Commission of the European Communities

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PROJECT NB 6756

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CAMAS

COMPUTER AIDED MIGRATION OF APPLICATIONS SYSTEM

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CAMAS-TR-2.2.4.6

F2SAD - prediction capabilities

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ACE - Univ. of Amsterdam - ESI SA - ESI GmbH - FEGS - PARSYTEC - Univ. of Southampton
Chapter 1
Introduction

This report describes an evaluation of the F2SAD tool with several well known basic algorithms. In this report we consider some sorting algorithms. With this report we respond to the specific request of the review commission to make a more detailed validation of the prediction capabilities of the UvA workbench tools.

The intention of this document is to provide confidence in the ability of the tools (that implement the models developed in SAD and PARASOL) to estimate the execution time of some well known algorithms. The algorithms described here have a performance behaviour that is common knowledge. Despite this fact, we are still able to come up with some points that are interesting and not plain textbook knowledge.

In this report, various figures depicting measured and predicted execution times of Fortran programs will pass. The main part of the annotated algorithms has been included in appendix A.

At the time of the CAMAS review 5 we will present additional results on numerical relaxation algorithms.
Chapter 2
Sorting algorithms

Sorting algorithms are symbolic algorithms. Rather than performing heavy computations they compare and manipulate (swap or reorder) data. Sorting algorithms are quite well understood in their complexity behaviour. Despite this fact, few textbooks do actually compare the execution time of the algorithms. Two algorithms can be of the same order of complexity and still differ in their performance because a different number of instructions is executed.

![Sorting algorithms chart]

Figure 2.1: This figure shows pure times of the sorting algorithms. The algorithms are applied to an array of uniform random numbers. It has been included here only to give an indication of the real execution time, since it is clearly not very informative when comparing the algorithms. All other figures therefore include either a logarithmic vertical axis and/or the execution time divided by the number of elements. This last type of figure works well to view the scalability of an algorithm and the crossover points for selecting between algorithms, but the overhead introduced by any algorithm is rather obscured. The logarithmic vertical axis plots still show the overhead, but the crossover points and behaviour is less visible. When viewing the other figures one has to keep in mind this figure, which tells you that the execution time of the sorting algorithms of the same complexity are actually very similar.

If we classify the sorting algorithms by their average-case time complexity, we can distinguish three classes. The exponential order $O(n^2)$, the logarithmic based sorting algorithms $n \log(n)$ and the linear time sorting algorithms $O(n)$. The exponential order algorithms are sometimes still (unjustifiably) used if a programmer is lazy and works on small arrays or under the disguise of a parallel computer. Shell-sort is such a variant used on parallel computers because of its parallel nature.

The most common sorting algorithms are the $O(n \log n)$ order sorting algorithms like quick-, heap- and mergesort. Although all three have the same average-case order of
2.1 Randomly distributed input data

<table>
<thead>
<tr>
<th>algorithm</th>
<th>average</th>
<th>worst-case</th>
<th>implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>bubblesort</td>
<td>$O(n^2)$</td>
<td>$\Theta(n^2)$</td>
<td>array (list also possible)</td>
</tr>
<tr>
<td>selectsor</td>
<td>$O(n^2)$</td>
<td>$\Theta(n^2)$</td>
<td>array (list also possible)</td>
</tr>
<tr>
<td>quicksort</td>
<td>$O(n \log n)$</td>
<td>$\Theta(n \log n)$</td>
<td>array</td>
</tr>
<tr>
<td>heapsort</td>
<td>$O(n \log n)$</td>
<td>$\Theta(n \log n)$</td>
<td>array</td>
</tr>
<tr>
<td>mergesort</td>
<td>$O(n \log n)$</td>
<td>$\Theta(n \log n)$</td>
<td>list</td>
</tr>
<tr>
<td>bucketsort</td>
<td>$O(n)$</td>
<td>$\Theta(n^2)$</td>
<td>list</td>
</tr>
</tbody>
</table>

Table 2.1: Sorting algorithms; their time complexity as average case time complexity $O(\ldots)$ and worst-case time complexity $\Theta(\ldots)$. The implementation can either be in linear array or a linked list form.

performance, they are inherently different. Quicksort has very bad worst-case performance, namely $\Theta(n^2)$. It is almost impossible to implement mergesort on arrays and the natural way to implement heapsort is on arrays. It is controversial whether quicksort is the fastest (on average), mergesort or heapsort. Comparing these algorithms is an interesting test case for our tool, F2SAD.

