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SUPPLEMENTARY MATERIAL "LOGLINEAR MODEL SELECTION AND HUMAN MOBILITY"

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1. Proof of Theorem 5.1. We take advantage of the theory on general classes of Markov birth-death processes from Preston (1977, Section 7 and 8). This class of Markov jump processes evolve in jumps which occur a finite number of times in any finite time interval. These jumps are of two types: (i) *birth* in which a single point is added, and the process jumps to a state that contains the additional point; and (ii) *death* in which one of the points in the current state is deleted, and the process jumps to a state with one less point. Preston (1977) shows that the process converges to a unique stationary distribution provided that the detailed balance conditions hold.

To define the balance conditions for our process, assume that at a given time, the process is in a graph state $G = (V, E)$ with $\theta_G \in \Theta_G$ as a vector of parameters. The process is characterized by the *birth rates* $B_e(G, \theta_G)$ for each $e \in \overline{E}$, the *death rates* $D_e(G, \theta_G)$ for each $e \in E$, and the birth and death *transition kernels* $K^{(G)}_{B_e}(\theta_G; \cdot)$ and $K^{(G)}_{D_e}(\theta_G; \cdot)$. Birth and death events occur as independent Poisson processes with rates $B_e(G, \theta_G)$ and $D_e(G, \theta_G)$ respectively. Given the birth of $e \in \overline{E}$ occurs, the probability that the following jump leads to a point in $\mathsf{F} \in \Theta_{G^{+e}}$ is

$$K^{(G)}_{B_e}(\theta_G; \mathsf{F}) = \frac{B_e(G, \theta_G)}{B(G, \theta_G)} \int_{\theta_e : \theta_G \cup \theta_e \in \mathsf{F}} b_e(\theta_e; \theta_G) \, \mathrm{d}\theta_e,$$

in which $B(G, \theta_G) = \sum_{e \in \overline{E}} B_e(G, \theta_G)$. Similarly, given the death of $e \in E$ occurs, the probability that the following jump leads to a point in $\mathsf{F} \in \Theta_{G^{-e}}$ is

$$K^{(G)}_{D_e}(\theta_G; \mathsf{F}) = \frac{D_e(G, \theta_G)}{D(G, \theta_G)} \mathsf{I}(\theta_{G^{-e}} \in \mathsf{F}), \tag{1.1}$$

in which $D(G, \theta_G) = \sum_{e \in E} D_e(G, \theta_G)$.

For this birth-death process, $\mathsf{P}(G, \theta_G \mid \mathbf{x})$ satisfies detailed balance con-

ditions if

$$\int_{\mathsf{F}} B(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\,\mathrm{d}\theta_G = \tag{1.2}$$
$$\sum_{e\in\overline{E}}\int_{\Theta_{G^{+e}}} D(G^{+e},\theta_{G^{+e}})K_{D_e}^{(G^{+e})}(\theta_{G^{+e}};\mathsf{F})\mathsf{P}(G^{+e},\theta_{G^{+e}} \mid \mathbf{x})\,\mathrm{d}\theta_{G^{+e}},$$

and

$$\int_{\mathsf{F}} D(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\,\mathrm{d}\theta_G = \tag{1.3}$$
$$\sum_{e\in E}\int_{\Theta_{G^{-e}}} B(G^{-e},\theta_{G^{-e}})K_{B_e}^{(G^{-e})}(\theta_{G^{-e}};\mathsf{F})\mathsf{P}(G^{-e},\theta_{G^{-e}} \mid \mathbf{x})\,\mathrm{d}\theta_{G^{-e}},$$

where $\mathsf{F}\subset\Theta_G$.

We check the first part of the detailed balance conditions (1.2) as follows

$$\begin{aligned}
LHS &= \int_{\mathsf{F}} B(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\,\mathrm{d}\theta_G\\
&= \int_{\Theta_G} \mathsf{I}(\theta_G\in\mathsf{F})B(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\,\mathrm{d}\theta_G\\
&= \int_{\Theta_G} \mathsf{I}(\theta_G\in\mathsf{F})\sum_{e\in\overline{E}} B_e(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\,\mathrm{d}\theta_G\\
&= \sum_{e\in\overline{E}}\int_{\Theta_G} \mathsf{I}(\theta_G\in\mathsf{F})B_e(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\,\mathrm{d}\theta_G\\
&= \sum_{e\in\overline{E}}\int_{\Theta_G} \mathsf{I}(\theta_G\in\mathsf{F})B_e(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})\left[\int_{\Theta_e} b_e(\theta_e;\theta_G)\,\mathrm{d}\theta_e\right]\mathrm{d}\theta_G\\
&\qquad\qquad [b_e \text{ must integrate to } 1]\\
&= \sum_{e\in\overline{E}}\int_{\Theta_G}\int_{\Theta_e} \mathsf{I}(\theta_G\in\mathsf{F})B_e(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})b_e(\theta_e;\theta_G)\,\mathrm{d}\theta_e\,\mathrm{d}\theta_G.
\end{aligned}$$

$$\begin{aligned}
RHS &= \sum_{e\in\overline{E}}\int_{\Theta_{G^{+e}}} D(G^{+e},\theta_{G^{+e}})K_{D_e}^{(G^{+e})}(\theta_{G^{+e}};\mathsf{F})\mathsf{P}(G^{+e},\theta_{G^{+e}} \mid \mathbf{x})\,\mathrm{d}\theta_{G^{+e}}\\
&\qquad\qquad [\text{equation (1.1) }]\\
&= \sum_{e\in\overline{E}}\int_{\Theta_{G^{+e}}} \mathsf{I}(\theta_G\in\mathsf{F})D_e(G^{+e},\theta_{G^{+e}})\mathsf{P}(G^{+e},\theta_{G^{+e}} \mid \mathbf{x})\,\mathrm{d}\theta_{G^{+e}}.
\end{aligned}$$

Therefore we have LHS=RHS provided that

$$B_e(G,\theta_G)\mathsf{P}(G,\theta_G \mid \mathbf{x})b_e(\theta_e;\theta_G) = D_e(G^{+e},\theta_{G^{+e}})\mathsf{P}(G^{+e},\theta_{G^{+e}} \mid \mathbf{x}).$$

Now, by integrating over $\theta_{G^{+e}} = \theta_G \cup \theta_e$ and knowing that the function $b_e(\theta_e;\theta_G)$ must integrate to 1 over Θ_e, we have

$$B_e(G)\mathsf{P}(G \mid \mathbf{x}) = D_e(G^{+e})\mathsf{P}(G^{+e} \mid \mathbf{x}),$$

which is the expression (5.4) in Theorem 5.1. In a similar way, it can be shown that the remaining part of the detailed balance conditions (1.3) also hold.

Table 4: Maximum cliques of the SA Twitter graph. The hub municipalities (JHB, EKU, TSH, ETH and CPT) appear in bold.

