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Localized $1\hbar\omega$ particle–hole strength in nuclei

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Abstract

A cross section enhancement, persisting to high momentum transfer, has been observed around $1\hbar\omega$ in excitation energy in spectra from inelastic proton-, alpha- and electron scattering on ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb . Data are presented for (p,p') at 201 MeV, (α, α') at 120 MeV and analyzed together with existing data obtained with 133.8 MeV polarized $(\vec{p}, p')$ on ^{116}Sn and electron-scattering data on ^{116}Sn and ^{196}Pt . Two different interpretations of the observed enhancement are discussed: that of the incoherent sum of all $1\hbar\omega$ cross section and alternatively the sum over the isoscalar normal modes of all multipolarities.

Keywords: NUCLEAR REACTIONS ^{90}Zr , ^{116}Sn , ^{196}Pt , ^{208}Pb (p,p') , $E = 201$ MeV; ^{116}Sn (polarized p,p'), $E = 133.8$ MeV; ^{90}Zr , ^{116}Sn , ^{196}Pt (α, α') , $E = 120$ MeV; measured $\sigma(\theta)$. ^{116}Sn , ^{196}Pt (e,e') , $E = 278$, 333 MeV; measured form factors. ^{90}Zr , ^{116}Sn , ^{196}Pt , ^{208}Pb deduced 0, $1\hbar\omega$ strength distribution. DWBA calculation and effective NN-force.

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1. Introduction

Particle-hole excitations in nuclei have been extensively investigated over the last two decades using several different probes. A prolific source of information on the properties of individual nuclear levels and on multipole-strength distributions has been provided notably by inelastic scattering of protons, electrons and alpha particles. The response of nuclei to these probes varies with the spin- and isospin of the particle-hole excitations. A well known class is formed by the various giant resonances, especially the collective nuclear excitations of low multipolarity. Their occurrence as localized strength distributions, associated with the response to low-rank tensor operators, has lead to their successful interpretation as collective surface- and density vibrations in the liquid-drop model. The other extreme is formed by the class of particle-hole states of stretched ($J = j_1 + j_2$) or nearly stretched spin. Such states, if viewed in a $0\hbar\omega$ or $1\hbar\omega$ shell-model basis, are often extremely simple, since few and sometimes just one particle-hole configuration exists of the spin-class under consideration. The most investigated stretched-spin states are those of unnatural parity [1,2].

Strength distributions for multipolarities intermediate between the giant resonances and the stretched-spin states have not been studied in the same detail. There exists some evidence on the low-energy octupole resonance (LEOR) [3] and hexadecapole strength has been located up to typically 6 MeV in a number of nuclei [4,5]. For higher multipolarities no systematics is available on their location and strength distribution. It is to be anticipated that these strengths do not show the same degree of collectivity as the low-multipolarity giant resonances [6]. On the other hand, a larger degree of fragmentation than is typical for the stretched-spin states must be expected. Common to all spin classes is the observation that generally not the full theoretically predicted cross section is observed and that their strength is fragmented. The localized strength of the isoscalar giant quadrupole resonance accounts typically for 50% of the energy-weighted sum rule (ESWR) for light and medium heavy ($A < 100$) nuclei, increasing to about 100% for heavy nuclei. For $L > 2$ the systematics is incomplete, but 50% or less of their ESWR [3] seems typical. The experimentally observed Gamow-Teller strength is only about 60% of its model-independent sum rule [7] and individual stretched spin states are quenched with respect to their single-particle estimates by factors of typically 0.2–0.6 [1,2,12]. The only exception seems to be the isobaric analog states, which both in the (p,n) and the (^3He ,t) reactions are identified with cross sections that correspond to their interpretation as “ideal” giant resonances [8,9]. Almost all spin classes are thus expected to have part of their cross section outside the region where they have been identified. In general this will be in tails that extend from the region of localization upwards in excitation energy.

Inelastic scattering spectra, at a momentum transfer $q \simeq 1.8 \text{ fm}^{-1}$, exciting the nucleus ^{116}Sn via the reactions (e,e') [10,11], (p,p') (Ref. [12] and this work) and (α , α') (this work) are shown in Fig. 1, together with a $^{115}\text{In}(\alpha,t)^{116}\text{Sn}$ spectrum [13]. The outstanding features in all these spectra are the enhancements of the cross section around excitation energies of 7 and 15 MeV. In transfer reactions these structures have been

Fig. 1. Spectra from different reactions leading to ^{116}Sn as the final nucleus: (a) (e,e') at 350 MeV, $\theta_{\text{lab}} = 61^\circ$, data NIKHEF-K [10,11]. (b) (p,p') at 133.8 MeV, $\theta_{\text{lab}} = 40^\circ$, data IUCF [12]. (c) (p,p') at 201 MeV, $\theta_{\text{lab}} = 32^\circ$, data IPN Orsay, this work. (d) (α,t) at 75 MeV, $\theta_{\text{lab}} = 10^\circ$, data KVI [13,58]. (e) (α,α') at 120 MeV, $\theta_{\text{lab}} = 21^\circ$, data KVI, this work.

demonstrated [14,15] to correspond to stripping into the next and the second-next higher major shells. In inelastic scattering such concentrated cross sections enhancements, persisting to high momentum transfer, have not been reported before. By analogy to the transfer reactions the lowest should be associated with a concentration of $1\hbar\omega$ excitation strength. It does not correspond to the well known giant resonances which, except for the LEOR, lie at higher excitation energies.

The second and broader enhancement around 15 MeV excitation energy must be interpreted as a concentration of $2\hbar\omega$ cross section and it includes also the $1\hbar\omega$ giant dipole resonance. This structure is not further analyzed in the present work.

In order to further investigate the $1\hbar\omega$ cross section concentration we performed inelastic proton-scattering experiments at 201 MeV on the nuclei ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb , and inelastic alpha-scattering experiments at 120 MeV on ^{90}Zr , ^{116}Sn , ^{196}Pt . The experiments cover a range in excitation energy up to 25 MeV.

We attempt in this work to give a quantitative account for the observed cross section by comparing with microscopic DWBA, based on one-body transition densities (OBTD's) that span the full $1\hbar\omega$ configuration space.

Two different approaches to account for the $1\hbar\omega$ cross section will be presented. In the first, the experimental cross section is compared to the incoherent sum of cross

sections for all simple $1\hbar\omega$ particle-hole excitations. This sum is an invariant for unitary transformations within the basis.

When schematically diagonalizing the matrices of each spin-parity class with a separable interaction, one state moves away from its unperturbed energy by the full trace of the matrix, either upwards, like the GDR, or downwards in the case of isoscalar natural-parity states. These collective states correspond closely to the responses of the nucleus to an external tensor field, and they are the normal modes of the liquid-drop model, cast into a microscopic form, by expanding them over the $1\hbar\omega$ basis states. In the second scenario the cross sections of the low-lying states and that of the bump region are analyzed in terms of these microscopic normal modes. In both approaches effective projectile-nucleon interactions are used.

In Section 2, details on the experiments are given. Section 3 describes the DWBA and the microscopic OBTD's in the incoherent-sum and the normal-mode formulations. The low-lying states in ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb are analyzed in Section 4 and the $1\hbar\omega$ region in Section 5.

We include in our analysis also electron scattering data on ^{196}Pt [16,17] up to $E_x = 18$ MeV. For ^{116}Sn [10,11] data up to this excitation energy exist for only two momentum transfers, one of which is shown in Fig. 1. Complete data are available below $E_x = 7$ MeV and these will be included in our analysis for low-lying states. Data from polarized proton scattering at 133.8 MeV, also for ^{116}Sn are available from earlier work [10,12] and have been included both in the analysis of the low-lying states and the bump region. A discussion is given in Section 6.

2. Experimental procedures and data analysis

The inelastic proton-scattering data at 201 MeV presented in this work were obtained at the synchrocyclotron of the Institut de Physique Nucléaire at Orsay (France) using the magnetic spectrograph Montpellier [18].

Self-supporting metallic targets of ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb were used. The dispersion of the beam can be matched to that of the spectrograph. The realization of this achromatic condition made it possible to obtain for thin targets an energy resolution $\Delta E/E \simeq 2 \times 10^{-4}$ at the focal plane of the spectrograph.

The spectrograph was used in two different modes. In the small-angle mode, with a Faraday cup inside the scattering chamber, the angular range 4° – 18° was covered. Beam currents of 4–50 nA were used in this configuration. In the large-angle mode data were taken from 20° to 56° . The beam was dumped in a Faraday cup located 8 m behind the target and beam currents up to 250 nA were used. In both settings the solid angle was 0.167 msr and the full horizontal opening angle was $\Delta\theta = 0.76^\circ$.

The electron-scattering experiments were carried out at the MEA accelerator [19] of NIKHEF-K with the QDD spectrometer. Data were taken on metallic self-supporting targets of ^{116}Sn and ^{196}Pt and cover a range of effective momentum transfer, q_{eff} , from 0.52 to 2.88 fm^{-1} for ^{116}Sn and from 0.36 to 2.43 fm^{-1} for ^{196}Pt . The system was

Table 1
Target thicknesses in mg/cm²

Isotope	(p,p')	(e,e')	(α , α')
⁹⁰ Zr	40.0		5.0
¹¹⁶ Sn	8.96	8.96	1.50
	22.1		
¹⁹⁶ Pt	7.1	7.1	3.50
	2.6		0.92
²⁰⁸ Pb	20.0		

operated in the dispersion-matching mode. A resolution of typically 7×10^{-5} was achieved.

Part of the electron-scattering data on low-lying states in ¹⁹⁶Pt [16,17,20] and ¹¹⁶Sn [10,11] has been published previously.

