approximation is consistent with the semiclassical calculation in the semiclassical region [3,4]. We see also an intermediate region which is almost flat. This region appears to correspond to the minimum of $\langle |\phi'|^2 \rangle$ in fig. 2. Figs. 3 and 4 show more detailed data at high temperature. We see here a T'^2 behavior which starts already just beyond the minimum of $\langle |\phi'|^2 \rangle$. According to fig 1, the rate at high temperature depends on the lattice spacing a' . With a gauge invariant lattice action for bosonic fields one expects the lattice spacing dependence to be like a series in a'^2 . This is confirmed in figs. 5 and 6 for two values of T' . A linear a'^2 approximation appears to hold for $a'^2 \lesssim 0.1$. We therefore fit the data in figs. 3 and 4 to the following ansatz,

$$\begin{aligned} F &= c_{00} + c_{02} a'^2 + (c_{20} + c_{22} a'^2) T'^2 \\ &= c_{00} + c_{02} \frac{a'^2 m_\phi^2}{2} \\ &\quad + 2 \left(c_{20} + c_{22} \frac{a'^2 m_\phi^2}{2} \right) \frac{(v^{-2} T')^2}{m_\phi^2}, \end{aligned} \quad (6)$$

and find

$$\begin{aligned} c_{00} &= (3.1 \pm 0.1) 10^{-3}, \quad c_{02} = (9.6 \pm 1.1) 10^{-3}, \\ c_{22} &= 0.20 \pm 0.02, \quad c_{20} = (-7 \pm 25) 10^{-5}. \end{aligned} \quad (7)$$

From this we see that the $v^{-4} T'^2$ behavior in eq. 2 is confirmed (contradicting [6]), with

Figure 4. F versus T'^2 for $a' = 0.32$ and the fit $F = 0.00409 + 0.0207 T'^2$.

$$\kappa(\xi, v) \approx 0.20 a^2 m_\phi^2, \quad \text{at } \xi = 4. \quad (8)$$

The coefficient of T^2 depends strongly on the lattice scale. Therefore, it is absolutely necessary to understand the a dependence of the coefficients in the effective action before we can say to have calculated κ .

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Figure 5. F versus a'^2 for $T' = 4/3$. The line represents the fit $F = 0.00307 + 0.387 a'^2$.

Figure 6. F versus a'^2 for $T' = 1$, and the fits $f = 0.00347 + 0.192 a'^2$ to the seven lowest data points, $g = 0.00373 + 0.171 a'^2$ to the five lowest data points and $h = 0.00367 + 0.169 a'^2 + 0.270 a'^4$ to all data. The insert zooms in on the region $a'^2 \lesssim 0.1$.