Id.	Size	Clique
1	9	**CPT EKU ETH JHB TSH** FS181 FS184 FS201 MAN
2	9	**CPT EKU ETH JHB TSH** FS194 KZN235 KZN237 KZN238
3	9	**CPT EKU ETH JHB TSH** BUF EC121 EC122 EC157
4	9	**CPT EKU ETH JHB TSH** BUF EC124 EC129 EC139
5	9	**CPT EKU ETH JHB TSH** BUF EC104 EC105 EC126
6	9	**CPT EKU ETH JHB TSH** FS161 FS182 MAN NC091
7	9	**CPT EKU ETH JHB TSH** KZN282 KZN284 KZN291 KZN292
8	9	**CPT EKU ETH JHB TSH** LIM331 LIM332 LIM333 LIM354
9	9	**CPT EKU ETH JHB TSH** LIM354 LIM355 LIM473 LIM476
10	9	**CPT EKU ETH JHB TSH** LIM354 LIM366 LIM367 LIM368
11	9	**CPT EKU ETH JHB TSH** NC071 NC072 NC073 WC053
12	9	**CPT EKU ETH JHB TSH** BUF EC104 EC105 NMA
13	9	**CPT EKU ETH JHB TSH** EC104 EC105 EC106 NMA
14	9	**CPT EKU ETH JHB TSH** BUF LIM354 MAN NMA
15	9	**CPT EKU ETH JHB TSH** BUF MAN MP326 NMA
16	9	**CPT EKU ETH JHB TSH** NW373 NW374 NW383 NW385
17	9	**CPT EKU ETH JHB TSH** NW381 NW382 NW383 NW392
18	9	**CPT EKU ETH JHB TSH** WC012 WC013 WC014 WC015
19	9	**CPT EKU ETH JHB TSH** WC014 WC015 WC023 WC024
20	9	**CPT EKU ETH JHB TSH** WC043 WC044 WC045 WC048
21	8	**CPT EKU ETH JHB TSH** KZN225 KZN226 KZN227
22	8	**CPT EKU ETH JHB TSH** NC062 NC064 WC011
23	8	**CPT EKU ETH JHB TSH** NC062 NC065 WC011
24	8	**CPT EKU ETH JHB TSH** WC011 WC012 WC013
25	8	**CPT EKU ETH JHB TSH** BUF EC122 EC123
26	8	**CPT EKU ETH JHB TSH** LIM351 LIM353 LIM354
27	8	**CPT EKU ETH JHB TSH** LIM344 LIM353 LIM354
28	8	**CPT EKU ETH JHB TSH** KZN271 KZN272 KZN276
29	8	**CPT EKU ETH JHB TSH** LIM362 LIM367 LIM368
30	8	**CPT EKU ETH JHB TSH** MP314 MP324 MP326
31	8	**CPT EKU ETH JHB TSH** KZN275 KZN281 KZN282
32	8	**CPT EKU ETH JHB TSH** FS191 FS196 MAN
33	8	**CPT EKU ETH JHB TSH** NC061 NC062 NC067
34	8	**CPT EKU ETH JHB TSH** FS183 FS184 FS185
35	8	**CPT EKU ETH JHB TSH** FS184 MAN NMA
36	8	**CPT EKU ETH JHB TSH** FS191 FS192 FS193
37	8	**CPT EKU ETH JHB TSH** FS191 FS192 MAN
38	8	**CPT EKU ETH JHB TSH** FS192 FS193 FS194
39	8	**CPT EKU ETH JHB TSH** FS194 FS195 FS205
40	8	**CPT EKU ETH JHB TSH** FS194 FS205 MP306
41	8	**CPT EKU ETH JHB TSH** FS194 GT423 MP306
42	8	**CPT EKU ETH JHB TSH** FS194 FS195 KZN238
43	8	**CPT EKU ETH JHB TSH** FS195 KZN238 KZN252
44	8	**CPT EKU ETH JHB TSH** FS201 FS203 MAN
45	8	**CPT EKU ETH JHB TSH** FS203 FS204 GT421
46	8	**CPT EKU ETH JHB TSH** FS203 GT421 MAN
47	8	**CPT EKU ETH JHB TSH** FS204 GT421 GT422
48	8	**CPT EKU ETH JHB TSH** GT421 MAN NMA
49	8	**CPT EKU ETH JHB TSH** GT481 NW372 NW373

Table 4 – continued from previous page

Id.	Size	Clique
50	8	**CPT EKU ETH JHB TSH** GT484 GT485 NW405
51	8	**CPT EKU ETH JHB TSH** GT484 NW403 NW405
52	8	**CPT EKU ETH JHB TSH** EC101 EC102 EC131
53	8	**CPT EKU ETH JHB TSH** EC101 EC131 NC072
54	8	**CPT EKU ETH JHB TSH** EC101 NC072 WC053
55	8	**CPT EKU ETH JHB TSH** EC121 EC137 EC157
56	8	**CPT EKU ETH JHB TSH** BUF EC104 EC129
57	8	**CPT EKU ETH JHB TSH** EC135 EC137 EC139
58	8	**CPT EKU ETH JHB TSH** BUF EC139 EC145
59	8	**CPT EKU ETH JHB TSH** EC142 EC145 FS163
60	8	**CPT EKU ETH JHB TSH** EC153 EC154 EC157
61	8	**CPT EKU ETH JHB TSH** EC153 EC157 KZN433
62	8	**CPT EKU ETH JHB TSH** EC154 EC155 EC157
63	8	**CPT EKU ETH JHB TSH** EC155 EC156 EC157
64	8	**CPT EKU ETH JHB TSH** EC156 EC157 EC442
65	8	**CPT EKU ETH JHB TSH** EC157 EC442 KZN433
66	8	**CPT EKU ETH JHB TSH** EC441 EC442 KZN433
67	8	**CPT EKU ETH JHB TSH** FS162 MAN NC072
68	8	**CPT EKU ETH JHB TSH** FS182 NC091 NW396
69	8	**CPT EKU ETH JHB TSH** KZN212 KZN213 KZN216
70	8	**CPT EKU ETH JHB TSH** KZN212 KZN213 KZN434
71	8	**CPT EKU ETH JHB TSH** KZN212 KZN434 KZN436
72	8	**CPT EKU ETH JHB TSH** KZN214 KZN216 KZN433
73	8	**CPT EKU ETH JHB TSH** KZN214 KZN433 KZN435
74	8	**CPT EKU ETH JHB TSH** KZN221 KZN225 KZN245
75	8	**CPT EKU ETH JHB TSH** KZN222 KZN235 KZN436
76	8	**CPT EKU ETH JHB TSH** KZN238 KZN241 KZN252
77	8	**CPT EKU ETH JHB TSH** KZN241 KZN242 KZN244
78	8	**CPT EKU ETH JHB TSH** KZN241 KZN252 KZN254
79	8	**CPT EKU ETH JHB TSH** KZN245 KZN286 KZN294
80	8	**CPT EKU ETH JHB TSH** KZN252 KZN253 KZN254
81	8	**CPT EKU ETH JHB TSH** KZN252 KZN253 KZN263
82	8	**CPT EKU ETH JHB TSH** KZN252 MP304 MP305
83	8	**CPT EKU ETH JHB TSH** KZN263 KZN265 KZN266
84	8	**CPT EKU ETH JHB TSH** KZN266 KZN282 KZN285
85	8	**CPT EKU ETH JHB TSH** KZN272 KZN275 KZN276
86	8	**CPT EKU ETH JHB TSH** KZN275 KZN276 KZN282
87	8	**CPT EKU ETH JHB TSH** KZN282 KZN284 KZN285
88	8	**CPT EKU ETH JHB TSH** KZN292 KZN293 KZN294
89	8	**CPT EKU ETH JHB TSH** LIM331 LIM333 LIM334
90	8	**CPT EKU ETH JHB TSH** LIM333 LIM334 LIM335
91	8	**CPT EKU ETH JHB TSH** LIM334 LIM335 MP325
92	8	**CPT EKU ETH JHB TSH** LIM335 MP321 MP325
93	8	**CPT EKU ETH JHB TSH** LIM343 LIM344 LIM345
94	8	**CPT EKU ETH JHB TSH** LIM343 LIM344 LIM354
95	8	**CPT EKU ETH JHB TSH** LIM355 LIM471 LIM473
96	8	**CPT EKU ETH JHB TSH** LIM471 LIM472 LIM473
97	8	**CPT EKU ETH JHB TSH** LIM471 LIM472 MP316
98	8	**CPT EKU ETH JHB TSH** LIM472 LIM473 LIM476
99	8	**CPT EKU ETH JHB TSH** LIM472 MP315 MP316
100	8	**CPT EKU ETH JHB TSH** MP301 MP302 MP313
101	8	**CPT EKU ETH JHB TSH** MP301 MP313 MP326
102	8	**CPT EKU ETH JHB TSH** MP312 MP313 MP326
103	8	**CPT EKU ETH JHB TSH** MP313 MP314 MP326
104	8	**CPT EKU ETH JHB TSH** MP314 MP321 MP326
105	8	**CPT EKU ETH JHB TSH** MP321 MP325 MP326
106	8	**CPT EKU ETH JHB TSH** NC073 NC075 NC076
107	8	**CPT EKU ETH JHB TSH** NC073 NC076 NC091
108	8	**CPT EKU ETH JHB TSH** NC085 NC087 NC091
109	8	**CPT EKU ETH JHB TSH** NC085 NC087 NC453
110	8	**CPT EKU ETH JHB TSH** NC085 NC086 NC452
111	8	**CPT EKU ETH JHB TSH** NC085 NC091 NC452
112	8	**CPT EKU ETH JHB TSH** NC085 NC452 NC453
113	8	**CPT EKU ETH JHB TSH** NC091 NC092 NC093
114	8	**CPT EKU ETH JHB TSH** NC091 NC093 NC094
115	8	**CPT EKU ETH JHB TSH** NC091 NC093 NW396
116	8	**CPT EKU ETH JHB TSH** NC091 NW396 NW404
117	8	**CPT EKU ETH JHB TSH** NC451 NC452 NC453
118	8	**CPT EKU ETH JHB TSH** NC451 NC452 NW392
119	8	**CPT EKU ETH JHB TSH** EC102 EC131 NMA
120	8	**CPT EKU ETH JHB TSH** EC106 EC108 NMA
121	8	**CPT EKU ETH JHB TSH** EC108 EC109 NMA
122	8	**CPT EKU ETH JHB TSH** EC109 NMA WC048
123	8	**CPT EKU ETH JHB TSH** KZN225 MAN NMA
124	8	**CPT EKU ETH JHB TSH** MAN NMA NW373
125	8	**CPT EKU ETH JHB TSH** MP312 MP326 NMA