The alpha-scattering experiments were performed at the KVI with momentum-analyzed beams of 120 MeV from the AVF cyclotron. Data were taken on ⁹⁰Zr, ¹¹⁶Sn and ¹⁹⁶Pt in the angular range 12°–60° using the QMG/2 magnetic spectrograph [21], equipped with a 120 cm long multi-wire drift-chamber detector [22]. Spectrograph solid angles of 7.1 msr ($\Delta\theta = 4^\circ$) and 10.0 msr ($\Delta\theta = 6^\circ$) were used. Typical overall energy resolutions of 50 keV were obtained.

The thicknesses of the targets, used in the different experiments are listed in Table 1.

Fig. 2 shows spectra from the (p,p') reaction taken at 29°. In all spectra a pronounced structure is observed, extending in ¹⁹⁶Pt and ²⁰⁸Pb from 4–8 MeV in excitation energy, in ¹¹⁶Sn from 5–10 MeV and in ⁹⁰Zr from 5–11 MeV. At around twice these energies a second and broader enhancement is observed for all nuclei.

In ²⁰⁸Pb the lowest structure is found to decompose into discrete peaks. Many of these are known from high-resolution experiments to be $1\hbar\omega$ particle-hole states, such as the high-spin states 14^- (6.738 MeV), 12^- (6.426 and 7.053 MeV), 12^+ (6.097 MeV) and 10^+ (4.895, 5.072 and 5.540 MeV). But also 3^- and 4^+ states have been identified in this region [5,23,24].

In Fig. 3 inelastic alpha-scattering and proton-scattering spectra on ⁹⁰Zr are displayed for comparable momentum transfers, $q_{\text{eff}} \simeq 1.65 \text{ fm}^{-1}$ and Fig. 4 shows electron-, proton- and alpha spectra for ¹⁹⁶Pt at the same momentum transfer.

It is compelling to identify the observed bump-like structures with particle-hole configurations of $1\hbar\omega$ and $2\hbar\omega$ character, respectively. The $1\hbar\omega$ bump is very similar in shape in the electron and proton spectra. On the other hand its appearance in the inelastic alpha-scattering spectrum is rather different: it is narrower and its centroid is found at a clearly lower excitation energy ($\simeq 4.5 \text{ MeV}$). In alpha-scattering at smaller angles it has been observed in different nuclei by Moss et al. [25,26], and has been identified with the LEOR.

The intervals that contain the assumed “ $1\hbar\omega$ -bumps”, indicated in Figs. 1–4, are chosen such that they begin just above the well-known low-lying states and extend to

Fig. 2. Spectra from inelastic proton scattering at 201 MeV and $\theta_{\text{lab}} = 29^\circ$ from ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb . The cross-section enhancements between the vertical dashed markers are associated with $1\hbar\omega$ excitations.

a local minimum around $E_x = 50 A^{-1/3} \text{ MeV}$, which in all cases marks the observed middle between the $1\hbar\omega$ - and the $2\hbar\omega$ -bumps.

The centroid energies of the bumps and the limits of the chosen intervals are given in Table 2.

3. One-body transition densities and DWBA

It is important at this point to note that virtually all cross section in the bump region stems from one-step reactions. In electron scattering the radiation-tails from states at lower excitation energies contribute less than 10% to the cross section in the energy

Fig. 3. Inelastic-scattering spectra off ^{90}Zr for (p,p') at 201 MeV (top) and (α,α') at 120 MeV (bottom) for comparable momentum transfer, $q \simeq 1.65 \text{ fm}^{-1}$.

Fig. 4. ^{196}Pt inelastic-scattering spectra for (e,e') , (p,p') and (α,α') at comparable momentum transfer, $q \simeq 1.65 \text{ fm}^{-1}$.

Table 2
Energy systematics of the $1\hbar\omega$ bump

Isotope	$\bar{E}$ [MeV]	$1\hbar\omega =$ $41 A^{-1/3} \text{ MeV}$	Low lim. ^a [MeV]	High lim. ^a [MeV]
⁹⁰ Zr	7.4	9.15	3.69	11.00
¹¹⁶ Sn	6.4	8.41	3.32	10.20
¹⁹⁶ Pt	5.6	7.06	2.35	8.81
²⁰⁸ Pb	5.5	6.92	>0	8.65

^a Boundaries used in analysis.

interval $E_x = 2.5\text{--}9$ MeV. For (p, p') the cross sections from two- and multistep processes in the same interval were estimated with the computer code of Bonetti and Chiesa [27] in the same way and with the same parametrizations for the NN force and the level densities as used by Condé et al. [28]. It was found that these processes contribute less than 10%.

Our analysis will be entirely in terms of the single collision approximation, employing one-body transition densities (OBTD's) and DWBA with an effective projectile–nucleon interaction.

3.1. Particle–hole and normal mode OBTD's

Under the assumption that the bump represents an important fraction of the available $1\hbar\omega$ cross section in an incoherent way, it is compared with the sum of the calculated cross sections for all individual $1\hbar\omega$ particle–hole excitations, both for neutrons and protons. For a given spin class, I^π , this sum takes the form:

$$\frac{d\sigma_{\text{DW}}(0 \rightarrow I^\pi)}{d\Omega} = \sum_{\text{ph}} v_{j_h}^2 u_{j_p}^2 \frac{d\sigma_{\text{DW}}(0 \rightarrow (j_p \otimes j_h^{-1}; I^\pi))}{d\Omega}. \quad (1)$$

Within the basis the oscillator sums of $B(E\lambda)$ - and $B(M\lambda)$ -values are invariants under unitary transformations. Under the conditions that the reaction mechanism is that of a single projectile–nucleon collision and that the Q -values of all states are taken degenerate, it is trivially proven that likewise the cross-section sum, defined in Eq. (1), is an invariant for each spin-parity class, I^π , separately.

The incoherent sum over all states with spin I^π will henceforth be denoted as:

$$\sum_{\text{ph}} = \sum_{I^\pi} \frac{d\sigma_{\text{DW}}(0 \rightarrow I^\pi)}{d\Omega}. \quad (2)$$

This sum contains about 350 separate DWBA calculations in the case of ²⁰⁸Pb. For non-magic nuclei, such as ¹⁹⁶Pt, the occupation probabilities, $v^2(j)$ of the hole orbital and the emptiness $u^2(j)$ of the particle shell are to be estimated from single-nucleon transfer reactions.

In the alternative scenario the bump is compared with the cross section associated with collective excitations. An idealized microscopic description of these is provided by

the normal modes (NM) [29], which are the responses of the nucleus to the action of simple isoscalar or isovector tensor operators, expanded over the $1\hbar\omega$ basis:

$$| \text{NM}_{T,LSIM} \rangle = N^{-1} \sum_{\text{ph}} v_{jh} u_{jp} | j_p, j_h^{-1}; IM \rangle \langle j_p, j_h^{-1}; JM || Q_{T,LSI} || 0 \rangle \quad (3)$$

with for the isoscalar normal modes:

$$Q_{T=0,L0I} = r^L Y_L(\hat{r}) \delta_{L,I} \quad (L \geq 2) \quad (4)$$

and

$$Q_{T=0,L1I} = r^L [\sigma \otimes Y_L]_I \quad (\text{all } L, I). \quad (5)$$

The $S = 0$ mode for $I^\pi = 1^-$, corresponding to the operator $Q_{T=0,101} = rY_1(\hat{r})$, represents the spurious centre-of-mass motion and is left out from the basis. Likewise the $I^\pi = 0^+$ mode $Q_{T=0,000} = Y_0(\hat{r})$ is left out, as it does not represent an excitation of the nucleus. It is customary, in the liquid-drop model, to describe 1^- and 0^+ excitations as responses to the operators $r^3Y_1(\hat{r})$ and $r^2Y_0(\hat{r})$, respectively, the latter corresponding to the breathing mode. These are, however, $3\hbar\omega$ - and $2\hbar\omega$ -excitations in a microscopic approach and will not be considered here.

The isovector normal modes are obtained by:

$$Q_{T=1,LSI} = Q_{T=0,LSI} t_z. \quad (6)$$

Here, the $1\hbar\omega$ mode for $(S = 0, L = 1)$, $Q_{T=1,101} = rY_1(\hat{r}) t_z$ does exist and is associated with the giant dipole resonance (GDR). The response of the operator $Q_{T=1,000} = Y_0(\hat{r}) t_z$ yields in part the ground state again, and leads also to an analog state with isospin ($T_> = (N - Z)/2 + 1$). Both the GDR and this analog state are located well above the excitation-energy interval studied in this work.

The factor N in Eq. (3) is chosen such that the normal mode exhausts the full multipole strength

$$S_{TLI} = \sum_M | \langle \text{NM}_{T,LSIM} | Q_{T,LSIM} | 0 \rangle |^2 = \sum_{\text{ph}} v_{jh}^2 u_{jp}^2 | \langle j_p, j_h || Q_{T,LSI} || 0 \rangle |^2 \quad (7)$$

of the operator $Q_{T,LSI}$ that is contained in the adopted 1p–1h basis. By this definition the normal modes are one-body transition-densities (OBTD's), not states.

It can readily be shown that any other OBTD, defined on the same particle–hole basis and orthogonal to the normal mode, carries zero strength.

Each spin-parity class, I^π , has e.g. four normal modes, which are for natural-parity states ($I = L$) characterized by $(S, T) = (0, 0), (1, 0), (0, 1)$ and $(1, 1)$ and for unnatural-parity states ($I = L \pm 1, S = 1$) by $(L, T) = (I - 1, 0), (I + 1, 0), (I - 1, 1)$ and $(I + 1, 1)$.

