UvA-DARE (Digital Academic Repository)

Mortality risk and mental disorders: longitudinal results from the Upper Bavarian Study

Fichter, M.M.; Rehm, J.; Elton, M.R.; Dilling, H.; Achatz, F.

DOI

[10.1017/S0033291700036199](https://doi.org/10.1017/S0033291700036199)

Publication date

1995

Published in

Psychological Medicine

[Link to publication](#)

Citation for published version (APA):

Fichter, M. M., Rehm, J., Elton, M. R., Dilling, H., & Achatz, F. (1995). Mortality risk and mental disorders: longitudinal results from the Upper Bavarian Study. *Psychological Medicine*, 25, 297-309. <https://doi.org/10.1017/S0033291700036199>

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

Solving stochastic optimization models with learning and rational expectations

Hans M. Amman^a, David A. Kendrick^{b,*}, Sudhakar Achath^b

^a*Department of Macroeconomics, University of Amsterdam, Roeterstraat 11, 1018 WB Amsterdam, Netherlands*

^b*Department of Economics, University of Texas, Austin, TX 78712, USA*

Received 15 July 1994; accepted 20 July 1994

Abstract

In this paper we present a general single-agent stochastic dynamic optimization model that is capable of dealing with learning and with rational expectations. We obtain the solution by solving at every time step the rational expectations model and then applying existing optimization schemes to derive the optimal solution.

Keywords: Learning; Stochastic control; Rational expectations; Forward variables

JEL classification: C63

1. Introduction

There are at least three different approaches to solving dynamic optimization models using rational expectations. The first method is stacking the variables of the model in the tradition of Theil, cf. Fisher et al. (1986). The second is the approach of Amman and Kendrick (1993), which adapts the deterministic model iterative procedure of Fair and Taylor (1983) for use with stochastic models. The third approach is to use the framework described by Vaughan (1970), Blanchard and Kahn (1980) and partly in McGrattan (1994), as a preliminary step to solving the model using rational expectations. The advantage of the Blanchard and Kahn (BK) method is the fact that it deals nicely with the boundary conditions that rational expectations impose on the model solution when dealing with finite time models. The BK method ‘discounts’ all future effects of expectational variables through the use of the unstable roots of the model under consideration. That means that it allows for an elegant implementation in either deterministic, assuming perfect foresight, or stochastic optimization models.

In this paper we extend the BK approach to dynamic stochastic optimization models. This is

* Corresponding author.

done in two steps. In the first step we solve a model using rational expectations by applying the Blanchard and Kahn method. This provides a set of equations which could be used in the second step in the stochastic optimization procedure of Tse and Bar-Shalom as described in Kendrick (1981). However, it is necessary to iterate on some elements of the Blanchard and Kahn results in order to gain an expectationally consistent solution. This iterative procedure is then applied to a small model in order to illustrate the combined BK-optimization procedure.

2. Problem statement and solution

The standard single-agent stochastic linear-quadratic optimization problem is written as: Find the set of admissible instruments $U = \{u_0, u_1, \dots, u_{T-1}\}$ to minimize the quadratic welfare loss function

$$W_T = E \left\{ L_T(x_T) + \sum_{t=0}^{T-1} L_t(x_t, u_t) \right\}, \quad (1)$$

with

$$L_T = \frac{1}{2}(x_T - \hat{x}_T)' Q_T (x_T - \hat{x}_T),$$

$$L_t = \frac{1}{2}(x_t - \hat{x}_t)' Q_t (x_t - \hat{x}_t) + (x_t - \hat{x}_t)' F_t (u_t - \hat{u}_t) + \frac{1}{2}(u_t - \hat{u}_t)' R_t (u_t - \hat{u}_t),$$

subject to the model

$$x_{t+1} = A_t(\theta_t) + B_t(\theta_t)u_t + C_t(\theta_t)z_t + \sum_{j=1}^k D_{j,t}(\theta_t)x_{t+j}^e + \varepsilon_t, \quad (2)$$

where $x_t \in R^n$, $u_t \in R^m$ and $z_t \in R^s$, $t \in \{0, 1, \dots, T\}$, a stochastic time-varying parameter vector $\theta_t \in R^r$ and with $E x_0 \in R^n$, $E \theta_0 \in R^r$ given. The vector x_t contains the model variables and the vector u_t the variables an economic agent can use for optimizing his or her welfare function. The matrices $A_t(\theta_t)$, $B_t(\theta_t)$, $C_t(\theta_t)$ and $D_{j,t}(\theta_t)$ are a function of the stochastic time-varying parameters contained in the model. Q_t , F_t and R_t are penalty matrices of which the first is assumed quasi-positive definite and the third positive definite. $\hat{x}_t \in R^n$ and $\hat{u}_t \in R^m$ are target values of the model and instruments, respectively.

