UvA-DARE (Digital Academic Repository)

Extraction of the gluon density of the proton at small x

M. Derrick et al., M.; Botje, M.A.J.; Chlebana, F.S.; Engelen, J.J.; de Kamps, M.; Kooijman, P.M.; Kruse, A.; Tiecke, H.G.J.M.; Verkerke, W.; Vreeswijk, M.; Wiggers, L.W.; de Wolf, E.; van Woudenberg, R.; Dake, A.P.; Bentvelsen, S.C.M.; de Jong, P.J.; O'Dell, V.; Tenner, A.G.

DOI

[10.1016/0370-2693\(95\)00021-C](https://doi.org/10.1016/0370-2693(95)00021-C)

Publication date

1995

Published in

Physics Letters B

[Link to publication](#)

Citation for published version (APA):

M. Derrick et al., M., Botje, M. A. J., Chlebana, F. S., Engelen, J. J., de Kamps, M., Kooijman, P. M., Kruse, A., Tiecke, H. G. J. M., Verkerke, W., Vreeswijk, M., Wiggers, L. W., de Wolf, E., van Woudenberg, R., Dake, A. P., Bentvelsen, S. C. M., de Jong, P. J., O'Dell, V., & Tenner, A. G. (1995). Extraction of the gluon density of the proton at small x . *Physics Letters B*, 345, 576. [https://doi.org/10.1016/0370-2693\(95\)00021-C](https://doi.org/10.1016/0370-2693(95)00021-C)

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

UvA-DARE is a service provided by the library of the University of Amsterdam (<https://dare.uva.nl>)

Extraction of the gluon density of the proton at x

ZEUS Collaboration

M. Derrick, D. Krakauer, S. Magill, B. Musgrave, J. Repond, J. Schlereth, R. Stanek,
R.L. Talaga, J. Thron

Argonne National Laboratory Argonne, IL, USA⁴¹

F. Arzarello, R. Ayad¹, G. Bari, M. Basile, L. Bellagamba, D. Boscherini, A. Bruni, G. Bruni,
P. Bruni, G. Cara Romeo, G. Castellini², M. Chiarini, L. Cifarelli³, F. Cindolo, F. Ciralli,
A. Contin, S. D'Auria, F. Frasconi, I. Gialas, P. Giusti, G. Iacobucci, G. Laurenti, G. Levi,
A. Margotti, T. Massam, R. Nania, C. Nemoz, F. Palmonari, A. Polini, G. Sartorelli,
R. Timellini, Y. Zamora Garcia¹, A. Zichichi

University and INFN Bologna, Bologna, Italy³¹

A. Bargende, J. Crittenden, K. Desch, B. Diekmann, T. Doeker, M. Eckert, L. Feld, A. Frey,
M. Geerts, G. Geitz⁴, M. Grothe, H. Hartmann, D. Haun⁵, K. Heinloth, E. Hilger, H.-P. Jakob,
U.F. Katz, S.M. Mari, A. Mass, S. Mengel, J. Mollen, E. Paul, Ch. Rembser, R. Schattevoy⁵,
D. Schramm, J. Stamm, R. Wedemeyer

Physikalisches Institut der Universität Bonn, Bonn, Germany²⁸

S. Campbell-Robsor, A. Cassidy, N. Dyce, B. Foster, S. George, R. Gilmore, G.P. Heath,
H.F. Heath, T.J. Llewellyn, C.J.S. Morgado, D.J.P. Norman, J.A. O'Mara, R.J. Tapper,
S.S. Wilson, R. Yoshida

H.H. Wills Physics Laboratory University of Bristol, Bristol, UK⁴⁰

R.R. Rau

Brookhaven National Laboratory Upton, L.I., USA⁴¹

M. Arneodo, L. Iannotti, M. Schioppa, G. Susinno

Calabria University, Physics Dept. and INFN, Cosenza, Italy³¹

A. Bernstein, A. Caldwell, J.A. Parsons, S. Ritz, F. Sciulli, P.B. Straub, L. Wai, S. Yang

Columbia University, Nevis Labs. Irvington on Hudson, N.Y., USA⁴²

P. Borzemiński, J. Chwastowski, A. Eskreys, K. Piotrkowski, M. Zachara, L. Zawiejski

Inst. of Nuclear Physics, Cracow, Poland³⁵

L. Adamczyk, B. Bednarek, K. Eskreys, K. Jeleń, D. Kisiielewska, T. Kowalski,
E. Rulikowska-Zarębska, L. Suszycki, J. Zając

Faculty of Physics and Nuclear Techniques, Academy of Mining and Metallurgy, Cracow, Poland³⁵

T. Kędzierski, A. Kotański, M. Przybycień

Jagellonian Univ., Dept. of Physics, Cracow, Poland³⁶

L.A.T. Bauerdick, U. Behrens, J.K. Bienlein, S. Böttcher⁶, C. Coldewey, G. Drews,
M. Flasiński⁷, D.J. Wilkinson, P. Göttlicher, B. Gutjahr, T. Haas, W. Hain, D. Hasell,
H. Heßling, H. Hultschig, Y. Iga, P. Joos, M. Kasemann, R. Klanner, W. Koch, L. Köpke⁸,
U. Kötz, H. Kowalski, W. Kröger⁹, J. Krüger⁵, J. Labs, A. Ladage, B. Löhr, M. Löwe,
D. Lüke, O. Mańczak, J.S.T. Ng, S. Nickel, D. Notz, K. Ohrenberg, M. Roco, M. Rohde,
J. Roldán¹⁰, U. Schneekloth, W. Schulz, F. Selenke, E. Stiliaris¹⁰, T. Voß, D. Westphal,
G. Wolf, C. Youngman

Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

H.J. Grabosch, A. Leich, A. Meyer, C. Rethfeldt, S. Schlenstedt

DESY-Zeuthen, Inst. für Hochenergiephysik, Zeuthen, Germany

G. Barbagli, P. Pelfer

University and INFN, Florence, Italy³¹

G. Anzivino, G. Maccarrone, S. De Pasquale, S. Qian, L. Votano

INFN, Laboratori Nazionali di Frascati, Frascati, Italy³¹

A. Bamberger, A. Freidhof, T. Poser¹¹, S. Söldner-Rembold, J. Schroeder, G. Theisen,
T. Trefzger

Fakultät für Physik der Universität Freiburg i.Br., Freiburg i.Br., Germany²⁸

N.H. Brook, P.J. Bussey, A.T. Doyle, I. Fleck, V.A. Jamieson, D.H. Saxon, M.L. Uitley,
A.S. Wilson

Dept. of Physics and Astronomy, University of Glasgow, Glasgow, UK⁴⁰

A. Dannemann, U. Holm, D. Horstmann, H. Kammerlocher¹¹, B. Krebs¹², T. Neumann,
R. Sinkus, K. Wick

Hamburg University, I. Institute of Exp. Physics, Hamburg, Germany²⁸

E. Badura, B.D. Burow, A. Fürtjes¹³, L. Hagge, E. Lohrmann, J. Mainusch, J. Milewski,
M. Nakahata¹⁴, N. Pavel, G. Poelz, W. Schott, J. Terron¹⁰, F. Zetsche

Hamburg University, II. Institute of Exp. Physics, Hamburg, Germany²⁸

T.C. Bacon, R. Beuselinck, I. Butterworth, E. Gallo, V.L. Harris, B.H. Hung, K.R. Long,
D.B. Miller, P.P.O. Morawitz, A. Prinias, J.K. Sedgbeer, A.F. Whitfield

Imperial College London, High Energy Nuclear Physics Group, London, UK⁴⁰

U. Mallik, E. McCliment, M.Z. Wang, S.M. Wang, J.T. Wu, Y. Zhang

University of Iowa Physics and Astronomy Dept., Iowa City, USA⁴¹

P. Cloth, D. Filges

Forschungszentrum Jülich, Institut für Kernphysik, Jülich, Germany

S.H. An, S.M. Hong, S.W. Nam, S.K. Park, M.H. Suh, S.H. Yon

Korea University, Seoul, South Korea³³

R. Imlay, S. Kartik, H.-J. Kim, R.R. McNeil, W. Metcalf, V.K. Nadendla

Louisiana State University, Dept. of Physics and Astronomy Baton Rouge, LA, USA⁴¹

F. Barreiro¹⁵, G. Cases, R. Graciani, J.M. Hernández, L. Hervás¹⁵, L. Labarga¹⁵, J. del Peso,

J. Puga, J.F. de Trocóniz

Univer. Autónoma Madrid, Depto de Física Teórica, Madrid, Spain³⁹

F. Ikraiam, J.K. Mayer¹⁶, G.R. Smith

University of Manitoba, Dept. of Physics, Winnipeg, Manitoba, Canada²⁶

F. Corriveau, D.S. Hanna, J. Hartmann, L.W. Hung, J.N. Lim, C.G. Matthews, P.M. Patel,

L.E. Sinclair, D.G. Stairs, M. St-Laurent, R. Ullmann, G. Zacek

McGill University, Dept. of Physics, Montreal, Quebec, Canada^{26,27}

V. Bashkirov, B.A. Dolgoshein, A. Stifutkin

Moscow Engineering Physics Institute, Moscow, Russia³⁷

G.L. Bashindzhagyan, P.F. Ermolov, L.K. Gladilin, Y.A. Golubkov, V.D. Kobrin,

V.A. Kuzmin, A.S. Proskuryakov, A.A. Savin, L.M. Shcheglova, A.N. Solomin, N.P. Zotov

Moscow State University, Institute of Nuclear Physics, Moscow, Russia³⁸

S. Bentvelsen, M. Botje, F. Chlebana, A. Dake, J. Engelen, P. de Jong¹⁷, M. de Kamps,

P. Kooijman, A. Kruse, V. O'Dell¹⁸, A. Tenner, H. Tiecke, W. Verkerke, M. Vreeswijk,

L. Wiggers, E. de Wolf, R. van Woudenberg

NIKHEF and University of Amsterdam, Netherlands³⁴

D. Acosta, B. Bylsma, L.S. Durkin, K. Honscheid, C. Li, T.Y. Ling, K.W. McLean,

W.N. Murray, I.H. Park, T.A. Romanowski¹⁹, R. Seidlein

Ohio State University, Physics Department Columbus, Ohio, USA⁴¹

D.S. Bailey, G.A. Blair²⁰, A. Byrne, R.J. Cashmore, A.M. Cooper-Sarkar, D. Daniels²¹,

R.C.E. Devenish, N. Harnew, M. Lancaster, P.E. Luffman²², L. Lindemann, J. McFall,

