UvA-DARE (Digital Academic Repository)

Medium and finite nuclear size effects in relativistic two-body currents

Brussaard, P.; Tjon, J.A.; Dekker, M.J.

DOI

[10.1103/PhysRevC.51.1393](https://doi.org/10.1103/PhysRevC.51.1393)

Publication date

1995

Published in

Physical Review C

[Link to publication](#)

Citation for published version (APA):

Brussaard, P., Tjon, J. A., & Dekker, M. J. (1995). Medium and finite nuclear size effects in relativistic two-body currents. *Physical Review C*, 51, 1393.
<https://doi.org/10.1103/PhysRevC.51.1393>

General rights

It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations

If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: <https://uba.uva.nl/en/contact>, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.

Medium and finite nuclear size effects in relativistic two-body currents

M.J. Dekker* and P.J. Brussaard

R.J. Van de Graaff Laboratory, University of Utrecht, P.O. Box 80.000, 3508 TA Utrecht, The Netherlands

J.A. Tjon

Institute for Theoretical Physics, University of Utrecht, P.O. Box 80.000, 3508 TA Utrecht, The Netherlands

(Received 17 August 1994)

We study the effects of medium modifications on the contribution of relativistic two-body meson exchange and isobar currents and real-pion production currents to the transverse response function of nuclei. In particular the consequences of rescattering of the Δ isobar, effective masses, and realistic momentum distributions are considered. It is found that the two-body isobar and real-pion production current account for most of the observed strength in the dip region. Our results do not support a large modification of the Δ -isobar mass. We compare our model with the Δ -hole model and in doing so confirm the microscopic picture of several of its phenomenological ingredients. Finally, the use of more realistic momentum distributions, obtained from relativistic mean field single-particle wave functions gives rise to results that are not much different from those obtained in the Fermi-gas model and compare fairly well with the data.

PACS number(s): 25.30.Fj, 24.30.-v

I. INTRODUCTION

In a previous paper [1], to be referred to as I, we discussed two-body meson exchange and isobar currents. The most interesting result of that analysis was the apparent importance of the two-body isobar current in and beyond the dip region. The treatment in that paper was limited, however, to the noninteracting relativistic Fermi-gas model. Several consequences of the limitations of this model as well as the absence of unitarity of the Δ -isobar propagator were discussed. In this paper we extend the analysis of two-body meson exchange and isobar currents and include unitarity (through rescattering), effective nucleon and Δ -isobar masses as a result of medium modifications and go beyond the Fermi-gas model when we consider relativistic single-particle wave functions. The conventions, notation, and meaning of (mathematical) symbols in this paper are the same as in I. In order to be able to focus on the new aspects of the work presented in this paper, we refer to I for these definitions and related clarifications.

We start to establish the consistency between the Δ -isobar propagator and two-body exchange currents, which gives rise to the rescattering (or unitarity) analysis of Sec. II. The consistency requirement is analogous to the one we discussed in I with respect to the isobar decay width in the real-pion production process. As a result we can make contact with an important theoretical model developed to describe the nuclear response in scattering processes in the Δ -resonance region: the Δ -hole model. The effect of the inclusion of the rescattering contribution on the (transverse) response functions is studied. In Sec. III we then consider a simple approach to interacting nuclear matter which gives rise to effective nucleon

and Δ -isobar masses and again study the effects on the response functions. Next we make a first step beyond the (interacting) Fermi-gas model of the nucleus. We reformulate our expressions such that they allow for the use of single-particle orbits from the Serot-Horowitz solution [2], in Hartree approximation, to the $\sigma\omega\rho$ model in momentum space. Finally, several of the issues discussed in this paper are combined to comment on results obtained recently by Rost, Price, and Shepard [3] with emphasis on the energy dependence of the Δ -isobar decay width.

II. RESCATTERING AND THE CONNECTION TO THE Δ -HOLE MODEL

In the discussion of the results of our calculations in I we noted that the presence of two channels that both show a strong resonance behavior, should be regarded as a warning that unitarity is violated in the model. This section is devoted to a discussion of this issue. It will provide a means to compare our model with another important model for calculations of the nuclear response to a virtual photon in the Δ -resonance region: the Δ -hole model [4].

A. Rescattering

To set the stage, let us try to be more precise in what we mean with unitarity in this case. It is beyond the scope of the present work to present a unitary model for pion production in the nuclear medium. Rather we are interested in the way the results we obtained previously change once a first step in this direction is made. The unitarity relation for the Δ propagator is given by the following equation (Dirac indices being suppressed):

$$\text{Im}(G_{\Delta}) = \overline{G}_{\Delta} \text{Im}(\Sigma) G_{\Delta}, \quad (1)$$

* Present address: Philips Research Laboratories, WA-03, Prof. Holstlaan 4, 5656 AA Eindhoven, The Netherlands.

where G_Δ is the solution of Dyson's equation

$$G_\Delta = G_\Delta^0 + G_\Delta \Sigma G_\Delta^0. \quad (2)$$

In I we observed that the sum of the contribution of the two isobar channels considered in our model (real-pion production and the $2p2h$ excitation via an intermediate Δ isobar) to the nuclear response can formally be written as

$$W^{\mu\nu} \propto \bar{\Gamma}^\nu \bar{G}_\Delta \{ \text{Im}(\Sigma_{2p1h}) + \text{Im}(\Sigma_{\pi N}) \} G_\Delta \Gamma^\mu, \quad (3)$$

where Γ^μ denotes the photon coupling. Here Σ_{2p1h} denotes the contribution of pion absorption to the self-energy of the Δ isobar [see diagram (a) of Fig. 1]. Up to now the Green function G_Δ has been constructed in I as a solution of Eq. (2) with the πN -bubble contribution to the self-energy [diagram (b) of Fig. 1] only, whereas it should be the solution to Dyson's equation where the self-energy is the sum of $\Sigma_{\pi N}$ and Σ_{2p1h} . These contributions correspond to terms in which the intermediate pion is repeatedly absorbed and emitted by particle-hole excitations.

It is precisely the effect of these real-pion production and two-body isobar current contributions we want to study in this section. There exists, however, a second reason why we believe that a study of Σ_{2p1h} is interesting: the results allow for a direct comparison with the more phenomenological part of the Δ -hole model. Without going into detail yet we note that the spreading potential

(a)

(b)

FIG. 1. Feynman diagrams for the $2p1h$ [diagram (a)] and the πN -bubble [diagram (b)] self-energy contribution for the Δ isobar.

in the latter is expected to originate microscopically from the $2p1h$ self-energy as depicted in the diagram of Fig. 1. We will therefore evaluate Σ_{2p1h} in the relativistic Fermi-gas model, modify the propagator accordingly, study the connection to the Δ -hole model, and consider finally the effects on our calculations.