The theoretical complexity limit to sorting is $O(n \log n)$, but also in this case the practical knowledge of contraints to the input of the sorting algorithms can be exploited. To this albeit linear order sorting algorithms have been developed. These include radix, counting and bucketsort, in which the latter is sometimes used as a classifier or find-algorithm. These sortings algorithms do not base their algorithm on comparisons, like the sorting algorithms earlier mentioned, but rather on classification or decision trees.

The predicted times in these sections were produced using F2SAD which will now also incorporate the Parasol II tool in the same program. The tool produces a time-complexity formula in which the machine constants and the algorithmic parameters are still abstract. When not specifed, we have used a Sun Classic LX model as test machine.

Memory : 32 MB  
Model : SPARCstation LX  
At frequency : 50 MHz  
CPU : Texas Instruments TMS390S10 (MicroSparc)  
Data cache : 2 Kb blocksize=16 1-way associative  
Instruction cache : 4 Kb blocksize=32 1-way associative

2.1 Randomly distributed input data

Figure 2.2 shows the measured execution timings of the exponential and $O(n \log n)$ performance as well as the predicted execution timing. The input to the sorting algorithms is uniformly randomly distributed. Also the parameters in the time complexity formula have been set in such a way that they reflect this condition.

The reason for taking random input data is obviously that sorting algorithms will than expose an almost average-case behaviour. Some algorithms have a worst-case behaviour which is of a different complexity order or, less dramatic, have different minor terms in the complexity formula.

How the parameters are set we will come to later, but first we have a look at the measured and estimated execution time and compare them. Figure 2.3 shows the expected and estimated execution time for the bucketsort algorithm, compared to a mergesort implementation, while figure 2.2 gives the same data for the other sorting algorithms.

Figure 2.4 shows the error of the predicted versus the expected value.
CHAPTER 2. SORTING ALGORITHMS RANDOMLY DISTRIBUTED INPUT DATA

Figure 2.2: Measured and predicted execution times for the select-, bubble-, quick-, heap- and mergesort under the constraint that the input data to the sorting program is randomly distributed. The actual numbers are given in table ??.
The figures on the left show the measured execution time, on the right is the predicted execution time.
2.1. RANDOMLY DISTRIBUTED INPUT DATA

Chapter 2. Sorting Algorithms

Figure 2.3: Measured and predicted execution times for the bucketsort algorithm with different numbers of buckets (16, 256, 1024 and 16384) compared against the underlying mergesort algorithm. The input data to the sorting programs is assumed to be uniform randomly distributed. The actual numbers are in table ???. The figures on the left show the measured execution time, on the right is the predicted execution time. The upper two graphs plot the time against a logarithmic axis, the lower graphs show the execution time spend (on average) per element in the array.

Figure 2.4: The error as a percentage of the expected value for the data shown in figures 2.2 and 2.3.
2.2 Setting the parameters

For the algorithmic parameters—the number of times loops and conditions are taken—the Parasol II toolset has basically two ways actualizing. One is by using a profile files to determine the parameters and the other is by defining them by hand. The programs which have been analyzed in this report have been deliberately analyzed by hand. Below we give an example of theoretical complexity parameters for the selection sorting algorithm.

2.2.1 Selection Sort

In appendix A, the algorithm considered here, can be found. The number of times the outer most loop at line 9 is executed is clearly $n - 1$, the size of the array to be sorted. The inner loop at line 12 has different properties. In the first iteration of the outermost loop it is executed $n - 2$ times, the second time $n - 3$ times continuing until it is executed only once. This leads to the summation:

$$\sum_{i=1}^{n-2} i = \frac{1}{2} (n - 2) ((n - 2) + 1) = \frac{1}{2} (n - 1) (n - 2)$$

Since the outer loop iterates $n - 1$ times, the inner loop will iterate $\frac{1}{2} (n - 2)$ times on average.

