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Structure functions of pseudoscalar mesons in the SU(3) NJL model

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Abstract

We compute the structure functions of the pion, kaon and η mesons using a SU(3) version of the NJL model with scalar and pseudoscalar couplings. By considering the absorptive part of the forward Compton scattering amplitude, the regularization can be treated without ambiguities and gauge invariance is preserved at every stage of the calculation. Using Pauli–Villars regularization, the structure functions scale and have proper normalization in the Bjorken limit. In the chiral limit, the simple result $q(x) = \bar{q}(x) = \theta(x)\theta(1-x)$ is obtained for the structure functions. The results are evolved to higher momentum using the Altarelli–Parisi equations for three flavours, and good agreement with the experimental data for the pion structure function is found.

1. The study of deep inelastic scattering (DIS) on hadronic targets is interesting because it provides information about the quark content of hadrons. In the Bjorken limit, the operator product expansion (OPE) can be used to disentangle the hard and soft pieces of the twist-two contributions to the scattering amplitude. The OPE requires an extrapolation from the region $|x| > 1$, where it converges, to the physical region $|x| < 1$. Thus, much of what is theoretically known about the structure functions comes from the study of the absorptive parts of the forward Compton scattering amplitude of virtual photons from the target of interest. In fact, such an analysis makes it possible to evaluate the structure functions in terms of the off-shell quark–target scattering amplitude [1], under the assumption that the quark–target scattering amplitude vanishes sufficiently fast as a function of the quark momentum. Although perturbative QCD allows one to predict the large Q^2 dependence of the hard piece, some particular model or lattice calculation is needed for the soft, non-perturbative piece in order to make definite predictions for the hadronic structure functions. In this context, it has been suggested [2] that quark models can be used to calculate the leading twist contribution to the structure functions. The structure functions obtained from quark models are assumed to be valid at a certain low energy scale, Q_0^2 , and the results are evolved up to the Q^2 -scale relevant for DIS experiments using the renormalization group equations [3,4]. This idea has been implemented to study deep inelastic scattering in several models [2,5–10]. In some cases, the quark–target scattering amplitude has been used with the additional inclusion of hadronic form factors, or assumptions about translational invariance or completeness of the intermediate states have been made. These assumptions generally violate the proper normalization of the structure functions, or equivalently, the gauge invariance of the Compton amplitude. It is then not clear whether the counterterms needed to restore gauge invariance are compatible with the assumptions underlying

the derivation of the quark–target amplitude from the Compton amplitude. This makes the calculation of the structure function within the model itself ambiguous.

Relativistic and gauge invariance impose very clear constraints on the structure functions regarding their support and normalization. On the other hand, a fully covariant description of most hadrons as bound states of quarks is still lacking. From this point of view, the low-lying pseudoscalar mesons are clearly distinguished from the rest of the hadronic spectrum. Although the role of chiral symmetry in deep inelastic scattering is in general not known, the Goldstone nature of the pseudoscalar octet implies that apart from soft pion corrections, many of their properties are determined by chiral symmetry. This becomes even more transparent in effective chiral quark models which implement dynamical chiral symmetry breaking, such as the Nambu–Jona-Lasinio model (NJL) [11,12]. In this model, the pion is a deeply bound $\bar{q}q$ state. Although the NJL model does not incorporate confinement, the lack of it seems to be irrelevant in the case of the pseudoscalar mesons. Thus, we hope to learn more about the role of chiral symmetry in deep inelastic scattering by studying the lightest mesons. In addition, the pion structure function has been experimentally determined from πN scattering using the Drell–Yan process [13], and the corresponding first two moments have been calculated on the lattice in the quenched approximation [14].

In the present work, we compute the structure functions of the lightest pseudoscalar mesons, i.e. π , K and η , using the NJL model with three flavours. We start with the imaginary part of the forward Compton scattering amplitude, which is regularized by means of the Pauli–Villars method. In the Bjorken limit, we find scaling, proper support and normalization of the structure functions. Since perturbative gluons are supposed to be integrated out in the NJL model, we generate them using the QCD evolution equations. As a side remark, we would like to emphasize that starting from the Compton amplitude rather than the quark–target amplitude is not a superfluous step at all. In fact, if such an amplitude does not vanish faster than a certain power of the quark momentum, the connection between both amplitudes does not hold in the usual sense. Our way of proceeding provides a regularized version of the quark–target formula in this model. Finally, let us mention that deep inelastic scattering from the pion has also been studied recently in a relativistic quark model [15] and in the NJL model [16]. In neither case, however, do the authors obtain *consistently* normalized structure functions due to a careless treatment of the Compton amplitude.