B. Evaluation of Σ_{2p1h} and Pauli blocking self-energy

In the previous discussion we deliberately ignored the Lorentz indices of the Δ -isobar self-energy and propagator. In principle the Lorentz structure of the Δ -isobar self-energy in a nuclear medium is one of the interests of relativistic nuclear physics [5]. The πN -bubble diagram gives, for instance, a scalar and a tensor part [6,7], if the spin structure is taken into account. Since the spin structure of Σ_{2p1h} equals the one of the πN -bubble diagram, one has also in this case a scalar and tensor part. Moreover, in view of the phenomenology of the Δ -hole model, in which a spin-orbit potential in the Δ -propagator plays an important role, a study of the full Lorentz structure might be instructive. In the present analysis, however, we evaluate a spin-averaged self-energy in the same way as we did for the πN bubble in I and which thereby becomes a scalar. Making the same assumptions as we made there, one can easily include this scalar self-energy in the scalar part of the propagator. The $2p1h$ self-energy, in accordance with the analysis in I, is given by

$$\begin{aligned} \frac{\Sigma_{2p1h}^{\beta\alpha}}{i} = & \int \frac{d^4k d^4q}{(2\pi)^8} \frac{-if_{\pi N\Delta}}{m_\pi} (-k_\delta) \mathcal{F}^{\beta\delta} iG_0^+(p-k) \frac{if_{\pi N\Delta}}{m_\pi} \\ & \times k_\epsilon \mathcal{F}^{\epsilon\alpha} \left(\frac{f_{\pi NN}}{m_\pi} \frac{i}{k^2 - m_\pi^2} \right)^2 (-1) \text{Tr}[\not{k} \gamma^5 iG_0^+(q) \\ & \times (-\not{k} \gamma^5) iG_0^+(k+q)] T^{\dagger a} T^b \text{Tr}[\tau^a \tau^b]. \end{aligned} \quad (4)$$

The extra minus sign in front of the trace is due to the closed fermion loop. As discussed above we calculate the spin-averaged self-energy

$$\Sigma_{2p1h} \equiv \frac{1}{4} \sum_{\sigma} \bar{u}_{\Delta\beta}(\mathbf{p}, W, \sigma) \Sigma_{2p1h}^{\beta\alpha} u_{\Delta\alpha}(\mathbf{p}, W, \sigma). \quad (5)$$

This expression for the $2p1h$ self-energy is only the lowest-order contribution in a consistent treatment of both $\Sigma_{\pi N}$ and Σ_{2p1h} . In such a treatment one constructs a medium-modified pion propagator as the solution of $G_\pi = G_\pi^0 + G_\pi^0 \Sigma_{ph} G_\pi$, where Σ_{ph} denotes the pion self-energy due to a fluctuation into a particle-hole pair. Then one could consider the modified πN -bubble diagram with the use of this dressed-pion propagator. The imaginary part of this diagram would then contain both the pion and the $2p1h$ -cut consistently. We do not take medium modifications of the pion propagator into account in the calculations of the two-body currents, however. At our level of approximation we may thus consider only the lowest-order contribution. As a result we do not consider the pion cut of the self-energy of Eq. (4). Carrying out the contour integrations over k_0 and q_0 we find

$$\begin{aligned}
\Sigma_{2p1h} = & 2 \left(\frac{f_{\pi NN} f_{\pi N\Delta}}{m_\pi^2} \right)^2 \int \frac{d\mathbf{k} d\mathbf{q}}{(2\pi)^6} \frac{M^3}{E_{\mathbf{q}} E_{\mathbf{q}+\mathbf{k}} E_{\mathbf{p}-\mathbf{k}}} \\
& \times \theta^>(\mathbf{p}-\mathbf{k}) \theta^>(\mathbf{q}+\mathbf{k}) \theta^<(\mathbf{q}) \frac{1}{4} \text{Tr}_1 \text{Tr}_2 \\
& \times \left(\frac{1}{(W - E_{\mathbf{p}-\mathbf{k}})^2 - E_{\mathbf{p}-\mathbf{k}}^2} \right)^2 \\
& \times \frac{1}{W + E_{\mathbf{q}} - E_{\mathbf{p}-\mathbf{k}} - E_{\mathbf{q}+\mathbf{k}} + i\epsilon} \quad (6)
\end{aligned}$$

with

$$\begin{aligned}
\text{Tr}_1 &= \text{Tr} \left(k_\delta \mathcal{F}^{\beta\delta} \Lambda^+(\mathbf{p}-\mathbf{k}) k_\epsilon \mathcal{F}^{\epsilon\alpha} \frac{\not{p} + W}{2W} P_{\alpha\beta}^{3/2}(W) \right), \\
\text{Tr}_2 &= \text{Tr} [\not{k} \gamma^5 \Lambda^+(\mathbf{q}) \not{k} \gamma^5 \Lambda^+(\mathbf{q}+\mathbf{k})].
\end{aligned}$$

The imaginary part of this self-energy can be readily found. The integrals are evaluated with Monte Carlo integration and converge very rapidly. The result is a function of p_0 and $\mathbf{p}$, the energy and three-momentum of the Δ isobar, separately. This reflects the fact that we deal with a self-energy in medium. In Fig. 2 (minus) $\text{Im}(\Sigma_{2p1h})$ is plotted as a function of these two variables in a contour plot. The dotted line denotes the position of the resonance ($p_0^2 - |\mathbf{p}|^2 = M_\Delta^2$). As is clear from the figure, $\text{Im}(\Sigma_{2p1h})$ is almost constant over a large energy and momentum range. We stress, however, once again that we discarded all off-shell ambiguities of the Δ isobar by projecting on generalized spinors with invariant mass W and averaging over spins. Thus the results far off shell should be taken with caution. Twice the resulting self-energy is comparable in size with the decay width: about 40 MeV compared to 115 MeV. One may therefore expect a serious reduction of the results of the calculation in the resonance region, as we will see shortly.

The real part of the $2p1h$ self-energy is much harder to determine and requires some numerical effort. This is due to the pole structure of the integrand. Three-

dimensional hypersurfaces are present on which the integrand develops linear and quadratic singularities. We applied subtraction techniques to regularize the integrand to a smooth function. The counterterms are chosen such that they can be integrated analytically in one of the integration variables. In the present case a nested (in two integration variables), threefold subtraction procedure is necessary. The result is shown in Fig. 3 with contour plots as a function of the energy and momentum of the Δ isobar. The dotted curve indicates the resonance position. A rather rich structure occurs along the curve $W^2 \equiv p_0^2 - |\mathbf{p}|^2 = (M + m_\pi)^2$ (the dashed curve), which presumably is related to the opening of the pion cut at this invariant energy. The calculations show a sharp ridge precisely at the position of the dashed curve.