C. Nath, A. Quadt, H. Uijterwaal, R. Walczak, F.F. Wilson, T. Yip

Department of Physics, University of Oxford, Oxford, UK⁴⁰

G. Abbiendi, A. Bertolin, R. Brugnera, R. Carlin, F. Dal Corso, M. De Giorgi, U. Dosselli,

S. Limentani, M. Morandin, M. Posocco, L. Stanco, R. Stroili, C. Voci

Dipartimento di Fisica dell' Università and INFN, Padova, Italy³¹

J. Bulmahn, J.M. Butterworth, R.G. Feild, B.Y. Oh, J.J. Whitmore²³

Pennsylvania State University, Dept. of Physics University Park, PA, USA⁴²

G. D'Agostini, M. Iori, G. Marini, M. Mattioli, A. Nigro, E. Tassi

Dipartimento di Fisica, Univ. 'La Sapienza' and INFN, Rome, Italy³¹

J.C. Hart, N.A. McCubbin, K. Prytz, T.P. Shah, T.L. Short

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, UK⁴⁰

E. Barberis, N. Cartiglia, T. Dubbs, C. Heusch, M. Van Hook, B. Hubbard, W. Lockman,

J.T. Rahn, H.F.-W. Sadrozinski, A. Seiden

University of California Santa Cruz, CA, USA⁴¹

J. Biltzinger, R.J. Seifert, A.H. Walenta, G. Zech

Fachbereich Physik der Universität-Gesamthochschule Siegen, Germany²⁸

H. Abramowicz, G. Briskin, S. Dagan²⁴, A. Levy²⁴

School of Physics, Tel-Aviv University, Tel Aviv, Israel³⁰

T. Hasegawa, M. Hazumi, T. Ishii, M. Kuze, S. Mine, Y. Nagasawa, T. Nagira, M. Nakao,

I. Suzuki, K. Tokushuku, S. Yamada, Y. Yamazaki

Institute for Nuclear Study, University of Tokyo, Tokyo, Japan³²

M. Chiba, R. Hamatsu, T. Hirose, K. Homma, S. Kitamura, S. Nagayama, Y. Nakamitsu

Tokyo Metropolitan University, Dept. of Physics, Tokyo, Japan³²

R. Cirio, M. Costa, M.I. Ferrero, L. Lamberti, S. Maselli, C. Peroni, R. Sacchi, A. Solano,

A. Staiano

Università di Torino, Dipartimento di Fisica Sperimentale and INFN, Torino, Italy³¹

M. Dardo

II Faculty of Sciences, Torino University and INFN - Alessandria, Italy³¹

D.C. Bailey, D. Bandyopadhyay, F. Benard, M. Brkic, M.B. Crombie, D.M. Gingrich²⁵,

G.F. Hartner, K.K. Joo, G.M. Levman, J.F. Martin, R.S. Orr, C.R. Sampson, R.J. Teuscher

University of Toronto, Dept. of Physics, Toronto, Ont., Canada²⁶

C.D. Catterall, T.W. Jones, P.B. Kaziewicz, J.B. Lane, R.L. Saunders, J. Shulman

University College London, Physics and Astronomy Dept., London, UK⁴⁰

K. Blankenship, J. Kochocki, B. Lu, L.W. Mo

Virginia Polytechnic Inst. and State University, Physics Dept. Blacksburg, VA, USA⁴²

W. Bogusz, K. Charchuła, J. Ciborowski, J. Gajewski, G. Grzelak, M. Kasprzak,

M. Krzyżanowski, K. Muchorowski, R.J. Nowak, J.M. Pawlak, T. Tymieniecka,

A.K. Wróblewski, J.A. Zakrzewski, A.F. Żarnecki

Warsaw University, Institute of Experimental Physics, Warsaw, Poland³⁵

M. Adamus

Institute for Nuclear Studies, Warsaw, Poland³⁵

Y. Eisenberg²⁴, C. Glasman, U. Karshon²⁴, D. Revel²⁴, A. Shapira

Weizmann Institute, Nuclear Physics Dept., Rehovot, Israel²⁹

I. Ali, B. Behrens, S. Dasu, C. Fordham, C. Foudas, A. Goussiou, R.J. Loveless, D.D. Reeder,
S. Silverstein, W.H. Smith

University of Wisconsin, Dept. of Physics Madison, WI, USA⁴¹

T. Tsurugai

Meiji Gakuin University, Faculty of General Education, Yokohama, Japan

S. Bhadra¹¹, W.R. Frisken, K.M. Furutani

York University, Dept. of Physics, North York, Ont., Canada²⁶

Received 1 November 1994

Editor: K. Winter

Abstract

The gluon momentum density $xg(x, Q^2)$ of the proton was extracted at $Q^2 = 20 \text{ GeV}^2$ for small values of x between 4×10^{-4} and 10^{-2} from the scaling violations of the proton structure function F_2 measured recently by ZEUS in deep inelastic neutral current ep scattering at HERA. The extraction was performed in two ways. Firstly, using a global NLO fit to the ZEUS data on F_2 at low x constrained by measurements from NMC at larger x ; and secondly using published approximate methods for the solution of the GLAP QCD evolution equations. Consistent results are obtained. A substantial increase of the gluon density is found at small x in comparison with the NMC result obtained at larger values of x .

¹ Supported by Worldlab, Lausanne, Switzerland.

² Also at IROE Florence, Italy.

³ Now at Univ. of Pisa, Italy.

⁴ On leave of absence.

⁵ Now a self-employed consultant.

⁶ Now at Tel Aviv Univ., Faculty of Engineering.

⁷ Now at Inst. of Computer Science, Jagellonian Univ., Cracow.

⁸ Now at Univ. of Mainz.

⁹ Now at Univ. of California, Santa Cruz.

¹⁰ Supported by the European Community.

¹¹ Now at DESY.

¹² Now with Herfurth GmbH, Hamburg.

¹³ Now at CERN.

¹⁴ Now at Institute for Cosmic Ray Research, University of Tokyo.

¹⁵ On leave of absence at DESY, supported by DGICYT.

¹⁶ Now at Univ. of Toronto.

¹⁷ Now at MIT, Cambridge, MA.

¹⁸ Now at Fermilab, Batavia, IL.

¹⁹ Now at Department of Energy, Washington.

²⁰ Now at RHBNC, Univ. of London, England.

²¹ Fulbright Scholar 1993–1994.

²² Now at Cambridge Consultants, Cambridge, UK.

²³ On leave and partially supported by DESY 1993–95.

²⁴ Supported by a MINERVA Fellowship.

²⁵ Now at Centre for Subatomic Research, Univ. of Alberta,

Canada and TRIUMF, Vancouver, Canada.

²⁶ Supported by the Natural Sciences and Engineering Research Council of Canada.

²⁷ Supported by the FCAR of Quebec, Canada.

²⁸ Supported by the German Federal Ministry for Research and Technology (BMFT).

²⁹ Supported by the MINERVA Gesellschaft für Forschung GmbH, and by the Israel Academy of Science.

³⁰ Supported by the German Israeli Foundation, and by the Israel Academy of Science.

³¹ Supported by the Italian National Institute for Nuclear Physics (INFN).

³² Supported by the Japanese Ministry of Education, Science and Culture (the Monbusho) and its grants for Scientific Research.

³³ Supported by the Korean Ministry of Education and Korea Science and Engineering Foundation.