By their expansion over a truncated basis, these are not strictly orthogonal. Their overlap is small when the number of basis states is large, which is the case in practice for all but the highest spins. When evaluating the full isoscalar cross section, and thus

Table 3
Occupancies of valence shells in ^{116}Sn and ^{196}Pt

^{116}Sn neutrons		^{196}Pt protons		^{196}Pt neutrons	
shell	ν^2 ^a	shell	ν^2 ^b	shell	ν^2 ^c
$3s_{1/2}$	0.32	$3s_{1/2}$	0.50	$3p_{1/2}$	0.50
$2d_{3/2}$	0.40	$2d_{3/2}$	0.50	$3p_{3/2}$	0.50
$1h_{11/2}$	0.22	$2d_{5/2}$	0.83	$2f_{5/2}$	0.67
$1g_{7/2}$	0.88	$1h_{11/2}$	1.00	$1i_{13/2}$	0.79
$2d_{5/2}$	0.74	$1g_{7/2}$	1.00	$2f_{7/2}$	1.00
				$1h_{9/2}$	1.00

^a From Van der Werf et al. [52].

^b Estimated from systematics [58].

^c Estimated from systematics [58].

allowing two normal modes, $\Psi_{I_1^\pi}$ and $\Psi_{I_2^\pi}$, per spin-parity class, double counting must be corrected for by multiplying the calculated cross sections for these normal modes by

$$c_{I^\pi} = \frac{1}{1 + \langle \Psi_{I_1^\pi} | \Psi_{I_2^\pi} \rangle}. \quad (8)$$

The sum of the normal-mode cross sections will then be written as

$$\sum_{\text{NM}, I^\pi} \text{NM} = \sum_{\text{NM}, I^\pi} c_{I^\pi} \frac{d\sigma(\text{NM}, I^\pi)}{d\Omega}. \quad (9)$$

We adopt a $0\hbar\omega + 1\hbar\omega$ basis for the normal modes to analyze both the low-lying collective states (Section 4) and the bump region (Section 5).

For the evaluation of the relevant OBTD's, doubly-closed shell configurations were assumed for ^{90}Zr and ^{208}Pb , while also the proton configuration of ^{116}Sn was taken as closed. Adopted occupancies for the non-closed valence shells in ^{116}Sn and ^{196}Pt are given in Table 3. The incoherent particle-hole and normal-mode cross sections do not depend strongly on the precise values of these occupancies, nor on the binding energies of the single-nucleon states. In choosing the $0+1\hbar\omega$ basis states, intruder single-particle orbitals like the $1h_{11/2}$ and the $1i_{13/2}$ have been considered to belong to the next-lower major shell, in accordance with their location in binding energy.

3.2. DWBA

The single particle-hole excitations of the incoherent sum model and the normal modes, expanded over a restricted particle-hole basis, are tractable for microscopic DWBA.

DWBA calculations for inelastic proton scattering were performed with the program DW81 [30]. During the progress of this work the program DWBA91 of Raynal [31] became available. For a number of selected cases it was checked that both programs gave identical results. Three different effective nucleon-nucleon interactions are used: the

density-dependent G -matrices of the Paris potential by Von Geramb and Nakano [32] and of the Bonn potential by Nakayama and Love [33], and the t -matrix of Franey and Love [34].

The optical-model parameters used for 201 MeV proton scattering are given in Table 4. The parameters for ^{90}Zr are taken the same as found for ^{88}Sr by Kouw et al. [35]. For the 201 MeV data on ^{116}Sn we use the values from the systematics of Nadasen et al. [36] and for the 133.8 MeV data those from Van der Werf et al. [12]. The parameters for ^{208}Pb were taken from Djalali et al. [37] and were also used for ^{196}Pt .

Inelastic alpha-scattering is not usually analyzed in terms of an effective projectile–nucleon interaction and the few existing attempts use mostly a gaussian potential. See e.g. Bertrand et al. [38]. Programs like DW81 and DWBA91 use, however, Yukawa potentials. We have therefore constructed an interaction tailored after the free ($p, ^4\text{He}$) elastic scattering cross section at the relevant center-of-mass energy. The details of this interaction are presented in Subsection 4.3.

The optical-model parameters for alpha scattering at 120 MeV can be found in Table 4. Those for ^{90}Zr are from the detailed work of Put and Paans [39]. For ^{116}Sn we use the parameters of Brissaud et al. [40], determined for ^{120}Sn at 166 MeV and for ^{196}Pt the parameters of Goldberg et al. [41], found for ^{208}Pb at 140 MeV.

The microscopic calculations for electron scattering were performed with the codes [42] WSAXE, for natural-parity states and WSAXM for unnatural-parity states.

4. Low-lying natural-parity states in ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb

4.1. Normal mode analysis of the 3^- states in ^{90}Zr and ^{208}Pb

In this section we analyze the prominent states of natural parity in the low-lying part of the spectrum, i.e. well below the bump regions indicated in Figs. 1–4. This analysis is made in terms of the normal-mode OBTD's. It serves to assess the aptness of the normal-mode approach in microscopic DWBA. For each multipolarity the normal-mode cross section located in the low-energy region will have to be excluded from the subsequent analysis of the bump region in Section 5.

As a preliminary test of the suitability of the normal-mode method, we consider first the excitation of the 3^- states at $E_x = 2.748$ MeV in ^{90}Zr and at 2.615 MeV in ^{208}Pb .

Fig. 5 shows in the top panels the differential cross sections for the lowest 3^- states in ^{90}Zr and ^{208}Pb for (p, p') scattering at 201 MeV (this work) with calculations for the $S = 0$ isoscalar normal mode based on the Paris and Bonn G -matrices and on the t -matrix of Franey and Love. The left bottom panel shows the polarization for ^{90}Zr at 185 MeV, with the data of Hagberg et al. [43]. The analyzing power for ^{208}Pb is shown in the right bottom-panel with (part of) the data of Lee et al. [44].

The performance of the normal mode OBTD is quite remarkable. It describes the cross sections quantitatively and reproduces their shapes. Also the polarization in ^{90}Zr is reproduced well, except for the depths of the minima. The same is true for the analyzing

Fig. 5. Differential cross section and polarization data for the 3^- states at 2.748 MeV in ^{90}Zr (left) and at 2.615 MeV in ^{208}Pb from (p,p'). The data are compared with ($S = 0, T = 0$) normal-mode calculations using three different effective NN-forces: the density-independent t -matrix of Franey and Love [34], and density-dependent G -matrix interactions, based on the Paris potential [32] and the Bonn potential [33]. Top: differential cross sections at 201 MeV from this work, left ^{90}Zr , right ^{208}Pb . Left bottom: the polarization data from Hagberg et al. [43] at 185 MeV. Right bottom: analyzing power data from Lee et al. [44] at 200 MeV.

power in ^{208}Pb , although the calculations get out of phase with the data at larger angles.

The transition charge density (TCD) for the transition to the 3^- state in ^{208}Pb has been measured by Goutte et al. [45] and is compared with normal-mode TCD's in Fig. 6. It is observed that effective charges are needed to describe the experimental shape and magnitude. Using the free proton and neutron charges causes the experimental $B(E3)$ value of $6.12 \times 10^5 e^2\text{fm}^6$ to be underestimated by an order of magnitude. It should be kept in mind that the normal-mode wave function by its construction carries all E3 strength contained in the $1\hbar\omega$ basis. Clearly then, the TCD of the 3^- state receives contributions from excitations into higher major shells. The use of effective charges is needed to make up for this deficit in collectivity.

When using free charges, neutron excitations do not enter in the longitudinal electron-

Fig. 6. Transition charge density for the 3^- ($S = 0, T = 0$) normal mode in ^{208}Pb for different effective charges and reduced Woods-Saxon well radii. The thick curve represents the experimental transition charge density [45].

scattering form factor. When given an effective charge, their contribution will not only affect the magnitude of the form factor, but also the shape. Putting $\delta e_p = \delta e_n \equiv \delta e$, a proper reproduction of the form factor requires $\delta e \simeq 0.9$.

This situation where proton scattering is reproduced quantitatively by a transition density, defined on an already large basis, but where yet electron scattering needs the use of effective charges does not stand alone. In fact, for low-lying collective states of natural parity, it appears to be a rule rather than an exception. This core polarization effect has been studied in detail for the $E_x = 3.522$ MeV 9^- state in ^{116}Sn and the $E_x = 9.70$ MeV 5^- state in ^{28}Si [12].

With the proviso that the use of effective charges may be necessary for electron scattering, the normal-mode approach seems, however, adequate for the analysis of the low-lying collective states of natural parity. Normal modes will in the following also be used to analyze the bump region ascribed to $1\hbar\omega$ excitations.

4.2. Proton scattering to low-lying natural-parity states

Differential cross sections for 201 MeV proton scattering are analyzed in this subsection with $S = 0$ isoscalar normal modes. Spin normal modes, carrying $S = 1$ and isovector ($T = 1$) normal modes, are expected to occur at higher excitation energies. An a posteriori test of the spin-singlet isoscalar nature of these transitions will be provided later on by the consistency of the differential cross sections to the same states from alpha-scattering, which is a selective $S = 0, T = 0$ probe. Yet, for a few transitions a comparison with the spin mode will be made here.

