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THE DECIDABILITY OF DEPENDENCY IN INTUITIONISTIC PROPOSITIONAL LOGIC

DICK DE JONGH AND L. A. CHAGROVA

Abstract. A definition is given for formulae $A_1, \dots, A_n$ in some theory T which is formalized in a propositional calculus S to be (in)dependent with respect to S . It is shown that, for intuitionistic propositional logic **IPC**, dependency (with respect to **IPC** itself) is decidable. This is an almost immediate consequence of Pitts' uniform interpolation theorem for **IPC**. A reasonably simple infinite sequence of **IPC**-formulae $F_n(p, q)$ is given such that **IPC**-formulae A and B are dependent if and only if at least one of the $F_n(A, B)$ is provable.

§1. Introduction. We denote the intuitionistic propositional calculus by **IPC**. Let us call formulae $A_1, \dots, A_n$ of some intuitionistic theory T **IPC-dependent over T** , or *dependent over T* for short, if for some **IPC**-formulae $F(p_1, \dots, p_n)$ we have $\vdash_T F(A_1, \dots, A_n)$, but $\nVdash_{\text{IPC}} F(p_1, \dots, p_n)$. Otherwise $A_1, \dots, A_n$ are called *independent*. In de Jongh [1982] the behavior of formulae of one propositional variable in intuitionistic arithmetic **HA** was discussed. The main result of that paper was that for arithmetic sentences A , if $\nVdash_{\text{HA}} \neg\neg A \rightarrow A$ and $\nVdash_{\text{HA}} \neg\neg A$, then A is independent over **HA** with respect to **IPC**. This result was generalized to formulae. We did not mention the fact that the result applies to the propositional calculus itself as well.

1.1. THEOREM. *If $\nVdash_{\text{IPC}} \neg\neg A \rightarrow A$ and $\nVdash_{\text{IPC}} \neg\neg A$, then A is independent over **IPC**.*

In fact, the proof in §2 of de Jongh [1982] applies immediately to this case. Naturally, for **IPC** there is no immediate reason to look for a more constructive proof, as we did for **HA** in the major part of that paper. A fortiori, of course, the result implies that dependency is decidable for the one-variable case: it can be checked whether an arbitrary formula A is dependent by checking whether $\neg\neg A \rightarrow A$ or $\neg\neg A$ is provable. We call Theorem 1.1 a *minimal provability result*: if anything nontrivial propositional is provable about A , then $\neg\neg A \rightarrow A$ or $\neg\neg A$ is. The result leads to a characterization of the monadic propositional functions F for which there exist A such that exactly $\vdash F(A)$. This result holds for **HA** as well as for **IPC**. We remind the reader of the definition for n propositional variables:

1.2. DEFINITION. *Exactly $\vdash F(A_1, \dots, A_n)$ iff $\vdash F(A_1, \dots, A_n)$ and, for all propositional G , $\vdash G(A_1, \dots, A_n) \Rightarrow \vdash_{\text{IPC}} F(p_1, \dots, p_n) \rightarrow G(p_1, \dots, p_n)$.*

This leads to the following classification of formulas of one propositional variable in **HA** as well as in **IPC**.

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1.3. THEOREM. *To each formula exactly one of the following cases applies (non-constructively, of course, in the case of **HA**),*

- (I) *exactly* $\vdash A$,
- (II) *exactly* $\vdash \neg A$,
- (III) *exactly* $\vdash \neg\neg A$,
- (IV) *exactly* $\vdash \neg\neg A \rightarrow A$,
- (V) *exactly* $\vdash A \rightarrow A$ (*A is independent*).

Examples exhibiting the five cases in **IPC** are respectively, (I) $p \rightarrow p$, (II) $p \wedge \neg p$, (III) $\neg\neg p \rightarrow p$, (IV) $\neg p$, (V) p .