Table 4 – continued from previous page

Id.	Size	Clique
126	8	**CPT EKU ETH JHB TSH** NW372 NW373 NW375
127	8	**CPT EKU ETH JHB TSH** NW382 NW383 NW384
128	8	**CPT EKU ETH JHB TSH** NW382 NW383 NW403
129	8	**CPT EKU ETH JHB TSH** NW383 NW392 NW394
130	8	**CPT EKU ETH JHB TSH** NW383 NW394 NW403
131	8	**CPT EKU ETH JHB TSH** NW383 NW384 NW405
132	8	**CPT EKU ETH JHB TSH** NW383 NW403 NW405
133	8	**CPT EKU ETH JHB TSH** NW396 NW403 NW404
134	8	**CPT EKU ETH JHB TSH** WC022 WC023 WC024
135	8	**CPT EKU ETH JHB TSH** WC022 WC023 WC025
136	8	**CPT EKU ETH JHB TSH** WC023 WC025 WC026
137	8	**CPT EKU ETH JHB TSH** WC023 WC024 WC031
138	8	**CPT EKU ETH JHB TSH** WC023 WC026 WC031
139	8	**CPT EKU ETH JHB TSH** WC024 WC031 WC032
140	8	**CPT EKU ETH JHB TSH** WC024 WC034 WC048
141	8	**CPT EKU ETH JHB TSH** WC031 WC032 WC033
142	8	**CPT EKU ETH JHB TSH** WC031 WC042 WC043
143	8	**CPT EKU ETH JHB TSH** WC041 WC045 WC052
144	8	**CPT EKU ETH JHB TSH** WC042 WC043 WC044
145	8	**CPT EKU ETH JHB TSH** WC044 WC047 WC048
146	8	**CPT EKU ETH JHB TSH** EC109 WC047 WC048
147	7	**CPT EKU ETH JHB TSH** NW392 NW397
148	7	**CPT EKU ETH JHB TSH** NC066 NC074
149	7	**CPT EKU ETH JHB TSH** LIM366 NW371
150	7	**CPT EKU ETH JHB TSH** KZN227 KZN434
151	7	**CPT EKU ETH JHB TSH** KZN261 KZN263
152	7	**CPT EKU ETH JHB TSH** KZN261 MP303
153	7	**CPT EKU ETH JHB TSH** LIM334 LIM341
154	7	**CPT EKU ETH JHB TSH** LIM341 LIM344
155	7	**CPT EKU ETH JHB TSH** EC136 EC138
156	7	**CPT EKU ETH JHB TSH** EC137 EC138
157	7	**CPT EKU ETH JHB TSH** NC082 WC011
158	7	**CPT EKU ETH JHB TSH** NC073 NC077
159	7	**CPT EKU ETH JHB TSH** FS162 NC078
160	7	**CPT EKU ETH JHB TSH** NC078 NC091
161	7	**CPT EKU ETH JHB TSH** NC067 NC082
162	7	**CPT EKU ETH JHB TSH** NC082 NC087
163	7	**CPT ETH JHB TSH** NC082 NC084 NC087
164	7	**CPT EKU ETH JHB TSH** EC136 EC139
165	7	**CPT EKU ETH JHB TSH** EC153 EC443
166	7	**CPT EKU ETH JHB TSH** EC443 EC444
167	7	**CPT EKU ETH JHB TSH** EC443 KZN216
168	7	**CPT EKU ETH JHB TSH** MP311 MP312
169	7	**CPT EKU ETH JHB TSH** MP311 MP314
170	7	**CPT EKU ETH JHB TSH** LIM361 LIM362
171	7	**CPT EKU ETH JHB TSH** LIM361 NW375
172	7	**CPT EKU ETH JHB TSH** EC442 EC444
173	7	**CPT EKU ETH JHB TSH** KZN222 KZN223
174	7	**CPT EKU ETH JHB TSH** KZN223 KZN237
175	7	**CPT EKU ETH JHB TSH** NC062 NC087
176	7	**CPT EKU ETH JHB TSH** NC065 NC066
177	7	**CPT EKU ETH JHB TSH** NW393 NW394
178	7	**CPT EKU ETH JHB TSH** NW393 NW404
179	7	**CPT EKU ETH JHB TSH** FS183 NW396
180	7	**CPT EKU ETH JHB TSH** FS185 NW403
181	7	**CPT EKU ETH JHB TSH** FS201 NW403
182	7	**CPT EKU ETH JHB TSH** FS203 NW405
183	7	**CPT EKU ETH JHB TSH** GT422 GT423
184	7	**CPT EKU ETH JHB TSH** GT423 MP307
185	7	**CPT EKU ETH JHB TSH** GT481 GT485
186	7	**CPT EKU ETH JHB TSH** EC101 WC044
187	7	**CPT EKU ETH JHB TSH** EC122 EC135
188	7	**CPT EKU ETH JHB TSH** EC131 EC139
189	7	**CPT EKU ETH JHB TSH** EC141 EC142
190	7	**CPT EKU ETH JHB TSH** EC141 EC156
191	7	**CPT EKU ETH JHB TSH** EC141 EC441
192	7	**CPT EKU ETH JHB TSH** EC104 EC142
193	7	**CPT EKU ETH JHB TSH** EC145 FS162
194	7	**CPT EKU ETH JHB TSH** FS161 NC075
195	7	**CPT EKU ETH JHB TSH** FS163 MAN
196	7	**CPT EKU ETH JHB TSH** KZN216 KZN225
197	7	**CPT EKU ETH JHB TSH** KZN216 KZN292
198	7	**CPT EKU ETH JHB TSH** KZN221 KZN293
199	7	**CPT EKU ETH JHB TSH** KZN222 KZN225
200	7	**CPT EKU ETH JHB TSH** KZN222 KZN292
201	7	**CPT EKU ETH JHB TSH** KZN225 KZN237