³⁴ Supported by the Netherlands Foundation for Research on Matter (FOM).

³⁵ Supported by the Polish State Committee for Scientific Research (grant No. 204209101).

³⁶ Supported by the Polish State Committee for Scientific Research (grant No. PB 861/2/91 and No. 2 2372 9102, grant No. PB 2 2376 9102 and No. PB 2 0092 9101).

³⁷ Partially supported by the German Federal Ministry for Research and Technology (BMFT).

1. Introduction

About half of the proton's momentum is carried by gluons. Despite this, the determination of the density $g(x, Q^2)$ of gluons in the proton as a function of Bjorken- x and the momentum transfer squared, $-Q^2$, has turned out to be a difficult task. Some of the more recent determinations of $g(x, Q^2)$ from a variety of processes can be found in [1]–[9]. The principal difficulty in measuring the gluon density in deep inelastic scattering is that the gluons do not contribute to the cross section in zeroth order QCD (Quark-Parton Model) where the structure functions scale, i.e. do not depend on Q^2 . In the next order (leading order : LO) scaling violations occur through gluon bremsstrahlung from quarks and quark pair creation from gluons. At small x , $x < 10^{-2}$, the latter process dominates the scaling violations [10]. This property can be exploited to extract the gluon density directly from the slope $dF_2/d\ln Q^2$ of the proton structure function F_2 using the approximations of Prytz in LO [11] and in next-to-leading order (NLO) [12]. A different method for the determination of $g(x, Q^2)$ at small values of x has recently been proposed by Ellis, Kunszt and Levin (EKL) [13] based on the solution of the QCD evolution (GLAP) equations [14] in moment space up to next-to-next-to-leading order (NNL). The first applications of the Prytz method in LO were performed by ZEUS [15] and H1 [16] using F_2 data from the 1992 running at HERA.

The new measurements of F_2 obtained by the ZEUS experiment in deep inelastic neutral current ep scattering from the 1993 data [17] provide much improved statistical precision and reduced systematic uncertainties with respect to the 1992 data. The data were taken with the ZEUS detector at HERA with 26.7 GeV electrons colliding on 820 GeV protons. The structure function F_2 was determined for Q^2 values between 7 and 10^4 GeV² and x values between 3×10^{-4} and

1.6×10^{-1} . The F_2 values given in [17] and used here were corrected for the small contribution from Z^0 exchange, and thus are values for photon exchange only. The F_2 measurements are in accord with the scaling violations predicted by QCD.

In the present paper these F_2 data have been used to extract the gluon density at $Q^2 = 20$ GeV² using the Prytz and EKL methods. A global QCD fit to F_2 to determine the gluon density has also been performed. The fit made use of the full NLO GLAP evolution equations in the $\overline{\text{MS}}$ renormalisation and factorisation scheme [18]. To constrain the fit at large x , data from NMC [19] were included.

The gluon momentum density is extracted in a region of x not previously covered. The universality of the gluon density extracted here from the scaling violations of F_2 and subsequently from other processes will provide an important test of perturbative QCD GLAP evolution.

2. Scaling violations of F_2 and the gluon density

The gluon density $g(x, Q^2)$ is defined such that $g(x, Q^2)dx$ gives the number of gluons in the proton with a momentum fraction between x and $x+dx$. The gluon momentum density is then given by $xg(x, Q^2)$. The GLAP evolution equation predicts, in leading order, for the scaling violation of F_2 , in the case of photon exchange

$$\begin{aligned} \frac{dF_2(x, Q^2)}{d\ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \left[\int_x^1 \frac{dz}{z} \left(\frac{x}{z}\right) P_{qq}\left(\frac{x}{z}\right) F_2(z, Q^2) \right. \\ &\quad \left. + 2 \sum_q e_q^2 \int_x^1 \frac{dz}{z} \left(\frac{x}{z}\right) P_{qg}\left(\frac{x}{z}\right) zg(z, Q^2) \right], \quad (1) \end{aligned}$$

where the sum runs over all active quark flavours and e_q is the charge of quark q , $\alpha_s(Q^2)$ is the strong coupling constant and P_{qq} , P_{qg} are the quark and gluon splitting functions. A similar evolution equation applies for the gluon density.

The Q^2 dependence of the parton densities can be calculated from the GLAP QCD evolution equations provided their x dependences are known at a fixed

³⁸ Supported by the German Federal Ministry for Research and Technology (BMBF), the Volkswagen Foundation, and the Deutsche Forschungsgemeinschaft.

³⁹ Supported by the Spanish Ministry of Education and Science through funds provided by CICYT.

⁴⁰ Supported by the Particle Physics and Astronomy Research Council.

⁴¹ Supported by the US Department of Energy.

⁴² Supported by the US National Science Foundation.

value of Q^2 . The x dependences are not predicted by these equations and have to be extracted from data. At low x the scaling violations are dominated by the contribution from the gluon density. This fact can be exploited to determine the gluon density directly from the measured Q^2 dependence of F_2 , using the approximate methods described below.

In the Prytz LO method the quark contribution, contained in the F_2 term in Eq. (1), are neglected and the LO result for P_{qg} is used. The gluon momentum density $zg(z, Q^2)$ is expanded as a Taylor series around $z = x/2$ leading to the result for four flavours⁴³:

$$xg(x, Q^2) \approx \frac{dF_2(x/2, Q^2)/d \ln Q^2}{(40/27)\alpha_s/4\pi}. \quad (2)$$

Note that the gluon density at x is calculated using the slope of F_2 at $x/2$. This method has been extended by Prytz to include the NLO corrections. The contribution from quarks was again neglected. The result for four flavours in the $\overline{\text{MS}}$ scheme is:

$$xg(x, Q^2) \approx \frac{dF_2(x/2, Q^2)/d \ln Q^2}{(40/27 + 7.96\alpha_s/4\pi)(\alpha_s/4\pi)} - \frac{(20/9)(\alpha_s/4\pi)N(x/2, Q^2)}{40/27 + 7.96\alpha_s/4\pi}. \quad (3)$$

The correction function $N(x/2, Q^2)$ depends on the gluon density at large x , $x \gtrsim 10^{-2}$, which is constrained by existing data. It has been estimated [12] using the MRSD₀ ($xg(x, Q_0^2) \sim x^0$ as $x \rightarrow 0$, for $Q_0^2 = 4 \text{ GeV}^2$) and MRSD₁ ($xg(x, Q_0^2) \sim x^{-1/2}$ as $x \rightarrow 0$) parameterisations [20] which span the ZEUS small x data on F_2 [17]. For $Q^2 = 20 \text{ GeV}^2$ and in the x range relevant to this analysis the two parameterisations give almost the same results for N [12].

