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Quantification

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DOI

[10.1007/978-1-4020-9729-4_425](https://doi.org/10.1007/978-1-4020-9729-4_425)

Publication date

2011

Document Version

Submitted manuscript

Published in

Encyclopedia of Medieval Philosophy

[Link to publication](#)

Citation for published version (APA):

Dutilh Novaes, C. (2011). Quantification. In H. Lagerlund (Ed.), *Encyclopedia of Medieval Philosophy: philosophy between 500 and 1500* (Vol. 2, pp. 1093-1096). Springer.
https://doi.org/10.1007/978-1-4020-9729-4_425

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Medieval Theories of Quantification*

Catarina Dutilh Novaes

Abstract

Medieval authors approached the semantic phenomenon now known as quantification essentially by means of the concept of supposition, more specifically with the different modes of personal supposition. The modes of personal supposition were meant to codify the quantificational behavior of what we now refer to as quantifier expressions, and what the medievals referred to as syncategorematic terms. Perhaps the best way to understand the medieval approach to these quantifier expressions by means of the notion of supposition is as a two-step procedure that explicates their meaning and behavior. First, the syntactical structure of the proposition, i.e. the presence and order of its syncategorematic terms, determines the kind of personal supposition that each categorematic term has. Then, the semantic definitions of each mode of personal supposition determine the effect of quantifying syncategoremata over the quantity of objects involved in the assertion of a proposition. This entry discusses both groups of rules, and the contrasting 13th century and 14th century approaches. The former is based on the verification of propositions and focuses on the semantics of quantifier expressions taken individually; the latter focuses on the inferential relations of ascent and descent between propositions with quantifying syncategorematic terms and singular propositions of the form 'This a is b', and on the study of the global quantificational effect of syncategorematic terms in wider propositional contexts.

The phrase 'medieval theories of quantification' is, properly speaking, an anachronism; medieval authors never used the term 'quantification' in this sense, and even though they did treat semantic phenomena similar to what we now refer to as quantification, their theories differ from modern theories of quantification in significant aspects, to the point that this approximation may even be unwarranted (Matthews, 1973). Nevertheless, their treatments of such phenomena are often insightful and sophisticated, justifying thus that we look at them from the perspective of modern theories of quantification, but provided that the term 'quantification' be understood very broadly.

Broadly understood, quantification can be defined as the construct or procedure by means of which one specifies the quantity of individuals of the domain of discourse that apply to or verify a given statement. Typical quantifier expressions are 'Some', 'All', 'None', and they usually determine the quantity of individuals involved in an assertion. Medieval authors approached quantification and quantifier expressions essentially by means of the concept of supposition, more specifically with the different modes of personal supposition.

Besides supposition, they also approached quantificational phenomena from the vantage point of their theories of syllogisms, following the traditional Aristotelian approach. However, it is widely acknowledged that medieval authors did not have much to contribute to Aristotle's theory of syllogism for assertoric propositions, and that their main contributions concern modal syllogism. Therefore, the innovations proposed by medieval authors with respect to quantification are not to be found in their theories of syllogism, but in this typical medieval development, theories of supposition.

The different modes of personal supposition are indeed the closest medieval counterpart of our theories of quantification. The modes of personal supposition were meant to codify

* Submitted to the *Encyclopedia of Medieval Philosophy* (ed. H. Lagerlund).

the quantificational behavior of what we now refer to as quantifier expressions, and what the medievals referred to as syncategorematic terms. For reasons of space, I here focus on three representative texts, William of Sherwood's *Introduction to Logic*, William of Ockham's *Sum of Logic* (part I), and Buridan's *Treatise on Supposition*.

Perhaps the best way to understand the medieval approach to these quantifier expressions by means of the notion of supposition is as a two-step procedure that explicates their meaning and behavior. First, the syntactical structure of the proposition, i.e. the presence and order of its syncategorematic terms, determines the kind of personal supposition that each categorematic term has. Then, the semantic definitions of each mode of personal supposition determine the effect of quantifying syncategoremata over the quantity of objects involved in the assertion of a proposition.

In other words, the various theories of supposition presented by medieval authors typically have two groups of rules for the modes of personal supposition: the syntactic rules mapping terms in propositional contexts provoked by quantifier expressions into modes of personal supposition; and the semantic rules mapping modes of personal supposition into specific semantic behaviors (see Ashworth, 1978). To illustrate this, let us first discuss the four Aristotelian classes of categorical propositions: universal affirmative (A), particular affirmative (I), universal negative (E) and particular negative (O); and provide the two kinds of rules for these propositional forms.¹

- (A) Every *a* is *b*.
- (E) No *a* is *b*.
- (I) Some *a* is *b*.
- (O) Some *a* is not *b*.

Syntactical rules. The syntactical rules for these four propositional forms are easily enumerable, but in practice the enumeration of rules becomes very long when authors attempt to cover a wider range of propositional forms. The rules below can be found in all of our authors:²

- The positive universal syncategorema 'Every' (*omnis*) causes the term immediately following it to have confused and distributive supposition (*a* in (A)), and the term mediately following it to have merely confused supposition (*b* in (A)).
- A negative term, 'No' (*nullus*) or 'not' (*non*), causes all terms to its right to have confused and distributive supposition (*a* and *b* in (E) and *b* in (O)).
- The particular universal syncategorema 'Some' (*aliquid*) causes the term immediately following it to have determinate supposition (*a* in (I) and (O)).

¹ But notice that, even at early stages of its development, supposition theory already recognized other quantifier expressions – unlike modern quantification theory, which started out with the existential and universal quantifiers and only later developed into a theory of generalized quantifiers. Notice also that, for medieval logicians (following Aristotle), all affirmative propositions have existential import, existential and (contrasting with the modern universal quantifier) universal propositions alike.