Table 4 – continued from previous page

Id.	Size	Clique
202	7	**CPT EKU ETH JHB TSH** KZN242 KZN263
203	7	**CPT EKU ETH JHB TSH** KZN244 KZN245
204	7	**CPT EKU ETH JHB TSH** KZN252 KZN282
205	7	**CPT EKU ETH JHB TSH** KZN262 KZN265
206	7	**CPT EKU ETH JHB TSH** KZN262 KZN272
207	7	**CPT EKU ETH JHB TSH** KZN262 KZN282
208	7	**CPT EKU ETH JHB TSH** KZN262 MP303
209	7	**CPT EKU ETH JHB TSH** KZN265 KZN276
210	7	**CPT EKU ETH JHB TSH** KZN266 KZN286
211	7	**CPT EKU ETH JHB TSH** KZN284 KZN286
212	7	**CPT EKU ETH JHB TSH** KZN434 KZN435
213	7	**CPT EKU ETH JHB TSH** LIM331 LIM345
214	7	**CPT EKU ETH JHB TSH** LIM366 MP316
215	7	**CPT EKU ETH JHB TSH** LIM472 MP313
216	7	**CPT EKU ETH JHB TSH** LIM476 MP321
217	7	**CPT EKU ETH JHB TSH** MP302 MP303
218	7	**CPT EKU ETH JHB TSH** MP302 MP307
219	7	**CPT EKU ETH JHB TSH** MP303 MP304
220	7	**CPT EKU ETH JHB TSH** MP305 MP306
221	7	**CPT EKU ETH JHB TSH** MP305 MP307
222	7	**CPT EKU ETH JHB TSH** MP307 MP312
223	7	**CPT EKU ETH JHB TSH** MP312 MP315
224	7	**CPT EKU ETH JHB TSH** NC086 NC092
225	7	**CPT EKU ETH JHB TSH** NC094 NW394
226	7	**CPT EKU ETH JHB TSH** KZN282 NMA
227	7	**CPT EKU ETH JHB TSH** WC012 WC022
228	7	**CPT EKU ETH JHB TSH** WC025 WC053
229	7	**CPT EKU ETH JHB TSH** WC026 WC034
230	7	**CPT EKU ETH JHB TSH** WC026 WC045
231	7	**CPT EKU ETH JHB TSH** WC033 WC034
232	7	**CPT EKU ETH JHB TSH** WC034 WC041
233	7	**CPT EKU ETH JHB TSH** WC034 WC042
234	7	**CPT EKU ETH JHB TSH** EC108 WC044
235	7	**CPT EKU ETH JHB TSH** EC106 WC047
236	7	**CPT EKU ETH JHB TSH** WC052 WC053
237	7	**CPT ETH JHB TSH** NC071 WC051 WC053
238	7	**CPT ETH JHB TSH** WC025 WC051 WC053
239	7	**CPT ETH JHB TSH** WC051 WC052 WC053
240	7	**CPT EKU JHB TSH** LIM335 Local MP325
241	6	**CPT ETH JHB TSH** KZN224 KZN225
242	6	**CPT ETH JHB TSH** KZN224 KZN436
243	6	**CPT ETH JHB TSH** NC077 NC084
244	6	**CPT ETH JHB TSH** NC078 NC084
245	6	**CPT ETH JHB TSH** NC066 WC051
246	6	**CPT EKU JHB TSH** GT421 Local
247	6	**CPT EKU JHB TSH** FS162 Local
248	6	**CPT EKU JHB TSH** KZN216 Local
249	6	**CPT EKU JHB TSH** KZN276 Local
250	6	**CPT EKU JHB TSH** Local NW375
251	6	**CPT EKU JHB TSH** Local WC024

Table 5: Summary geographic and demographic information 213 South African municipalities. Population data extracted from the 2016 Community Survey, Statistics South Africa. Retrieved from https://interactive2.statssa.gov.za/webapi.

Id.	Municipality name	Province	Area (km^2)	Population	Density
EC101	Dr Beyers Naude	Eastern Cape	28,653	82,197	2.9
EC104	Makana	Eastern Cape	4,376	82,060	18.8
EC105	Ndlambe	Eastern Cape	1,841	63,180	34.3
EC121	Mbhashe	Eastern Cape	3,303	277,250	84
EC122	Mnquma	Eastern Cape	3,137	246,813	78.7
EC131	Inxuba Yethemba	Eastern Cape	11,663	70,493	6
EC137	Engcobo	Eastern Cape	2,484	162,014	65.2
EC141	Elundini	Eastern Cape	5,019	144,929	28.9
EC142	Senqu	Eastern Cape	7,329	140,720	19.2
EC153	Ngquza Hill	Eastern Cape	2,477	303,379	122.5
EC154	Port St Johns	Eastern Cape	1,291	166,779	129.2
EC155	Nyandeni	Eastern Cape	2,474	309,702	125.2
EC156	Mhlontlo	Eastern Cape	2,880	189,176	65.7
EC157	King Sabata Dalindyebo	Eastern Cape	3,019	488,349	161.8
EC441	Matatiele	Eastern Cape	4,352	219,447	50.4
FS161	Letsemeng	Free State	9,828	40,044	4.1

Table 5 – continued from previous page

Id.	Municipality name	Province	Area (km^2)	Population	Density
FS162	Kopanong	Free State	15,645	49,999	3.2
FS163	Mohokare	Free State	8,776	35,840	4.1
FS181	Masilonyana	Free State	6,618	62,770	9.5
FS182	Tokologo	Free State	9,326	29,149	3.1
FS183	Tswelopele	Free State	6,524	47,373	7.3
FS184	Matjhabeng	Free State	5,690	429,113	75.4
FS185	Nala	Free State	4,129	78,515	19
FS191	Setsoto	Free State	5,431	117,362	21.6
FS192	Dihlabeng	Free State	4,868	140,044	28.8
FS193	Nketoana	Free State	5,611	64,893	11.6
FS194	Maluti a Phofung	Free State	4,338	353,452	81.5
FS195	Phumelela	Free State	8,196	50,054	6.1
FS196	Mantsopa	Free State	4,291	53,525	12.5
FS201	Moqhaka	Free State	7,925	154,732	19.5
FS203	Ngwathe	Free State	7,055	118,907	16.9
FS204	Metsimaholo	Free State	1,717	163,564	95.3
FS205	Mafube	Free State	3,971	57,574	14.5
GT481	Mogale City	Gauteng	1,342	383,864	286
GT485	Rand West City	Gauteng	1,115	265,887	238.5
GT484	Merafong City	Gauteng	1,631	188,843	115.8
TSH	City of Tshwane	Gauteng	6,298	3,275,152	520
KZN212	Umdoni	KwaZulu-Natal	994	144,551	145.5
KZN214	uMuziwabantu	KwaZulu-Natal	1,089	108,576	99.7
KZN216	Ray Nkonyeni	KwaZulu-Natal	1,487	348,533	234.4
KZN238	Alfred Duma	KwaZulu-Natal	3,764	356,274	94.6
KZN237	Inkosi Langalibalele	KwaZulu-Natal	3,399	215,182	63.3
KZN235	Okhahlamba	KwaZulu-Natal	3,971	135,132	34
KZN241	Endumeni	KwaZulu-Natal	1,610	76,639	47.6
KZN242	Nqutu	KwaZulu-Natal	1,962	171,325	87.3
KZN252	Newcastle	KwaZulu-Natal	1,856	389,117	209.7
KZN253	Emadlangeni	KwaZulu-Natal	3,539	36,869	10.4
KZN254	Dannhauser	KwaZulu-Natal	1,707	105,341	61.7
KZN261	eDumbe	KwaZulu-Natal	1,943	89,614	46.1
KZN262	uPhongolo	KwaZulu-Natal	3,110	141,247	45.4
KZN263	Abaqulusi	KwaZulu-Natal	4,314	243,795	56.5
KZN265	Nongoma	KwaZulu-Natal	2,182	211,892	97.1
KZN266	Ulundi	KwaZulu-Natal	3,250	205,762	63.3
KZN271	Umhlabuyalingana	KwaZulu-Natal	4,977	172,077	34.6
KZN272	Jozini	KwaZulu-Natal	3,442	198,215	57.6
KZN275	Mtubatuba	KwaZulu-Natal	1,970	202,176	102.6
KZN284	uMlalazi	KwaZulu-Natal	2,214	223,140	100.8
KZN286	Nkandla	KwaZulu-Natal	1,828	114,284	62.5
KZN291	Mandeni	KwaZulu-Natal	545	147,808	271
KZN292	KwaDukuza	KwaZulu-Natal	735	276,719	376.5
KZN293	Ndwedwe	KwaZulu-Natal	1,093	143,117	131
KZN294	Maphumulo	KwaZulu-Natal	896	89,969	100.4
KZN436	Dr Nkosazana Dlamini Zuma	KwaZulu-Natal	3,602	118,480	32.9
KZN434	Ubuhlebezwe	KwaZulu-Natal	1,669	118,346	70.9
LIM331	Greater Giyani	Limpopo	4,172	256,127	61.4
LIM332	Greater Letaba	Limpopo	1,891	218,030	115.3
LIM333	Greater Tzaneen	Limpopo	2,897	416,146	143.7
LIM334	Ba-Phalaborwa	Limpopo	7,489	168,937	22.6
LIM335	Maruleng	Limpopo	3,563	99,946	28.1
LIM355	Lepele-Nkumpi	Limpopo	3,484	235,380	67.6
LIM361	Thabazimbi	Limpopo	11,190	96,232	8.6
LIM362	Lephalale	Limpopo	13,794	140,240	10.2
LIM366	Bela-Bela	Limpopo	3,406	76,296	22.4
LIM367	Mogalakwena	Limpopo	6,156	325,291	52.8
LIM471	Ephraim Mogale	Limpopo	2,011	127,168	63.2
LIM472	Elias Motsoaledi	Limpopo	3,713	268,256	72.2
LIM473	Makhuduthamaga	Limpopo	2,110	284,435	134.8
MP301	Chief Albert Luthuli	Mpumalanga	5,559	187,629	33.7
MP302	Msukaligwa	Mpumalanga	6,016	164,608	27.4
MP303	Mkhondo	Mpumalanga	4,882	189,036	38.7
MP304	Dr Pixley Ka Isaka Seme	Mpumalanga	5,227	85,395	16.3
MP305	Lekwa	Mpumalanga	4,557	123,419	27.1
MP306	Dipaleseng	Mpumalanga	2,645	45,232	17.1
MP307	Govan Mbeki	Mpumalanga	2,955	340,091	115.1
MP311	Victor Khanye	Mpumalanga	1,568	84,151	53.7
MP312	Emalahleni	Mpumalanga	2,678	455,228	170
MP313	Steve Tshwete	Mpumalanga	3,976	278,749	70.1
MP314	Emakhazeni	Mpumalanga	4,736	48,149	10.2
MP315	Thembisile	Mpumalanga	2,384	333,331	139.8
MP316	Dr JS Moroka	Mpumalanga	1,416	246,016	173.7
MP321	Thaba Chweu	Mpumalanga	5,719	101,895	17.8
MP324	Nkomazi	Mpumalanga	4,787	410,907	85.8