It is straightforward to evaluate the Pauli blocking effect on the self-energy of the Δ isobar due to the πN -bubble diagram. It equals minus the expression in free space, with the integration restricted to the Fermi sphere. Note that by doing this, we have chosen as reference frame the nuclear matter frame. If one still wants to evaluate the resulting integral in the frame where the Δ isobar has three-momentum zero, one must boost the expression to this frame. As a result the Fermi sphere will be deformed. We choose, however, to evaluate it in the nuclear matter frame. The resulting integrals again depend on both the energy and three-momentum of the Δ isobar. For the real parts the subtraction techniques mentioned earlier, are relatively easy to apply: the pole “hypersurfaces” in this case are, at most, two points. The results for the imaginary and real parts are shown in Figs. 4 and 5, respectively. A careful analysis for low momenta shows that the imaginary part behaves as a step function in the point $(p_0, |\mathbf{p}|) = (1266, 0)$ MeV. This is the value for which the decay momentum k in the center-of-mass system exceeds the Fermi momentum of 1.3 fm^{-1} . Below this threshold, for a Δ isobar at rest, the decay is always Pauli blocked, above never. A reflection of this phenomenon can already be observed in the increase of the density of contour lines in that region.

FIG. 2. Contour plot of (minus) the imaginary part of the $2p1h$ contribution to the Δ -isobar self-energy as a function of the energy and three-momentum transfer of the Δ isobar. The dotted curve indicates the position of the resonance: $p_0^2 - |\mathbf{p}|^2 = M_\Delta^2$. The contours are labeled in MeV.

FIG. 3. Results as in Fig. 2, but now for the real part of the $2p1h$ contribution to the Δ -isobar self-energy. The dashed curve indicates the points where $W = M + m_\pi$.

C. Comparison with the Δ -hole model and effect on the response functions

At this point we can make rather nice comparisons with the phenomenological results obtained in the Δ -hole model by Koch *et al.* [4]. In [4] expectation values for the real and imaginary parts of the spreading potential V_{sp} in the doorway states are tabulated. They obtain $\text{Im}(V_{sp}) \simeq -25$ MeV, almost independent of the photon three-momentum (which is directly related to the three-momentum of the Δ isobar in our case). This compares rather nicely with our results. We would like to stress once more that in their case these values follow partly from phenomenology, whereas in the present case we calculate them microscopically. For $\text{Re}(V_{sp})$ they obtain approximately +12 MeV, again almost independent of the photon momentum, whereas we obtain values that vary between +9 and +14 MeV in the relevant momentum range. For the imaginary part of the Pauli blocking the results compare well. For the real part, however,

both magnitude and sign differ. On the other hand, our results for the real part at the resonance compare well with those obtained in [7], where Pauli blocking effects on the Δ -isobar propagator are studied in the $\sigma\omega$ model.

In terms of Eq. (1) the Δ -hole model can be schematically denoted by the left-hand side, whereas our approach is based on the right-hand side. Apart from the relativistic aspects of our model, the main difference is that we include a microscopic picture for the spreading potential. We have seen that indeed the $2p2h$ reaction channel plays an important role in the total response function. The present analysis provides a justification for the claim made in [4] that the isobar current depicted in Fig. 6 is the dominant contribution to the spreading potential. In the phenomenological approach to the spreading potential, the particle-hole interaction may be mediated by other mesons than the pion only. In that respect an analysis along our lines including the ρ meson would be interesting.

From the present discussion it is clear that the iso-

FIG. 4. Contour plot of the imaginary part of the Pauli blocking contribution to the Δ -isobar self-energy as a function of the energy and three-momentum transfer of the Δ isobar. The dotted curve indicates the position of the resonance: $p_0^2 - |\mathbf{p}|^2 = M_\Delta^2$. The contours are labeled in MeV. The Fermi momentum is 1.3 fm^{-1} .

FIG. 5. Results as in Fig. 4, but now for (minus) the real part of the Pauli blocking contribution to the Δ -isobar self-energy. The Fermi momentum is 1.4 fm^{-1} .

bar model, which at first sight appears to be a one-body model, may contain implicit two-body processes via self-energies in the Δ -isobar propagator. This shows that one has to be careful with calculations that involve the Δ isobar in the fixed-width approximation (like in [3]). In the latter treatment much interesting physics can be hidden behind such an approximation. This we will discuss in more detail in Sec. V.

It is straightforward to extend the calculations of the two-body knockout and real-pion production contribution to the transverse response function as described in I, to include the rescattering and Pauli blocking contributions in the Δ -isobar propagator. The energy-dependent width $\Gamma(s)$ is simply replaced by $\Gamma_{\text{tot}}(s, \mathbf{p}, M, M_{\Delta})$, where we mention M and M_{Δ} to remind the dependence of the various contributions to Γ_{tot} on the masses. The real part can be incorporated via a change of p_0 in the denominator of the Δ -isobar propagator to $p_0 - \text{Re}(\Sigma_{\text{Pauli}} + \Sigma_{2p1h})$. The latter has hardly any effect on the calculations though. Note that according to Eq. (3) Γ_{tot} has to be used in the $2p2h$ channel as well as the real-pion production channel.

In Figs. 7 and 8 we show the results for the calculations for kinematics I ($|\mathbf{q}| = 550 \text{ MeV}/c$) and II ($|\mathbf{q}| = 1140 \text{ MeV}/c$). No parameter has been changed with respect to the calculations in I. From these results several observations can be made. First, the inclusion of Γ_{tot} reduces the response in the resonance region considerably, both for the real-pion production and the $2p2h$ contribution. This is a consequence of the fact that $2\Sigma_{2p1h}$ is comparable in size to the free width $\Gamma(s)$, especially below the resonance. For kinematics I the results we obtain after the inclusion of the rescattering contribution are thus considerably below the data, both in the dip region and resonance region. We stress, however, that this inclusion is necessary since it is based on the consistency requirements we discussed earlier. Finally in Fig. 9 we show the results of the calculations for a fixed momentum transfer of $0.1 \text{ (GeV}/c)^2$. As is clear from the figure, our calculations underestimate the data once the Δ -isobar-self-energy due to rescattering has been taken into account.

This seems to be a general feature of models that account for the various possible reaction channels that involve the Δ isobar in a unitary way (like the Δ -hole model). The implications of this fact are not clear.

We conclude that although our model at present does not provide a fully satisfactory description of the data in the dip region and Δ -resonance region, it does establish that a large part of the inclusive response in the dip region is due to two-body knockout. Moreover it confirms the assumption of the microscopic origin of the spreading potential in the Δ -hole model. The values for Σ_{2p1h} we obtained justify that part of the phenomenology of the Δ -hole model.

III. EFFECTIVE MASSES

We now study the effects on the response functions of a simple mean-field approach to interacting nuclear matter,

FIG. 6. Diagram representing the two-body isobar current which provides (in the terminology of the Δ -hole model) the dominant contribution to the spreading potential. The wiggly line denotes the photon, the dashed line the exchanged pion, and the solid lines the two nucleons.