In the EKL method the gluon momentum density and F_2 are assumed to behave as $x^{-\omega_0}$, which leads to the following form for the scaling violation of F_2 :

$$d\Sigma/d \ln Q^2 = P^{FF}(\omega_0)\Sigma(x, Q^2) + P^{FG}(\omega_0)g(x, Q^2) \quad (4)$$

with

$$\Sigma(x, Q^2) = \frac{1}{\langle e_q^2 \rangle} \frac{F_2(x, Q^2)}{x}, \quad (5)$$

where $\langle e_q^2 \rangle$ is the average of the squares of the quark charges (5/18 for four flavours). The non-singlet contributions are neglected. The evolution kernels P^{FF} and P^{FG} , calculated in the $\overline{\text{MS}}$ scheme, are expanded up to third order in α_s (NNL):

$$P^{FF}(\omega_0) \approx \alpha_s p_0^{FF} + \alpha_s^2 p_1^{FF} + \alpha_s^3 p_2^{FF} \quad (6)$$

$$P^{FG}(\omega_0) \approx \alpha_s p_0^{FG} + \alpha_s^2 p_1^{FG} + \alpha_s^3 p_2^{FG}. \quad (7)$$

The NLO and LO results are obtained by keeping in Eqs. (6) and (7) the terms up to $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s)$, respectively. The coefficients p_i^{FF} and p_i^{FG} depend on the parameter ω_0 . They are tabulated in [13] for a range of ω_0 values. The actual value of ω_0 must be extracted from the data. The quark contribution is contained in the first term of Eq. (4) and hence, in contrast to the Prytz method, is included in the EKL method. The expression for the gluon momentum density, for four flavours, is:

$$xg(x, Q^2) = \frac{18/5}{P^{FG}(\omega_0)} \times \left[\frac{dF_2(x, Q^2)}{d \ln Q^2} - P^{FF}(\omega_0)F_2(x, Q^2) \right]. \quad (8)$$

In the EKL method, in contrast to the Prytz method, the gluon density at x is calculated using the structure function F_2 and its logarithmic slope at the same value of x .

3. Extraction of the gluon density

3.1. Prytz and EKL methods

The F_2 values from ZEUS obtained with the double-angle method (DA) were used [17]. Since we chose to extract the gluon density at $Q^2 = 20 \text{ GeV}^2$, we considered only those x bins which have data both below and above $Q^2 = 20 \text{ GeV}^2$ and where the errors allow a significant determination of the slope (see Fig. 1).

The value of the structure function F_2 and its slope $dF_2/d \ln Q^2$ were determined for each x bin using a linear fit of the form:

$$F_2(x, Q^2) = c_1(x) + c_2(x) \ln(Q^2/20 \text{ GeV}^2). \quad (9)$$

⁴³ In this and subsequent formulae, α_s is to be taken as a function of Q^2 .

ZEUS 1993

Fig. 1. The F_2 structure function measurements of ZEUS together with the linear fits using Eq. (9), (solid lines), and the result from the global NLO fit, (dashed-dotted lines). The error bars show the statistical (inner error) and systematic error added together in quadrature. The arrow indicates the Q^2 value at which the slope and F_2 are determined in order to extract the gluon distribution using the approximate methods.

The fit parameters are $c_1 = F_2(x, Q^2 = 20 \text{ GeV}^2)$ and $c_2 = dF_2/d\ln Q^2$. To obtain the central values of c_1 and c_2 and their statistical errors, fits were performed which included the statistical errors of F_2 only. These fits give a good representation of the data as shown by the full lines in Fig. 1. The systematic errors of F_2 were evaluated in [17] by determining the bin-by-bin shifts in F_2 from the central value when changing the selection and analysis procedure (in total 14 changes). The systematic uncertainties of the fit parameters were estimated as follows. The sets of shifted F_2 values with

their statistical error were used to obtain 14 sets of fit parameters c_1, c_2 . The positive (negative) deviations from the central values of c_1 and c_2 were added in quadrature to give the positive (negative) systematic uncertainties of the fit parameters. The values of the fit parameters c_1 and c_2 and their errors are given in Table 1. Note that one of the systematic checks addressed the uncertainty of the unfolding method. Performing the unfolding using MRSD'_0 distributions (flat gluon) instead of MRSD'_- (singular gluon) changed the logarithmic slope, c_2 , by less than 0.03, and was included

Table 1

Values and errors of F_2 and of the slope $dF_2/d \ln Q^2$ at $Q^2 = 20$ GeV^2 obtained from fits to the ZEUS data according to Eq. (9), where the first error is statistical and the second systematic.

x	$F_2(x, Q^2 = 20 \text{ GeV}^2)$	$dF_2(x)/d \ln Q^2$
8.5×10^{-4}	$1.25 \pm 0.02 \pm 0.06$	$0.45 \pm 0.03 \begin{smallmatrix} +0.05 \\ -0.10 \end{smallmatrix}$
1.55×10^{-3}	$0.96 \pm 0.02 \pm 0.04$	$0.30 \pm 0.03 \begin{smallmatrix} +0.09 \\ -0.03 \end{smallmatrix}$
2.68×10^{-3}	$0.83 \pm 0.01 \pm 0.05$	$0.25 \pm 0.02 \begin{smallmatrix} +0.07 \\ -0.09 \end{smallmatrix}$
4.65×10^{-3}	$0.66 \pm 0.02 \pm 0.06$	$0.23 \pm 0.03 \begin{smallmatrix} +0.02 \\ -0.05 \end{smallmatrix}$

in the systematic error. In [17] F_2 was also determined for a limited region of x and Q^2 using electron variables only. The slopes determined from these data are consistent with the values quoted in Table 1.