The prominent states of normal parity selected for this analysis are listed in Table 5,

Table 5

Low-lying states in ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb and their fraction of the ($S = 0, T = 0$) normal-mode cross section in 201 and 133.8 MeV (p, p')

^{90}Zr			^{116}Sn			^{196}Pt			^{208}Pb		
I^π	E_x [MeV]	$\sigma/\sigma_{\text{NM}}$	I^π	E_x [MeV]	$\sigma/\sigma_{\text{NM}}$	I^π	E_x [MeV]	$\sigma/\sigma_{\text{NM}}$	I^π	E_x [MeV]	$\sigma/\sigma_{\text{NM}}$
2^+	2.186	0.60	2^+	1.294	2.0	2^+	0.356	10	3^-	2.615	1.0
5^-	2.319	0.5	3^-	2.266	1.0	4^+	0.877	0.5	5^-	3.197	0.34
3^-	2.748	1.0	5^-	2.366	0.3	5^-	1.271	0.15	5^-	3.710	0.14
4^+	3.077	0.33	4^+	2.392	0.37	7^-	1.374	0.12	7^-	4.037	0.44
6^+	3.448	0.10	4^+	2.529	0.26	3^-	1.447	0.18	2^+	4.085	2.5
8^+	3.589	0.02	4^+	2.802	0.20	4^+	1.888	0.4	4^+	4.323	1.8
2^+	3.843	0.24	7^-	2.908	0.15	3^-	2.431	0.11	6^+	4.422	1.5
			4^+	3.046	0.23	3^-	2.638	0.13	8^+	4.610	1.0
			7^-	3.210	0.15				3^-	4.703	0.07
									3^-	5.245	0.08
									3^-	5.348	0.10
									4^+	5.690	0.5
									6^+	5.993	0.7

which gives the ratios of the experimental and the calculated normal-mode cross sections, $(d\sigma/d\Omega)_{\text{exp}}/(d\sigma/d\Omega)_{\text{NM}}$.

In all cases the Paris G -matrix was used. For $S = 0, T = 0$ normal modes the differences with the Bonn G -matrix and the Franey–Love t -matrix are small and the conclusions of this part of the analysis do not alter when using one of the latter forces. We will come back to differences between the potentials later on.

In Fig. 7 experimental and calculated differential cross sections are shown for ^{90}Zr , ^{116}Sn and ^{196}Pt .

4.2.1. $^{90}\text{Zr}(p, p')^{90}\text{Zr}^*$ at 201 MeV

The shapes of the differential cross sections of the 2^+ states at 2.186 and 3.843 MeV are well described with the $S = 0, T = 0$ normal mode. Together they exhaust 85% of the normal-mode cross section.

The cross section of the 3^- state at 2.748 MeV is reproduced by the normal mode without any renormalization, while no significant cross section resides on other low-lying 3^- states. This is similar to the situation in ^{208}Pb .

The cross section of the 4^+ state at 3.077 MeV requires a reduction factor of 0.33. The fit that includes this factor is shown in Fig. 7 as a dashed curve. Similarly the cross section of the 5^- state at 2.319 MeV is reduced by a factor 0.5 with respect to the normal-mode cross section and the 6^+ and 8^+ states exhibit an even larger reduction (see Table 5).

The even-spin states have been interpreted by Scott et al. [46] as belonging to a $\pi(1g_{9/2})^2$ multiplet. Their differential cross sections are displayed in Fig. 8.

Assuming the 8^+ state to be a purely $\pi(1g_{9/2})^2$ and neglecting a possible $(1g_{7/2}, 1g_{9/2})$ component and the possible presence of cross section on higher 8^+ states [47],

Fig. 7. Differential cross sections for inelastic proton scattering at 201 MeV to low-lying collective states in ^{90}Zr , ^{116}Sn and ^{196}Pt . The solid curves represent ($S = 0, T = 0$) normal-mode calculations, using the Paris G -matrix. The dashed curves include the renormalization factors given in Table 5.

one finds that its cross section leads to an estimate of the ground-state occupancy of the $\pi(1g_{9/2})$ orbit of $v_{1g_{9/2}}^2 = 0.15$ [48].

The occupation numbers for proton orbitals around the Fermi level in ^{90}Zr , have been calculated including short-range correlations [49]. This calculation yields occupation numbers between 0.86 for the $2p_{1/2}$ - and 0.96 for the $1f_{5/2}$ orbital below the Fermi surface and values of 0.08 and 0.04 for the $1g_{9/2}$ and $1g_{7/2}$ above the Fermi surface.

From the observed Gamow–Teller strength in a recent $^{90}\text{Zr}(n,p)^{90}\text{Y}$ experiment [28] an occupancy of $v^2(1g_{9/2}) = 0.093 \pm 0.006$ was deduced for the $\pi(1g_{9/2})$ shell. In a recent $^{90}\text{Zr}(e,e'p)$ experiment, Den Herder et al. [50] locate $\sum C^2S(1g_{9/2}) = 0.69 \pm 0.12$ up till 21 MeV excitation energy in the nucleus ^{89}Y . This value is consistent with the (n,p) result, since systematics on single-particle strengths indicates that about 30% of the spectroscopic strength is moved to higher excitation energies by short-range and tensor correlations.

In the calculations of the cross section, based on the $\pi(1g_{9/2})^2$ configurations, the largest value for the occupation of 0.093 was used. This results in a quenching factor, Q , of approximately $0.093(1 - 0.093) = 0.084$ [51].

Including this reduction factor we calculated the cross sections for pure $\pi(1g_{9/2})^2$ configurations. The results are shown in Fig. 8 as dashed curves for the observed 2^+ , 4^+ , 6^+ and 8^+ states. It is evident that these cross sections fall short of reproducing the data by an order of magnitude, thus invalidating the assumption of pure $\pi(1g_{9/2})^2$ configurations. In the same figure fits with normal-mode OBTD's are shown as solid

Fig. 8. Differential cross sections for transitions to positive-parity states in ^{90}Zr from (p,p') at 201 MeV. Solid curves are for $(S=0, T=0)$ normal-mode calculations, with renormalization factors as given in Table 5. The dashed curves are calculations for pure $\pi(1g_{9/2})^2$ configurations, taking into account a ground state occupancy of 0.093 (see text).

curves, with scaling factors as given in Table 5.

4.2.2. $^{116}\text{Sn}(p,p')^{116}\text{Sn}^*$ at 201 and 133.8 MeV

Differential cross sections for inelastic proton scattering at 201 MeV to low-lying states in ^{116}Sn are shown in Fig. 7, middle panel. As for ^{90}Zr , prominent cross sections in the low-lying region reside on states of natural parity. Since the neutron shells are only partly filled, the normal-mode OBTD's are constructed in a $0\hbar\omega + 1\hbar\omega$ basis, using occupation numbers from Van der Werf et al. [52]. The cross section of the 2^+ state at 1.294 MeV is underestimated by a factor 2 by the normal mode, reflecting that this transition receives important contributions from yet higher major shells. Its shape is, however, reproduced reasonably.

The cross section of the 3^- state at 2.266 MeV is reproduced quantitatively, as was also the case for ^{90}Zr and ^{208}Pb .

Several 4^+ states are excited and most of the observed cross section is found on the states at 2.392, 2.529, 2.802 and 3.046 MeV, adding up to 1.06 times the normal-mode cross section.

Only one prominent 5^- state is observed, at 2.366 MeV. It carries a fraction 0.3 of the normal-mode cross section. Similarly the summed cross section of the 7^- states at

Fig. 9. Differential cross sections (left) and analyzing powers (right) for inelastic $(\bar{p}, p')$ scattering at 133.8 MeV to low-lying states in ^{116}Sn . The summed cross sections of the 4^+ states at 2.392, 2.529, 2.802 and 3.046 MeV and the 7^- states at 2.901 and 3.210 MeV are plotted as filled circles and the cross sections for the 2.392 MeV 4^+ and 2.908 MeV 7^- states as open circles. Unrenormalized ($S=0, T=0$) normal mode calculations are shown for the density-dependent G -matrix force, based on the Paris G -matrix [32] (solid curves) and for the Franey-Love t -matrix force [34] (dash-dotted curve). The dashed curves give the results for the Paris G -matrix including the normalization factors from Table 5.

2.908 and 3.210 MeV is smaller by a factor 0.3.

Inelastic-scattering data at 133.8 MeV with polarized protons are available from an earlier experiment [10]. In Fig. 9 the differential cross sections and analyzing powers are shown for the same states as given in Fig. 7 for the 201 MeV data. The solid curves indicate again the full normal-mode cross sections. The dashed curves are obtained with the same renormalization factors that were found from the 201 MeV data. For comparison the full normal-mode cross sections obtained with the Franey-Love

interaction [34] are given. The analyzing powers are reasonably described, though not as well as the differential cross sections. In general the minima in the data are more pronounced than in the calculations. This feature is to be attributed, as in the case of the 3^- state in ^{208}Pb , to contributions from the interior of the transition densities. The normal mode, by its construction, adds the different contributing amplitudes in such a way that they add coherently at the nuclear surface and beyond. The cancellation in the interior region, due to destructive interference of amplitudes with different numbers of nodes, is better the larger the basis from which the transition density is constructed.

4.2.3. $^{196}\text{Pt}(p,p')^{196}\text{Pt}^*$ at 201 MeV

The right-most panel of Fig. 7 shows the differential cross sections from 201 MeV proton scattering to the 2^+ , 3^- , 4^+ , 5^- and 7^- states in ^{196}Pt . The results resemble those for the corresponding states in ^{116}Sn , with the following differences: The cross section of the first 2^+ state is ten times larger than the normal-mode cross section, whereas it is only twice as large in the case of ^{116}Sn . This high degree of collectivity is also reflected in the $B(\text{E}2)$ value of $1.49(21) \times 10^4 e^2\text{fm}^4$ [17,20], which is also ten times larger than the normal-mode value. Since in turn, the normal mode exhausts the full E2-strength contained in a $1\hbar\omega$ basis, the transition density of the 2^+ state appears to receive coherent contributions from many higher shells. Another difference is the fragmentation of the 3^- cross section. While in ^{90}Zr and ^{116}Sn only the lowest 3^- state is prominent and has a cross section equal to the normal-mode value, in ^{196}Pt three about equally intense 3^- states are observed, which together carry less than the normal-mode cross section. For the 4^+ states, on the other hand, the situation is reversed: where in ^{116}Sn most of the normal-mode cross section is about evenly shared by four states, there are only two such 4^+ states, at 0.877 and 1.888 MeV, in ^{196}Pt .