FIG. 7. Comparison of the combined transverse response for the model with self-energies (solid line), of the quasifree knockout (dash-dotted line), the pion production (dash-triple dotted line), and the full two-body knockout (dashed line) with the data of kinematics I for ^{56}Fe . Also displayed is the static-limit result for the two-body knockout contribution (long-dashed line). Finally, the dotted line shows the combined response one would obtain replacing the full two-body with the SL two-body results. The data are taken from [8].

where all interactions manifest themselves in the emergence of quasiparticles with an effective mass M^* . For nuclear matter there exists a scheme to incorporate interactions in the relativistic description of the system in a self-consistent way: Dirac-Brückner theory (DB). Work along these lines has been carried out by Ter Haar and Malfliet [11] and by Amorim and Tjon [12]. In these studies special attention was paid to the relativistic aspects of the scheme: the influence of negative-energy states and the Lorentz structure of the nucleon self-energy. Although in the end one should apply an analysis like the one described above in our case, at present we adopt a much more modest approach to the incorporation of interactions.

Our approach [13] is motivated by work of Wehrberger, Bedau, and Beck [14,15]. They extend the Walecka $\sigma\omega$ model to include the Δ isobar, which is coupled to the scalar field σ and vector field ω . In Hartree (mean-field) approximation one obtains a self-consistent solution for the (classical) σ and ω fields, $\langle\phi_s\rangle$ and $\langle V^0\rangle$, respectively.

These then determine the modified nucleon propagator. The whole scheme can be described on shell in terms of quasiparticles with an effective mass $M^* = M + \Sigma_s = M + g_s^N \langle\phi_s\rangle$ and corresponding effective Dirac spinors. For the Δ isobar they assume that the couplings with the scalar and vector fields, g_s^Δ and g_v^Δ , are equal in strength as those of the nucleon to these fields. Since little is known about these couplings, the physical justification for this assumption is limited [15]. As our starting point we recall the expression for the nucleon self-energy in the nuclear matter frame

$$\Sigma(p) = \Sigma_s(p) - \Sigma_0(p)\gamma^0 + \gamma \cdot \mathbf{p} \Sigma_v(p). \quad (7)$$

The strength and dependence of these self-energies on the Fermi momentum is one of the prime results of the DB approach to interacting nuclear matter. Here, however, we treat these as given, momentum-independent scalars, to be treated as essentially free parameters, and to be compared after the fact with the results of microscopic analyses like the DB approach. Since in these

FIG. 8. Comparison of the results of the model with self-energies with the data, for kinematics II. The various curves have the same meaning as in Fig. 7. The data are taken from [9].

FIG. 9. Results as in Fig. 7 (inclusion of $2p1h$ and Pauli blocking self-energies), but now for the data of [10] where the four-momentum transfer is kept fixed.

analyses $|\mathbf{p}|\Sigma_v$ is generally small compared to Σ_s and Σ_v , we will neglect this term. The interacting one-particle Green function can be approximated by a noninteracting one with the replacements $M \rightarrow M^* = M + \Sigma_s$ and $p_0 \rightarrow p_0^* = p_0 + \Sigma_0$. For the latter we have the on-shell relation: $p_0^* = \sqrt{M^{*2} + \mathbf{p}^2} \equiv E_{\mathbf{p}}^*$. It is therefore convenient to write the interacting Green function [leaving out the negative-energy part by using the positive-energy projection operator $\Lambda^+(\mathbf{p})^*$] as

$$G^+ = \frac{M^*}{E_{\mathbf{p}}^*} \Lambda^+(\mathbf{p})^* \left(\frac{\theta^>(\mathbf{p})}{p_0^* - E_{\mathbf{p}}^* + i\epsilon} + \frac{\theta^<(\mathbf{p})}{p_0^* - E_{\mathbf{p}}^* - i\epsilon} \right). \quad (8)$$

Following Wehrberger and Beck we assume that the Δ -isobar mass can be modified in the same way as the nucleon, i.e., we add to the Rarita-Schwinger (RS) wave equation the above defined self-energy (without the Σ_v term), define the usual modified momentum $p^* = (p_0 + \Sigma_0, \mathbf{p})$, and change the RS constraints accordingly. The on-shell solution to these equations are the usual RS spinors, with the replacements $M_\Delta \rightarrow M_\Delta + \Sigma_s \equiv M_\Delta^*$ and $E_{\mathbf{p}}^\Delta \rightarrow E_{\mathbf{p}}^{\Delta*} = \sqrt{\mathbf{p}^2 + M_\Delta^{*2}}$. The interacting RS propagator is then equal to the original one with the same replacements (with the inclusion of the $p_0 \rightarrow p_0 + \Sigma_0$). It is then easy to repeat the derivations of the expressions of the respective contributions to $W^{\mu\nu}$, as described in I, for the case of interacting-nucleon Green functions and Δ -isobar propagators. The argument of the energy-conserving delta function becomes $E_{\mathbf{p}_1}^* + E_{\mathbf{p}_2}^* + \bar{\omega} - E_{\mathbf{p}_1'}^* - E_{\mathbf{p}_2'}^*$ with $\bar{\omega} = \omega - [\Sigma_0(p_1') - \Sigma_0(p_1)] - [\Sigma_0(p_2') - \Sigma_0(p_2)]$. This allows for an interpretation of the phenomenological binding-energy shift as resulting from the differences of self-energies at different particle momenta. We will neglect the momentum dependence of self-energies, however, and treat the energy shift as a phenomenological parameter, which we adjust to fit the position of the quasifree peak. In [16] a Hartree-Fock analysis for the Walecka model has been carried out, which indeed indicates that the momentum dependence is weak. Since only on-shell effective-nucleon projection operators ap-

pear and since we use the same mean-field (momentum-independent) self-energies for nucleon and Δ isobar, the Σ_0 term does not appear explicitly in the calculations. Moreover, the Δ -isobar decay width is adjusted to account for the interacting-nucleon propagator that appears in the diagram of Fig. 4 of I.

With a given self-energy Σ_s the modified masses M^* and M_Δ^* are determined. In the calculations of this section we use

$$M^* = 0.80M, \quad M_\Delta^* = 0.85M_\Delta,$$

and an energy shift $\Delta\omega \equiv \omega - \bar{\omega} = 20$ MeV per particle-hole pair. In Fig. 10 we show the results of the calculations with effective masses for kinematics I. The quasifree knockout process is represented by the dash-dotted line, the full two-body knockout contribution by the short dashed line and the static limit (SL) of the latter by the long-dashed line. The contribution due to real-pion production is represented by the dash-triple-dotted line and the sum of all three contributions is denoted by the solid line whereas the sum obtained with the SL result is given by the dotted line. Figure 11 shows the same for kinematics II. In general we observe trends that could be expected from simple kinematics: the resonance and quasifree responses move toward larger energy transfer and the width of the responses becomes considerably larger.