Table 5 – continued from previous page

Id.	Municipality name	Province	Area (km^2)	Population	Density
NW371	Moretele	North West	1,498	191,306	127.7
NW372	Local Municipality of Madibeng	North West	3,720	536,110	144.1
NW373	Rustenburg	North West	3,416	626,522	183.4
NW374	Kgetlengrivier	North West	3,973	59,562	15
NW375	Moses Kotane	North West	5,726	243,648	42.5
NW381	Ratlou	North West	4,884	106,108	21.7
NW382	Tswaing	North West	5,875	129,052	22
NW383	Mafikeng	North West	3,646	314,394	86.2
NW384	Ditsobotla	North West	6,387	181,865	28.5
NW385	Ramotshere Moiloa	North West	7,323	157,690	21.5
NW392	Naledi	North West	7,030	68,803	9.8
NW393	Mamusa	North West	3,614	64,000	17.7
NW394	Greater Taung	North West	5,639	167,827	29.8
NW396	Lekwa-Teemane	North West	3,654	56,025	15.3
NW397	Kagisano/Molopo	North West	23,827	102,703	4.3
NW403	City of Matlosana	North West	3,602	417,282	115.8
NW404	Maquassi Hills	North West	4,671	82,012	17.6
NC061	Richtersveld	Northern Cape	9,608	12,487	1.3
NC062	Nama Khoi	Northern Cape	17,990	46,512	2.6
NC064	Kamiesberg	Northern Cape	14,208	9,605	0.7
NC065	Hantam	Northern Cape	39,085	21,540	0.6
NC066	Karoo Hoogland	Northern Cape	30,230	13,009	0.4
NC067	Khi-Ma	Northern Cape	15,715	12,333	0.8
NC071	Ubuntu	Northern Cape	20,393	19,471	1
NC072	Umsobomvu	Northern Cape	6,813	30,883	4.5
NC073	Emthanjeni	Northern Cape	13,472	45,404	3.4
NC074	Kareeberg	Northern Cape	17,701	12,772	0.7
NC075	Renosterberg	Northern Cape	5,529	11,818	2.1
NC076	Thembelihle	Northern Cape	8,023	16,230	2
NC077	Siyathemba	Northern Cape	14,727	23,075	1.6
NC078	Siyancuma	Northern Cape	16,753	35,941	2.1
NC082	Kai !Garib	Northern Cape	26,377	68,929	2.6
NC084	!Kheis	Northern Cape	11,107	16,566	1.5
NC085	Tsantsabane	Northern Cape	18,290	39,345	2.2
NC086	Kgatelopele	Northern Cape	2,478	20,691	8.3
NC091	Sol Plaatjie	Northern Cape	3,145	255,041	81.1
NC092	Dikgatlong	Northern Cape	7,316	48,473	6.6
NC453	Gamagara	Northern Cape	2,648	53,656	20.3
WC011	Matzikama	Western Cape	12,981	71,045	5.5
WC012	Cederberg	Western Cape	8,007	52,949	6.6
WC013	Bergrivier	Western Cape	4,407	67,474	15.3
WC022	Witzenberg	Western Cape	10,753	130,548	12.1
WC023	Drakenstein	Western Cape	1,538	280,195	182.2
WC025	Breede Valley	Western Cape	3,834	176,578	46.1
WC026	Langeberg	Western Cape	4,518	105,483	23.3
WC033	Cape Agulhas	Western Cape	3,471	36,000	10.4
WC034	Swellendam	Western Cape	3,835	40,211	10.5
WC041	Kannaland	Western Cape	4,765	24,168	5.1
WC042	Hessequa	Western Cape	5,733	54,237	9.5
WC043	Mossel Bay	Western Cape	2,001	94,135	47
WC044	George	Western Cape	5,191	208,237	40.1
WC045	Oudtshoorn	Western Cape	3,540	97,509	27.5
WC047	Bitou	Western Cape	992	59,157	59.6
WC048	Knysna	Western Cape	1,109	73,835	66.6
WC051	Laingsburg	Western Cape	8,784	8,895	1
WC052	Prince Albert	Western Cape	8,153	14,272	1.8
WC053	Beaufort West	Western Cape	21,917	51,080	2.3
NC451	Joe Morolong	Northern Cape	20,180	84,201	4.2
NC452	Ga-Segonyana	Northern Cape	4,495	104,408	23.2
KZN213	Umzumbe	KwaZulu-Natal	1,221	151,676	124.2
KZN276	Big Five Hlabisa	KwaZulu-Natal	3,466	116,622	33.6
KZN227	Richmond	KwaZulu-Natal	1,231	71,322	57.9
KZN433	Greater Kokstad	KwaZulu-Natal	2,680	76,753	28.6
KZN435	Umzimkhulu	KwaZulu-Natal	2,436	197,286	81
NC093	Magareng	Northern Cape	1,546	24,059	15.6
NC094	Phokwane	Northern Cape	828	60,168	72.7
WC024	Stellenbosch	Western Cape	831	173,197	208.4
WC031	Theewaterskloof	Western Cape	3,259	117,167	36
EC442	Umzimvubu	Eastern Cape	2,579	199,620	77.4
EC444	Ntabankulu	Eastern Cape	1,385	128,848	93.1
EC443	Mbizana	Eastern Cape	2,415	319,948	132.5
EC123	Great Kei	Eastern Cape	1,700	31,692	18.6
EC124	Amahlathi	Eastern Cape	4,505	101,826	22.6
KZN221	uMshwathi	KwaZulu-Natal	1,866	111,645	59.8
KZN244	Msinga	KwaZulu-Natal	2,375	184,494	77.7
ETH	eThekwini	KwaZulu-Natal	2,556	3,702,231	1,448.50