For the extraction of the gluon density the value of α_s was taken from a NLO fit [21] to F_2 data from SLAC [22] and BCDMS [3] which yields $\alpha_s = 0.180 \pm 0.008$ at $Q^2 = 50 \text{ GeV}^2$, corresponding to $\Lambda_{\overline{\text{MS}}}^{(4)} = 263 \pm 42 \text{ MeV}$ and $\alpha_s(M_Z^2) = 0.113 \pm 0.005$. This value of α_s agrees with the one given by the PDG [23]. Evolution to $Q^2 = 20 \text{ GeV}^2$ yields the value $\alpha_s = 0.203 \pm 0.010$. For EKL, a choice has to be made for the value of ω_0 . A fit using the F_2 form assumed by EKL, i.e. $F_2 = bx^{-\omega_0}$, to the ZEUS data at fixed values of Q^2 between 12 and 35 GeV^2 , gives values for ω_0 between 0.3 and 0.4. The global fit (see below) gives a value of $\omega_0 \sim 0.4$ at $Q^2 = 20 \text{ GeV}^2$. We have used a value of $\omega_0 = 0.4$ in the EKL analysis.

From the fitted parameters and the value of α_s (and of ω_0 for EKL) the gluon density can be evaluated. Uncertainties in the extracted gluon distribution arise not only from experimental sources but also from theoretical ones. In particular the error on the value of α_s results in a variation of $\pm 6\%$ in $xg(x)$ for both methods in LO and NLO. For the EKL method we find that the largest uncertainty in the normalisation of the extracted gluon distribution arises from the choice of ω_0 . Varying ω_0 from 0.5 to 0.3 results in a 40% decrease in $xg(x)$. This sensitivity becomes even stronger in NNLO. Neglecting the quark contribution to $dF_2/d \ln Q^2$ leads to an overestimate of the extracted gluon distribution. It amounts to about 5% for the kinematic region under study for the Prytz NLO method [12]. Further, from model calculations using the parton density functions MRSD₀' and MRSD₁', it was shown that the Prytz NLO method reproduces the

input gluon density to within 10% [12]. Thus, to this level, the method is insensitive to the power behaviour of the gluon density at low x . The results from the two methods are discussed in Section 3.3.

3.2. Global fit

A GLAP QCD fit using all ZEUS 1993 F_2 data was performed [24]. In order to constrain the fit at larger values of x , NMC data on F_2 from proton and deuteron measurements with $Q^2 > 4 \text{ GeV}^2$ were included [19]. The parton momentum distributions were parameterised as functions of x at a fixed value of $Q^2 = Q_0^2$, using the functional forms from [20]:

$$\begin{aligned} xg(x, Q_0^2) &= A_g x^{\delta_g} (1-x)^{\eta_g} \\ xq_{SI}(x, Q_0^2) &= A_{SI} x^{\delta_{SI}} (1-x)^{\eta_{SI}} (1 + \epsilon_{SI} \sqrt{x} + \gamma_{SI} x) \\ xq_{NS}(x, Q_0^2) &= A_{NS} x^{\delta_{NS}} (1-x)^{\eta_{NS}}. \end{aligned} \quad (10)$$

Here xg denotes the gluon momentum density, xq_{SI} the singlet quark distribution (the sum of quarks and anti-quarks, i.e. the sum of the valence and sea quark distributions) and xq_{NS} the ($u-d$) non-singlet distribution (the difference of up and down quarks and anti-quarks). The non-singlet distribution is constrained by the difference between proton and deuteron F_2 measurements. In the fit, a value of $Q_0^2 = 7 \text{ GeV}^2$ was chosen; this is the lowest Q^2 covered by the ZEUS F_2 data. The threshold values for charm and bottom quark production were set at $Q_{th,c}^2 = 4 \text{ GeV}^2$ and $Q_{th,b}^2 = 25 \text{ GeV}^2$; above these thresholds the charm and bottom quarks were assumed to contribute fully. At $Q^2 = 4 \text{ GeV}^2$ the strange quark distribution was assumed to be given by $xq_s = 0.2 xq_{sea}$ [25] and the sea quark distribution was obtained by subtracting the valence distribution from the singlet distribution. Apart from α_s , the only external input to the fit is therefore the valence quark distribution at $Q^2 = 4 \text{ GeV}^2$ which was taken from the parton distribution set MRSD₀' and which has the same functional form as the singlet distribution given above.

The parton densities were evolved in Q^2 using the GLAP equations in LO [14] and NLO [18] in the $\overline{\text{MS}}$ scheme. Values for F_2 were calculated, to the corresponding order [26], from the evolved parton distributions at the x and Q^2 of each data point. Optimal values for the parameters in Eq. (10) were obtained

ZEUS 1993

Fig. 2. A comparison of the ZEUS and NMC F_2 data with the result from the global NLO GLAP fit as a function of x for the Q^2 values (GeV^2) of [17]. The error bars show the statistical (inner error) and systematic error added together in quadrature.

from a χ^2 minimisation procedure. The normalisation A_R of the gluon was constrained by the requirement that the total momentum fraction carried by quarks and gluons added up to unity. There were thus 10 parameters left free in the fits. The value of α_s quoted in the previous section was used in the fits. The Q^2 evolution of α_s in LO and NLO was performed using the renormalisation group equation with the quark mass thresholds treated as given in [27]. In the computation of the χ^2 , only statistical errors were taken into account resulting in a χ^2 of 97 for the 56 ZEUS data points and 283 for the 184 NMC data points. The values of the parameters obtained from the NLO fit are given in Table 2 as the central values. Applying the constraint $\delta_R = \delta_{SI}$ does not change the χ^2 of the fit appreciably. In contrast, imposing the constraint $\delta_R = 0$ (i.e. assuming a constant xg as $x \rightarrow 0$ at $Q_0^2 = 7 \text{ GeV}^2$) increases the χ^2 contribution of the

ZEUS data to 130, whilst the χ^2 contribution of the NMC points increases only to 299.