The cross sections of the 5^- state at 1.271 MeV and the 7^- state at 1.374 MeV have reduction factors comparable to those in ^{116}Sn .

4.2.4. $^{208}\text{Pb}(p,p')^{208}\text{Pb}^*$ at 201 MeV

Differential cross sections for proton scattering off ^{208}Pb are shown in Fig. 10. The analysis of the 3^- state at 2.615 MeV has been shown before to agree with the $S = 0$, $T = 0$ normal mode. Three more 3^- states are known below 5.5 MeV and their summed cross section amounts to another 25% of the normal mode. For comparison the $S = 1$, $T = 0$ normal mode cross section is shown in the figure and found to be an order of magnitude smaller.

The summed cross sections on the 5^- states and that on the 7^- state are about half those of the $S = 0$, $T = 0$ normal modes, consistent with the picture for the other nuclei, discussed above. For the 7^- state also the $S = 1$, $T = 0$ normal mode is given for comparison. It does in this case agree in cross section. The analysis of the alpha scattering data in Subsection 4.3 will, however, indicate the scalar-isoscalar nature of this excitation. The right-hand panel of Fig. 10 shows the differential cross sections for the low-lying states of natural parity and even spin. The 2^+ and the summed 4^+ and 6^+

Fig. 10. Differential cross sections for transitions to low-lying natural-parity states in ^{208}Pb from (p,p') at 201 MeV, summed together for each spin-parity class. The solid curves represent $(S=0, T=0)$ normal-mode calculations. The dashed curves include the normalization factors given in Table 5. For the 3^- , 7^- and 8^+ states the unrenormalized results for $(S=1, T=0)$ normal-modes are indicated as dash-dotted curves.

cross sections are underestimated by a factor of 2 to 2.5 but agree well in shape. The 8^+ state at 4.610 MeV is quantitatively reproduced at forward angles, but the shape of the calculated angular distribution deviates around the second maximum and beyond.

4.3. Alpha scattering to low-lying states

Inelastic alpha-scattering experiments were performed at 120 MeV on the nuclei ^{90}Zr , ^{116}Sn and ^{196}Pt . An analysis along the same lines as done in the previous section for proton scattering requires that the absolute differential cross sections be compared with those of the normal modes in a microscopic formulation. One thus must adopt an effective alpha-nucleon interaction to allow for microscopic DWBA. As before, we use the code DW81. It is then required that the effective projectile-nucleon force be expressed as a superposition of Yukawa-type potentials. The simplest spin- and isospin-independent force is a single central Yukawa. Choosing its range $R = 1.414$ fm, V_0 is adjusted such that it reproduces quantitatively the cross sections of the lowest 3^- states in ^{90}Zr and ^{116}Sn and the sum of the lowest three 3^- states in ^{196}Pt . This fixes V_0 to a value of 71 MeV.

Fig. 11. Elastic proton- ^4He scattering cross section at 31 MeV from Ref. [53]. The solid line represents a fit with a complex 3-Yukawa t -matrix without exchange term. The dotted line gives this t -matrix, scaled to fit the lowest 3^- states in ^{90}Zr , ^{116}Sn and ^{196}Pt with $S = 0, T = 0$ normal-mode OBTD's. The dashed line is a real single-Yukawa t -matrix that also fits these 3^- states.

It is interesting to see to which extent this effective t -matrix describes the free proton-alpha interaction. Fig. 11 shows the p - ^4He elastic cross section, measured at the same centre-of-mass energy by Thompson et al. [53].

The single-Yukawa t -matrix that fits the 3^- states in ^{90}Zr , ^{116}Sn and ^{196}Pt (dashed line) is seen to give too high a free cross section. The uprise of the data points towards larger angles is naturally reproduced in an exact solution of the potential scattering problem, also using a single-Yukawa scattering potential. In a single collision picture it is associated with the exchange term. If the single Yukawa is, however, meant to be the t -matrix as required in DW81, an exchange term for the alpha-nucleon interaction is meaningless and the direct term alone cannot give the observed rise at backward angles.

We have attempted, in a non-exhaustive search, to find a complex 3-range Yukawa fit of the t -matrix that would reproduce to some extent the increase of the free cross section towards larger angles. The result, for which the cross section is shown in Fig. 11 as the solid curve, has the form:

$$V_{\text{eff}}(r) = \sum_k (V_k + iW_k) \frac{\exp(-r/R_k)}{r/R_k} \quad (10)$$

with parameters $R_1 = 1.4$ fm, $V_1 = -50$ MeV, $W_1 = -14.3$ MeV, $R_2 = 0.6$ fm, $V_2 = -400$ MeV, $W_2 = -300$ MeV, $R_3 = 0.2$ fm, $V_3 = 7500$ MeV and $W_3 = 7500$ MeV.

A best fit of the cross sections of the lowest 3^- states in ^{90}Zr and ^{116}Sn and the sum of the lowest three 3^- states in ^{196}Pt requires this t -matrix to be scaled up by a factor 1.8.

Fig. 12. Inelastic alpha-scattering cross sections for low-lying states in ^{90}Zr , ^{116}Sn and ^{196}Pt . The solid curves are unrenormalized ($S = 0, T = 0$) normal-mode calculations, using the effective $3Y \alpha - N$ force derived from inelastic $p\text{-}^4\text{He}$ scattering, scaled up by a factor 1.8. The dashed curves include the normalization factors given in Table 5.

In Fig. 12 the differential cross sections for the same states as studied in proton scattering (see Fig. 7) are shown. The solid curves represent the calculated normal-mode differential cross sections for the scaled complex 3-Yukawa t -matrix, without further normalization. The results for the real single-Yukawa t -matrix are nearly identical for angles larger than 10° and are not shown separately.

The dashed curves have been obtained by applying the normalizations obtained from proton scattering. In all cases the shapes of the angular distributions are correct.

The agreement in shape and absolute value for all cross sections is striking. Only for the highest multiplicities, the 7^- states in ^{116}Sn and the 5^- and 7^- states in ^{196}Pt , the calculated cross sections underestimate the data. We suggest that this may be due to the neglect of a spin-orbit term in the effective alpha- p t -matrix. This term cannot be included since the LS force of a spin-1/2 with a spin-0 particle cannot be cast in the form, required by the program DW81, which is written for spin-1/2 particles.

Though the construction of the effective alpha- p interaction is highly phenomenological, it is gratifying that it does not severely deviate from the free interaction. Whether the increase in strength by 80%, needed to fit the inelastic scattering data, is to be expected on theoretical grounds as arising from in-medium effects, is not known.

We conclude that the microscopic description of the alpha scattering in terms of an effective t -matrix and normal modes seems adequate to serve in the later analysis of the $1\hbar\omega$ bump in Section 5.

4.4. Electron scattering to low-lying states

The data on low-lying states in ^{196}Pt have been published earlier. The 4^+ states have been analyzed in the framework of the interacting boson model including the g-boson [16]. An analysis in terms of a particle-phonon coupling model including core excitations of many multiplicities has been given by Ponomarev et al. [17].

In this subsection we present an analysis of data on ^{116}Sn and ^{196}Pt in terms of the $S = 0, T = 0$ normal modes to further assess its consistency with the proton- and alpha-scattering data analyses. As observed in the analysis of the 3^- state at 2.615 MeV in

Fig. 13. Electron-scattering form factors for low-lying natural-parity states in ^{116}Sn and ^{196}Pt . The solid curves have been obtained for $(S = 0, T = 0)$ normal modes without renormalization, but with effective charges $e_p = 1.5e$ and $e_n = 0.5e$. The dashed curves are renormalized with the factors listed in Table 5. Dash-dotted curves, for the 3^- states only, indicate the normal mode form factors obtained without effective charges.

^{208}Pb , made at the beginning of Section 4, the most collective isoscalar $S = 0$ excitations of low multipolarity may have $B(\text{EL})$ values that exceed well the strength contained in the $0\hbar\omega + 1\hbar\omega$ basis. It is then necessary to introduce effective charges. In order to avoid a proliferation of parameters, we will, as in our analysis of the 3^- state in ^{208}Pb , use only one additive charge δe such that $e_{\text{eff}}(\text{p}) = e + \delta e$ and $e_{\text{eff}}(\text{n}) = \delta e$. This will constitute a meaningful approach to the later analysis of the $1\hbar\omega$ bump in Section 5, if the analysis of the collective low-lying states shows that the value of δe does not behave erratically. In Fig. 13 the form factors for the low-lying states in ^{116}Sn and ^{196}Pt are shown. The calculations were done with the code WSAXE [42]. A common additive charge $\delta e = 0.5e$ fits the lowest 3^- state in ^{116}Sn and the sum of the three lowest 3^- states in ^{196}Pt .

Form factors obtained with free charges are indicated by dash-dotted curves and are seen to seriously underestimate the form factor data.

For all multiplicities the form factors for the unnormalized normal modes, using the same additive effective charge of $0.5e$, are indicated as solid curves. Dashed curves include the normalization factors found from the analysis of the proton-scattering data and are shown when renormalization is required. It is observed from Fig. 13 that the agreement is fair, both in shape and in absolute scale. For the analysis of the bump region it seems therefore justified to use the additive effective charge of $0.5e$ for all multiplicities.

5. The $1\hbar\omega$ excitation energy region

Inelastic proton-scattering spectra on ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb at $E_p = 201$ MeV are shown in Fig. 14. The clear enhancement, visible in each spectrum between the dashed markers, is associated with $1\hbar\omega$ excitations. The centroids of these structures appear at excitation energies some 1.5 MeV lower than the usual estimate of $41 A^{-1/3}$ MeV. In the electron-scattering spectra they appear at the same position as in the proton spectra and are similar in shape. However, in the alpha-scattering spectra (Figs. 1, 4 and 5), the corresponding bump lies significantly lower.