The specific value for the nucleon effective mass is rather arbitrary. It has been chosen such that a reasonable comparison with the data was obtained. Changes in this value affect the results considerably. One has to bear in mind that the model applies formally to nuclear matter, whereas we compare with data for ^{56}Fe . Typical values for the effective nucleon mass in nuclear matter calculations are on the order of $0.6M$. One may assume that in the interior of a heavy nucleus the hadron density is not too different from nuclear matter (and there are many indications that this is the case) such that there the local value for M^* is about $0.6M$. This value will approach the free value M in the surface region of the nucleus. In such a local-density approximation the effective masses and the Fermi momentum depend on the

FIG. 10. Results obtained with the use of effective masses for kinematics I. The values adopted in these calculations are $M^* = 0.80M$ and $M_\Delta^* = 0.85M_\Delta$. The various curves have the same meaning as in Fig. 7.

radius r . If we compare our value for M^* , which gave reasonable results for ^{56}Fe , with such a local-density approach, we should realize that it is a density-averaged quantity. As such the obtained value seems to be quite reasonable.

Including the rescattering and Pauli blocking contributions in the Δ -isobar propagator (where the self-energies are now calculated with the use of effective masses) we obtain the results displayed in Fig. 12 for kinematics I and Fig. 13 for kinematics II. Comparison with data clearly does not favor an effective Δ -isobar mass. Its main effect is to shift the Δ -isobar peak to larger energy transfer. Once the rescattering is taken into account, this leads to a large underestimation of the data. On the basis of the present analysis of the contribution of the Δ isobar to the transverse response functions we may conclude that a large Δ -isobar mass modification due to interaction with the mean fields in the nucleus is not allowed. Note that the latter conclusion does not exclude a large modification of the nucleon mass.

IV. FINITE NUCLEI

Several times we pointed out that our model contains one major approximation: the treatment of the nucleus as an infinite system. This allowed us to use plane waves as basis states. Furthermore, interactions could be incorporated in a rather transparent way by the modification of the one-particle Green function. In this section we address the question how to improve this treatment. Since one does not want to lose the relativistic aspects of the model, one has to turn to relativistic models of nuclear structure. During the past 15 yr several models of this kind have been developed. We cannot even give a modest account of these developments that does justice to these endeavors and their results. We continue here with an approach based on the work of Horowitz and Serot [2]. They obtained single-particle wave functions in coordinate space in the self-consistent mean-field approximation to the $\sigma\omega\rho$ model. These wave functions

FIG. 11. Results as in Fig. 10 but now for kinematics II.

FIG. 12. Results as in Fig. 7 (inclusion of $2p1h$ and Pauli blocking self-energies), but with the use of effective masses.

in particular describe satisfactorily a number of static properties of closed-shell nuclei.

A. Quasifree scattering

Let us first discuss the simplest contribution of our model: quasifree scattering. We work in momentum space, assume that the virtual-photon absorption proceeds through the one-body current, and neglect final-state interactions. The initial state is taken to be a single Slater determinant of occupied single-particle orbits. The latter are obtained as the Fourier transform of the Horowitz-Serot wave functions of [2] in coordinate space. We denote these wave functions by ψ_α , where α refers to the set of quantum numbers that specify the single-particle state. These wave functions are spinors in Dirac space. To exploit this we introduce the density operator

$$\hat{n}_\alpha(\mathbf{p}) \equiv \psi_\alpha(\mathbf{p}) \bar{\psi}_\alpha(\mathbf{p}). \quad (9)$$

For the explicit form of the momentum density operator we refer to Appendix A. The contribution of the quasifree knockout to the nuclear response tensor is then given by

$$W^{\mu\nu} = \frac{1}{\pi} \sum_\alpha \int \frac{d\mathbf{p}}{(2\pi)^3} \text{Tr}[\bar{J}^\nu \Lambda^+(\mathbf{p}') J^\mu \hat{n}_\alpha(\mathbf{p})] \times \delta(M + \epsilon_\alpha + \omega - E_{\mathbf{p}'}). \quad (10)$$

Here ϵ_α represents the binding energy of the state and $\mathbf{p}'$ the momentum of the nucleon in the final state. The summation runs over all occupied (closed) shells in the ground state of the nucleus. In [17] a completely equivalent derivation is presented.

First we study the effect of two further approximations. We replace the operator in Dirac space by a free-spinor positive-energy projection operator

$$\sum_\alpha \hat{n}_\alpha(\mathbf{p}) \simeq \sum_\alpha n_\alpha(\mathbf{p}) \Lambda^+(\mathbf{p}) \equiv n(\mathbf{p})_B \Lambda^+(\mathbf{p}). \quad (11)$$

Furthermore, the binding-energy correction in the

FIG. 13. Results as in Fig. 8 (inclusion of $2p1h$ and Pauli blocking self-energies), but with the use of effective masses.

FIG. 14. Comparison of the results for the calculation of the quasifree response with the full momentum density operator (solid line) and those obtained with the approximate momentum density operator (MDC, see text) (dashed line). Results for ^{40}Ca (left panel) and ^{56}Fe (right panel).

energy-conserving delta function, which is different for each single-particle state, is replaced by an average value. We then obtain the standard expression (like in [19]) with a few modifications. First, most importantly, the Fermi-gas momentum distribution $\theta^<(\mathbf{p})$ has been replaced by the baryon density $n(\mathbf{p})$. Second, there is yet no Pauli blocking for the nucleon in the final state present. This can be achieved by multiplying the obtained expression with $[1 - n(\mathbf{p})]$. Finally some normalizations have to be taken into account. In Fig. 14 we compare the results of the calculation of the quasifree response for both ^{40}Ca and ^{56}Fe ¹ for kinematics I with (dashed line) and without (solid line) the last two approximations. The latter type of calculations we refer to as momentum density calculations (MDC). For the binding energy in the case of the MDC we take the (weighted) average that can be obtained from the binding energies of the individual orbitals and which we denote by $\bar{\epsilon}_B$. For ^{40}Ca it is about 24 MeV. Although there are differences between the two calculations they justify a treatment where one neglects the full operator structure of the momentum density operator and performs an MDC. Since this simplifies the modifications in the two-body case substantially, we make the approximation as given in Eq. (11) for the two-body contributions. There are no fundamental reasons to make this approximation other than economic ones: we do not have to repeat the algebraic manipulation part of the calculations of the two-body response functions and the numerical part remains tractable. The modification of the expressions in I for the two-body current contribution to $W^{\mu\nu}$ is then simply given by (recall that in this case a binding energy correction was used of 35 MeV per particle hole pair which we denote by ϵ_{FG})

$$\theta^<(\mathbf{p}_1)\theta^<(\mathbf{p}_2)\theta^>(\mathbf{p}'_1)\theta^>(\mathbf{p}'_2)$$

$$\rightarrow n(\mathbf{p}_1)n(\mathbf{p}_2)[1 - n(\mathbf{p}_1)][1 - n(\mathbf{p}_2)] , \quad (12)$$

$$\delta(E_{\mathbf{p}_1} + E_{\mathbf{p}_2} + q_0 - 2\epsilon_{\text{FG}} - E_{\mathbf{p}'_1} - E_{\mathbf{p}'_2})$$

$$\rightarrow \delta(2M + q_0 - 2|\bar{\epsilon}_B| - E_{\mathbf{p}'_1} - E_{\mathbf{p}'_2}) . \quad (13)$$

We wish to point out that for the two-body case the expressions above implicitly assume a factorization of the two-body density matrix: $n(\mathbf{p}_1, \mathbf{p}_2) = n(\mathbf{p}_1)n(\mathbf{p}_2)$. This is certainly a strong assumption. Correlations in the nuclear ground state, for instance, due to the hard core in the nucleon-nucleon potential, will considerably modify the two-body density matrix, especially when it comes to the high momentum components. One therefore does a much better job if one uses a (relativistic) two-body spectral function which contains these correlation effects. To our knowledge a relativistic two-body spectral function has not yet been developed. It would be very worthwhile to do so, but this is outside the scope of the present work. We proceed with the factorized form and it is now straightforward to perform calculations of the various contributions to the transverse response in the case of the modified momentum distributions.