The conditional on line 13 is true whenever an element $a_i$ in the list $a_0, a_1 \ldots a_n$ is smaller than all its predecessors ($a_{i-1}, a_{i-2} \ldots a_0$) For $i = 0$ the conditional is always true, for $i = 1$ with a uniform random list this will be $\frac{1}{2}$, for $i = 2$ it will be $\frac{1}{4}$. We will not go into any detail, but the the idea between this logic is that each predecessor has a probability of $\frac{1}{2}$ to be smaller and in this way each predecessor will half the chance that the element $a_i$ is smaller than all its predecessors.

Each element $a_i$ will be subject to the conditional $i$ times, which leads to the following formula for the chance that the condition evaluates to true:

$$\frac{\frac{1}{2} + \frac{1}{3} + \ldots +\frac{1}{n}}{n - i}$$

The nominator approaches 2 thus we get:

$$\sum_{i=0}^{n-1} \frac{2}{n - i} = 2 \ln(n) + C$$

In which we can ignore $C$, and since there are $n$ elements we have to divide this by $n$

To recapulate we are left with the following parameters:

| N.1 | n - 1               | (the outer loop) |
| N.2 | 0.5*(n-2)           | (the inner loop) |
| P.1 | 2*ln(n) / n         | (the constant)   |
2.3 A NOTE ON LINEAR SORTING

As was mentioned above the linear order sorting algorithms use some other sorting algorithm to sort the classes they have build. As we have seen, the overhead and the usage of an other algorithm do not make it attractive for sorting purposes, since it is only very slightly better than the underlying sort. But, the linear order sorting algorithms have also a very different purpose. If it is necessary to classify the input in ranges, resulting in a list of only roughly sorted lists, there is no need for the underlying comparison sort mechanism. And therfor these algorithms have their separate usefulness, especially in parallel computers in which data has be redistributed. The bucket method can be used to classify the data, and to distribute each class to a processor.
Appendix A
Source code

This appendix includes all the source code of the algorithms studied in this report. The main program is not included since it is generated in order to provide multiple input data sets.

A.1 Bubblesort

```
SUBROUTINE bubblesort(asize, a)
IMPLICIT NONE
INTEGER asize
DOUBLE PRECISION a
DIMENSION a(*)
DOUBLE PRECISION swap
INTEGER i, size
LOGICAL flag

size = asize
flag = .FALSE.
DO 20, i=1, size-1
  IF(a(i) .GT. a(i+1)) THEN
    PRINT *, i, a(i), a(i+1)
    swap = a(i)
    a(i) = a(i+1)
    a(i+1) = swap
    flag = .TRUE.
  END IF
20 CONTINUE
size = size - 1
IF(flag) GOTO 10
END
```

A.2 Selectsort

SUBROUTINE selectsort(size, a)
IMPLICIT NONE
INTEGER size
DOUBLE PRECISION a
DIMENSION a(*)
DOUBLE PRECISION smallest
INTEGER i, j, index

DO 20, i=1, size-1
  index = i
  smallest = a(index)
  DO 10, j=i+1, size
    IF(smallest .GT. a(j)) THEN
      index = j
      smallest = a(index)
    END IF
  10 CONTINUE
  a(index) = a(i)
  a(i) = smallest
20 CONTINUE
END
A.3 Heapsort

The heapify routine is the key to the heapsort algorithm. The parameters to the heapify routine are an array \( A \) and an index \( i \) into that array. The precondition for the heapify routine is that the left binary subtree and the right binary subtree are both heaps. \( A(i) \) however may be larger than the elements in both subtrees, thus violating the heap property. The heapify routine will "sift down" this element \( A(i) \) and by this way both subtrees and \( A(i) \) will become one larger heap.