In this note we will show that for n variables the decidability of dependency is a consequence of Pitts' uniform interpolation theorem (Pitts, [1992]). Moreover, we will give an analogue for two propositional variables of Theorem 1.1. A general analogue for Theorem 1.3 seems much harder (see de Jongh and Visser [1993], however, for some results). We will not go into that here except to remark that in the case of arithmetic there are easy analogues of Theorem 1.3 for restricted cases, e.g. if one restricts oneself to Π_1^0 -sentences. In the monadic case we have, for Π_1^0 -sentences A , exactly $\vdash A$, or exactly $\vdash \neg A$, or exactly $\vdash \neg\neg A \rightarrow A$ (i.e., in particular, a Π_1^0 -sentence is never an independent one). In the binary case, if not $\vdash A$, $\vdash \neg A$, $\vdash B$, or $\vdash \neg B$, then exactly

$$\vdash (\neg\neg A \rightarrow A) \wedge (\neg\neg B \rightarrow B),$$

or

$$\vdash (A \rightarrow B) \wedge (\neg\neg A \rightarrow A) \wedge (\neg\neg B \rightarrow B),$$

or

$$\vdash (B \rightarrow A) \wedge (\neg\neg A \rightarrow A) \wedge (\neg\neg B \rightarrow B),$$

or

$$\vdash (A \leftrightarrow B) \wedge (\neg\neg A \rightarrow A) \wedge (\neg\neg B \rightarrow B).$$

The only nontrivial relationship between Π_1^0 -sentences is apparently the one of implication, as it is in classical arithmetic.

We thank Albert Visser for many discussions on the subject.

§2. Decidability of dependency over IPC. Pitts [1992] proved, among other things, that, for any **IPC**-formula $A(\vec{p}, \vec{r})$, one can find a formula $E_{\vec{r}}A(\vec{p}, \vec{r})$, not containing $\vec{r}$, such that, for any formula $B(\vec{p})$,

$$A(\vec{p}, \vec{r}) \vdash_{\text{IPC}} B(\vec{p}) \Leftrightarrow E_{\vec{r}}A(\vec{p}, \vec{r}) \vdash_{\text{IPC}} B(\vec{p}).$$

Consider the formulae $A_1, \dots, A_n$ in the variables $\vec{r}$. We will show that these formulae are independent iff $\vdash E_{\vec{r}}((A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n))$. As **IPC** is decidable, this will do.

First assume $A_1, \dots, A_n$ to be independent. From Pitts' theorem it follows that

$$\begin{aligned} & \vdash (A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n) \rightarrow B(\vec{p}) \\ & \Leftrightarrow \vdash E_{\vec{r}}((A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n)) \rightarrow B(\vec{p}). \end{aligned}$$

On the other hand,

$$\vdash B(\vec{A}) \Leftrightarrow \vdash (A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n) \rightarrow B(\vec{p}).$$

Taking $E_{\vec{r}}((A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n))$ for the $B(\vec{p})$ makes the right-hand side of the first equivalence true, so $\vdash B(\vec{A})$; by independence of $A_1, \dots, A_n$ it follows that $\vdash B(\vec{p})$, which means that

$$\vdash E_{\vec{r}}((A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n)).$$

Next assume $\vdash E_{\vec{r}}((A_1 \leftrightarrow p_1) \wedge \dots \wedge (A_n \leftrightarrow p_n))$. The right-hand side of the first equivalence then reduces to $\vdash B(\vec{p})$, i.e. $\vdash B(\vec{A})$ iff $\vdash B(\vec{p})$.

One might be of the opinion that A and B would more properly be defined as being dependent if there is some F such that $\vdash F(A, B)$, but there is no pair G, H such that both $\vdash G(A)$ and $\vdash H(B)$ hold, and $G(p), H(q) \vdash F(p, q)$. With this alternative definition e.g. $\neg p$ and q would be independent, while under Definition 1.2 they are dependent, since $\neg \neg \neg p \rightarrow \neg p$ is provable. (This alternative was suggested to us by V. Shavrukov.) It will be clear that decidability follows for the alternative definition just as well by a proof as the above. It seems to us that both definitions describe relevant concepts. A more extensive investigation of related matters is given in de Jongh and Visser [1993].