Fig. 2 shows the F_2 data from ZEUS and NMC as a function of x for the Q^2 values of ZEUS [17] together with the result of the NLO fit. For comparison with the linear fits used in Section 3.1, Fig. 1 shows the result of the global NLO fit (dashed-dotted lines) together with the ZEUS F_2 data for fixed x as a function of Q^2 . The NLO fit provides a good description of the ZEUS and NMC data, including the data from NMC measured at Q^2 values different from the ones shown in Fig. 2.

3.3. Results

In LO the Prytz method offers a simple estimate of the LO gluon momentum density (see Eq. (2)). Fig. 3 compares the result from this method with the

ZEUS 1993

Fig. 3. The gluon momentum density as a function of x at $Q^2 = 20 \text{ GeV}^2$ determined from the ZEUS data using the methods of Prytz and EKL in LO. The error bars show the experimental statistical (inner error) and systematic error added together in quadrature. Also shown, as a solid curve, is the LO GLAP global fit.

one from the LO global GLAP fit. Good agreement between the two methods is observed. Also shown in Fig. 3 are the results obtained from the EKL method in LO, which are consistent with those of the other two methods.

Fig. 4 shows the gluon momentum densities obtained in NLO. Good agreement between the results of the three methods is observed. The shaded band in Fig. 4 indicates the uncertainty of the gluon density from the global GLAP fit as estimated by adding the statistical and systematic errors in quadrature. The systematic errors were estimated as follows. The ZEUS data were offset by each systematic change (see [17]) in turn and the fit re-done. Likewise the NMC data were offset by their systematic errors, properly taking into account the correlations as described in [19]. Normalisation errors of 3.5% on the ZEUS data and of 1.6 (2.6)% on the 90 (280) GeV NMC data sets were included. The resulting deviations above (below) the central value were added in quadrature to yield the upper (lower) systematic error. It is found that the systematic uncertainties in the NMC data give the dominant contribution. In order to indicate the equivalent

ZEUS 1993

Fig. 4. The gluon momentum density as a function of x at $Q^2 = 20 \text{ GeV}^2$ determined from the ZEUS data using the methods of Prytz and EKL in NLO. The error bars show the experimental statistical (inner error) and systematic error added together in quadrature. The solid line shows the result of the NLO GLAP global fit with the associated error indicated by the shaded region. The gluon parameterisations of MRSD' and MRSD_0 are shown as dash-dotted lines. The hatched region shows the gluon distribution determined by the NMC experiment.

uncertainty in the fitted parameters, the parameters describing the upper and lower extremes of the error band shown in Fig. 4 are given in Table 2. Using the ZEUS F_2 data obtained with the electron method gave consistent results. The errors shown in Fig. 4 do not include the uncertainty due to α_s ; using the error quoted previously results in a variation of about 6% in the gluon distribution at $Q^2 = 20 \text{ GeV}^2$ and $x \lesssim 10^{-2}$.

Additional sources of uncertainty in the global fit were investigated. Repeating the fit without imposing the momentum sum rule had a negligible effect on the gluon distribution. The uncertainty due to the assumed strange quark content of the proton, estimated by repeating the analysis with $q_s = 0.1q_{\text{sea}}$ and $q_s = 0.3q_{\text{sea}}$ (instead of $q_s = 0.2q_{\text{sea}}$), was found to be negligible. This also shows that the global fit is insensitive to changes in the input valence quark distribution. Starting the evolution from $Q_0^2 = 4$ and 20 GeV^2 instead of 7 GeV^2 did not significantly change the extracted

Table 2

Parameters obtained at $Q^2 = 7 \text{ GeV}^2$ from the global fits discussed in the text.

Parameter	NLO		
	Lower	Central	Upper
A_{NS}	13.03	5.70	3.39
δ_{NS}	1.74	1.32	1.05
η_{NS}	4.63	3.62	2.95
A_{SI}	0.49	0.47	0.47
δ_{SI}	-0.24	-0.26	-0.28
η_{SI}	3.36	3.29	3.28
ϵ_{SI}	2.55	2.45	2.26
γ_{SI}	5.91	6.38	6.84
A_R	1.11	1.01	1.08
δ_R	-0.25	-0.35	-0.39
η_R	6.44	5.22	4.81

gluon distribution. The influence of higher twist contributions, as given in [8], to the Q^2 evolution of F_2 was found to be negligible. The determination of the gluon density at low Q^2 is known to be sensitive to the exact treatment of the charm threshold. Small changes of the order of 5% were observed at low x when different treatments of the charm threshold behaviour [28] were used. Changing the bottom production threshold to $Q^2 = 100 \text{ GeV}^2$ had a small effect. Possible shadowing corrections to the NMC deuterium data [29] were investigated and found to have no effect on the extracted gluon distribution. Using F_2 values determined with the large rapidity gap events [30,31] excluded from the data sample also had a small effect ($\lesssim 10\%$) on the gluon distribution.

The gluon momentum densities obtained from the approximate methods, shown in Fig. 4, agree well with the result from the global fit. In the case of EKL this is true only if ω_0 has the value of 0.4, as determined from the global fit at $Q^2 = 20 \text{ GeV}^2$. From the global fit the scaling violations in the x region of the ZEUS data are, as expected, dominated by the contribution from the gluon. The agreement between the results from the Prytz method and the global fit justify the approximations made in the Prytz method. Furthermore, it lends support to the simple functional form for xg used in the global fit, since the Prytz method does not rely on a specific functional form for xg .

Also shown in Fig. 4 as a hatched region is the result for xg obtained by NMC for $x \gtrsim 10^{-2}$. A sub-

stantial rise in xg is seen as x decreases from the region of the NMC result to the low x region of the ZEUS data. For completeness we show in Fig. 4 the parameterisations MRSD_0' and MRSD_0'' . The gluon density from MRSD_0'' is consistent with the central value of the global fit whereas MRSD_0' is below the lower limit of the error band. The parameterisations GRV(HO) [32], CTEQ2M [33] and MRSA [34] (not shown) agree closely with the central value of the fit.

