Mesonic correlation functions in the NJL-model with vector mesons

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Mesonic correlation functions in the NJL model with vector mesons

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Abstract

We compute mesonic correlation functions in Euclidean coordinate space within the Nambu and Jona-Lasinio model with explicit scalar, pseudoscalar, vector and axial-vector couplings. After bosonization and application of the Pauli-Villars regularization method, we verify that the relevant dispersion relations are satisfied in the model, and use them to evaluate the point to point correlators in several channels, namely $\sigma, \pi, p$ and $A$. The results are not very sensitive to the particular value of the vector coupling constant, $G_v$. We also discuss and compare with the corresponding calculations in lattice QCD in the quenched approximation and the random instanton liquid model.

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Hadronic point-to-point correlators in Euclidean coordinate space appear to be an appropriate tool to describe quantitatively the QCD dynamics at all scales \cite{11}. At very short distances, their behaviour is characterized by asymptotic freedom, and deviations from it can be systematically implemented by means of the operator product expansion parameterized in terms of gluon and quark condensates. At very long distances, the spontaneous breaking of chiral symmetry seems to dominate, in accordance with the intuitive picture of constituent dynamical quarks. Their behaviour at intermediate scales is difficult to describe theoretically and, although they can be parameterized in terms of resonances, the correlation functions at this scale have so far eluded systematic theoretical study. Recently, there has been progress in providing a numerical understanding of the hadronic correlators at all scales, both within quenched lattice QCD \cite{21}, and in a random instanton liquid model (RIM) \cite{3}. The agreement between these calculations and phenomenological studies \cite{11} is fairly good, although discrepancies do exist. Integrating out the perturbative gluonic fluctuations by means of cooling techniques \cite{4} has provided a understanding of the agreement between the lattice and RIM results. These calculations are quite time consuming, and the question naturally arises to what extent they can be reproduced within a more modest framework, for example in hadronic models. One should say, however, that the Euclidean correlators are not directly comparable to experiment. One has to proceed through dispersion relations and subsequent Fourier transformation from momentum space, where the experiment is done, to Euclidean coordinate space, where the
theory can be most easily handled. From a practical and numerical point of view, it is not clear what degree of uncertainty is introduced by such a scheme.

In the present letter we address the calculation of point-to-point mesonic correlation functions in Euclidean coordinate space within an effective chiral quark model of the Nambu and Jona-Lasinio (NJL) type [5-7], and compare with the quenched lattice QCD results [2] and the RIM results [3]. Although a direct Fourier transformation of the correlation functions from momentum space to coordinate space is possible, we adopt the scheme of proceeding through the dispersion relations. This has the advantages of greatly simplifying the numerical work and of allowing a more direct comparison with the parameterizations of the correlation functions used in Refs. [2,3]. As the NJL model with vector mesons has been claimed to be a low-energy approximation to QCD [8-10], we want to answer the question in what channels and at what energies or distances can QCD be modelled by an effective four-quark point interaction. A useful feature of our approach is that it allows one to go from coordinate to momentum space in a straightforward manner, and hence the results can be more easily interpreted. However, the model is not renormalizable and requires using a finite cut-off regularization, and it does not incorporate confinement.

The NJL correlation functions have already been considered in the literature [11-14], but, with the exception of finite temperature results [7], have not been directly confronted with the quenched lattice QCD results or the RIM results. In this letter, we undertake this comparison by Fourier transforming the known NJL correlation functions to Euclidean coordinate space. For this purpose, and to keep the interpretation as simple as possible, it is convenient to use a regularization method which fulfills dispersion relations [7]. In fact, dispersion relations have been used to compare the NJL model with QCD sum rules [15,16], but their validity was not checked. A side result of the present work is to show that the dispersion relations do not automatically hold in all (finite cut-off) regularizations. Indeed, we find that the widely used Proper-Time regularization does not fulfill them, in contrast to the Pauli-Villars method. In addition, we address the question whether the Euclidean correlation functions can be used to fix the value of the vector coupling constant, $G_V$, since it has been argued based on finite temperature results [7], QCD sum rules [15], and the Georgi vector limit [17] that it should vanish.