B. Results of the momentum distribution calculations

Before we present the results of our calculations, one more modification should be mentioned: the imaginary part of the $2p1h$ self-energy is also recalculated with the momentum density described above. The effect on this self-energy is very small, however. It is typically 1 or 2 MeV smaller, which has only a very small effect on the final calculations. This result is not to be expected *a priori*. It supports the widespread approach to calculate self-energies of this type in a Fermi-gas or local-density approach.

The results we obtained for a MDC in the case of both

¹ ^{56}Fe is not a doubly magic nucleus. To be able to use the wave functions of Horowitz and Serot we use the wave function of ^{56}Ni instead.

FIG. 15. Results for ^{56}Fe obtained with the use of the momentum density as obtained from the Horowitz-Serot wave functions [2] in momentum space for kinematics I.

kinematics I and II for ^{56}Fe are shown in Figs. 15 and 16, and for ^{40}Ca in the case of kinematics I in Fig. 17. If we compare these results with the one obtained in the Fermi-gas calculations (Figs. 7 and 8) there are little changes due to the finite-size effects, especially in the two-body knockout and the real-pion production calculations. This is an indication that the Fermi-gas picture we applied thus far provides rather adequate results in the case of the contributions that involve the Δ isobar. The results are in fair agreement with the data. One expects at high energy transfer other resonances or reaction channels to become important. These should account for the large discrepancy between the calculations and the data of kinematics II which starts around 700 MeV. We wish to conclude that the present momentum density calculations provide a good account of those contributions to the transverse nuclear response where the excitation of a Δ isobar is involved.

V. Δ -ISOBAR DECAY WIDTH REVISITED

In I we stressed the importance of the energy dependence of the Δ -isobar decay width. Moreover, we claimed that the results obtained by Rost, Price, and Shepard in [3], which lead them to the result that phenomenological one-body currents could well account for the response functions in the dip and resonance region, are seriously influenced by the fixed width approximation they make. It is the aim of this section to clarify these points.

Although a completely equivalent analysis in momentum space of their coordinate space calculation is difficult to perform, we will here present one that contains many of the main ingredients.

The starting point of their coordinate space calculation for the nuclear response is the following expression for the hadronic tensor:

FIG. 16. Results as in Fig. 15 but now for kinematics II.

FIG. 17. Results as in Fig. 15 but now for ^{40}Ca .

$$W^{\mu\nu} = \frac{1}{\pi} \text{Im} \left(\sum_h \int d\mathbf{x} d\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{y}} \{ \bar{U}_h(\mathbf{x}) \bar{J}^\nu \right. \\ \times G_F(\mathbf{x}, \mathbf{y}, \epsilon_h + \omega) J^\mu U_h(\mathbf{y}) + \bar{U}_h(\mathbf{y}) \bar{J}^\nu \\ \left. \times G_F(\mathbf{y}, \mathbf{x}, \epsilon_h - \omega) J^\mu U_h(\mathbf{x}) \} \right), \quad (14)$$

where G_F may denote either the one-body nucleon or the one-body Δ -isobar Green function which is obtained as the solution of

$$(\omega\gamma_0 + i\boldsymbol{\gamma} \cdot \nabla - M - \Sigma_H(\mathbf{x})) G_N(\mathbf{x}, \mathbf{y}) = \delta(\mathbf{x} - \mathbf{y})$$

for the nucleon Green function or as the solution of

$$[(\omega + i\frac{1}{2}\Gamma)\gamma_0 + i\boldsymbol{\gamma} \cdot \nabla - M_\Delta - \Sigma_H(\mathbf{x})] G_\Delta(\mathbf{x}, \mathbf{y}) \\ = \delta(\mathbf{x} - \mathbf{y})$$

for the Δ -isobar Green function, respectively. The latter is thus treated as a spin- $\frac{1}{2}$ particle. The current operator should of course be chosen accordingly. $U_h(\mathbf{x})$ denotes the hole (initial nucleon) wave function, for which they use the solution of the $\sigma\omega\rho$ model in the Hartree approximation, as obtained by Horowitz and Serot [2]. In this solution one obtains a self-consistent Hartree mean field which is denoted here by Σ_H . Equation (14) is only valid in the plane-wave impulse approximation and if one assumes a simple Slater determinant for the nuclear wave function. If the Hartree mean field is taken constant (independent of $\mathbf{x}$) and $\mathbf{q} = \mathbf{p}' - \mathbf{p}$ is substituted, the equivalent expression of Eq. (14) in momentum space is given by

$$W^{\mu\nu} = \frac{1}{\pi} \sum_h \text{Im} \left(\int \frac{d\mathbf{p}}{(2\pi)^3} \left[\bar{U}_h(\mathbf{p}) \bar{J}^\nu G_F(\mathbf{p}', \epsilon_h + \omega) \right. \right. \\ \left. \left. \times J^\mu U_h(\mathbf{p}) + (q_\mu \rightarrow -q_\mu) \right] \right), \quad (15)$$

where the solution in momentum space for the Green functions is given by

$$G_N(p) = \frac{1}{\not{p} - M - \Sigma_H + i\epsilon} \quad (16)$$

for the nucleon and by

$$G_\Delta(p) = \frac{1}{(p_0 + i\frac{1}{2}\Gamma)\gamma_0 - \boldsymbol{\gamma} \cdot \mathbf{p} - M_\Delta - \Sigma_H} \quad (17)$$

for the Δ isobar. Note that if we would replace the hole-state wave functions $U_h(p)$ by free Dirac spinors (with the appropriate normalization), we would recover the expression one obtains in the Fermi-gas model. Due to the approximation of the $\mathbf{x}$ independence of the Hartree field, the present momentum space analysis is not completely equivalent with the one in [3]. The qualitative differences we will show shortly do not depend on this approximation, however. Since they treat the Δ isobar as a spin- $\frac{1}{2}$ particle, they must adjust the coupling vertex and choose to approximate it by