The rational expectations are captured in the fourth right-hand-side term of Eq. (2). The vector x_{t+j}^e contains the (subjective) expected values of the model variables by the agent(s) at period $t+j$ as formed at period t . k is the maximum expectational lead. Under the Rational Expectations Hypothesis (REH) this reduces to

$$x_{t+1} = A_t(\theta_t) + B_t(\theta_t)u_t + C_t(\theta_t)z_t + \sum_{j=1}^k D_{j,t}(\theta_t)E x_{t+j} + \varepsilon_t, \quad (3)$$

or in the deterministic case with perfect foresight and known parameters to

$$x_{t+1} = A_t + B_t u_t + C_t z_t + \sum_{j=1}^k D_{j,t} x_{t+j}. \quad (4)$$

The stochastic time-varying parameter vector θ_t cannot only be uncertain, but may also follow a first-order Markov process:

$$\theta_{t+1} = \Omega_t \theta_t + \eta_t, \quad (5)$$

where Ω_t is a Markov transition matrix and $\eta_t \in R^r$ is a random vector. By re-estimating the stochastic parameter vector at every time step the optimization procedure would permit learning about the changes in the model. As the θ_t feeds into the $D_{j,t}$ matrices, the optimization procedure also has the opportunity to learn about the response of the variables to changes in the expectational variables.¹ The vectors $\varepsilon_t, \eta_t, x_0, \theta_0$ are each assumed to be independent normally distributed random vectors with known means and covariances, so $x_0 \sim N(E x_0, \Sigma_0^{xx})$, $\theta_0 \sim N(E \theta_0, \Sigma_0^{\theta\theta})$, $\varepsilon_t \sim N(0, \Sigma^{\varepsilon\varepsilon})$ and $\eta_t \sim N(0, \Sigma^{\eta\eta})$. Furthermore, Σ_0^{xx} is the covariance matrix of the initial variables of the model. $\Sigma_0^{\theta\theta}$ is the covariance matrix for initial period parameter estimates, $\Sigma^{\varepsilon\varepsilon}$ is the covariance matrix for the model random shocks, and $\Sigma^{\eta\eta}$ is the covariance matrix for Markov disturbances. $E x_0, E \theta_0, \Sigma_0^{\eta\eta}, \Sigma_0^{xx}$ and $\Sigma_0^{\theta\theta}$ are assumed to be known. If we set $\Omega_t = I$ and $\Sigma^{\eta\eta} = 0$ we have a simple OLS estimation problem for the parameters. If $\Omega_t = I$ and $\Sigma^{\eta\eta} > 0$, the parameters follow a random walk. The matrices Ω_t and $\Sigma^{\eta\eta}$ can be estimated using a Full Information Likelihood Estimation procedure, Cuthbertson et al. (1992).

In order to compute the admissible set of instruments we have to eliminate the rational expectation variables from the model. For this we use the BK method. Eq. (2) may be transformed into BK's first linear form, cf. Chow (1984), in the following way.

$$\begin{aligned} \begin{bmatrix} x_{t+1} \\ E_{t+1}x_{t+2} \\ \vdots \\ E_{t+1}x_{t+k} \end{bmatrix} &= \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & & 0 \\ \vdots & & & & \\ -D_{k,t}^{-1}A_t & D_k^{-1} & -D_{k,t}^{-1}D_{1,t} & & -D_{k,t}^{-1}D_{k-1,t} \end{bmatrix} \begin{bmatrix} x_t \\ x_{t+1} \\ E_{t+1}x_{t+2} \\ \vdots \\ E_{t+1}x_{t+k-1} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ \vdots \\ -D_{k,t}^{-1}B_t \end{bmatrix} u_t + \begin{bmatrix} 0 \\ \vdots \\ -D_{k,t}^{-1}C_t \end{bmatrix} z_t, \end{aligned} \quad (6)$$

assuming that the $D_{k,t}$ matrix is invertible.² Applying the BK notation this simplifies to

$$\begin{bmatrix} x_{t+1} \\ E_{t+1}p_{t+2} \end{bmatrix} = \hat{A} \begin{bmatrix} x_t \\ p_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma_{u,t} \end{bmatrix} u_t + \begin{bmatrix} 0 \\ \gamma_{z,t} \end{bmatrix} z_t, \quad (7)$$

with

$$E_{t+1}p_{t+2} = \begin{bmatrix} E_{t+1}x_{t+2} \\ \vdots \\ E_{t+1}x_{t+k} \end{bmatrix}, \quad \gamma_{u,t} = \begin{bmatrix} 0 \\ \vdots \\ -D_{k,t}^{-1}B_t \end{bmatrix} \quad \text{and} \quad \gamma_{z,t} = \begin{bmatrix} 0 \\ \vdots \\ -D_{k,t}^{-1}C_t \end{bmatrix}.$$

¹ This captures a part of the Lucas (1976) critique when dealing with macro models.

² For applied models this may be a strong assumption.