We restrict our study to the two lightest ($u, d$) flavours, the generalization to SU(3) being straightforward. The NJL Lagrangian in Minkowski space is

$$\mathcal{L}_{NJL} = \bar{q}(i\gamma^\mu - \hat{M}_0)q + \frac{1}{2}G_S\{[(\bar{q}q)^2 + (\bar{q}\gamma_5 q)^2] - \frac{1}{2}G_V[(\bar{q}\gamma_\mu q)^2 + (\bar{q}\gamma_5 \gamma_\mu q)^2]\}, \quad (1)$$

where $q = (u, d)$ represents a quark spinor with $N_c$ colours and two flavours and $\hat{M}_0 = \text{diag}(m_u, m_d)$ stands for the current, or bare, quark mass matrix. The calculation of the momentum-space correlation functions using various methods has appeared many times in the literature [11-14]. We use the bosonization procedure [18] with Pauli-Villars regularization [19], and work in the $N_c \to \infty$ limit, or at the one quark-loop level [20,21]. We consider the following (normal ordered) currents

$$J_\sigma(x) = \bar{q}(x)q(x), \quad J_\mu^a(x) = \bar{q}(x)i\gamma_\mu \gamma^a q(x),$$

$$J_\rho^a(x) = \frac{1}{2} \bar{q}(x)\gamma_\mu \gamma_5 \gamma^a q(x), \quad (J_\rho^a)_\mu(x) = \frac{1}{2} \bar{q}(x)\gamma_\mu \gamma_5 \gamma^a q(x), \quad (2)$$

and the correlation functions in Minkowski space (with $m_u = m_d$ and for the third isospin component, $a = 3$)

$$i\langle 0|T\{J_\sigma(x)J_\sigma(0)\}|0\rangle = \int \frac{d^4q}{(2\pi)^4} \exp(iq \cdot x)\Pi_\sigma(q),$$

$$i\langle 0|T\{J_\mu^a(x)J_\mu^a(0)\}|0\rangle = \int \frac{d^4q}{(2\pi)^4} \exp(iq \cdot x)\Pi_{\mu^a}(q),$$

$$i\langle 0|T\{J_\rho^a(x)J_\rho^a(0)\}|0\rangle = \int \frac{d^4q}{(2\pi)^4} \exp(iq \cdot x)\Pi_{\rho^a}(q),$$
The polarization operators are

\[
K_\pi(q) = 4N_c \left[ q^2 F(q^2) + 2I_2 \right] = K_{\pi}(q) + 16N_c M^2 F(q^2),
\]

\[
K_\mu^\nu(q) = 4N_c q^2 S(q^2) \left( -\eta^\mu{}^\nu + \frac{q^\mu q^\nu}{q^2} \right), \quad K_A^\mu(q) = K_A(q) g^\mu{}^\nu + K_B(q) \left( -\eta^\mu{}^\nu + \frac{q^\mu q^\nu}{q^2} \right),
\]

\[
K_A(q) = 2M K_{\pi A}(q) = 16N_c M^2 F(q^2), \quad K_B(q) = 4N_c q^2 S(q^2),
\]

with the one-loop Pauli-Villars regularized integrals given by

\[
F(q^2) = -i \int \frac{d^4 k}{(2\pi)^4} \int_0^1 \left[ \frac{1}{-k^2 - x(1 - x)q^2 + M^2 + \Lambda_i^2 - i\epsilon} \right],
\]

\[
S(q^2) = -i \int \frac{d^4 k}{(2\pi)^4} \int_0^1 \left[ \frac{(2x - 1)^2 - 1}{-k^2 - x(1 - x)q^2 + M^2 + \Lambda_i^2 - i\epsilon} \right],
\]

\[
I_2 = -i \int \frac{d^4 k}{(2\pi)^4} \sum_i c_i \frac{1}{-k^2 + M^2 + \Lambda_i^2 - i\epsilon}.
\]

The two point functions obtained by solving the Bethe-Salpeter equation [6] are then

\[
\Pi_{\pi}(q) = K_{\pi}(q) \left. \frac{1}{1 - G_S K_{\pi}(q)} \right.,
\]

\[
\Pi_\pi(q) = \frac{K_{\pi}(q) \left[ 1 + G_V K_A(q) - q^2 G_V K^2_{\pi A}(q) \right]}{\left[ 1 + G_V K_A(q) \right] \left[ 1 - G_S K_{\pi}(q) \right] + q^2 G_S G_V K^2_{\pi A}(q)},
\]

\[
\Pi_\mu^\nu(q) = \frac{K_B(q)}{1 - G_V K_B(q)} \left( -\eta^\mu{}^\nu + \frac{q^\mu q^\nu}{q^2} \right), \quad \Pi_A^\mu(q) = -\Pi_A(q) g^\mu{}^\nu + \Pi_B(q) \frac{q^\mu q^\nu}{q^2},
\]