\[
\begin{align*}
&1. \quad l \leftarrow \text{Left}(i) \\
&2. \quad r \leftarrow \text{Right}(i) \\
&3. \quad \text{IF } (l \leq \text{HeapSize}[A] \text{ and } A[l] > A[i]) \\
&4. \quad \quad \text{then } \text{largest} \leftarrow l \\
&5. \quad \quad \text{else } \text{largest} \leftarrow i \\
&6. \quad \text{IF } (r \leq \text{HeapSize}[A] \text{ and } A[r] > A[\text{largest}]) \\
&7. \quad \quad \text{then } \text{largest} \leftarrow r \\
&8. \quad \quad \text{ENDIF} \\
&9. \quad \text{IF } (\text{largest} \neq i) \\
&10. \quad \quad \text{then exchange } A[i] \leftrightarrow A[\text{largest}] \\
&11. \quad \quad \text{Heapify}(A[\text{largest}]) \\
&12. \quad \quad \text{ENDIF} \\
&13. \quad \text{ENDIF} \\
&14. \quad \text{GOTO 10} \\
&15. \quad \text{END IF} \\
&16. \quad \text{END}
\end{align*}
\]

Most parameters of the heapsort are

\[
\begin{align*}
&1. \quad \text{a}(i) = a(\text{largest}) \\
&2. \quad a(\text{largest}) = \text{swap} \\
&3. \quad i = \text{largest} \\
&4. \quad \text{GOTO 10} \\
&5. \quad \text{END IF} \\
&6. \quad \text{END}
\end{align*}
\]
A.3. HEAPSORT  

APPENDIX A. SOURCE CODE

32  
for \( i \leftarrow \frac{\text{Size}[n]}{2} \) downto 1  
do  
  Heapify(A, i)

33  
SUBROUTINE buildheap(size, a)  
IMPLICIT NONE  
DOUBLE PRECISION a  
INTEGER size  
DIMENSION a(*)  
INTEGER i  

34  
DO 10, i=size/2, 1, -1  
  CALL heapify(size, a, i)

10  CONTINUE  
END

47  
BuildHeap(A)  
for \( i \leftarrow \text{length}[A] \) downto 2  
do  
  exchange \( A[1] \leftrightarrow A[i] \)  
  decrease \( \text{HeapSize} \) by 1  
  Heapify(A, 1)

46  
SUBROUTINE heapsort(asize, a)  
IMPLICIT NONE  
INTEGER asize  
DOUBLE PRECISION a  
DIMENSION a(*)  
DOUBLE PRECISION swap  
INTEGER i, size  

55  
size = asize  
CALL buildheap(size, a)  
DO 10, i=size, 2, -1  
  swap = a(1)  
  a(1) = a(i)  
  a(i) = swap  
  size = size - 1  
  CALL heapify(size, a, 1)

10  CONTINUE  
END

12
A.4 Mergesort

```
SUBROUTINE mergelsort(a, lstptr, head, tail)
IMPLICIT NONE
INTEGER stacksize
PARAMETER (stacksize = 256)
INTEGER lstptr, head(*), tail(*)
DOUBLE PRECISION a(*)
INTEGER stackindex, stack(stacksize)
INTEGER size, list1, list2, run, hsize, x, y, z

x = 0
y = 0
z = 0
size = 0
run = lstptr
10 IF(run .GT. 0) THEN
  size = size + 1
  run = tail(run)
  GOTO 10
END IF
sv = size

1 IF(size .LE. 1) GO TO 4

list1 = lstptr
list2 = lstptr
hsize = size/2
20 IF(hsize .GT. 0) THEN
  hsize = hsize - 1
  list2 = tail(list2)
  GOTO 20
END IF

stack(stackindex) = size
stack(stackindex+1) = list2
stack(stackindex+2) = 1

GO TO 1

list1 = lstptr
stackindex = stackindex - 3
size = stack(stackindex)
list2 = stack(stackindex+1)
GO TO 1

list2 = lstptr
```

A.4. MERGESORT

APPENDIX A. SOURCE CODE

IF(a(head(list1)) .LT. a(head(list2))) THEN
  lstptr = list1
ELSE
  lstptr = list2
END IF

run = 0
30 IF(list1 .GT. 0 .AND. list2 .GT. 0) THEN
  z = z + 1
  IF(a(head(list1)) .LT. a(head(list2))) THEN
    IF(run .GT. 0) THEN
      tail(run) = list1
    ELSE
      lstptr = list1
    END IF
    run = list1
    list1 = tail(list1)
  ELSE
    IF(run .GT. 0) THEN
      tail(run) = list2
    ELSE
      lstptr = list2
    END IF
    run = list2
    list2 = tail(list2)
  END IF
  GOTO 30
ENDIF