It is clear, of course, that, for any propositional logic S for which a uniform interpolation theorem holds, dependency is decidable for the logic itself. In fact the uniform interpolation theorem has been proved for the provability logic **L** by Shavrukov [1993], completely independently of Pitts' result and by a completely different proof. Hence, dependency is decidable for **L**. Unfortunately, for most modal logics not even a standard interpolation theorem holds (see e.g. Maksimova [1982]), so for many logics a completely different method will have to be found if one wants to study the problem.

§3. A minimal provability result for two variables. We first recall some facts about Kripke models for intuitionistic propositional logic.

(i) For each A of **IPC**, if $\nVdash_{\text{IPC}} A$, there is a finite tree-ordered Kripke model $\mathbf{K} = \langle W, \leq, \Vdash \rangle$ such that $\mathbf{K} \not\models A$ (we write $\mathbf{K} \models B$ to mean that $w \Vdash B$ for all $w \in W$). (There is no essential reason to restrict oneself to tree-ordered Kripke models, but they are more easily described.)

(ii) We write $w \uparrow$ for $\{w' \in W \mid w \leq w'\}$. The model $\mathbf{K}$ restricted to $w \uparrow$ is called a *generated submodel* of $\mathbf{K}$.

(iii) A *p-morphism* from a Kripke model $\mathbf{K} = \langle W, \leq, \Vdash \rangle$ to a Kripke model $\mathbf{K}' = \langle W', \leq', \Vdash' \rangle$ is a surjection $\phi: W \rightarrow W'$ such that:

- (a) $w \leq w' \Rightarrow \phi(w) \leq' \phi(w')$,
- (b) $\phi(w) \leq' \phi(w') \Rightarrow \exists w'' \geq w (\phi(w'') = \phi(w))$,
- (c) for all $w \in W$ and all propositional variables p , $\phi(w) \Vdash' p \Leftrightarrow w \Vdash p$.

It is easily shown then that (c) applies to all formulas.

(iv) A finite tree-ordered Kripke model is called *irreducible* if all its p -morphic images to tree-ordered Kripke models are isomorphic. We will call such a model a T -model for short.

(v) (de Jongh [1970]) For each T -model $\mathbf{K}$ there are formulas C_K and D_K such that

- (a) $\mathbf{K} \models C_K$, but $\mathbf{K} \not\models D_K$.
- (b) For each T -model $\mathbf{L}$ such that $\mathbf{L} \models C_K$, $\mathbf{L}$ is isomorphic to a generated submodel of $\mathbf{K}$ ($\mathbf{L} \preceq \mathbf{K}$).
- (c) For each T -model $\mathbf{L}$, such that $\mathbf{L} \not\models D_K$, $\mathbf{K} \preceq \mathbf{L}$.

Strongly related to the models used here are the n -universal models used by Rybakov [1992] that were introduced by Shehtman [1978]. The fact that Shehtman and Rybakov do not restrict themselves to trees is not essential. Any of our models has a p -morphism onto a unique generated submodel of their infinite model, and, vice versa, any generated submodel of Rybakov's $I(n)$ can be made into a T -model by a standard treeification procedure.

(vi) If we consider a Kripke model for the language consisting of the two propositional variables p and q , the values of p and q at the root of a model $\mathbf{K}$ are respectively i and j , and the generated submodels corresponding to the immediate successors of the root are $\mathbf{L}_1, \dots, \mathbf{L}_n$, then we denote $\mathbf{K}$ by

Each T -model with a domain of more than one element has such a form with $\mathbf{L}_1, \dots, \mathbf{L}_n$ irreducible and none of the $\mathbf{L}_j$ isomorphic to a generated submodel of any of the others. In case $n = 1$, the root of $\mathbf{L}_1$ has a forcing relation distinct from ij . All finite T -models can be obtained from the four irreducible p - q -models with one-element domains: 00, 01, 10, 11 by repeatedly adjoining roots with proper valuations to finite sets of $\preceq$ -incomparable T -models already obtained.