\[
\Pi_A(q) = \frac{K_B(q) - K_A(q)}{1 - G_V [K_B(q) - K_A(q)]},
\]

\[
\Pi_B(q) = \frac{1 - G_S K_{\pi}(q)}{K_{\pi}(q) \left[ 1 + G_V K_A(q) \right] - q^2 G_V K^2_{\pi A}(q)} \left[ 1 + G_V [K_A(q) - K_B(q)] \right] \Pi_{\pi}(q),
\]

which for \( G_S = G_V = 0 \) obviously reduce to the polarization operators. In what follows we will consider the combinations \( \Pi_\rho(q) = (\Pi_\rho)_\mu^\nu \) and similarly for the \( A_1 \) channel.

The correlation function in Euclidean coordinate space is defined as [1]

\[
\Pi(x) = \int \frac{d^4 p}{(2\pi)^4} \exp(ip \cdot x) \Pi(p),
\]

where \( \Pi(p) \) represents the two point function in the Euclidean region. If \( \Pi(p) \), Eq. (7), satisfies a dispersion relation with a finite number subtractions, then one has (up to delta type functions at \( x = 0 \))

\[
\Pi(x) = \text{Res} \left[ \Pi(m_M^2) D(m_M, x) + \frac{1}{\pi} \int_0^\infty \text{Im} \left( \Pi(s) \cos D(\sqrt{s}, x) \right) ds \right],
\]

where

\[
\Pi(m_M^2) = \frac{1}{\pi} \int_0^\infty D(s, x) ds.
\]
where \( D(m, x) \) is the free massive bosonic propagator \([1]\). This is the method used in Refs. \([1-3]\) to relate the experimentally measurable absorptive part to the theoretically computed Euclidean correlator and the same method will be applied in our case. This requires the explicit verification of the dispersion relations for the two point functions, and implies that singularities in the first Riemann sheet can only appear along the real negative \( Q^2 \)-semiaxis.

It is not difficult to check analytically that the functions \( F \) and \( S \) do satisfy an unsubtracted dispersion relation, and hence the polarization operators, \((4)\), satisfy a once subtracted one. We have checked numerically up to a Euclidean momentum of \( Q^2 = 10 \text{ GeV}^2 \) that within the Pauli-Villars regularization scheme all correlation functions under consideration do indeed fulfill, up to at most one subtraction, the dispersion relations. This is a nice feature as it allows for a more transparent connection between momentum and coordinate space, and shows that our finite cut-off regularization preserves causality. However, the Pauli-Villars regulators introduce a non-trivial energy dependence which, for \( s > 4(M^2 + \Lambda^2) \), violate unitarity since the imaginary part becomes negative for these high values of the c.m. energy. In contrast, the conventional Proper-Time regularization applied directly to the Euclidean action seems to preserve unitarity, but causality is violated\(^1\).

The non-vanishing residues in Eq. \((9)\) are (primes denote derivation with respect to squared meson mass, \(m_{M}^2\))

\[
\text{Res } \Pi_\pi(m^2_\pi) = \frac{1}{2NcG\phi} \left( \frac{1 + 16NcM^2G\phi F(m^2_\pi)}{G\phi [m^2_\pi F'(m^2_\pi)]'} - 4mG\phi F'(m^2_\pi) \right),
\]

\[
\text{Res } \Pi_\rho(m^2_\rho) = \frac{3}{4G\phi} \frac{S'Nc}{S(m^2_\rho)m^2_\rho'}, \quad \text{Res } \Pi_A(m^2_\pi) = 4f^2f^2 m^2_\pi,
\]

with the positions of the poles given by

\[
m^2_\pi = \frac{m}{M} \frac{1}{4NcF(m^2_\pi)G\phi} \left[ 1 + 16NcM^2G\phi F(m^2_\pi) \right],
\]

\[
m^2_\rho = \frac{1}{4NcG\phi S(m^2_\rho)},
\]

provided \(m_\rho\) and \(m_\pi\) are smaller than \(2M\).