2. The SU(3) NJL Lagrangian in Minkowski space is given by [11,12]

$$\mathcal{L}_{\text{NJL}} = \bar{q}(i\rlap{/}\partial - \hat{M}_0)q + \frac{G_S}{2} \sum_{a=0}^8 [(\bar{q}\lambda_a q)^2 + (\bar{q}\lambda_a i\gamma_5 q)^2], \quad (1)$$

where $q = (u, d, s)$ represents a quark spinor with N_c colours and three flavours. The λ 's are the Gell-Mann matrices of the U(3) flavour group, $\hat{M}_0 = \text{diag}(m_u, m_d, m_s)$ stands for the current quark mass matrix, and G_S is the coupling constant. In the limiting case of vanishing current quark masses, the NJL action is invariant under the global $U(3)_R \otimes U(3)_L$ group of transformations¹. For the rest of the paper we neglect isospin breaking effects, i.e. we set $m_u = m_d$.

The vacuum to vacuum transition amplitude in the presence of external bosonic (s, p, v, a) fields of the NJL Lagrangian can be written, after bosonization [20], integrating out the quarks, and Pauli–Villars regularization [21], as the path integral

$$Z[s, p, v, a] = \langle 0 | T \exp\left(i \int d^4x \{\bar{q}[\psi + \not{p}\gamma_5 - (s + i\gamma_5 p)]q\}\right) | 0 \rangle = \int DS DP \exp(iS). \quad (2)$$

The normal parity (γ_5 -even) contribution to the effective action is

¹ To account for the $U_A(1)$ anomaly, we consider the Witten–Veneziano [17,18] term in the strong coupling limit. In this case, the η is a non-mixing η_8 state. A full discussion of other breakings, introduction of vector couplings, and more details of our calculation will be presented elsewhere [19].

$$S_{\text{even}} = -\frac{iN_c}{2} \sum_i c_i \text{tr} \log(\mathbf{D}\mathbf{D}_S + \Lambda_i^2 + i\epsilon) - \frac{1}{(4G_S + i\epsilon)} \int d^4x \text{tr}(S^2 + P^2), \quad (3)$$

where the Dirac operators

$$\begin{aligned} i\mathbf{D} &= i\cancel{d} - \hat{M}_0 - (S + i\gamma_5 P) + \psi + \cancel{d}\gamma_5 - (s + i\gamma_5 p), \\ i\mathbf{D}_S &= i\cancel{d} - \hat{M}_0 - (S - i\gamma_5 P) + \psi - \cancel{d}\gamma_5 - (s - i\gamma_5 p), \end{aligned} \quad (4)$$

have been introduced. (S, P) are dynamical, internal bosonic SU(3) fields. Two subtractions are needed to regularize the quadratic divergence, and we take the limit $\Lambda_1 \rightarrow \Lambda_2 = \Lambda$ for the Pauli–Villars regulators. In this limit, we obtain the identity $\sum_i c_i f(\Lambda_i^2) = f(0) - f(\Lambda^2) + \Lambda^2 f'(\Lambda^2)$ (see Refs. [22,23] for more details as well as the connection to proper-time regularization). Any mesonic correlation function can be obtained from this gauge-invariantly regularized effective action by suitable functional differentiation with respect to the relevant external fields. In practice, we work in the limit $N_c \rightarrow \infty$, or equivalently at the one quark loop level.