Let us now suppose that we have that $\not\models D_K(A_1, A_2)$. Then, in a sense, the model $\mathbf{K}$ is “available” for A_1, A_2 , because any countermodel to $D_K(A_1, A_2)$ (and such a model has to exist) has to contain $\mathbf{K}$ in its valuations for A_1, A_2 (if necessary after taking a p -morphic image and treeification). Any counterexamples to formulas which can be given on $\mathbf{K}$ or its generated submodels then give rise to underivable formulas as well.

If we have a finite set of D_L 's which are not derivable for A_1, A_2 , then we may also construct models by taking the set of the $\mathbf{L}$'s and adjoining a root below them. If it happens to be the case that the forcing on the root is automatically 00, then the model thus obtained is a model that gives rise to underivable formulas in its turn. This is so, if among the old roots at least one value 00 occurs or if both 10 and 01 occur. More exactly, if such a case applies and the model $\mathbf{K}$ arises in the construction from the models $\mathbf{L}_1, \dots, \mathbf{L}_n$ and $D_{\mathbf{L}_1}(A_1, A_2), \dots, D_{\mathbf{L}_n}(A_1, A_2)$, are not derivable in **IPC**, then neither is $D_K(A_1, A_2)$. In this case we will denote

the newly obtained model by $(L_1, \dots, L_n)^+$ and say that $(L_1, \dots, L_n)^+$ has been obtained by *rooting* from $L_1, \dots, L_n$.

Let us recall the one-variable case:

$$K_0 = 1, \quad K_1 = 0, \quad K_2 = \begin{matrix} 1 \\ \uparrow \\ 0 \end{matrix}.$$

And, in general for any $n \geq 0$,

This means that all K_n can be constructed from $X = \{K_0, K_1, K_2\}$ by the second method of rooting models. Also, K_1 is a generated submodel of K_2 . The proof of Theorem 1.1 is then actually contained in the above sketch, applied to the one-variable case.

In the two-variable case a set X of Kripke models which is sufficient for the construction of all models is the set of all T -models with 00 *only* occurring at the root. All T -models with 00 occurring at the root can be obtained from X by repeatedly rooting models, and all other T -models are generated submodels of models in this set.

A simpler such set, however, is the following set X^* :

Let us denote by K_n^1 the model K_n preceded by 1 everywhere, i.e. K_n with 0 replaced by 10 and 1 by 11. Similarly K_n^2 will denote K_n followed by 1 everywhere, i.e. K_n with 0 replaced by 01 and 1 by 11. Now take

$$X^* = \left\{ \begin{matrix} K_n^i \\ \uparrow \\ 00 \end{matrix} \mid n \in \mathbb{N}, i = 1, 2 \right\} \cup \left\{ \begin{array}{ccc} & K_n^i & K_{n+1}^i \\ & \swarrow & \nearrow \\ & 00 & \end{array} \mid n \in \mathbb{N}, i = 1, 2 \right\}.$$

To show that this set suffices it is sufficient to generate the original set X from X^* by taking generated submodels and rooting them. Take an arbitrary member

of X . If a root of one of the L_i is 11, then that $L_i = K_0^1 = K_0^2$. In general, any T -model with a root 11, 10, or 01 is a K_n^i . If among the L_i no root 01 occurs, then all the L_i 's are K_n^1 's and we actually have one of the cases $(K_n^1)^+$ or $(K_n^1, K_{n+1}^1)^+$; similarly, if no root 10 occurs. If both 01 and 10 occur on the roots of the L_i , then $ij = 00$ is forced and the model is obtained by rooting the L_i ; and the L_i themselves are generated submodels of models $(K_n^1)^+$ and $(K_n^1, K_{n+1}^1)^+$.