$$J_\Delta^\mu = \bar{c} M_\Delta |\mathbf{q}| \gamma^\mu \gamma^5.$$

Here the coupling $\bar{c}$ is given by Eq. (7.12) of I and one expects this approximation to be certainly unrealistic in the resonance region when $q_0 \simeq |\mathbf{q}|$. It is now straightforward to evaluate the response function for the isobar contribution. We use once more the momentum space density operator of Eq. (9) and obtain

$$W^{\mu\nu} = \frac{2}{3} \bar{c}^2 M_\Delta^2 \sum_h \int \frac{d\mathbf{p}}{(2\pi)^3 2E_{\mathbf{p}+\mathbf{q}}^{\Delta*}} \\ \times \frac{1}{\pi} \frac{\Gamma/2}{(M^* - |\epsilon_h| + \omega - E_{\mathbf{p}+\mathbf{q}}^{\Delta*})^2 + \frac{1}{4}\Gamma^2} \\ \times \mathbf{q}^2 \text{Tr}[\gamma^\mu \gamma^5 (\not{p} + M_\Delta^*) \gamma^\mu \gamma^5 \hat{n}_h(k)] \quad (18)$$

with $M_\Delta^* = M_\Delta + \Sigma_H$ and $\hat{p} = E_{\mathbf{p}}^{\Delta*} = \sqrt{\mathbf{p}^2 + M_\Delta^{*2}}$. The factor $\frac{2}{3}$ accounts for the spin- $\frac{3}{2}$ projection operator that is lost due the effective spin- $\frac{1}{2}$ treatment of the Δ isobar in this analysis.

In Fig. 18 we show the results of the isobar contribution

FIG. 18. Comparison of the contribution to the transverse response function of ^{40}Ca for kinematics I from the Δ isobar according to Rost, Price, and Shepard [3] in the fixed-width approximation (solid line) with the contribution that is obtained when the energy dependence of the free decay width is accounted for (dotted line). The dashed curve shows the results for the full width obtained in Sec. II.

to the transverse nuclear response for ^{40}Ca at $|\mathbf{q}| = 550$ MeV. For purpose of comparison with [3] we also plot the response on a logarithmic scale. The solid line is obtained with the fixed-width approximation ($\Gamma = 110$ MeV) and clearly extends into the low energy transfer region. If one takes the energy dependence of the width that we obtained in I into account, however, this tail disappears. This important qualitative effect has strong bearings on the conclusion drawn by Rost *et al.* that one-body processes would suffice to describe the nuclear response in the dip region. We believe that the present analysis shows that this no longer holds once the energy dependence of the width or equivalently the allowed phase space of the produced pion is taken into account. To show the effect of the contribution of the two-body isobar currents in the present framework, we finally depict the results we obtain when we use the full width as found in Sec. II by the dashed curve. The latter calculation does show a low energy tail in the Δ -isobar response, but this tail is essentially of two-body origin. From the above results we may conclude that the energy dependence of the Δ -isobar decay width must be taken into account in any approach to pion electroproduction to obtain realistic results.

VI. CONCLUSIONS

In this paper we reported on a study of medium modifications of the contribution of relativistic meson exchange and isobar currents to the response of nuclei in electron scattering at intermediate energies. We now summarize the conclusions which emerge from this work.

Throughout this paper we paid attention to the question whether a relativistic treatment of two-body currents can provide a satisfactory explanation of the experimentally observed strength in the dip region. The results presented in Sec. IV, where consistency between the treatment of the real-pion production and isobar currents is achieved (by the inclusion of rescattering) and

realistic momentum distributions have been used, show that the two-body currents provide about one-third of the strength in the dip region. A large part of this contribution is due to a resonance behavior of the (two-body) isobar current. Our relativistic treatment was essential to describe this resonance behavior correctly. In the results of the combined model most of the strength in the dip region is accounted for.

To put our results in better perspective it is, at this stage, useful to summarize the main shortcomings of our model. An important aspect of the physics of electron scattering at intermediate energies is the effect of short-range correlations in the nuclear ground state. As we discussed in Sec. IV A these are absent in our mean-field treatment. A second ingredient that our model is lacking is the inclusion of final-state interactions (FSI). These are known to considerably influence both the longitudinal and transverse response functions (see, for instance, the work of Chinn *et al.* [18]). As a consequence caution should be taken when a detailed quantitative comparison is made between our results and the experimental data. There is however no reason to expect that the two ingredients mentioned above will affect the important qualitative features of our unitary model presented in this paper.

From the rescattering analysis mentioned above, an interesting comparison with the Δ -hole model could be made. Our results support the proposed microscopic picture of the phenomenological spreading potential: it results from pion absorption in the nuclear medium. The calculated values for both the real and imaginary parts of our "rescattering self-energy" Σ_{2p1h} compare well with those reported for the spreading potential.

We studied the effect of medium modifications of both the nucleon and Δ -isobar masses due to couplings to the scalar and vector mean fields. These couplings were taken equally strong for the nucleon and the Δ isobar. When the rescattering was taken into account, the results of the calculation did not support a large reduction of the Δ -isobar mass. From this we may conclude that a large

coupling of the Δ isobar to the scalar mean field that is responsible for the effective nucleon mass, is not probable. The results presented in Sec. IV for the response function at high momentum transfer are in fair agreement with the data. At this energy scale, meson, nucleon, and Δ -isobar degrees of freedom can provide a correct account of the physics involved. We may ascribe the discrepancy between the data and the results of our model at energy transfers above 700 MeV to the excitation of higher nucleon resonances or final-state interaction effects, which we did not take into account in our model. Finally we stressed the importance and physical implications of the energy dependence of the Δ -isobar propagator in relation to the results obtained in [3].

ACKNOWLEDGMENTS

This work was supported in part by the Stichting voor Fundamenteel Onderzoek der Materie (FOM), which is sponsored by the Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO). Part of the numerical calculations were performed on the Cray YMP of the Amsterdam Computer Centre (SARA), with financial support from the “Stichting Nationale Computer Faciliteiten” (NCF). A detailed comparison between part of our results and those of C.R. Chinn and J.W. van Orden was made possible by their kind help, which is gratefully acknowledged.