3. To fix the parameters in the Pauli–Villars regularized NJL model, we consider several mesonic properties. The effective potential leads to dynamical chiral symmetry breaking, thereby yielding dynamical quark masses (M_u, M_d, M_s) and condensates given by

$$\frac{\langle \bar{u}u \rangle}{M_u - m_u} = \frac{\langle \bar{d}d \rangle}{M_d - m_d} = \frac{\langle \bar{s}s \rangle}{M_s - m_s} = -\frac{1}{2G_S}, \quad (5)$$

with the condensate given by the integral

$$\langle \bar{q}_\alpha q_\alpha \rangle = -4N_c M_\alpha I_2^\alpha = 4N_c M_\alpha i \sum_i c_i \int \frac{d^4k}{(2\pi)^4} \frac{1}{-k^2 + M_\alpha^2 + \Lambda_i^2 - i\epsilon}. \quad (6)$$

Calculation of the relevant correlation functions yields the following expressions for the pseudoscalar meson masses:

$$\begin{aligned} m_\pi^2 &= \frac{2I_2^u}{F_{uu}(m_\pi^2)} \frac{m_u}{M_u - m_u}, \\ m_K^2 &= \frac{1}{F_{us}(m_K^2)} \left(\frac{m_u}{M_u - m_u} I_2^u + \frac{m_s}{M_s - m_s} I_2^s \right), \\ m_\eta^2 &= \frac{2}{F_{uu}(m_\eta^2) + 2F_{ss}(m_\eta^2)} \left(\frac{m_u}{M_u - m_u} I_2^u + 2 \frac{m_s}{M_s - m_s} I_2^s \right), \end{aligned} \quad (7)$$

which, in the SU(3) limit, reproduce the Gell-Mann–Okubo formula, $4m_K^2 - m_\pi^2 = 3m_\eta^2$. The pseudoscalar decay constants are

$$\begin{aligned} f_\pi &= 4N_c M_u F_{uu}(m_\pi^2) g_{\pi uu}, \\ f_K &= 2N_c [(M_u - M_s) T_{us}(m_K^2) + (M_u + M_s) F_{us}(m_K^2)] g_{Kus}, \\ f_\eta &= 2N_c [g_{\eta uu} F_{uu}(m_\eta^2) + g_{\eta ss} F_{ss}(m_\eta^2)], \end{aligned} \quad (8)$$

with the meson–quark–quark coupling constants given by

$$\frac{1}{g_{P\alpha\beta}^2} = 4N_c \frac{d}{dp^2} [p^2 F_{\alpha\beta}(p^2)] \Big|_{p^2=m_p^2}. \quad (9)$$

Here, we have introduced the following short-hand notations

$$T_{\alpha\beta}(p^2) = \int_0^1 dx (2x-1) F_{\alpha\beta}(p^2, x), \quad F_{\alpha\beta}(p^2) = \int_0^1 dx F_{\alpha\beta}(p^2, x), \quad (10)$$

where, in terms of the Pauli–Villars regularized one-loop integrals,

$$F_{\alpha\beta}(p^2, x) = -i \int \frac{d^4k}{(2\pi)^4} \sum_i c_i \frac{1}{[-k^2 - x(1-x)p^2 + (1-x)M_\alpha^2 + xM_\beta^2 + \Lambda_i^2 - i\epsilon]^2}. \quad (11)$$

$F_{\alpha\beta}$ satisfies the symmetry relation $F_{\alpha\beta}(p^2, x) = F_{\beta\alpha}(p^2, 1-x)$. The parameters are fixed as usual; we adjust the cut-off to reproduce the physical weak pion decay constant, $f_\pi = 93.3$ MeV, and the physical pion ($m_\pi = 139.6$ MeV) and kaon ($m_K = 494$ MeV) masses. This leaves only one free parameter, which we choose to be the constituent up quark mass, M_u . The values of f_K , m_η and f_η are then predicted. For $M_u = 280$ MeV, we obtain $m_u = 7$ MeV, $m_s = 175$ MeV, $M_s = 527$ MeV, $\Lambda = 870$ MeV, $m_\eta = 501$ MeV (exp. 549 MeV), and $f_K = 102$ MeV (exp. 113 MeV). We will mainly take this value of M_u in our calculations since it maximizes the current quark mass contributions to the structure functions.

