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Rydberg dressing of a one-dimensional Bose-Einstein condensate

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We study the influence of Rydberg-dressed interactions in a one-dimensional (1D) Bose-Einstein condensate (BEC). We show that a 1D geometry offers several advantages over a three-dimensional geometry for observing BEC Rydberg dressing. The effects of dressing are studied by investigating collective BEC dynamics after a rapid switch-off of the Rydberg dressing interaction. The results can be interpreted as an effective modification of the s-wave scattering length. We include this modification in an analytical model for the 1D BEC and compare it to numerical calculations of Rydberg dressing under realistic experimental conditions.

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Ultracold quantum gas experiments allow for an extremely precise control of interatomic interactions. Strong interactions at the atomic level enable in principle the creation of strongly correlated many-body systems, where the tunability gives them an important advantage over their solid-state equivalents. The short-range interactions between ground-state atoms can be controlled by Feshbach resonances, which resulted, e.g., in the demonstration of the BCS–Bose–Einstein condensate (BEC) type superfluid crossover. However, while the interactions can be made very strong by going to the unitarity regime, it is still under debate whether this quantum gas can be considered as a strongly correlated system. Strong correlations will be evident when the interaction is both strong and long range, i.e., the range of the interaction exceeds the average interparticle separation. Rydberg atomsContextualize...
resulting interaction potential between dressed atoms has the form \( W(r) = \beta^4 \frac{\epsilon_0}{R_B^2} \) [12]. In practice, the blockade radius \( R_B \) is in the micrometer range.

While the Rydberg-dressed interaction can be strong because of the high value of \( \epsilon_0 \), for larger densities (typical for a BEC) it saturates around the characteristic density \( \rho_B = 1/\beta^2 V_B \), where there is one excited Rydberg atom per Rydberg blockade volume \( V_B \). This leads to an overall offset of the chemical potential of the BEC, but only to a small modification of the shape compared to no dressing. Only for low relative density, \( \rho < \rho_B \), is there a significant alteration of the BEC shape due to dressing. In 3D, as considered in Ref. [31], one has \( V_B = \frac{4}{3} \pi R_B^3 \) and for typical parameters \( \rho_B \) is lower than that of typical densities of 3D BECs. In contrast, in the 1D case we consider here, one has a 1D blockade volume of \( V_B = 2 R_B \), leading to a different scaling of \( \rho_B \). We find that for practical experimental conditions the entire 1D BEC can have a density below \( \rho_B \).

We now give a quantitative description of the effects of the Rydberg dressing on a (effectively) 1D BEC. We assume a cigar-shaped BEC, in a trap with a radial trapping frequency \( \omega_r \), much larger than the axial trapping frequency \( \omega_0 \) and a chemical potential of \( \mu < \hbar \omega_r \). We start from the generalized Gross-Pitaevskii (GP) equation \( \hbar \frac{\partial}{\partial t} \psi(x) = \left[-\frac{\hbar^2 \partial^2}{2m} + g_0 N |\psi(x)|^2 + \frac{\mu}{2} + V_{MF} \right] \psi(x) \), where \( N \) is the particle number and \( g_0 = 2 \alpha J \omega_0 \) is the 1D mean-field coupling parameter. In the mean-field regime, and assuming the radial size of the BEC is much smaller than \( R_B \) (and at sufficiently low densities where collective effects can be neglected), \( V_{MF} = N \int W(x) |\psi(x')|^2 dx' \), which is treated in the same way as recently shown in Rydberg physics [10,15]. If we assume that the length of the 1D BEC is much larger than the Rydberg blockade radius \( R_B \), so that the density is constant over this radius, we may approximate the \( V_{MF} \) term as \( V_{MF} = N |\psi(x)|^2 \int W(x') dx' \). Contrary to the bare van der Waals interaction, the combination of short-range saturation and a long-range \( 1/r^6 \) tail of the two-body dressed interaction assures that the above integral is finite, and therefore gives a correction to the 1D mean-field coupling \( g_0 \) of

\[
g_{eff} = g_0 + \frac{2}{3} \pi \frac{C_6}{R_B^6} \beta^4 = g_0 + \frac{\pi}{2} \frac{\hbar \Omega^4}{12 |\Delta|^3} R_B, \tag{1}
\]

and corresponds to a linear increase of \( \partial_\rho E \) with density in Fig. 1. Note that when these conditions are applied to a 3D BEC, we similarly find \( \delta_{eff}^{3D} = g_0^{3D} + \frac{\pi^2}{12} \frac{\hbar \Omega^4}{\rho B} R_B^6 \) which has already been derived in Refs. [12,30,31].