Finally, we have also considered what one might call “dispersive regularization”. This corresponds to evaluating the polarization functions \(F(q)\) and \(S(q)\) in the Euclidean region by means of once subtracted dispersion relations,

\[
F(Q^2) = F(0) + \frac{Q^2}{\pi} \int_0^\infty \frac{\text{Im } F(s)}{s(s + Q^2)} ds.
\]

thereby introducing unknown constants \(F(0)\) and \(S(0)\), but eliminating an explicit cut-off dependence. This regularization has the advantage of reproducing the free quark result at high Euclidean momenta for the polarization operators of Eq. \((4)\), and hence looks like asymptotic freedom and it also preserves unitarity. It should be noted, however, that the correlation functions, Eq. \((7)\), do not approach the free result at high momenta unless the coupling constants in that channel are zero. The “dispersive regularization” procedure corresponds to renormalizing the Lagrangian by subtracting counterterms not present in the original Lagrangian \([18]\). Unfortunately, it also yields a vacuum instability due to the appearance of a tachyonic pole \([22,23]\) within the loop expansion independently of the value of \(G\phi\). In fact, this was the original motivation to consider regularizations with an explicit cut-off dependence. In any case, it should be noted that the produced quarks are constituent quarks and not bare quarks, and thus what happens at energies as large as \(s = 4(M^2 + \Lambda^2)\)

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\(^1\) After completion of this work we have been informed by W. Broniowski that there even appear branch cuts in the complex \(Q^2\) plane.
Fig. 1. Euclidean point to point correlation functions, $P(x)$, in the scalar-isoscalar ($\sigma$), pseudoscalar-isovector ($\pi$), vector-isovector ($\rho$) and axial-isovector ($A_1$) channels, respectively, as a functions of the distance, $x$, in fm. The NJL results are presented for the values of the constituent quark masses $M = 250$ MeV (dotted curve), $350$ MeV (full curve) and $450$ MeV (dashed curve). Also shown are the RIM results (circles) [3] and the quenched lattice QCD results (crosses) [2].

is beyond the range of applicability of the model. Moreover, for these energies perturbative QCD is already applicable, so that the complete absorptive part might be modelled as [7]

$$\text{Im} \Pi(s) = \text{Im} \Pi_{\text{NJL}}(s) \Theta \left[4(A^2 + M^2) - s\right] + \text{Im} \Pi_{\text{QCD}}(s) \Theta \left[s - 4(A^2 + M^2)\right].$$

where $\text{Im} \Pi_{\text{QCD}}$ is proportional to the c.m. energy $s$ [1].

To analyze the correlation functions, it is necessary to fix the parameters of the model, namely $G_S$, $G_V$, $\Lambda$ and $M_0$. These can be mapped into $M$, $m_\pi$, $f_\pi$ and $m_\rho$ (provided the $\rho$ meson is truly bound). If one fixes $m_\pi = 140$ MeV, $f_\pi = 93$ MeV and $m_\rho = 770$ MeV, then the only free parameter is the constituent quark mass $M$. If the $\rho$ is not bound, several methods have been proposed to compute its mass [12–14,24]. In this case, we choose to treat $G_V$ as an additional parameter. The resulting Euclidean correlation functions are normalized to the ones corresponding to free massless quarks (note that our currents have a different normalization than those in Ref. [3]),

$$2\Pi_\sigma^0(x) = 2\Pi_\pi^0(x) = -\Pi_\rho^0(x) = -\Pi_{A_1}^0(x) = \frac{4N_c}{\pi^4x^6},$$

and are shown in Fig. 1 for the values of the constituent quark mass of $M = 250$ MeV (10.7), $350$ MeV (10.3) and $450$ MeV (16.0), and in the scalar ($\sigma$), pseudoscalar ($\pi$), vector ($\rho$) and axial-vector ($A_1$) channels.
respectively. The corresponding values of $G_V$, in units of GeV$^{-2}$, are shown in parentheses. The RIM results and quenched lattice QCD results are also shown. It should be noted that in the $\pi$ and $\rho$ channels above 0.5 fm there appear systematic discrepancies between the lattice and RIM results. However, in Ref. [3] these discrepancies are not considered to be a serious problem since the correlation functions are normalized to the free massless ones which fall by several orders of magnitude in the interval 0.5 fm < x < 1.5 fm. Our results tend to be closer to the RIM results than to the lattice results, which is not unreasonable since neither the RIM nor the NJL model incorporate confinement. In the pion channel the agreement is quite good for $M = 350$ MeV above 0.3 fm, while in the vector channel there is qualitative agreement with the RIM and lattice results only above 0.8–1.0 fm. The qualitative agreement at intermediate and large distances is an indication that the model provides a reasonable description of the coupling of the resonance to the corresponding bilinear quark operator. In the $\pi$ channel, we may directly compare our coupling to those found in Refs. [1–3] using $\lambda^2_\pi = \text{Res} \ \Pi_\pi/2$. We obtain $\sqrt{\lambda_\pi} = 897, 599$ and 761 MeV for $M = 250, 350$, and 450 MeV, respectively, compared to the “experimental” value of $\approx 480 \pm 110$ MeV [1]. As the $\sigma$ and $A_1$ are unbound and the $\rho$ is unbound for too small constituent quark mass, a direct comparison of couplings in these channels is difficult. We note, however, that based solely on the correlation function in the interval considered here and in Refs. [2,3], it is difficult to determine if the $\rho$ is truly bound. If the $\rho$ is unbound, the large distance asymptotic behaviour of the ratio $\Pi_\rho(x)/\Pi_\rho^0(x)$ changes from $x^{3/2}\exp(-m_\rho x)$ to $x^3\exp(-2Mx)$. Thus, the difference is a power $3/2$ in the Euclidean distance $x$, and in the range of $x$ values considered here, such an effect can hardly be seen. A systematic trend observed in all our curves is the suppression at small Euclidean distance $x$. This is a rather general feature present in any finite cut-off regularization and indicates that the important property of asymptotic freedom is lacking. As previously argued, this is consistent with the fact that the asymptotic states, corresponding to constituent quarks and not to bare quarks, should be suppressed. Actually, it is amusing that direct use of Eq. (13) provides a remarkable description of these correlation functions over the entire $x$ range considered here.