5.1. Analysis of inelastic proton data in the $1\hbar\omega$ region

The right-hand panel of Fig. 14 shows the angular distributions for the regions between the dashed markers. The lower markers have been chosen in each nucleus (except for ^{208}Pb) so as to exclude from the interval the prominent low-lying collective states, which have been analyzed separately in Section 4 in terms of $S = 0, T = 0$ normal modes. The upper marker has been put at an excitation energy of $50 A^{-1/3}$ MeV for all four nuclei and marks the local minimum between the $1\hbar\omega$ and $2\hbar\omega$ bumps. By this choice the interval excludes most of the known giant monopole, dipole and quadrupole resonances.

Calculations of the contributions from two-step processes in this excitation energy domain [27,28,54] show that they are negligible, and will at most amount to a few

Fig. 14. Left panel: spectra for 201 MeV proton scattering off ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb at $\theta_{\text{lab}} = 29^\circ$. Right panel: Differential cross sections for the $1\hbar\omega$ -bump regions, indicated in the spectra by vertical dashed markers.

percent of the total cross section.

The angular distributions in Fig. 14 show a featureless monotonously decreasing behavior. This is consistent with a composition from a wide range of angular momenta. The analysis of the cross sections in the adopted $1\hbar\omega$ interval is given in terms of the incoherent particle-hole sum and in terms of isoscalar normal modes.

5.1.1. Incoherent sum of $1p1h$ configurations

The view behind this approach is that the full cross section contained within a basis of $1p1h$ final states, can, regardless of the particular linear orthogonal combinations in which they appear as eigenstates, be obtained as the incoherent sum, $\sum \text{ph}$, defined in Subsection 3.1.

The spin-parity class $I^\pi = 1^-$ needs a special treatment to cope with the spurious centre-of-mass motion. This motion is represented by the normal mode of the operator rY_1 . Also the GDR, the response of rY_1t_z , is excluded. The incoherent sum of particle-hole cross section has been evaluated over all states in the $1\hbar\omega$ basis that are orthogonal to these normal modes.

In as far as $1\hbar\omega$ $1p1h$ cross section has been found to reside on low-lying states outside the interval, the corresponding cross section has also to be subtracted out. Thus, using the fractions $W_{I\pi} = \sigma/\sigma_{\text{NM}}$ of the normal-mode cross sections as found on these states in the previous section (Table 5) to a maximal value of $W_{I\pi} = 1$, one has:

$$\sum' \text{ph} = \sum \text{ph} - \sum_{\text{NM}, I\pi} W_{I\pi} \frac{d\sigma(\text{NM}, I\pi)}{d\Omega} \quad (11)$$

Fig. 15. The differential cross section of the $1\hbar\omega$ -bump region in ^{208}Pb , for 201 MeV proton scattering, compared with the incoherent sum Σ_{ph} (left) and the sum of $T = 0$ normal modes Σ_{NM} (right). Calculations are shown for the Franey-Love t -matrix force [34], the Bonn G -matrix [33] and for the Paris G -matrix [32].

to compare the cross sections within the “ $1\hbar\omega$ interval” with.

In Fig. 15, left panel, we show the result of this approach for ^{208}Pb . In this case, where the bump region appears still as a collection of individual states, the distinction between a bump region and a low excitation-energy region is not made and the summation includes all excited states up to $E_x = 8.65$ MeV.

The incoherent summation of the angular distribution contains the more than 350 possible particle-hole states. Curves are shown for three different effective interactions, the FL85 [34] interaction, the Bonn G -matrix [33] and the Paris G -matrix [32]. For all three interactions the total cross section is too high and the slope of the angular distribution does not correspond to that of the data. The results with the FL85 interaction and the Bonn G -matrix are very similar, but markedly different from that of the Paris G -matrix. The latter reproduces much better the slope of the experimental data.

Kelly et al. [55], have found, in an analysis of inelastic proton scattering from ^{16}O , that the Paris-Hamburg interaction gives a better description of the cross sections and especially the analyzing powers for natural-parity states than does the Nakayama-Love parametrization of the Bonn potential.

Our own analysis, in Section 4, of low-lying natural-parity states shows that the cross sections for these states may differ indeed in shape, but hardly in magnitude. This is

in agreement with the analysis of Kelly et al. However, the difference in the summed incoherent cross sections between, on the one hand the Paris potential and, on the other, the FL85 force and the Bonn potential is far greater. It is not only the slope that differs, but also the magnitude of the cross section. The same pattern as shown in Fig. 15 (left) for the total incoherent cross section is found to persist in all spin-parity classes separately, except for the very highest spins.

5.1.2. Sum of normal modes

The fact that the experimental angular distribution is steeper, than that from the incoherent sum over all spins, might imply that the response of the nucleus is softer than that of an incoherent sum of $1\hbar\omega$ excitations. The earlier observation that the centroid energy of the bump was lower than $1\hbar\omega$ seems to support this. The alternative approach will therefore be to compare the bump cross section to that of a sum of normal modes.

Normal modes were introduced in Section 3 as the responses to operators $r^L Y_L(\hat{r})$ and $r^L [\sigma \otimes Y_L(\hat{r})]_J$. By this definition these excitations have isoscalar character.

Our analysis will for the moment be in terms of isoscalar normal modes only, since isovector excitations are expected to lie higher in excitation energy. We will, however, discuss this point later on.

As for the incoherent summation over particle-hole states, the cross section of the bump region should be compared with the sum over normal modes, from which the cross section, located on the low-lying collective states, has been subtracted:

$$\sum' \text{NM} = \sum \text{NM} - \sum_{\text{NM}, I\pi} w_{I\pi} \frac{d\sigma(\text{NM}, I\pi)}{d\Omega}. \quad (12)$$

Again the $I^\pi = 1^-$ class needs a special treatment. After taking away from the particle-hole basis the spurious center-of-mass motion and the GDR, no meaningful collective 1^- state exists. Yet, the remaining cross section is important at small angles, in the case of alpha scattering even dominant. We therefore adopt, both in the sums $\sum \text{ph}$ and $\sum \text{NM}$, this remaining incoherent cross section as the 1^- contribution.

In the right panel of Fig. 15 the $^{208}\text{Pb}(p, p')$ data are compared to the sum over all isoscalar normal-mode cross sections, corrected for their overlaps as described in Subsection 3.1.

The agreement of the Paris–Hamburg interaction is good. The FL85 and the Bonn interactions overshoot the data by a factor 2, but otherwise describe the slope much better than in the case of the incoherent sum over particle-hole states.

In Fig. 16 the contributions of the separate normal modes are shown, indicating that the difference between the Paris potential on the one hand and the FL85 and Bonn interactions on the other is mostly due to a much smaller contribution of the $S = 1$, $L = J$ (spin-flip natural-parity) normal mode in the former. The differences for the $S = 0$, $L = J$ normal modes are less, as already noted in the analysis of the low-lying natural-parity states in Section 4.

Fig. 16. The same data as in Fig. 16. Calculations of $T = 0$ normal-mode sums are shown for the three NN forces, split out into the classes $(S = 0, J = L)$, $(S = 1, J = L)$, $(S = 1, J = L - 1)$ and $(S = 1, J = L + 1)$.

5.1.3. Comparison between the incoherent sum and the normal-mode sum

The incoherent and normal-mode cross sections have been calculated for the four nuclei, ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb , using the Paris–Hamburg G -matrix and are shown in Fig. 17.

The picture that emerges is quite consistent: the normal-mode cross sections reproduce well both the magnitudes and the slopes of the differential cross sections.

The incoherent $1p1h$ curves do not have the right slope for ^{208}Pb , as shown before, and the same is true for ^{196}Pt . For ^{116}Sn and ^{90}Zr on the other hand their slopes are nearer to those of the data.

In all cases the incoherent sum overestimates the data by more than a factor 2, consistent with the notion that the missing fraction of the $1p1h$ cross sections is to be found above the adopted intervals.

The effect of subtracting out the cross section of the normal modes, associated with low-lying states below the interval of the bump, is never large and affects only the smaller angles.

The major difference between the two approaches is that the normal modes are surface-peaked in r -space by the canceling of the many amplitudes with different numbers of radial nodes in the interior region. This suggests that the high-momentum cross section, present in the incoherent particle hole sum, $\sum ph$ must be moved away upwards from the $1\hbar\omega$ region, leaving the collective normal modes behind.

5.1.4. Role of the effective NN interaction

At this point the differences between the effective nucleon–nucleon interactions, available for such a microscopic analysis, are noteworthy. The best agreement in magnitude and shape are obtained for the density dependent Paris–Hamburg G -matrix interaction [32]. The results for the G -matrix version of the Bonn potential, due to Nakayama

Fig. 17. Differential cross sections from (p,p') at 201 MeV for the $1\hbar\omega$ region in ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb . Left: the incoherent particle-hole sum, without and with correction for the cross section on lower-lying states, indicated respectively as Σ_{ph} and Σ'_{ph} . Right: as the left panel, but for the $T = 0$ normal mode-sums Σ_{NM} and Σ'_{NM} . All calculations are made with the Paris G -matrix.

and Love [33] and the t -matrix of Franey and Love [34] give clearly higher cross sections and do not agree with the data in slope.

Though the results of the Paris-Hamburg potential seem to do better, it remains a bit of a puzzle, why this is so. The major difference between the Paris potential on the one hand and the Bonn potential and the FL85 interaction on the other, is that the Paris potential includes to some extent off-shell effects of the t -matrix by averaging over the partial-wave matrix elements. As a consequence of this averaging, Von Geramb's interaction does not have the correct limit when the Fermi momentum approaches zero, i.e., does not reproduce correctly the free NN scattering t -matrix. This is particularly true in the tensor components which are much stronger in the Paris G -matrix than in the other two interactions. Off-shell effects have been ignored in the construction of the Bonn and the FL85 interactions.