Next, to allow for higher linear densities where collective effects can play a role, we adapt the treatment of Ref. [12] to 1D, again using the assumption that the radial size of the BEC is much smaller than \( R_B \). In this way, we obtain an energy density for the internal degrees of freedom \( \epsilon_{val} \) at linear density \( \rho \),

\[
\epsilon_{val}(\theta, \xi) = \frac{\hbar \Delta \rho}{2} \cos 2\theta - \frac{\hbar \Omega \rho}{2} \sin 2\theta + \frac{\lambda}{2} \sin^4 \theta \frac{C_6 \rho^2}{\xi^5}, \tag{2}
\]

having as variational parameters the Rydberg mixing angle \( \theta \) and the correlation length \( \xi \). Note that in 1D the last term scales as \( \xi^{-5} \), compared to \( \xi^{-3} \) in 3D. The correlation length \( \xi \) is constrained to the blockade radius \( R_B \) for low densities and to the average distance between Rydberg atoms, \( 1/f_\rho \), for higher densities; \( \xi \approx \min(R_B, 1/f_\rho) \), with \( f = \sin^2 \theta \) the Rydberg excitation fraction. In this approach, minimizing the energy density with respect to \( \theta \) yields the energy functional \( E[\rho] \), and the derivative \( \partial_\rho E[\rho] \) (see solid lines in Fig. 1) is inserted as \( V_{MF} \) into the generalized GP equation. In the low-density limit of Eq. (2), one has \( \xi = R_B = \theta \) and approaches \( \beta \); comparing the result to the Rydberg part of the two-body limit Eq. (1) allows us to determine the value \( \lambda = 2\pi/3 \) in Eq. (2).

As a specific example, we consider dressing the \(^{87}\text{Rb} \) ground state with the \( n = 35S \) Rydberg state (which has \( C_6/\hbar = 2\pi \times 0.1891 \text{ GHz} \cdot \mu \text{m}^6 \) [37]) and the detuning \( \Delta = -2\pi \times 100 \text{ kHz} \), leading to the blockade radius \( R_B = 3.13 \mu \text{m} \). (Note for that \( n < 50 \) Rydberg dressing will not be affected by molecular dimer excitations, as the splitting between the dimer and the atomic excitation line is always greater than the typical linewidth of \( < 1 \text{ MHz} \) [38]). The resulting energy functional is shown in Fig. 1 for various Rabi frequencies parametrized by \( \beta \) and as a function of the linear density \( \rho \). For the parameters and densities in Fig. 1 we find \( R_B < 1/f_\rho \), and thus we may take \( \xi = R_B \) throughout. It is worth emphasizing here that the saturation in the energy functional as the density increases is much less severe in 1D when compared to 3D and that the densities of 1D BECs in Fig. 1 are in the range that is typical for those achieved on atom chips [34–36].

A useful approximate description can be obtained as follows: the energy functional \( E[\rho] \) is qualitatively described...
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$$F[\rho] = \epsilon_0 - \epsilon_0 \left(1 + \frac{2}{3} \pi C \frac{\rho}{\rho_B}\right)^{-1/C},$$

(3)

which fulfills the two limiting cases: (i) it saturates for $\rho \gg \rho_B$ at a constant value of $\epsilon_0 = \hbar|\Delta|^2$ [12]; (ii) it is linear with density for small densities, $F \approx \pi/12 \hbar \Omega^2 / |\Delta|^2 R_B \rho$ [see Eq. (1)]. The free parameter $C$ is numerically obtained by minimizing the root mean square of the difference between $F$ and $\partial_F \rho[\rho]$; these results are also shown in Fig. 1 as dashed lines. The linearization of Eq. (3) around the trap center allows us to extract the effective mean-field coupling constant $g_{\text{eff}}$. Because the Rydberg dressing will only have a small effect on the Thomas-Fermi profile, as is shown in the following paragraphs, we are able to express the dressing effect as an effective change of the trapping frequency. This is done by equating all the different potential energy terms in the GP equation:

$$g_0 \rho + m \omega_0^2 x^2/2 + F[\rho] \approx g_0 \rho + m \omega_{\text{eff}}^2 x^2/2$$

$$= g_{\text{eff}} \rho + m \omega_{\text{eff}}^2 x^2/2,$$

(4)

with $\rho$ being the undressed Thomas-Fermi profile. After linearization of $F$ around the trap center, we find the effective mean-field coupling constant including the saturation effects of dressing to be

$$g_{\text{eff}} = g_0 + \frac{\hbar \Omega^2}{12 |\Delta|^2 R_B} \left(1 + \frac{\pi C \frac{\rho_0}{\rho_B}}{3}\right)^{-1/C-1},$$

(5)

where $\rho_0$ is the BEC peak density and $C$ is a variational parameter for Eq. (3). One can observe that the two-body limit is obtained for vanishing densities, as well as for $\beta < 0.018$ (see Fig. 1).