4. The hadronic tensor, $W_{\mu\nu}(p, q)$, can be obtained from the imaginary part of the forward Compton amplitude for virtual photons as follows:

$$\begin{aligned} W_{\mu\nu}(p, q) &= \frac{1}{2\pi} \text{Im} T_{\mu\nu}(p, q) \\ &= W_1(q^2, p \cdot q) \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{W_2(q^2, p \cdot q)}{m_p^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right), \end{aligned} \quad (12)$$

where

$$T_{\mu\nu}(p, q) = i \int d^4x \exp(iq \cdot x) \langle \pi(p) | T \{ J_\mu^{\text{em}}(x) J_\nu^{\text{em}}(0) \} | \pi(p) \rangle. \quad (13)$$

Here, $m_p^2 = p^2$ is the mass of the pseudoscalar meson and q is the momentum of the virtual photon. In the limit $N_c \rightarrow \infty$, only one-loop quark diagrams survive. To preserve gauge invariance, one has to consider in addition to the usual handbag diagrams, the process $\pi\gamma \rightarrow \pi \rightarrow \pi\gamma$, which involves the off-shell electromagnetic form factor of the pseudoscalar meson. Using the Pauli–Villars regularized effective action, (3), the latter process is indeed a higher twist contribution. This point is not entirely trivial since we are using a theory with a finite cut-off and naive counting rules do not necessarily work. In the Bjorken limit, we obtain after straightforward manipulations ($x = -q^2/(2p \cdot q)$),

$$W_{\mu\nu}(p, q) = \frac{1}{2\pi} \text{Im} T_{\mu\nu}(p, q) \rightarrow F(x) \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \frac{1}{q^2} \left(p_\mu - \frac{q_\mu}{2x} \right) \left(p_\nu - \frac{q_\nu}{2x} \right) \right], \quad (14)$$

with

$$F(x) = \frac{1}{2} \sum_{i=u,d,s} e_i^2 [\bar{q}_i(x) + q_i(x)]. \quad (15)$$

The π , K and η structure functions may be conveniently written as

$$\begin{aligned}
u_\pi(x) &= \bar{d}_\pi(1-x) = 4N_c g_{muu}^2 \frac{d}{dp^2} [p^2 F_{uu}(p^2, x)] \Big|_{p^2=m_\pi^2}, \\
u_K(x) &= \bar{s}_K(1-x) = 4N_c g_{mus}^2 \frac{d}{dp^2} [p^2 F_{us}(p^2, x)] \Big|_{p^2=m_K^2}, \\
u_\eta(x) &= \bar{u}_\eta(x) = d_\eta(x) = \bar{d}_\eta(x) = 4N_c \left(\frac{1}{g_{\eta uu}^2} + \frac{2}{g_{\eta ss}^2} \right)^{-1} \frac{d}{dp^2} [p^2 F_{uu}(p^2, x)] \Big|_{p^2=m_\eta^2}, \\
s_\eta(x) &= \bar{s}_\eta(x) = 8N_c \left(\frac{1}{g_{\eta uu}^2} + \frac{2}{g_{\eta ss}^2} \right)^{-1} \frac{d}{dp^2} [p^2 F_{ss}(p^2, x)] \Big|_{p^2=m_\eta^2},
\end{aligned} \tag{16}$$

in the interval $0 < x < 1$. All other distribution functions are exactly zero, in accord with the fact that we do not have gluons or sea quarks in the model. As one can see, the pion and kaon distribution functions are normalized to one. For the η , only the total sum over flavours is normalized to one. Furthermore, using (11) and (16) the momentum sum rule can be easily verified. Notice that in the chiral limit, both the pion and kaon structure functions coincide and give a constant equal to one, independent of the parameters and cut-off. Our result for the pion structure function is different from previous calculations [15,16]. In one case [15], the authors formally work in the infinite cut-off limit and reinterpret the light-cone pion wave function in terms of a non-relativistic quark model. This is certainly inconsistent with the original Compton amplitude with which the authors started. In another case [16], the cut-off is introduced ad hoc and in a way that the symmetry under the operation $x \rightarrow 1-x$ is lost, implying the unphysical result that the up and down quark momentum fractions are different even for $m_u = m_d$. Moreover, in both cases the normalization is introduced by hand, and the momentum sum rule is not satisfied. We would like to emphasize that our calculation is clear-cut; once the effective action has been gauge-invariantly regularized, the rest follows.