Finally, we present in Fig. 2 our results for $G_V = 0$ and for the quark masses considered above. The values of $\sqrt{\lambda_\pi}$ are $4/0, 392, $ and $378$ MeV for $M = 250, 350$, and 450 MeV, respectively. Apparently, it is hard to decide whether a vanishing vector coupling constant yields a better overall description of the Euclidean correlation functions than a non-vanishing one. Therefore, we conclude that by looking at the Euclidean correlation functions, even for $x$ greater than 1 fm, no definite statement about the value of $G_V$ can be made. This result is also in accordance with the asymptotic behaviour for large Euclidean momenta. In our regularization we find that the vector polarization operator behaves as

$$\Pi_\rho(q^2 = -Q^2) = \frac{N_c}{6\pi^2} \frac{1}{Q^4} \left( \sum_i c_i \Lambda_i^4 + 6 \sum_i c_i \Lambda_i^4 \log(\Lambda_i^2/Q^2) \right) + \ldots,$$

which, up to a slowly varying logarithm, represents a power correction. Recall that in QCD sum rules, power corrections are the high energy trace of non-perturbative effects. This is no exception, as we claim that the NJL model should aim to describe non-perturbative effects only (which is the spirit of Eq. (13)). Let us also mention that the low energy properties of the pion are not very sensitive to the value of $G_V$ [20,21], in qualitative agreement with our findings.

Our conclusion disagrees with the ones found in Refs. [15] and [16] where QCD sum rules were used to elucidate a realistic value for $G_V$. Whereas in Ref. [15] it is found that $G_V \ll G_S$, in Ref. [16] a finite value is obtained, $G_V \sim G_S$. These statements depend strongly on the regularization and also on the region of momenta under study. In Ref. [15] a dispersive regularization has been implicitly used and comparison has been made at very high momenta. On the contrary, in Ref. [16] only a region of small momenta was considered using both Pauli-Villars and Proper-Time regularizations. In our case, we see that at high Euclidean momenta $\Pi_\rho$ is insensitive to the value of $G_V$, and thus supports the view of Ref. [16] that the low energy region should be used to determine $G_V$. Nevertheless, as can be seen from the figures, the $\pi$ channel seems to prefer a small
value of $G_V$, whereas the $\rho$ and $A_1$ channels prefer a larger $G_V$ value.

Summarizing, in the present work we have computed the mesonic correlation functions in Euclidean coordinate space within the NJL model with vector mesons. It is clear that by doing so we can only learn about the simplifications and limitations of this model as compared to lattice QCD and the random instanton model. It is nevertheless a success of the NJL model that this study can be carried out, in contrast to most other effective hadronic models. In view of the discrepancies between the RIM and lattice results, the description of the correlation functions at intermediate and large distances should be considered satisfactory for the channels considered. The presence of a finite cut-off, however, contradicts the observed behaviour both in lattice and instanton calculations for $x < 0.5$ fm. We have also found no compelling reason to set the vector coupling constant equal to or different from zero. This is confirmed by looking at the vector correlation function at large Euclidean momenta, and is consistent with previous findings based on the study of low energy pion properties.

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