APPENDIX A: MOMENTUM DENSITY OPERATOR

In [2] Horowitz and Serot solve the $\sigma\omega\rho$ model self-consistently in the Hartree mean-field approximation for spherically symmetric closed-shell nuclei. One then has to solve the Dirac equation with a central potential. The general form of these solutions can be written as

$$\psi_{nlj\mu}(\mathbf{r}) = \frac{1}{r} \begin{pmatrix} iG_{nlj}(r)\phi_{\kappa\mu} \\ -F_{nlj}(r)\phi_{-\kappa\mu} \end{pmatrix} \quad (\text{A1})$$

with the central-field spinors

$$\phi_{\kappa\mu} = \sum_{ms} \langle lm \frac{1}{2} s | j\mu \rangle Y_{lm}(\Omega_r) \chi_s.$$

Here Y_{lm} denotes a spherical harmonic function and χ_s a Pauli spinor. The coupling coefficients are Clebsch-Gordan coefficients. Dirac’s quantum number κ is the eigenvalue of the operator $-\gamma_0(\boldsymbol{\sigma} \cdot \mathbf{L} + 1)$ so that $\gamma_0(\boldsymbol{\sigma} \cdot \mathbf{L} + 1)\phi_{\kappa\mu} = -\kappa\phi_{\kappa\mu}$. In terms of κ , the quantum numbers l and j are given by $l = \kappa$ for $\kappa > 0$, $l = -(\kappa + 1)$ for $\kappa < 0$, and $j = |\kappa| - \frac{1}{2}$. The angular structure of the solutions is general for every solution to the Dirac equation with a central potential; the radial functions $G_{nlj}(r)$ and $F_{nlj}(r)$ contain the actual solution for the model under consideration. For these functions we take those of Horowitz and Serot. The Fourier transform $\psi_{nlj\mu}(\mathbf{k})$ of these wave functions can be readily found

$$\begin{aligned} \psi_{nlj\mu}(\mathbf{k}) &= \int d\mathbf{r} e^{-i\mathbf{k} \cdot \mathbf{r}} \psi_{nlj\mu}(\mathbf{r}) \\ &= (-i)^l \begin{pmatrix} iG_{nlj}(k) \\ i\boldsymbol{\sigma} \cdot \hat{\mathbf{k}} F_{nlj}(k) \end{pmatrix} \phi_{\kappa\mu}(\Omega_k) \end{aligned} \quad (\text{A2})$$

with

$$\begin{aligned} G_{nlj}(k) &= 4\pi \int dr r j_l(kr) G_{nlj}(r), \\ F_{nlj}(k) &= \begin{cases} -4\pi \int dr r j_{l+1}(kr) F_{nlj}(r) & \text{for } \kappa < 0 \\ 4\pi \int dr r j_{l-1}(kr) F_{nlj}(r) & \text{for } \kappa > 0 \end{cases} \end{aligned}$$

and $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$. This expression is obtained after the expansion of the plane wave in spherical harmonics and spherical Bessel functions $j_l(kr)$

$$e^{-i\mathbf{k} \cdot \mathbf{r}} = \sum_{lm} 4\pi (-i)^l j_l(kr) Y_{lm}^*(\Omega_r) Y_{lm}(\Omega_k)$$

and use of the orthogonality of the spherical harmonics. Furthermore the relation $\{\boldsymbol{\sigma} \cdot \hat{\mathbf{k}}, \boldsymbol{\sigma} \cdot \mathbf{L} + 1\} = 0$ allows one to change, in the lower component, $\phi_{-\kappa\mu}(\Omega_k)$ to $-\boldsymbol{\sigma} \cdot \hat{\mathbf{k}} \phi_{\kappa\mu}(\Omega_k)$

FIG. 19. Baryon density in momentum space obtained from the coordinate space wave functions of Horowitz and Serot for ^{40}Ca and ^{56}Ni . The latter is used to describe the density of ^{56}Fe .

In the calculations of the response functions we need the momentum density operator

$$\hat{n}_\alpha(\mathbf{k}) \equiv \sum_\mu \psi_{\alpha\mu}(\mathbf{k}) \bar{\psi}_{\alpha\mu}(\mathbf{k}), \quad (\text{A3})$$

where α represents nlj . Without proof (see, e.g., in [20] Eq. (A11)) we state

$$\sum_\mu \phi_{\kappa\mu} \phi_{\kappa\mu}^* = \frac{2j+1}{8\pi} I \quad (\text{A4})$$

with I the identity operator in Pauli spin space. We can then write $\hat{n}_\alpha$ in terms of γ matrices as

$$\hat{n}_\alpha(\mathbf{k}) \equiv \not{p}_\alpha + M_\alpha \quad (\text{A5})$$

with

$$p_{0\alpha}(k) = \frac{2j+1}{16\pi} [G_\alpha^2(k) + F_\alpha^2(k)],$$

$$\mathbf{p}_\alpha = \frac{2j+1}{8\pi} \hat{\mathbf{k}} F_\alpha(k) G_\alpha(k),$$

$$M_\alpha(k) = \frac{2j+1}{16\pi} [G_\alpha^2(k) - F_\alpha^2(k)].$$

Finally we need the baryon density $n_B(k)$

$$n_B(k) \equiv \sum_{\text{occupied } \alpha} p_{0\alpha}(k). \quad (\text{A6})$$

The baryon density we obtain this way is plotted for the case of ^{40}Ca (dashed line) and ^{56}Ni (solid line) in Fig. 19. The normalization has been chosen such that $\int dk k^2 n_B(k) = A$, where A denotes the atomic mass number.

-
- [1] M.J. Dekker, P.J. Brussaard, and J.A. Tjon, Phys. Rev. C **49**, 2650 (1994).
 - [2] C.J. Horowitz and B.D. Serot, Nucl. Phys. **A368**, 503 (1981).
 - [3] E. Rost, C.E. Price, and J.R. Shepard, Phys. Rev. C **47**, 2250 (1993).
 - [4] J.H. Koch and N. Ohtsuka, Nucl. Phys. **A435**, 765 (1985).
 - [5] F. De Jong and R. Malfliet, Phys. Rev. C **46**, 2567 (1992); F. de Jong, Ph.D. thesis, University of Groningen (unpublished).
 - [6] E. Oset and L.L. Salcedo, Nucl. Phys. **A468**, 631 (1987).
 - [7] K. Wehrberger and R. Wittman, Nucl. Phys. **A513**, 603 (1990).
 - [8] Z.E. Meziani *et al.*, Phys. Rev. Lett. **54**, 1233 (1985).
 - [9] J.P. Chen *et al.*, Phys. Rev. Lett. **66**, 1283 (1991).
 - [10] D.T. Baran *et al.*, Phys. Rev. Lett. **61**, 400 (1988).
 - [11] B. ter Haar and R. Malfliet, Phys. Rep. **149**, 207 (1987).
 - [12] A. Amorim and J.A. Tjon, Phys. Rev. Lett. **68**, 772 (1992).
 - [13] M.J. Dekker, P.J. Brussaard, and J.A. Tjon, Phys. Lett. B **289**, 255 (1992).
 - [14] K. Wehrberger and F. Beck, Phys. Lett. B **270**, 1 (1991).
 - [15] K. Wehrberger, C. Bedau, and F. Beck, Nucl. Phys. **A504**, 797 (1989).
 - [16] C.J. Horowitz and B.D. Serot, Nucl. Phys. **A399**, 529 (1983).
 - [17] C.R. Chinn, Ph.D. thesis, University of Maryland (unpublished).
 - [18] C.R. Chinn, A. Picklesimer, and J.W. van Orden, Phys. Rev. C **40**, 790 (1989).
 - [19] E.J. Moniz, Phys. Rev. **184**, 1154 (1969).
 - [20] J.A. Tjon and S.J. Wallace, Phys. Rev. C **36**, 1085 (1987).