To get a generalization of Theorem 1.1 it suffices to ascribe D -formulae to the T -models $(K_n^1)^+$ and $(K_n^1, K_{n+1}^1)^+$. These could be found by applying a general

method (de Jongh [1970], Rybakov [1992]), but the following formulae are nicer:
For $(\mathbf{K}_{n+1}^1)^+$:

$$g_n(q) \wedge ((p \rightarrow g_{n+1}(q)) \rightarrow p) \rightarrow p := h_{n+1}(p, q),$$

for $(\mathbf{K}_0^1)^+$:

$$\neg\neg q \wedge (p \leftrightarrow q) \rightarrow p := h_0(p, q),$$

and for $(\mathbf{K}_n^1, \mathbf{K}_{n+1}^1)^+$:

$$g_{n+2}(q) \wedge (p \leftrightarrow f_{n+1}(q)) \rightarrow p := k_n(p, q).$$

Here $g_n(q)$ and $f_n(q)$ are such that, for any 1-variable T -model $\mathbf{L}$, $\mathbf{L} \models g_n(q)$ iff not $\mathbf{K}_n \preceq \mathbf{L}$, i.e., for $n \geq 1$, iff $\mathbf{L} \preceq \mathbf{K}_n$, and $\mathbf{L} \models f_{n+1}(q)$ iff not $\mathbf{K}_n \preceq \mathbf{L}$ or not $\mathbf{K}_{n+1} \preceq \mathbf{L}$, i.e., for $n \geq 1$, iff $\mathbf{L} \preceq \mathbf{K}_n$ or $\mathbf{L} \preceq \mathbf{K}_{n+1}$.

Thus, the following theorem has been proved.

3.1. THEOREM. *If there is no $n \in \mathbb{N}$ for which $\vdash_{\text{IPC}} g_n(B) \wedge ((A \rightarrow g_{n+1}(B)) \rightarrow A) \rightarrow A$ or $\vdash_{\text{IPC}} g_{n+2}(B) \wedge (A \leftrightarrow f_{n+1}(B)) \rightarrow A$, or $\vdash_{\text{IPC}} g_n(A) \wedge ((B \rightarrow g_{n+1}(A)) \rightarrow B) \rightarrow B$ or $\vdash_{\text{IPC}} g_{n+2}(A) \wedge (B \leftrightarrow f_{n+1}(A)) \rightarrow B$ is provable in IPC and $\nVdash_{\text{IPC}} \neg\neg B \wedge (A \leftrightarrow B) \rightarrow A$, then A and B are independent over IPC.*

It is to be noted that, just like Theorem 1.1, this theorem immediately applies to **HA**, since rooting the models is applicable in the case of **HA** by adjoining the standard model $\mathbb{N}$ to the new root (see Smoryński [1973]). That Theorem 3.1 in a sense best possible can be demonstrated by showing that $h_n(p, q)$ and $k_n(p, q)$ are exactly provable. (Again this then applies to **HA** as well, now by the uniform version of the arithmetic completeness of IPC over **HA**; see Smoryński [1973].) The case of $h_0(p, q)$ is subsumed in the proof.

3.2. THEOREM. (a) *The formula $h_{n+1}(p, q)$ is exactly provable for $(g_n(q) \wedge ((p \rightarrow g_{n+1}(q)) \rightarrow p)) \vee p$ and q .*

(b) *The formula $k_n(p, q)$ is exactly provable for $(g_{n+2}(q) \wedge (p \leftrightarrow f_{n+1}(q))) \vee p$ and q .*

PROOF. (a) Actually, we will show in general that $C \wedge ((p \rightarrow D) \rightarrow p) \rightarrow p$, where C and D do not contain p , is exactly provable with the substitution $(C \wedge ((p \rightarrow D) \rightarrow p)) \vee p$ for p and the identity for the other variables.