In order to further quantify the effects of Rydberg dressing on a 1D BEC, we now consider a more specific scenario. We assume a BEC of $N = 200, 1000, 2000$ $^8$Rb atoms with an s-wave scattering length of $a_s = 99 a_0$ (where $a_0$ is the Bohr radius) confined in a cylindrical harmonic trap with radial frequency $\omega_\perp = 2\pi \times 3000$ Hz and axial frequency $\omega_z = 2\pi \times 30$ Hz. These parameters correspond to the mean-field coupling strength $g_0 = 2.08 \times 10^{-38}$ J m with $E_0 = \hbar \omega_0 l_0 = \sqrt{\hbar / m \omega_0} = 1.97 \mu$m, and $l_0 = \omega_0^{-1} = 5.3$ ms as energy, length, and time units, respectively. In the absence of dressing, the corresponding BEC half-length along the axial direction and the peak density are $10.6 \mu$m and $28 \mu$m$^{-1}$ for $N = 200$, $18.2 \mu$m and $81 \mu$m$^{-1}$ for $N = 1000$, and $23 \mu$m and $128 \mu$m$^{-1}$ for $N = 2000$.

The effects of the Rydberg dressing can now be characterized as follows. For different particle numbers $N$ and dressing parameters $\beta$ (keeping $\Delta = -2 \pi \times 100$ kHz fixed) we first calculate the ground state $\psi_0$ of the generalized GP equation, using $\partial_F \rho$ obtained from Eq. (2). Next, we calculate the time evolution of this initial wave packet upon a sudden switch-off of the Rydberg dressing and numerically calculate the relative change of the BEC size defined as $\kappa(t) = R(t)/R_0$, where $R_0$ is the initial dressed BEC half-length in the axial direction and $R(t)$ is the half-length in the axial direction obtained from fitting the parabola to the undressed cloud at time $t$.

This nonadiabatic switch-off of the dressing results in the excitation of a BEC breathing mode. The dynamics of a BEC can be described by the collective motion of atoms with the time-dependent density $\rho(x,t) \propto \rho_0 [x/\lambda(t)]/\lambda(t)$, where $\rho_0(x)$ is the initial BEC density and the scaling parameter $\lambda(t)$ obeys $\frac{\partial^2 \lambda}{\partial t^2} = \frac{\omega_{\text{eff}}^2}{\pi^2} - \omega_{\text{eff}}^2 \lambda$ [39]. A solution of this equation is periodic, with an amplitude parametrized by $\omega_{\text{eff}}$ and the frequency $\omega_{\text{eff}} \approx \sqrt{300}$. For fixed $N$, $n$, and $\beta$ we calculate $\kappa(t)$, which we then use to find $\omega_{\text{eff}}$ by minimizing the root mean square of the difference between $\lambda(t)$ and $\kappa(t)$. Figure 2 presents $\kappa$ (black solid lines) and $\lambda$ (red dashed lines) for fixed numbers of atoms $N = 200, 1000$, and $2000$ and fixed dressing parameters $\beta = 0.03, 0.05, 0.07$, and $0.09$, with corresponding critical densities $\rho_B$ equal to $177, 64, 33$, and $20 \mu$m$^{-1}$, respectively. Knowing $\omega_{\text{eff}}$ we can extract the effective density-dependent change of the mean-field interaction strength $g_{\text{eff}}/g_0 = (\omega_{\text{eff}}/\omega_0)^2$. Figure 3 presents the change of the effective trapping frequency and of the

![FIG. 2. Relative change of the BEC size after sudden switch-off of the dressing lasers ($\kappa$) for different BEC particle numbers and different dressing parameters. Red dashed lines correspond to the self-similar density evolution ($\lambda$).](image1)

![FIG. 3. Relative change of the effective trapping frequency $\omega_{\text{eff}}$ (top panel) and relative change of the mean-field coupling $g_{\text{eff}}$ (bottom panel) for different atom numbers $N$ for the fixed dressing parameter (curves from top to bottom: $\beta = 0.03, 0.05, 0.07$, and $0.09$). Dashed lines correspond to Eq. (5).](image2)
effective mean-field interaction strength as a function of the particle number \( N \), for the fixed dressing parameter \( \beta \), which agrees very well with the analytical expression of Eq. (5).