4. Conclusion

The gluon momentum density, $xg(x, Q^2)$, was extracted at $Q^2 = 20 \text{ GeV}^2$ down to x values of $\sim 4 \times 10^{-4}$ using a NLO GLAP global fit to the ZEUS F_2 data. The global fit was constrained at larger x using F_2 data from NMC for $Q^2 > 4 \text{ GeV}^2$. A good description of the data was obtained. Two approximate methods were also used to extract the gluon distribution in NLO from the scaling violations in the F_2 data of ZEUS alone. Both methods agree with the global GLAP fit. The scaling violations are dominated by the contribution from the gluon. A substantial increase in the gluon momentum density is observed as x decreases from the region $x \gtrsim 10^{-2}$ studied by the NMC experiment to the x region, $4 \times 10^{-4} < x < 10^{-2}$, covered by this analysis.

Acknowledgements

The experiment was made possible by the inventiveness and the diligent efforts of the HERA machine group who continued to run HERA most efficiently during 1993. The design, construction, and installation of the ZEUS detector has been made possible by the ingenuity and dedicated effort of many people who are not listed as authors. Their contributions are acknowledged with great appreciation. The strong support and encouragement of the DESY Directorate has been invaluable. Discussions with R.K. Ellis, Z. Kunszt and E. Levin on the EKL method are gratefully acknowledged.

References

- [1] CDHS Collab., H. Abramowicz et al., *Z. Phys.* C12 (1982) 289.
- [2] CHARM Collab., F. Bergsma et al., *Phys. Lett.* B123 (1983) 269.
- [3] BCDMS Collab., A.C. Benvenuti et al., *Phys. Lett.* B223 (1989) 485 and 490; *Phys. Lett.* B237 (1990) 592 and 599.
- [4] E. Anassontzis et al., *Sov. J. Nucl. Phys.* 51 (1990) 836.
- [5] NMC Collab., D. Allasia et al., *Phys. Lett.* B258 (1991) 493.
- [6] EMC Collab., J. Ashman et al., *Z. Phys.* C56 (1992) 21.
- [7] UA2 Collab., J. Alitti et al., *Phys. Lett.* B299 (1993) 174.
- [8] NMC Collab., M. Arneodo et al., *Phys. Lett.* B309 (1993) 222.
- [9] UA6 Collab., G. Ballocci et al., *Phys. Lett.* B317 (1993) 250.
- [10] A.M. Cooper-Sarkar et al., *Z. Phys.* C39 (1988) 281.
- [11] K. Prytz, *Phys. Lett.* B311 (1993) 286.
- [12] K. Prytz, *Phys. Lett.* B332 (1994) 393.
- [13] R.K. Ellis, Z. Kunszt and E.M. Levin, *Nucl. Phys.* B420 (1994) 517.
- [14] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* 15 (1972) 438, 675;
L.N. Lipatov, *Sov. J. Nucl. Phys.* 20 (1975) 94;
G. Altarelli and G. Parisi, *Nucl. Phys.* B126 (1977) 298.
- [15] J.F. Martin, *Proceedings of the XVI International Symposium on Lepton and Photon Interactions*, Cornell University, Ithaca NY (1993) 119.
G. Wolf, DESY 94-022 (1994);
M. Röco, PhD Thesis, University of Iowa, DESY Internal Report F35D-94-04, (1994).
- [16] H1 Collab., J. Abt et al., *Phys. Lett.* B321 (1994) 161.
- [17] ZEUS Collab., M. Derrick et al., DESY 94-143 (1994), submitted to *Z. Phys. C*.
- [18] W. Furmanski and R. Petronzio, *Phys. Lett.* B97 (1980) 437;
G. Curci, W. Furmanski and R. Petronzio, *Nucl. Phys.* B175 (1980) 27.
- [19] NMC Collab., P. Amaudruz et al., *Phys. Lett.* B295 (1992) 159 and preprint CERN-PPE/92-124.
- [20] A.D. Martin, W.J. Stirling and R.G. Roberts, *Phys. Lett.* B306 (1993) 145; *Phys. Lett.* B309 (1993) 492 (erratum).
- [21] M. Virchaux and A. Milsztajn, *Phys. Lett.* B274 (1992) 221.
- [22] L.W. Whitlow et al., *Phys. Lett.* B282 (1992) 475.
- [23] Particle Data Group, L. Montanet et al., *Phys. Rev.* D50 (1994) 1173.
- [24] The fit was performed using two independent programs, the first being an upgrade of that used by BCDMS [21] and NMC [8] and the second one newly developed by ZEUS.
- [25] CCFR Collab., C. Foudas et al., *Phys. Rev. Lett.* 64 (1990) 1207;
S.A. Rabinowitz et al., *Phys. Rev. Lett.* 70 (1993) 134.
- [26] W. Furmanski and R. Petronzio, *Z. Phys.* C11 (1982) 293.
- [27] V.J. Maricano, *Nucl. Phys. (Proc. Suppl.)* B11 (1989) 5, Section 2.1.
- [28] M. Glück, E. Hoffmann and E. Reya, *Z. Phys.* C13 (1982) 119;
M. Glück, E. Reya and H. Stratmann, *Nucl. Phys.* B422 (1994) 37.
- [29] V. Barone et al., *Z. Phys.* C58 (1993) 541.
- [30] ZEUS Collab., M. Derrick et al., *Phys. Lett.* B315 (1993) 481.
- [31] ZEUS Collab., M. Derrick et al., *Phys. Lett.* B332 (1994) 228.
- [32] M. Glück, E. Reya and A. Vogt, *Phys. Lett.* B306 (1993) 391.
- [33] CTEQ Collab., J. Botts et al., *Phys. Lett.* B304 (1993) 159.
- [34] A. D. Martin, W. J. Stirling and R. G. Roberts, University of Durham preprint DTP/94/34 (and RAL 94-055) June 1994.