5. With the parameters fixed as described above, we show in Fig. 1 our computed pion structure function for a constituent up quark mass of $M_u = 280$ MeV, together with the calculation of Ref. [16]. Following the suggestion of Ref. [2], we assume our result to be valid at some small Q_0 scale, and evolve the computed structure functions using the Altarelli–Parisi equations [4]. This certainly makes sense in our model as the valence quarks carry all the momentum at Q_0 . The sea quark and the gluons are then generated by QCD evolution. In addition, since we want to compare to experimental data near the charm threshold, $Q^2 \approx 4$ GeV² [13], we have found it more appropriate to do the evolution with three flavours, and have matched $\alpha_s(N_f=3)$ to $\alpha_s(N_f=4)$ at 4 GeV². The evolved result for the pion structure function is also shown in Fig. 1, together with the experimental fit of Ref. [13]. To determine the evolution ratio, $r = \log(Q_0^2/\Lambda_{\text{QCD}}^2)/\log(Q^2/\Lambda_{\text{QCD}}^2)$, we match the valence quark momentum fraction of our calculation to that determined by Ref. [13] at $Q^2 = 4$ GeV². We find $r = 0.15$, which, for $\Lambda_{\text{QCD}} = 0.226$ GeV, yields $Q_0 = 0.312$ GeV. This value is surprisingly close to the constituent quark mass, M_u , and agrees with the Q_0 value found for the nucleon structure function [9,10]. We have found that finite pion mass corrections are negligible. Our results for the kaon u and s distributions before and after evolution are depicted in Fig. 2. The difference to the pion case mainly arises from the relatively large kaon mass compared to the up and strange constituent quark masses. In Fig. 3 we show the ratio of the up valence quark distribution in the kaon compared to the corresponding quantity in the pion at $Q^2 = 20$ GeV². We do not show the structure function for the η , but simply mention that for $M_u = 280$ MeV, we get 71% of up and down quarks and 29% of strange quarks. We have also checked that our results are rather insensitive to changes in the constituent up quark mass in the region between 250 and 450 MeV.

An interesting aspect of the present investigation is the influence of the regularization method. We have found that the conventional proper-time method [NJL] does not regularize the imaginary part of the Compton amplitude, leading to a scaling violation in the model proportional to $\log(Q^2/M^2)$ in the Bjorken limit. Such behaviour is also obtained in the Pauli–Villars scheme in the regime $4M^2 \ll Q^2 < 4(M^2 + \Lambda^2)$. This scaling violation, however, does not coincide with the QCD result, since there are no explicit gluonic degrees of freedom in the model, and hence

