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Dynamical Triangulation with Fluctuating Topology

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We consider a dynamical triangulation model of euclidean quantum gravity where the topology is not fixed. This model is equivalent to a tensor generalization of the matrix model of two dimensional quantum gravity. A set of moves is given that allows Monte Carlo simulation of this model. Some preliminary results are presented for the case of four dimensions.

1. Introduction

In the dynamical triangulation model of quantum gravity one usually considers systems where the spacetime topology is fixed (although the topology of space is allowed to change with time). In a path integral formulation of quantum gravity, summing over the topologies seems a natural thing to do.

Because the typical curvature fluctuations become larger at smaller scale, allowing an arbitrary topology will result in a very complicated structure at the Planck scale [1]. Such a spacetime which is full of holes is commonly called spacetime foam. It was estimated by Hawking [2] that the dominant contribution to the path integral would come from spacetimes where the Euler characteristic χ is of the order of the volume of the spacetime in Planck units.

2. Definition of the model

The partition function of the model in d dimensions is

$$Z(\kappa, N_d) = \sum_{\mathcal{T}(N_d)} \exp(\kappa N_{d-2}). \quad (1)$$

This expression is the same as for a fixed topology, but here the sum is over all possible ways to glue a fixed number N_d of d -simplices together, maintaining orientation (i.e. only identify $d-1$ dimensional faces with opposite orientation). Because the faces have to be glued in pairs, the number of simplices N_d must be even in odd dimensions.

At this stage we include disconnected configurations. Because the action is a sum of the actions of the connected components, the Boltzmann weight factorizes, which means that the local physics will not change. However, including these configurations will tell us something about the chance to obtain a particular size of connected component. A universe which is most likely to be split up into many small parts seems an unlikely candidate for the real world.

Naively gluing together simplices will not result in manifolds, but only in pseudomanifolds. Several ways to deal with this problem are conceivable. First, as nobody knows what spacetime looks like at the Planck scale, one could argue that this is not a problem. Second, one can (at least for $d \leq 4$) locally deform the resulting pseudomanifold to turn it back into a manifold by removing a small region around the singular points and pasting in a regular region. This can be done while changing the total curvature only by an arbitrarily small amount.

Third, these configurations might be unimportant in the limit $\kappa \rightarrow \infty$ (which will have to be taken, see below). E.g. in three dimensions for each fixed N_0 (and volume N_3) the number of edges (which couples to κ) is maximal if and only if the configuration is a real (i.e. non-pseudo) manifold. See [4] for an explanation. This is less clear in four dimensions, though, because a similar reasoning only results in a closed neighbourhood of a point being bounded by a simplicial manifold (as opposed to a pseudomanifold), but not necessarily a sphere.

At low κ the connected configurations will contribute most as they have the highest entropy. At high κ the disconnected configuration will contribute most as they have the lowest action. It is a priori not clear whether this change occurs gradually or whether there is a phase transition. Suppose for a moment that there is a sharp crossover at some κ^c depending on the volume. Because the number of connected configurations rises faster with the volume than the number of disconnected ones, the value of κ^c will increase logarithmically with the number of simplices. This means that in a possible continuum limit, the value of κ will have to be taken to infinity.

3. Tensor model

We can formally write down a tensor model which is a generalization of the well known one matrix model of two dimensional quantum gravity (see e.g. [5] and references therein). The partition function of this tensor model is written in terms of a k -dimensional tensor M of rank d . The tensor M is invariant under an even permutation of its indices and goes to its complex conjugate under an odd permutation of its indices. The partition function for three dimensions is

$$Z = \int dM_{abc} \exp \left(-\frac{1}{6} M_{abc} M_{abc}^* + \right. \quad (2)$$

$$\left. g M_{abc} M_{ade}^* M_{bdf} M_{cef}^* \right), \quad (3)$$

and for four dimensions it is

$$Z = \int dM_{abcd} \exp \left(-\frac{1}{24} M_{abcd} M_{abcd}^* + \right. \quad (4)$$

$$\left. g M_{abcd} M_{aefg}^* M_{behf} M_{cfgh}^* M_{dgij} \right). \quad (5)$$

Using the properties of M , the action can easily be seen to be real. The generalization to more dimensions should be obvious from these expressions.

These expressions are only formal, because the interaction term can be negative for any g and is of higher order than the gaussian term. Therefore, these integrals will not converge. However, if we expand these expressions in g , each term of the expansion is well defined and the dual of each

of its Feynman diagrams is a DT configuration of size N_d equal to the order of g used. Each propagator carries two sets of d indices. Each vertex of the tensor model corresponds to a d -simplex, and each propagator between them corresponds to the identification of two $d-1$ dimensional faces of simplices. The sets of indices at each end of a propagator must be an odd permutation of each other, making sure that the simplicial complex has an orientation. The contribution of a particular diagram is

$$g^{N_d} k^{N_{d-2}} = \exp(\ln(g)N_d + \ln(k)N_{d-2}), \quad (6)$$

which is precisely proportional to the Boltzmann weight of the DT configuration according to the Regge-Einstein action, with the identification $\kappa = \ln k$.

A similar model using hypercubes was introduced in [3]. The model described above has been discussed for three dimensions in [4]. A different generalization of the matrix model where the dimension of the matrix couples to the number of points in the DT configuration (which means one does not get the Regge-Einstein action in more than 3 dimensions) has been discussed in [6].

4. Monte Carlo simulation

A move can most easily be described in the tensor model formulation. It consists of first cutting two propagators and then randomly reconnecting them. Due to the orientability, there are $d!/2$ ways to connect two propagators. In the dynamical triangulation model, this corresponds to cutting apart the simplices at each side of two of the $d-1$ dimensional faces and pasting these faces together.

Unlike the case of fixed topology with (k, l) moves, one can easily see that these moves are ergodic. The number of moves needed to get from one configuration to another is $O(N_d)$, raising none of the computability problems associated with the fixed topology [7]. The non-existence of a classification of four-topologies and their unrecognizability is usually mentioned as a problem for the summation over topologies. In dynamical triangulation, however, it seems to be more a problem for fixing the topology.

Figure 1. The number of connected components as a function of κ in the four dimensional model.

We use the standard Metropolis test to accept or reject the moves. Because (again unlike the fixed topology case) the number of possible moves does not depend on the configuration, detailed balance is easily obtained.

One could restrict the simulation to connected configurations by checking connectedness for each move accepted by the Metropolis test. This might be rather slow, because this is not a local test. Also, although it seems very plausible, it is not clear whether this would be ergodic in the space of connected configurations.

5. Results

We have simulated this model in four dimensions at a volume $N_4 = 500$. We used a hot start, that is a configuration with completely random connections. This would be an equilibrium configuration at $\kappa = 0$.

The number of connected components is plotted in figure 1. We see that at low κ the average number of components is almost one, while at high κ the average number of components is almost equal to the maximum number of $N_d/2$. Already at this low number of simplices, the change between few and many components looks quite sharp.

Unfortunately, the acceptance rates for these

moves are quite low for κ near κ^c , of the order of 0.1%. One of the reasons for this is that the proposed moves are not local in the sense that they can connect any two points in the complex. This means that simulating much larger systems will probably not be feasible with only these moves.

6. Discussion

The main problem to investigate is the existence of a sensible continuum limit. Although, due to the factorially increasing number of configurations, the grand canonical partition function in the normal definition does not exist, this does not exclude that the local behaviour of the system might show scaling for large volumes.

In two dimensions a limit of the grandcanonical partition function, the so-called double scaling limit, is known [8]. In this limit we also see that the coupling κ has to be taken to infinity.

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