Table 5 – continued from previous page

Id.	Municipality name	Province	Area (km^2)	Population	Density
KZN226	Mkhambathini	KwaZulu-Natal	868	57,075	65.7
KZN225	The Msunduzi	KwaZulu-Natal	751	679,039	904.1
KZN222	uMngeni	KwaZulu-Natal	1,520	109,867	72.3
KZN224	Impendle	KwaZulu-Natal	1,610	29,526	18.3
KZN281	Mfolozi	KwaZulu-Natal	1,300	144,363	111.1
KZN282	uMhlathuze	KwaZulu-Natal	1,233	410,465	332.8
KZN285	Mthonjaneni	KwaZulu-Natal	1,639	78,883	48.1
EC106	Sundays River Valley	Eastern Cape	5,995	59,793	10
EC108	Kouga	Eastern Cape	2,670	112,941	42.3
EC109	Kou-Kamma	Eastern Cape	3,642	43,688	12
NMA	Nelson Mandela Bay	Eastern Cape	1,957	1,263,051	645.4
BUF	Buffalo City	Eastern Cape	2,750	834,997	303.6
EC126	Ngqushwa	Eastern Cape	2,115	63,694	30.1
MP325	Bushbuckridge	Mpumalanga	10,248	546,215	53.3
EC135	Intsika Yethu	Eastern Cape	2,873	152,159	53
EC136	Emalahleni	Eastern Cape	3,484	124,532	35.7
EC138	Sakhisizwe	Eastern Cape	2,318	63,846	27.5
WC014	Saldanha Bay	Western Cape	2,015	111,173	55.2
WC015	Swartland	Western Cape	3,707	133,762	36.1
WC032	Overstrand	Western Cape	1,675	93,407	55.8
CPT	City of Cape Town	Western Cape	2,446	4,005,016	1,637.60
LIM351	Blouberg	Limpopo	9,540	172,601	18.1
LIM353	Molemole	Limpopo	3,628	125,327	34.5
LIM354	Polokwane	Limpopo	5,054	797,127	157.7
LIM368	Modimolle/Mookgophong	Limpopo	10,367	107,699	10.4
LIM476	Greater Tubatse/Fetakgomo	Limpopo	5,693	489,902	86
LIM341	Musina	Limpopo	10,347	132,009	12.8
LIM343	Thulamela	Limpopo	2,642	497,237	188.2
LIM344	Makhado	Limpopo	7,605	416,728	54.8
LIM345	New	Limpopo	5,003	347,974	69.6
NC087	Dawid Kruiper	Northern Cape	44,231	107,161	2.4
MP326	Mbombela	Mpumalanga	7,141	695,913	97.4
NW405	Ventersdorp/Tlokwe	North West	6,398	243,527	38.1
MAN	Mangaung	Free State	9,886	787,803	79.7
EC145	Walter Sisulu	Eastern Cape	13,269	87,263	6.6
EC139	Enoch Mgijima	Eastern Cape	13,584	267,011	19.7
EC129	Raymond Mhlaba	Eastern Cape	6,357	159,515	25.1
KZN245	Umvoti	KwaZulu-Natal	2,705	122,423	45.3
KZN223	Mpofana	KwaZulu-Natal	1,757	37,391	21.3
GT423	Lesedi	Gauteng	1,484	112,472	75.8
GT422	Midvaal	Gauteng	1,722	111,612	64.8
GT421	Emfuleni	Gauteng	966	733,445	759.3
EC102	Blue Crane Route	Eastern Cape	11,068	36,063	3.3
JHB	City of Johannesburg	Gauteng	1,645	4,949,347	3,008.80
EKU	Ekurhuleni	Gauteng	1,975	3,379,104	1,710.60

2. Additional maps, figures and tables.

References.

Preston, C. J. (1977). Spatial birth-and-death processes. *Bulletin of the International Statistical Institute 46*, 371–391.

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TABLE 1

Results from the simulation study from Section 6. Binary tables with $p \in \{10, 20\}$ variables and $n \in \{200, 500, 1000\}$ samples were generated from graphical models defined by three types of graphs: "Random", "Cluster", and "Scale-free". This table reports means and standard deviations of the F_1-score (6.1) and SHD (6.2) measures for the accuracy in recovering the structure of the true graph across 50 replicate binary tables generated for every combination of p, n and graph type. The best performing models in terms of the F_1-score and SHD are shown in boldface.

		F_1-score			SHD		
p	n	BDMCMC	HC(or)	HC(and)	BDMCMC	HC(or)	HC(and)
				Random			
10	200	**0.7**(0.12)	0.68(0.11)	0.57(0.11)	**8.2**(2.9)	8.7(2.9)	10.6(3)
	500	**0.8**(0.1)	0.78(0.1)	0.7(0.1)	**5.8**(2.5)	6.2(2.6)	8(2.6)
	1000	**0.87**(0.08)	0.86(0.08)	0.8(0.09)	**3.9**(2.2)	4(2.1)	5.6(2.2)
20	200	**0.7**(0.08)	0.69(0.07)	0.58(0.09)	17.5(4.6)	**17.3**(3.9)	21.5(5.7)
	500	**0.8**(0.08)	0.79(0.08)	0.7(0.09)	**11.9**(4.2)	12.5(4.5)	16.6(4.4)
	1000	**0.85**(0.07)	**0.85**(0.07)	0.78(0.08)	**8.9**(4)	9.3(3.8)	12.6(4)
				Cluster			
10	200	**0.76**(0.13)	0.75(0.13)	0.66(0.14)	**4.5**(2)	4.6(1.8)	5.8(2)
	500	**0.86**(0.1)	0.83(0.13)	0.75(0.15)	**2.7**(1.7)	3.3(2.1)	4.6(2.4)
	1000	**0.91**(0.09)	0.9(0.09)	0.87(0.09)	**1.7**(1)	1.8(1.1)	2.4(1.3)
20	200	0.69(0.13)	**0.71**(0.11)	0.63(0.14)	14.8(10.2)	**11.7**(4.2)	13.5(4.9)
	500	**0.86**(0.08)	0.84(0.08)	0.76(0.11)	**5.9**(2.8)	6.7(3.2)	9.5(4.1)
	1000	**0.93**(0.06)	0.92(0.06)	0.87(0.08)	**3.3**(2.7)	3.6(2.7)	5.9(2.9)
				Scale-free			
10	200	**0.67**(0.12)	0.66(0.1)	0.56(0.1)	**8.5**(2.6)	**8.5**(2.1)	10(1.9)
	500	**0.73**(0.11)	**0.73**(0.11)	0.62(0.12)	**6.9**(2.4)	**6.9**(2.2)	8.8(2.2)
	1000	0.8(0.07)	**0.81**(0.08)	0.7(0.1)	5.3(1.7)	**5.2**(1.9)	7.4(1.9)
20	200	**0.63**(0.13)	**0.63**(0.13)	0.53(0.09)	21.3(12.7)	**19.1**(5.6)	21.4(3.7)
	500	**0.74**(0.08)	**0.74**(0.09)	0.63(0.08)	14(3.6)	**13.8**(3.8)	17.7(3.2)
	1000	**0.78**(0.09)	**0.78**(0.09)	0.7(0.09)	11.8(4.3)	**11.4**(4.3)	14.7(3.9)