Once the model is in first-order form we can apply the BK method for solving first-order systems with rational expectations. Applying the *Jordan canonical form method*,³ dropping the time index for the matrices, we get

$$x_{t+1}^{v+1} = \hat{A}x_t^{v+1} + \hat{B}u_t^{v+1} + \hat{c}_t^v, \quad (8)$$

where

$$\begin{aligned} \hat{A} &= B_{11}J_1B_{11}^{-1}, \\ \hat{B} &= -(B_{11}J_1C_{12} + B_{12}J_2C_{22})C_{22}^{-1}J_2^{-1}C_{22}\gamma_u, \\ \hat{c}_t^v &= -(B_{11}J_1C_{12} + B_{12}J_2C_{22})C_{22}^{-1} \sum_{i=0}^{\infty} \{J_2^{(-i-1)}C_{22}(\gamma_2E_tz_{t+i} + \gamma_uu_{t+i+1}^v)\}. \end{aligned} \quad (9)$$

Thus, in moving from Eq. (2) to Eq. (8) we have used the BK method to reduce the economic model to a form without rational expectations. Now that we have the model in this form it is easy to set up an iterative scheme to solve our optimization model. We set the instruments at the first iteration step to zero, $\forall t \ u_t^0 = 0$, we can solve the optimization model using (8). This produces a new set of instruments, $\forall t \ u_t^1$, allowing us to iterate on Eq. (8) with $\hat{c}_t^1$. This iterative procedure is repeated until the values of the model x_t^v converge. Clearly there is a boundary condition in (9) on the $u_t, t \in \{T, \dots\}$. However, if the saddle-point property holds, the model has a unique solution $\{x^*, u^*\}$ and this unique solution can be used for the boundary condition.

3. An example

To illustrate the algorithm, we provide a numerical example. Consider a simple deterministic macro model with output y_t , consumption c_t , investment i_t , government expenditures g_t , and taxes τ_t . The corresponding model vector has the form $x_t = [y_t \ c_t \ i_t \ g_t \ \tau_t]'$ and the instrument vector has the form $u_t = [g_{t+1}]$. The problem can then be stated as: Find for the model

$$\begin{aligned} y_{t+1} &= c_{t+1} + i_{t+1} + g_{t+1}, & c_{t+1} &= 0.8(y_t - \tau_t) + 200, & i_{t+1} &= 0.2E_ty_{t+2} + 100, \\ g_{t+1} &= u_t, & \tau_{t+1} &= 0.25y_{t+1}, \end{aligned}$$

with $x_0 = [1500 \ 1100 \ 400 \ 0 \ 375]'$, a set of admissible controls $U = \{u_0, \dots, u_9\}$ to minimize the quadratic welfare loss function

$$W_{10} = \frac{1}{2}(y_{10} - 1600)^2 + \frac{1}{2} \sum_{t=0}^9 [(y_t - 1600)^2 + g_t^2]. \quad (10)$$

Applying the algorithm from the previous section we obtain the following (Table 1).

³ In order to compute the actual numeric solution of a model we must assume that the matrix $\hat{A}$, can be diagonalized.

Table 1

Solution of optimization model by using Blanchard and Kahn method

t	0	1	2	3	4	5	6	7	8	9	10
y	1500	1571	1589	1593	1595	1595	1595	1595	1595	1593	1583
g	53	28	21	20	19	19	19	19	19	16	

Notes: $W_{10} = 8936$, $y^* = 1596.15$ and $g^* = 19.23$

4. Summary

In this paper we have presented a single-agent stochastic dynamic optimization model that is capable of dealing with rational expectations and learning. We obtained a solution by solving at every time step the rational expectations model and then applying existing stochastic optimization schemes to compute the solution. This procedure adds little or no overhead to existing non-experimental models, if the model involved can be put into the Blanchard and Kahn framework.

References

- Amman, H.M. and D.A. Kendrick, 1993, Forward looking behavior and learning in stochastic control, *International Journal of Supercomputer Applications* 7, 201–211.
- Blanchard, O.J. and C.M. Kahn, 1980, The solution of linear difference models under rational expectations, *Econometrica* 48, 1305–1311.
- Chow, G., 1984, *Econometrics* (McGraw-Hill, New York).
- Cuthbertson, K., S.G. Hall and M.P. Taylor, 1992, *Applied Econometric Techniques* (Philip Allan, Hemel Hempstead).
- Fair, R.C. and J.B. Taylor, 1983, Solution and maximum likelihood estimation of dynamic rational expectations models, *Econometrica* 51, 1169–1185.
- Fisher, P.G., S. Holly and A.J. Hughes Hallet, 1986, Efficient solution techniques for dynamic non-linear rational expectations models, *Journal of Economic Dynamics and Control* 10, 139–145.
- Kendrick, D.A., 1981, *Stochastic control for economic models* (McGraw-Hill, New York).
- Lucas, R.E., 1976, Econometric policy evaluation: A critique, in: K. Brunner and A.H. Meltzer, eds., *The Phillips curve and labor markets*, 19–46, *Supplementary Series to the Journal of Monetary Economics*.
- McGrattan, E.R., 1994, A note on computing competitive equilibria in linear models, *Journal of Economic Dynamics and Control* 18, 149–160.
- Vaughan, D.R., 1970, A non recursive algorithm solution for the discrete riccati equation, *IEEE Transactions on Automatic Control* 15, 597–599.