IF(list1 .GT. 0) THEN
  IF(run .GT. 0) THEN
    tail(run) = list1
  ELSE
    lstptr = list1
  ENDIF
ELSE IF(list2 .GT. 0) THEN
  IF(run .GT. 0) THEN
    tail(run) = list2
  ELSE
    lstptr = list2
  END IF
END IF

IF(size .EQ. 1) tail(lstptr) = 0

F2C isn't able to process vector-if statements, that is why the following IF-statement is commented out and two replacement IF's are dropped in.

y = y + 1
IF(stack(stackindex-1).EQ.1) GO TO 2
x = x + 1
IF(stack(stackindex-1).EQ.2) GO TO 3
END
A.5 Quicksort

SUBROUTINE quicksort(asize, a)
  INTEGER stacksize
  PARAMETER (stacksize = 256)
  INTEGER asize
  DOUBLE PRECISION a(*)
  DIMENSION a(*)
  DOUBLE PRECISION aux
  INTEGER start, size, front, back, stack, idx
  DIMENSION stack(stacksize)

  size = asize
  idx = 0
  start = 1
  IF(size .GT. 1) THEN
    front = start+1
    back = start+size-1
    IF(front .LE. back) THEN
      IF (a(start) .GT. a(front)) THEN
        front = front + 1
      ELSE IF(a(start) .LE. a(back)) THEN
        back = back - 1
      ELSE
        aux = a(front)
        a(front) = a(back)
        a(back) = aux
        END IF
      GOTO 30
    END IF
    IF(front .NE. start+1) THEN
      aux = a(start)
      a(start) = a(front-1)
      a(front-1) = aux
      stack(idx+1) = front - start - 1
      stack(idx+2) = start
      idx = idx + 2
    END IF
    size = size - back + start - 1
    start = front
    GOTO 20
  END IF

  IF(front .LE. back) THEN
    IF (a(start) .GT. a(front)) THEN
      front = front + 1
    ELSE IF(a(start) .LE. a(back)) THEN
      back = back - 1
    ELSE
      aux = a(front)
      a(front) = a(back)
      a(back) = aux
      END IF
    GOTO 30
  END IF

  IF(front .NE. start+1) THEN
    aux = a(start)
    a(start) = a(front-1)
    a(front-1) = aux
    stack(idx+1) = front - start - 1
    stack(idx+2) = start
    idx = idx + 2
  END IF
  size = size - back + start - 1
  start = front
  GOTO 20
END IF

IF(idx .GT. 0) THEN
  idx = idx - 2
  size = stack(idx+1)
  start = stack(idx+2)
  GOTO 20
END IF

END
A.6. Bucketsort

```fortran
SUBROUTINE bucketlsort(a, lstptr, head, tail)

IMPLICIT NONE

DOUBLE PRECISION a(*)
INTEGER lstptr, head(*), tail(*)
INTEGER numbuckets
PARAMETER (numbuckets = 16384)
INTEGER buckets(numbuckets), bucketnum, i, aux

DO 10, i=1, numbuckets
   buckets(i) = 0
10 CONTINUE

20 IF(lstptr .GT. 0) THEN
   bucketnum = INT(a(head(lstptr)) * numbuckets) + 1
   aux = tail(lstptr)
   tail(lstptr) = buckets(bucketnum)
   buckets(bucketnum) = lstptr
   lstptr = aux
   GO TO 20
20 END IF

DO 30, i=1, numbuckets
   CALL mergelsort(a, buckets(i), head, tail)
30 CONTINUE

lstptr = 0
DO 40, i=1, numbuckets
   IF(buckets(i) .GT. 0) THEN
      IF(lstptr .EQ. 0) THEN
         lstptr = buckets(i)
      ELSE
         tail(aux) = buckets(i)
      END IF
      aux = buckets(i)
   END IF
   IF(tail(aux)) THEN
      aux = tail(aux)
      GO TO 50
   END IF
40 CONTINUE

END
```