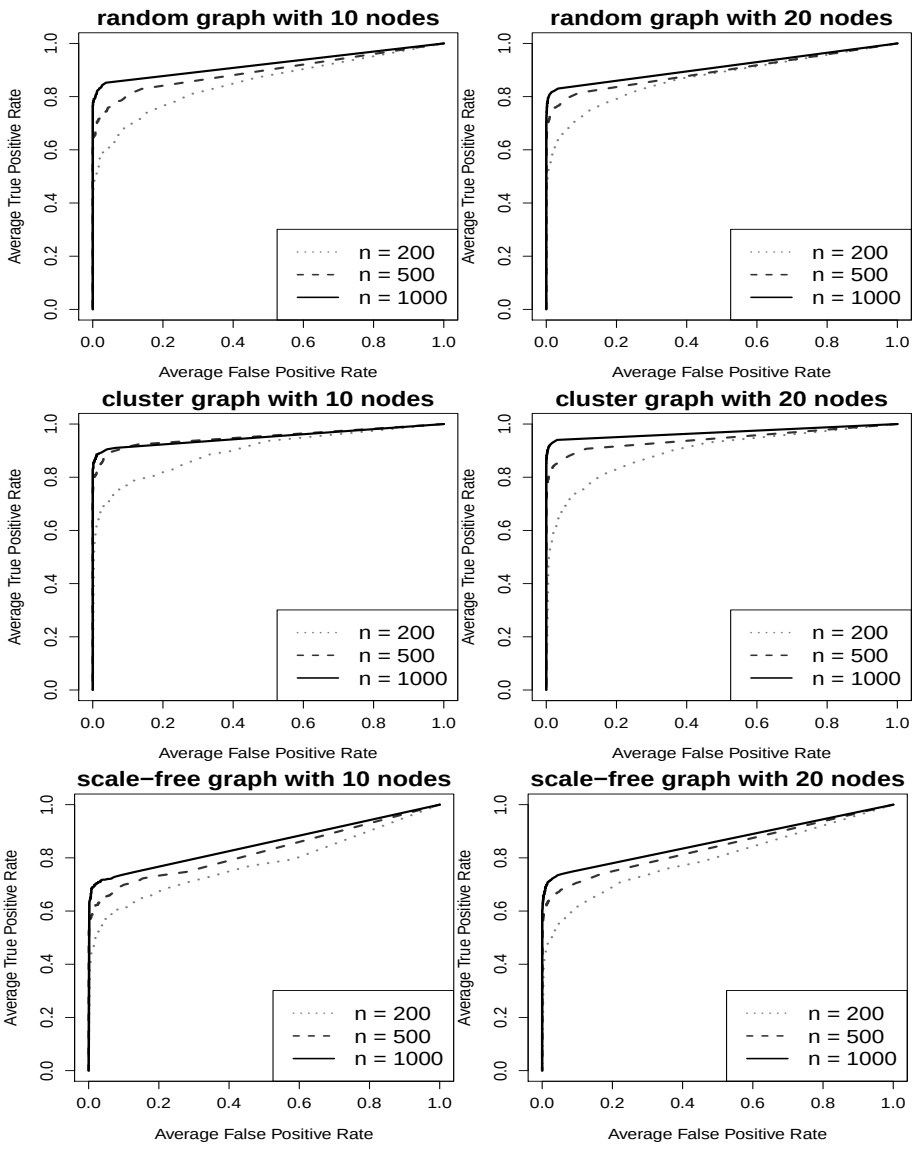

FIG 1. *ROC curves showing the performance of the BDMCMC algorithm in recovering the structure of the true graph in the simulation study from Section 6. Binary tables with* $p \in \{10, 20\}$ *variables and* $n \in \{200, 500, 1000\}$ *samples were simulated from graphs of three types ("Random", "Cluster", and "Scale-free").*

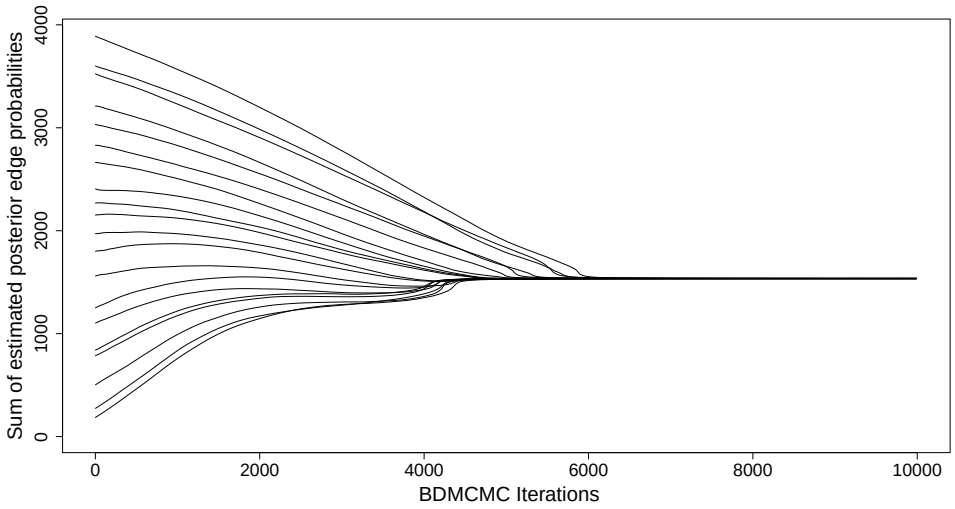

FIG 2. *Convergence plot of the BDMCMC algorithm for the Twitter data introduced in Section 4 and analyzed in Section 7. The plot shows the sum of the estimated posterior edge inclusion probabilities (y axis) against iteration number (x axis) from* 20 *runs of the BDMCMC algorithm starting from* 20 *different graphs sampled from graph space* $\mathcal{G}_{214}$ *with different number of edges.*

TABLE 2

Summary geographic and demographic information about the 10 municipalities linked by an edge with the Local (yes/no) variable in the SA Twitter graph that have an aOR smaller than 1. Population data extracted from the 2016 Community Survey, Statistics South Africa. Retrieved from https://interactive2.statssa.gov.za/webapi.

Id.	Municipality name	Province	Area (km^2)	Population	Density	aOR (95% CI)
FS162	Kopanong	Free State	15,645	49,999	3.2	0.259 (0.237,0.284)
KZN216	Ray Nkonyeni	KwaZulu-Natal	1,487	348,533	234.4	0.731 (0.6769839,0.790)
LIM335	Maruleng	Limpopo	3,563	99,946	28.1	0.251 (0.225,0.280)
NW375	Moses Kotane	North West	5,726	243,648	42.5	0.692 (0.640,0.748)
KZN276	Big Five Hlabisa	KwaZulu-Natal	3,466	116,622	33.6	0.482 (0.419,0.555)
WC024	Stellenbosch	Western Cape	831	173,197	208.4	0.760 (0.726,0.796)
MP325	Bushbuckridge	Mpumalanga	10,248	546,215	53.3	0.413 (0.380,0.448)
CPT	City of Cape Town	Western Cape	2,446	4,005,016	1,637.60	0.354 (0.346,0.362)
JHB	City of Johannesburg	Gauteng	1,645	4,949,347	3,008.80	0.920 (0.898,0.942)
EKU	Ekurhuleni	Gauteng	1,975	3,379,104	1,710.60	0.804 (0.781,0.827)

TABLE 3

Summary geographic and demographic information about the 2 municipalities linked by an edge with the Local (yes/no) variable in the SA Twitter graph that have an aOR greater than 1. *Population data extracted from the* 2016 *Community Survey, Statistics South Africa. Retrieved from* https://interactive2.statssa.gov.za/webapi.

Id.	Municipality name	Province	Area (km^2)	Population	Density	aOR (95% CI)
TSH	City of Tshwane	Gauteng	6,298	3,275,152	520	2.347 (2.260,2.437)
GT421	Emfuleni	Gauteng	966	733,445	759.3	5.414 (4.714,6.218)

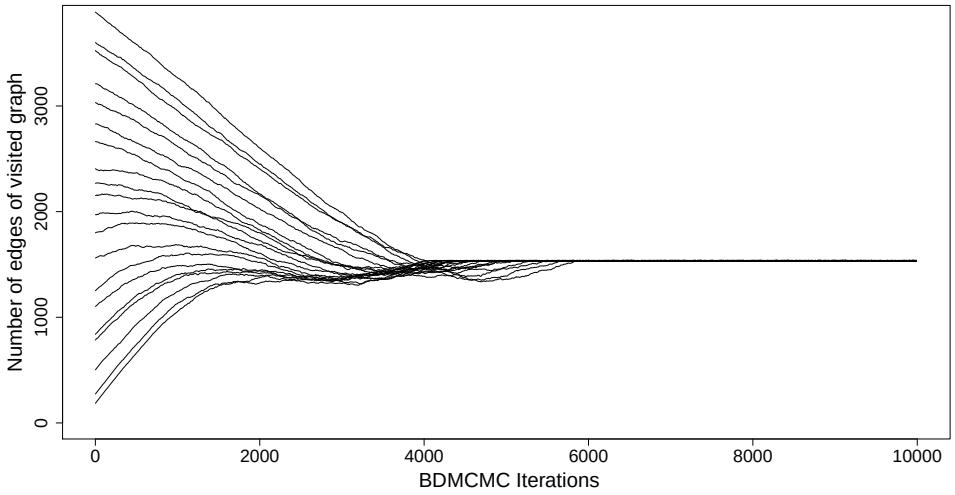

FIG 3. *Convergence plot of the BDMCMC algorithm for the Twitter data introduced in Section 4 and analyzed in Section 7. The plot shows the number of edges included in the sampled graphs (y axis) against iteration number (x axis) from* 20 *runs of the BDMCMC algorithm starting from* 20 *different graphs sampled from graph space* $\mathcal{G}_{214}$ *with different number of edges.*