An important consideration is the decay associated with the Rydberg dressing, including the effects of radiative decay of the intermediate state in the two-photon coupling scheme. In practice the atomic system is a three-level system with an atomic ground state \( |g\rangle \) coupled to a Rydberg \( S \)-state \( |r\rangle \) via an intermediate state \( |e\rangle \). The Rabi frequency and detuning for the transition from \( |g\rangle \) to \( |e\rangle \) are \( \Omega_2 \) and \( \Delta_1 \) while similarly for the transition from the intermediate state \( |e\rangle \) to the Rydberg state \( |r\rangle \) they are denoted as \( \Omega_3 \) and \( \Delta_2 \). The intermediate level \( |e\rangle = |S\rangle \) is far detuned, i.e., \(|\Delta_1|, |\Delta_2| \gg \Omega_1, \Omega_2, \Gamma_{5P_{1/2}}\) and can be adiabatically eliminated. This effectively reduces the three-level system to a two-level system with two-photon Rabi frequency \( \Omega = \sqrt{\Omega_2^2/\Delta_1 + \Delta_2} \) and total detuning \( \Delta = \Delta_1 + \Delta_2 \), with \(|\Delta| \ll |\Delta_1|, |\Delta_2|\).

As an example, we can take the red (780-nm wavelength) laser parameters as having a waist of \( r_{\text{red}} = 500 \mu \text{m} \) with a power of \( P_{\text{red}} = 10 \mu \text{W} \) and the blue (480-nm wavelength) laser parameters as having a waist of \( r_{\text{blue}} = 90 \mu \text{m} \) with a maximum power of \( P_{\text{blue}} = 50 \text{ mW} \) with a fixed intermediate state detuning of \( \Delta_1 = 2\pi \times 1.5 \text{ GHz} \). Keeping the total detuning \( \Delta = -2\pi \times 100 \text{ kHz} \) constant as before, we change the dressing parameter \( \beta \) via the Rabi frequency \( \Omega_2 \). We calculate the lifetime of the Rydberg-dressed state by considering a weak admixture to the intermediate state \( |S\rangle \) with the decay rate \( \Gamma_{5P_{1/2}} \) and an admixture of the Rydberg state \( |r\rangle \) with the decay rate \( \Gamma_{nS} \), which results in an effective decay rate smaller than the effective rate \( \Gamma_{\text{eff}} = \beta^2 \Gamma_{nS} + (\Omega_1/2\Delta_1)^2 \Gamma_{5P_{1/2}} \) (Note that the effective decay rate is smaller when collective effects play a role because in that case one should replace \( \beta^2 \) with \( f^2 \) and \( f < \beta^2 \)). The lifetime \( \Gamma_{nS}^{-1} \) for the \( n = 3\) is \( 2\pi \times 6.1 \text{ MHz} \) and \( \Omega_1/2\Delta_1 = 18 \times 10^{-4} \). The corresponding minimum lifetimes for \( \beta = 0.03, 0.05, 0.07, \) and 0.09 are then 6.3, 4.5, 3.1, and 2.2 ms, respectively. From these results, it is clear that it is realistic to observe Rydberg dressing in an experiment: for instance, for the parameters \( N = 1000 \) and \( \beta = 0.07 \), the resulting change in BEC size during the breathing is about 8%.

It should be possible to reach the dressed ground state by turning on the dressing lasers during the very final stages of evaporative cooling. An alternative approach is to consider adiabatic turn-on and rapid switch-off of the dressing; this is beyond the scope of the present paper. An advantage of the above scheme is that the resulting breathing occurs in the undressed BEC, so that the limited lifetime of the dressed state is not an issue during the breathing.

The Rydberg-mediated control over the interactions we have discussed here offers an important alternative to previously considered schemes. For instance, Feshbach resonances allow for time-dependent nonlinear dynamics as they can be utilized for a periodic modification of the mean-field coupling. This was proposed by Saito and Ueda [41], who considered a sinusoidal time-dependent modulation of the coupling constant, and by Kevrekidis et al. [42], who considered a block type of periodic modulation of the mean-field coupling constant. An intriguing opportunity offered by the dressing is that by spatially modulating the dressing lasers (e.g., in a standing wave) it would be possible to create a spatial modulation of the interaction strength, as well as a temporal modulation, which can be rapidly switched.

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