² Sherwood, *Introduction to Logic*, §5.13.1; Ockham, *Summa Logicae I* chaps. 71-74; Buridan, *Summulae de Suppositionibus*, chaps. 4.3.7 and 4.3.8.

- In the absence of syncategorematic terms immediately preceding a term, and of universal terms affecting a term mediately, a term has determinate supposition (*b* in (I)).

Semantic rules. Authors account for the semantic behavior of the various modes of personal supposition in different ways, in particular with a clear cleavage between 13th century and 14th century approaches. In the 13th century, with Peter of Spain, William of Sherwood and Lambert of Auxerre, there was a tendency towards defining the modes of personal supposition in terms of the verification of the proposition or the supposition of its terms:

- Supposition is determinate when the locution can be expounded by means of some single thing. Which is the case when the word supposits for some single thing.³
- Supposition is distributive when [the word] supposits for many in such a way as to supposit for any.⁴
- A term has merely confused supposition in a categorical proposition when it can be taken there for several of its supposita, not necessarily for all.⁵

By contrast, in the 14th century with Walter Burley, William of Ockham and John Buridan, it became customary to define the modes of personal supposition in terms of ‘ascent and descent’, that is, in terms of the inferential relations that do or do not obtain between a proposition and the singular propositions falling under it, of the form ‘This *a* is *b*’ (see Priest and Read, 1977; Spade, 1996, chap. 9).

Let (S) and (Q) stand for any syncategorematic terms, and the general form of a proposition P be ‘(Q) *a* is (S) *b*’. The generic definitions of the modes of personal supposition in terms of ascent and descent can be formulated as:⁶

- A term *a* has determinate supposition in P => A disjunction of propositions of the form ‘This *a* is (S) *b*’ can be inferred from P but a conjunction of propositions of the form ‘This *a* is (S) *b*’ cannot be inferred from P.
- A term *a* has confused and distributive supposition in P => A conjunction of propositions of the form ‘This *a* is (S) *b*’ can be inferred from P.
- A term *a* has merely confused supposition in P⁷ => A proposition with a disjunctive term of the form ‘This *a*, or that *a* etc... is (S) *b*’ can be inferred from P, but neither a disjunction nor a conjunction of propositions of the form ‘This *a* is (S) *b*’ can be inferred from P.

The same applies *mutatis mutandis* to the predicate term.

³ Sherwood, *Introduction to Logic*, §5.2.

⁴ Sherwood, *Introduction to Logic*, §5.2.

⁵ For want of a satisfactory formulation of merely confused personal supposition in our authors, this is Parsons’ (1997, 45) ‘generic’ version.

⁶ See Ockham *Summa Logicae* I, chap. 70; Buridan, *Summulae de Suppositionibus*, chaps. 4.3.5 and 4.3.6.

⁷ Among the (A), (E), (I) and (O) propositional forms, merely confused supposition occurs only in predicate position (in (A) propositions). But more generally, it can occur also in subject position, such as in exceptive propositions of the form ‘Only *a* is *b*’.

By applying the two groups of rules successively (first the syntactical rules and then the semantic rules), one obtains the desired result, i.e. an account of the quantity of individuals involved in a given assertion, and thus of the semantics of quantifier expressions. For example, in 'Every man is an animal', 'man' has confused and distributive supposition and 'animal' has merely confused supposition, according to the syntactical rules for 'every'. According to the semantic rules, this proposition asserts that 'man' supposits for all of the individuals falling under it (men) and that 'animal' supposits for several individuals, but not (necessarily) for all of those falling under it.

Terrence Parsons (1997) has made the compelling suggestion that the differences between the 13th and 14th century approaches can also be explained as the distinction between the study of the semantics of quantifier expressions taken individually vs. the study of global quantificational effect in wider propositional contexts. Indeed, 14th century authors had a keen interest in the effect of nested quantifier expressions, such as the effect of a negation over an affirmative universal quantifier. Take 'Not every man is an animal': according to the 13th century authors, 'man' would have distributive and confused supposition, since it is preceded by 'every'. But for 14th century authors, the negation preceding 'every' would have the effect of suppressing its distributive effect, so that 'man' would no longer have distributive and confused supposition but rather determinate supposition (see Karger, 1993; Dutilh Novaes, forthcoming). In sum, "[w]hat distinguishes the earlier theory from the later one is whether the mode of supposition of a term in a proposition is something that that term retains when its proposition is embedded in further contexts." (Parsons, 1997; 43)

Further developments. For reasons of space, I can only present the rough lines of the approach to quantification based on supposition. But medieval authors developed this approach in several different directions, such as: the definition of valid inferences among different categorical propositions (see Karger, 1993; Dutilh Novaes, 2004); an analysis of multiple quantification (of subject and predicate) and of other quantifier expressions (see Ashworth, 1978); discussions on what are now known as anaphoric pronouns (see Parsons, 1994). Here I have discussed 13th and 14th century authors only, but 15th and 16th century authors refined the framework even further, dealing in particular with the difficulties that emerged from the earlier theories (see Ashworth, 1974; Ashworth, 1978; Karger, 1997).

The modes of personal supposition have been a topic of heated debate in the literature, but a consensus as to their purpose and some of the technical details concerning them has not yet been reached. It is clear that they can be said to be a general theory of quantification, but one must bear in mind that the overall approach is fundamentally different from modern post-Fregean theories of quantification.

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Cross-reference: William of Sherwood; Peter of Spain; William of Ockham; John Buridan; Supposition theory; Syncategoremata; Properties of terms.