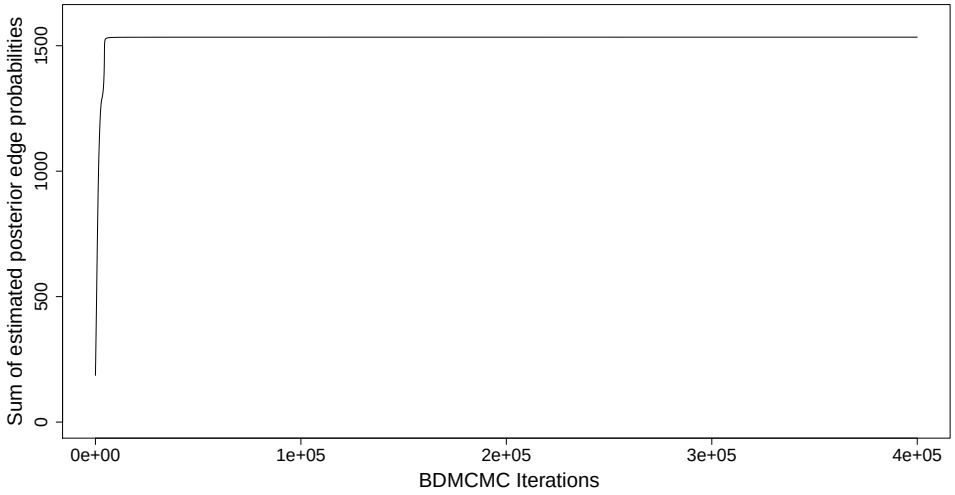

FIG 4. *Convergence plot for a longer run of 400,000 iterations of the BDMCMC algorithm for the Twitter data introduced in Section 4 and analyzed in Section 7. The plot shows the sum of the estimated posterior edge inclusion probabilities (y axis) against iteration number (x axis).*

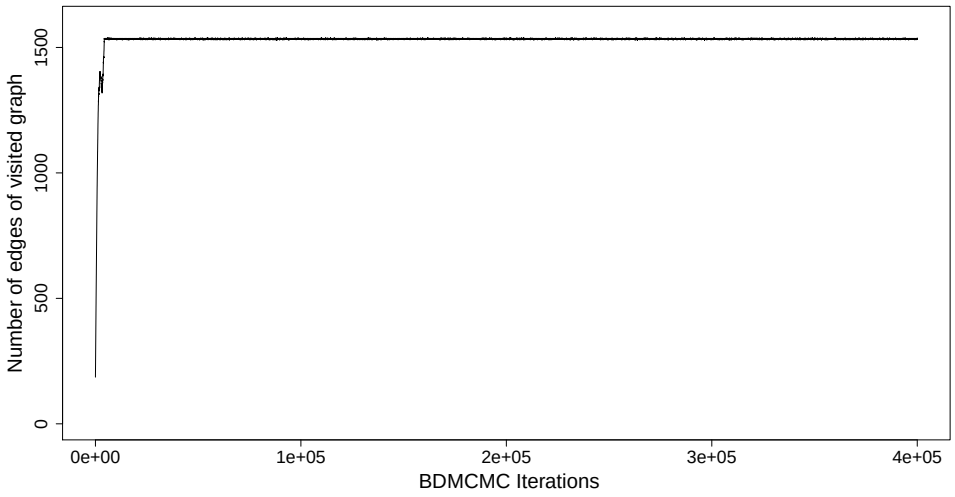

FIG 5. *Convergence plot for a longer run of* $400,000$ *iterations of the BDMCMC algorithm for the Twitter data introduced in Section 4 and analyzed in Section 7. The plot shows the number of edges included in the sampled graphs (y axis) against iteration number (x axis).*

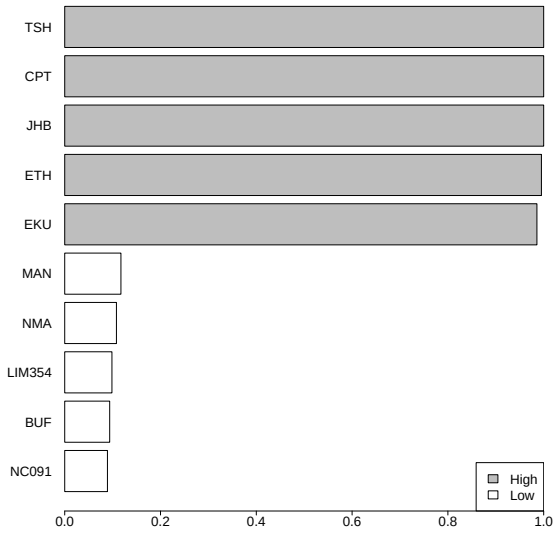

FIG 6. *Barplot showing the top* 10 *municipalities with the largest degree in the SA Twitter graph. The five hubs are shown in gray.*

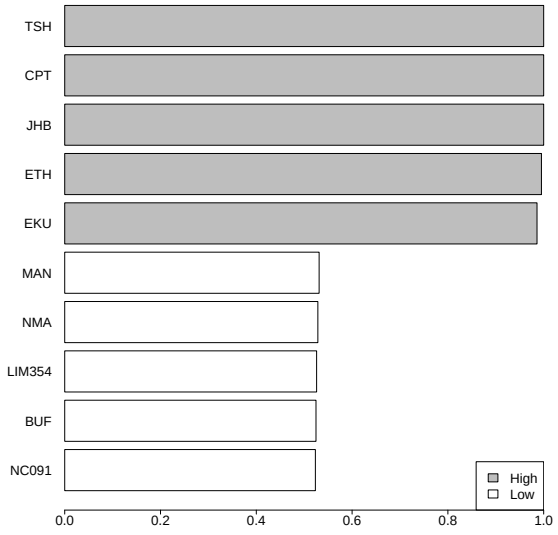

FIG 7. *Barplot showing the top* 10 *municipalities with the largest closeness in the SA Twitter graph. The five hubs are shown in gray.*

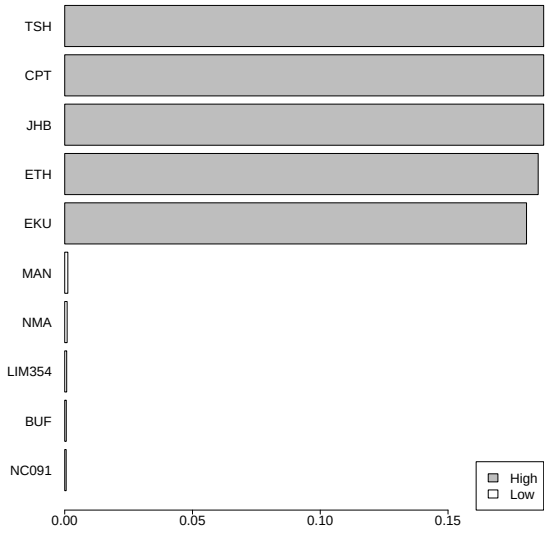

FIG 8. *Barplot showing the top* 10 *municipalities with the largest betweenness in the SA Twitter graph. The five hubs are shown in gray.*

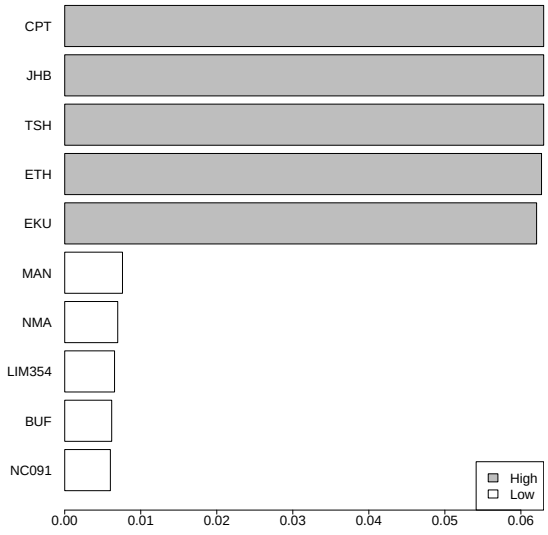

FIG 9. *Barplot showing the top* 10 *municipalities with the largest page rank in the SA Twitter graph. The five hubs are shown in gray.*