It should be emphasized here that the differences between the interactions are largely due to the different approximations that have been used in constructing effective interactions by different groups and not their intrinsic differences.

Empirically we are able to localize the main difference between the Paris G -matrix

Fig. 18. Differential cross section of the $1\hbar\omega$ region in ^{90}Zr from (α, α') at 120 MeV. Left: the uncorrected particle-hole and normal-mode sums and the separate contributions of the multipoles $L = 1-4$. Right: the normal-mode sum $\Sigma' \text{NM}$, which is corrected for the normal-mode cross section observed below the chosen interval and the sum $\Sigma'' \text{NM}$, which in addition contains the additional quadrupole and octupole (LEOR) cross section, known to reside inside the interval. All calculations were made with the complex 3Y force, discussed in the text.

and the two other interactions in the cross sections from $(S = 1, I = L)$ transitions (Fig. 16).

5.2. Comparison with different probes

For ^{90}Zr , ^{116}Sn and ^{196}Pt data have also been taken for inelastic alpha-scattering at 120 MeV. Electron scattering data are available for ^{196}Pt . This allows for a further test of the normal-mode and incoherent sum descriptions of the cross sections in the bump region.

The effective alpha-nucleon interaction used in the DWBA calculations was established from an analysis of the low-lying states in these nuclei and has the form of a complex 3-range central Yukawa potential, as discussed in Subsection 4.3.

5.2.1. $^{90}\text{Zr}(\alpha, \alpha')$

In Fig. 18 (left) the differential cross section for inelastic alpha scattering is shown for the bump region in ^{90}Zr , together with the calculated curves for $S = 0, T = 0$ normal modes and for the incoherent sum over 1p1h cross sections. The difference between

the two curves is minor and this is valid not only for their sum over the different spin classes, but for each spin class separately. The fact that all cross section of a given spin resides on the normal mode forms therefore a justification of the customary procedure, where $BE(\lambda)$ values are derived from an assumed proportionality with inelastic alpha-scattering cross sections.

In view of this near-identity of the incoherent ph-sums and the normal modes, we will hereafter show only the results for the latter.

The contributions of the full $L = 1-4$ cross sections are shown in the left panel. They indicate that the total cross section is dominated by the lower multipolarities to a much larger extent than in the case of the (p,p') reaction.

From the work of Poelheken et al. [56,57] we know that a considerable amount of isoscalar 1^- cross section lies inside the adopted interval and we have included their 0° data point in Fig. 18.

The right-hand panel of Fig. 18 shows the normal-mode sum, indicated as a singly-primed sum, for which the cross sections on low-lying states have been subtracted out.

Guided by the results of the (p,p') reaction we have assumed the 3^- strength to be exhausted by the lowest state at 2.748 MeV and have therefore excluded it from the bump region. However, from the work of Moss et al. [25,26], who studied inelastic alpha scattering at very forward angles, it is known that the low-energy octupole resonance (LEOR) lies inside the interval that we study here, with an unweighted strength equal to that of the low-lying 3^- state at 2.748 MeV.

Similarly Fujita et al. [23] find inside the $1\hbar\omega$ interval an amount of E2 strength of about 90% of that residing on the 2.186 MeV state. We observe in the present work the 2^+ state at 3.843 MeV, which carries the largest single E2-strength next to the 2.186 MeV state.

The effect of adding back these cross sections for the quadrupole and octupole excitations, in proportion to their observed strengths, is shown by the solid curve in the right-hand panel of Fig. 18, and is labeled Σ'' . It approaches closely the experimental data points.

For the (p,p') reaction the corresponding cross section is intermediate between the full normal-mode sum (Σ NM) and the sum, corrected for the normal-mode cross section on low-lying states (Σ' NM) and is not shown separately in Fig. 17.

5.2.2. $^{116}\text{Sn}(\alpha, \alpha')$ and $^{116}\text{Sn}(\vec{p}, p')$

The data for the (α, α') reaction and the (p,p') reaction with polarized protons at 133.8 MeV are shown in Fig. 19. The excitation-energy interval studied in the latter experiment does not extend over the full bump region and the cross section has been scaled up, invoking a similarity in shape with the 201 MeV data.

The top panel shows the three normal-mode sums, Σ , Σ' and Σ'' . For the second, the normal-mode cross sections have been subtracted out for low-lying states with weight factors as given in Table 5. For the doubly-primed sum the additional contribution of the

Fig. 19. Top: differential cross section of the $1\hbar\omega$ region in ^{116}Sn from (α, α') with the normal mode sums ΣNM , $\Sigma'\text{NM}$ and $\Sigma''\text{NM}$. Middle: differential cross section of the same region from $(\bar{p}, p')$ at 133.8 MeV, with the sums $\Sigma'\text{ph}$ and $\Sigma'\text{NM}$ obtained with the Paris G -matrix. Bottom: analyzing power from 133.8 MeV proton scattering as above.

LEOR has been summed back. Its strength is, as for ^{90}Zr , equal to that of the lowest 3^- state [26] and its cross section corresponds therefore to that of the full normal mode.

For the (p, p') cross section at 133.8 MeV, the picture is identical to that at 201 MeV. The shapes of the incoherent-sum and the normal-mode calculations are in fair agreement with the data and the incoherent sum overshoots them by the same factors as in the 201 MeV case. Only the singly-primed cross sections, corrected for the low-lying states, are presented.

The analyzing power of the bump is shown in the lower panel of Fig. 19. The agreement of both the incoherent $1p1h$ -sum and the normal-mode sum (Σ') is good, in the case of the normal-mode sum even quantitative. The difference between the predictions is, however, not large enough to make a decision against the incoherent-sum scenario.

Fig. 20. Electron-scattering (top) and (α, α') (bottom) differential cross sections for the $1\hbar\omega$ region in ^{196}Pt with incoherent particle-hole and normal-mode sums.

5.2.3. $^{196}\text{Pt}(\alpha, \alpha')$ and $^{196}\text{Pt}(e, e')$

The alpha-scattering data on ^{196}Pt are shown in the lower panel of Fig. 20. The weight factors, applied for the cross sections on low-lying states are listed in Table 5. It should be noted that only 42% of the 3^- normal-mode strength is subtracted out in this case. Moss et al. [26] do not give the strength of the LEOR in the case of ^{196}Pt . However, the LEOR appears to be a general feature of all nuclei. From their work, the strength of the LEOR is found systematically with a cross section that is close to that of the 3^- normal mode. We have added a curve, labeled by Σ'' , to indicate the effect of adding back this cross section. This curve, hardly different from the uncorrected sum, (Σ), shows good agreement with the data.

The (e, e') data from NIKHEF-K were available as raw spectra, not corrected for radiation tails. Comparison of these spectra with unfolded spectra, showed that the radiation tails of excitations outside the excitation energy region of the bump amounts

to at most a few percent for the total measured cross section. Therefore, no background due to processes other than direct excitation has been subtracted, and the continuum observed in this excitation energy region is considered to be due entirely to unresolved excitations of a 1p1h nature.

The top panel of Fig. 20 shows the electron-scattering form factor. It was found in Subsection 4.4 that the form factors of the low-lying states could be well described by normal-mode transition densities. However, it was observed that effective charges were needed to obtain the same normalizations as in proton scattering and values of $e_{\text{eff}}(p) = 1.5e$ and $e_{\text{eff}}(n) = 0.5e$ have been used.

The incoherent sum of 1p1h form factors is shown in the figure for these effective charges. It overshoots the data at large momentum transfer. The use of free charges is definitely inadequate and the corresponding incoherent sum severely underestimates the data.

The sums $\sum'$ and $\sum''$ of all $T = 0$ normal-mode form factors are shown in Fig. 20, again for the adopted effective charges. The magnitudes come out too low by a factor 2, while the slopes are correct.

6. Summary, discussion and conclusions

6.1. The $1\hbar\omega$ region

In this work we present data available to us from the inelastic scattering reactions (p, p') , (α, α') and (e, e') on the nuclei ^{90}Zr , ^{116}Sn , ^{196}Pt and ^{208}Pb .

The data give evidence for the systematic occurrence, in all nuclei, of a cross-section concentration around an excitation energy slightly lower than $1\hbar\omega$, which persists up till high momentum transfer.

The experimental cross sections, integrated over an interval around this bump, have been compared with the incoherent sum over all microscopic cross sections of the single particle excitations that span a $0+1\hbar\omega$ basis. For the inelastic proton scattering data the incoherent sum of 1p1h excitations is found to exceed the data by a factor 2–2.5. Quite reasonably, this suggests that the remaining $0+1\hbar\omega$ cross section must extend upwards from the interval of the peak cross section.

However, some subtle amendments to this scenario are needed: we find that the peaks of the cross sections appear to be shifted downward in energy, compared with the unperturbed $1\hbar\omega$ estimate (see Table 2) and that the slopes of the (p, p') cross sections are steeper than the incoherent 1p1h sum would predict them to be.

These observations suggest the presence of some degree of collectivity. Indeed, the well-known collective states of natural parity, lower in excitation energy than the bump region, are also to be associated with $0+1\hbar\omega$ strength. It has been demonstrated in Section 4 that these low-lying states of natural parity can be conveniently analyzed in terms of isoscalar normal modes, coherent linear superpositions of the basis states that exhaust the full strength of an isoscalar multipole or spin-multipole operator. As

Fig. 21. The incoherent particle-hole sum and the $T = 0, 1$ and $0+1$ normal-mode sums for (p, p') at 201 MeV off ^{208}Pb . All calculations have been made with the Paris G -matrix.

an alternative to the incoherent-sum picture of the $0+1\hbar\omega$ region, we have therefore attempted a description in terms of a sum over isoscalar normal modes, not only for the lower spins, but for all spin-parity classes. This gives a near quantitative description for the (p, p') cross sections, provided that the Paris G -matrix interaction is used.