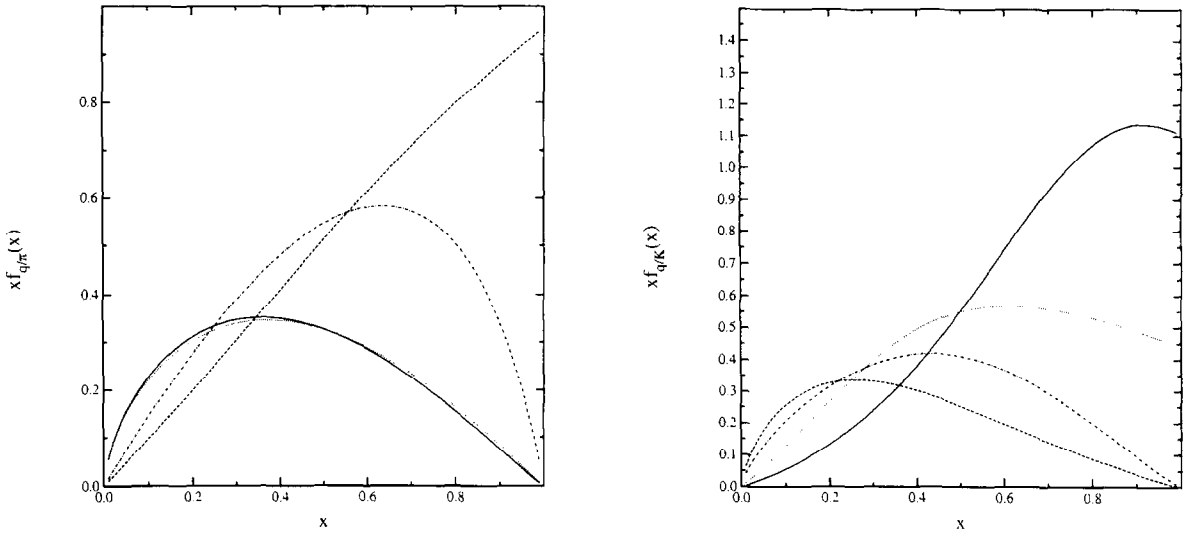


Fig. 1. Valence quark distribution function for the pion, $xu_{\pi}(x) = x\bar{d}_{\pi}(x)$. The dashed line represents the present calculation. The dash-dotted line is the calculation of Ref. [16]. The full line represents our evolved valence quark distribution up to $Q^2 = 4 \text{ GeV}^2$ with three flavours. The fit to the experimental result as given in Ref. [13] is represented by the dotted line.

Fig. 2. Quark distribution functions for the kaon. The dotted and dashed lines represent the up quark distribution before and after evolution, whereas the solid and dash-dotted lines stand for the \bar{s} before and after evolution respectively.

cannot be considered as an effective way of mimicking QCD evolution. In this sense, we do not consider proper-time regularization suitable for the study of the leading twist contribution to the structure function. In particular, Eqs. (16) cannot be obtained within a conventional proper-time scheme.

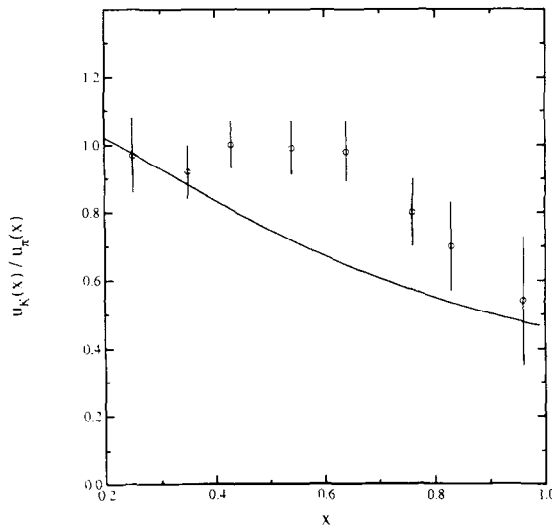


Fig. 3. Ratio of the u valence quark distribution of the kaon and the pion, $u_K(x)/u_{\pi}(x)$, at $Q^2 = 20 \text{ GeV}^2$. The experimental data are taken from Ref. [24].

6. In the present paper, we have evaluated the structure functions of the pseudoscalar mesons within a SU(3) NJL model. In contrast to models of other hadrons, this model provides a fully covariant description of pseudoscalar mesons as deeply bound $\bar{q}q$ states. Therefore, we do not expect a better theoretical understanding of a deep inelastic process than in the case of the pseudoscalar mesons. Most of their low energy properties are governed by the spontaneous breaking of chiral symmetry, and confinement effects are not expected to play an essential role. Desirable features of a structure function are proper normalization, proper support, scaling and $F_1(x) = 2xF_2(x)$, as dictated by gauge invariance, relativistic invariance, scale invariance and the fact that quarks are spin one-half particles. Our calculation automatically fulfills all these requirements, and leads to the remarkable result that in the chiral limit the pion and kaon structure functions are a constant equal to one. In fact, after QCD evolution we are able to fit the experimental data for the pion structure function rather well. We have also found that the finite pion mass corrections induce negligible corrections, whereas the effects of the finite kaon mass corrections are, after evolution, at the 10% level. Our calculation can also be extended to the octet of vector mesons which, due to the $A-\pi$ mixing, ought to change the x dependence of the pseudoscalar meson distribution functions away from the chiral limit. Work along these lines is presently in progress. Finally, let us mention that our calculation strongly suggests that the present results in the chiral limit are not specific to the NJL model, and could possibly be derived within a more general framework. We believe that this point deserves further investigation, and might help to understand the role of chiral symmetry in deep inelastic scattering.

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