In Fig. 21 the sums of all isoscalar and all isovector normal-mode cross sections are shown for inelastic proton scattering at 201 MeV from ^{208}Pb . They are of the same magnitude and add up to nearly the full incoherent sum.

We infer from these analyses that, whilst the incoherent $0+1\hbar\omega$ sum model must be considered as globally valid, the cross sections lowest in excitation energy tend to appear as isoscalar normal modes and it is this part of the cross section that stands out as a bump over the spectrum. Indeed, the isovector cross section is expected to lie higher in excitation energy as exemplified by the most collective of them, the GDR, which is situated at $E_x = 13.5$ MeV in ^{208}Pb .

The isoscalar cross sections, associated with the lower multipolarities notably $I^\pi = 2^+$ and 3^- , are in some cases found with cross sections and strengths that exceed not only the normal mode values, but also the cross section (or strength) contained in the full basis. The most drastic example is the lowest 2^+ state at 0.356 MeV in ^{196}Pt , that has ten times the normal-mode cross section.

Likewise, in all nuclei that we studied here, the lowest 3^- state is found with a cross section that the normal mode predicts, but an equally large cross section must

be attributed to the LEOR. This phenomenon points to a core polarization effect that causes cross section (and strength) from configurations beyond the $0+1\hbar\omega$ basis to add significantly to these low-lying states. This core polarization becomes especially evident in the analysis of the electron scattering data on ^{116}Sn and ^{196}Pt , where effective charges were needed to account for the observed magnitudes of the form factors. The effect may be larger yet than assumed in the present analysis. Though consistent in shape, the normal mode calculations (Fig. 20) still underestimate the form factor of the $1\hbar\omega$ region by a factor 2.

Finally, the present analysis may have some relevance to the issue of chaos in nuclei [59]. The $1\hbar\omega$ incoherent cross section sum is independent of the interaction matrices for each spin class and these matrices could as well be random for that sake. On the other hand, normal modes reflect a separability of the matrices that arises when the residual particle–hole interaction is of the multipole–multipole type. While for high spins the occurrence of strong collectivity has been established for multi-particle multi-hole excitations forming rotational bands, the observations of collectivity for $1p1h$ excitations have mostly been restricted to lower multipolarities. The agreement with the normal mode analysis at higher momentum transfer, contrasted by the wrong slope of the incoherent sum model in the analysis of the (p,p') and (e,e') data, suggests that collectivity exist in the $1\hbar\omega$ region also for the higher spins.

6.2. Inelastic alpha-scattering

We have used an effective alpha–nucleon interaction to enable the use of a microscopic description of the (α, α') reaction, thereby making its analysis compatible with that of the (p,p') and (e,e') reactions. This effective alpha–nucleon interaction has been derived from elastic proton– ^4He scattering at the same centre-of-mass energy, as described in Subsection 4.3 and scaled by a factor 1.8 to give the same normalization for the lowest 3^- states as found from the (p,p') reaction. In view of the fact that the density corrections in the NN force are already important, this rescaling should not worry us too much. It is rewarding to find that, once this scaling is made, all normalizations for the low-lying states come out the same as for the (p,p') reaction.

The (α, α') cross sections decrease with increasing multipolarity much faster than those of the (p,p') reaction. It is therefore in particular the (α, α') data from which one can conclude the presence, in the $1\hbar\omega$ region, of additional low-multipolarity cross section, besides the known LEOR.

The calculations suggest that at 0° the (α, α') cross section is almost entirely due to the isoscalar dipole. The full non-spurious 1^- cross section in the $1p1h$ basis is found to correspond within errors to the cross section observed by Poelheken et al. [56,57] in the case of ^{90}Zr .

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References

- [1] F. Petrovich et al., *Phys. Lett. B* 91 (1980) 27.
- [2] R.A. Lindgren et al., *Nuclear structure at high spin, excitation and momentum transfer*, ed. H. Nann, AIP Conf. Proc. vol. 142 (New York, 1985) p. 133.
- [3] A. van der Woude, *Electric and magnetic resonances in nuclei*, ed. J. Speth (World Scientific, Singapore, 1991) p. 99.
- [4] M. Pignanelli et al., *Nucl. Phys. A* 540 (1992) 27.
- [5] Y. Fujita et al., *Phys. Rev. C* 40 (1989) 1595.
- [6] J. Speth and A. van der Woude, *Rep. Prog. Phys.* 44 (1981) 719.
- [7] C. Gaarde, *J. de Phys.* 45 (1984) C4-115.
- [8] B.D. Anderson et al., *Phys. Rev. C* 43 (1989) 1630.
- [9] S.Y. van der Werf et al., *Nucl. Phys. A* 496 (1989) 305.
- [10] S.Y. van der Werf et al., *Phys. Lett. B* 166 (1986) 372.
- [11] P. de Heijer, Master's thesis, University of Amsterdam (1988).
- [12] S.Y. van der Werf et al., *Phys. Rev. C* 36 (1987) 1796.
- [13] J.M. Schippers, Ph.D. Thesis, University of Groningen (1988).
- [14] S. Galès et al., *Phys. Rev. Lett.* 48 (1982) 1593.
- [15] J.M. Schippers et al., *Nucl. Phys. A* 548 (1992) 271.
- [16] W.T.A. Borghols et al., *Phys. Lett. B* 152 (1985) 330.
- [17] V. Yu. Ponomarev et al., *Nucl. Phys. A* 549 (1992) 180.
- [18] M. Morlet and A. Willis, Institut de Physique Nucléaire-Orsay Internal, report IPNO-PHN-7915 (1979).
- [19] C. de Vries et al., *Nucl. Instr. Meth.* 223 (1984) 1.
- [20] W.T.A. Borghols, Master's thesis, University of Amsterdam (1983).
- [21] A.G. Drentje, H.A. Enge and S.B. Kowalski, *Nucl. Instr. Meth.* 122 (1974) 485.
- [22] J.M. Schippers, W.T.A. Borghols and S.Y. van der Werf, *Nucl. Instr. Meth. A* 247 (1986) 467.
- [23] Y. Fujita et al., *Phys. Rev. C* 32 (1985) 425.
- [24] Y. Fujita et al., *Phys. Rev. C* 45 (1992) 993.
- [25] J.M. Moss et al., *Phys. Rev. Lett.* 37 (1976) 816.
- [26] J.M. Moss et al., *Phys. Rev. C* 18 (1978) 741.
- [27] R. Bonetti and C. Chiesa, Multistep code (unpublished).
- [28] H. Condé et al., *Nucl. Phys. A* 545 (1992) 785.
- [29] A. Bohr and B.R. Mottelson, *Nuclear structure*, vol. 2 (Benjamin, New York, 1975).
- [30] J.R. Comfort, Program DW81, updated 1983, extended version of DWBA70, R. Schaeffer and J. Raynal.
- [31] J. Raynal, Program DWBA91, distributed through NEA Data Bank under ID: 1209/02.
- [32] H.V. von Geramb and K. Nakano, *Interactions between medium energy nucleons in nuclei*, ed. H.O. Meyer, AIP Conf. Proc., vol. 97 (New York, 1983) p. 44.
- [33] K. Nakayama and W.G. Love, *Phys. Rev. C* 38 (1988) 51.
- [34] M.A. Franey and W.G. Love, *Phys. Rev. C* 31 (1985) 488.
- [35] L.R. Kouw, H.P. Blok and J. Blok, *Nucl. Phys. A* 481 (1988) 13.

- [36] A. Nadasen et al., Phys. Rev. C 23 (1981) 1023.
- [37] C. Djalali, N. Marty, M. Morlet and A. Willis, Nucl. Phys. A 380 (1982) 42.
- [38] F.E. Bertrand et al., Phys. Rev. C 22 (1980) 1832.
- [39] L.W. Put and A.M.J. Paans, Phys. Lett. B 49 (1974) 266.
- [40] I. Brissaud et al., Phys. Rev. C 6 (1972) 595.
- [41] D.A. Goldberg et al., Phys. Rev. C 7 (1976) 1938.
- [42] J. Heisenberg, Programs WSAXE and WSAXM.
- [43] E. Hagberg, A. Ingemarsson and B. Sundqvist, Phys. Scripta 3 (1971) 245.
- [44] L. Lee et al., Nucl. Phys. A 492 (1989) 607.
- [45] D. Goutte et al., Phys. Rev. Lett. 45 (1980) 1618.
- [46] A. Scott et al., Indiana University Cyclotron Facility, Ann. Rep. (1977) p. 99.
- [47] T.E. Milliman et al., Phys. Rev. C 32 (1985) 805.
- [48] J. Heisenberg et al., Phys. Rev. C 29 (1984) 97.
- [49] M.G.E. Brand et al., Nucl. Phys. A 531 (1991) 253.
- [50] J.W.A. den Herder et al., Nucl. Phys. A 490 (1988) 507.
- [51] C.N. Papanicolas, V.R. Pandharipande and J. Wambach, Phys. Rev. Lett. 53 (1984) 1133.
- [52] S.Y. van der Werf et al., Nucl. Phys. A 289 (1977) 141.
- [53] G.E. Thompson, M.B. Epstein and T. Sawada, Nucl. Phys. A 142 (1970) 571.
- [54] R.D. Smith and M. Bozoian, Phys. Rev. C 39 (1989) 1751.
- [55] J.J. Kelly et al., Phys. Rev. C 39 (1989) 1222.
- [56] T.D. Poelhekkens, Ph.D. Thesis, University of Groningen (1989).
- [57] T.D. Poelhekkens et al., Phys. Lett. B 278 (1992) 423.
- [58] N. Blasi, Ph.D. Thesis, University of Groningen (1984).
- [59] O. Bohigas and H.A. Weidenmüller, Ann. Rev. Part. Sci. 38 (1988) 421.