We first show that the required formula is actually provable. We apply the easily verified IPC-equivalence of $A \rightarrow B$ to $((A \rightarrow B) \rightarrow A) \rightarrow B$. Let us write p^* for $(C \wedge ((p \rightarrow D) \rightarrow p)) \vee p$. Then $p^* \rightarrow D$ is equivalent to $(C \wedge ((p \rightarrow D) \rightarrow p) \rightarrow D) \wedge (p \rightarrow D)$ and hence to $p \rightarrow D$.

Therefore, $(p^* \rightarrow D) \rightarrow p^*$ is equivalent to $(p \rightarrow D) \rightarrow (C \wedge ((p \rightarrow D) \rightarrow p)) \vee p$ and hence implies $(p \rightarrow D) \rightarrow p$. Thus, $C \wedge ((p^* \rightarrow D) \rightarrow p^*)$ implies p^* .

Next we have to show that no stronger formulae are provable. For that it is sufficient to note that in any Kripke model validating $C \wedge ((p \rightarrow D) \rightarrow p) \rightarrow p$, changing the valuation of p to that of $(C \wedge ((p \rightarrow D) \rightarrow p)) \vee p$ will leave all forcing relations as they are. This is obvious, because in any such Kripke model $(C \wedge ((p \rightarrow D) \rightarrow p)) \vee p$ is actually equivalent to p .

(b) We will show in general that $C \wedge (p \leftrightarrow D) \rightarrow p$, where C and D do not contain p , is exactly provable with the substitution $(C \wedge (p \leftrightarrow D)) \vee p$ for p and the identity for the other variables. Obviously, the second part of the proof is the same as in (a), so it is sufficient to show that the relevant formula is provable. Let

us write p^* for $(C \wedge (p \leftrightarrow D)) \vee p$. Then $p^* \leftrightarrow D$ implies $p \rightarrow D$ as well as $D \rightarrow (C \wedge (p \leftrightarrow D)) \vee p$. The latter implies $D \rightarrow p$. So, $C \wedge (p^* \leftrightarrow D)$ implies p^* . $\square$

REFERENCES

- [1970] D. H. J. DE JONGH, *A characterization of the intuitionistic propositional calculus*, in Kino, Myhill, and Vesley [1970] pp. 211–217.
- [1982] ———, *Formulas of one propositional variable in intuitionistic arithmetic*, in Troelstra and van Dalen [1982] pp. 51–64.
- [1993] D. H. J. DE JONGH and A. VISSER, *Embeddings of Heyting algebras*, **ILLC Prepublications ML-93-14**, ILLC, University of Amsterdam, Amsterdam.
- [1970] A. KINO, J. MYHILL, and R. E. VESLEY (EDITORS), *Intuitionism and proof theory*, North-Holland, Amsterdam.
- [1982] L. L. MAKSIMOVA, *Interpolation properties of superintuitionistic, positive and modal logics*, **Intensional logic: Theory and Applications**, Acta Philosophica Fennica, vol. 35, pp. 70–78; **Societas Philosophica Fennica**, Helsinki.
- [1992] A. PITTS, *On an interpretation of second order quantification in first order intuitionistic propositional logic*, this JOURNAL, vol. 57, pp. 33–52.
- [1992] V. V. RYBAKOV, *Rules of inference with parameters for intuitionistic logic*, this JOURNAL, vol. 57, pp. 912–923.
- [1993] V. SHAVRUKOV, *Subalgebras of diagonalizable algebras of theories containing arithmetic*, **Dissertationes Mathematicae/Rozprawy Matematyczne**, vol. 323.
- [1978] V. B. SHEHTMAN, *Rieger-Nishimura lattices*, **Soviet Mathematics Doklady**, vol. 19, pp. 1014–1018.
- [1973] C. SMORYŃSKI, *Applications of Kripke models*, in Troelstra [1973] pp. 324–391.
- [1973] A. S. TROELSTRA (EDITOR), *Metamathematical investigations of intuitionistic arithmetic and analysis*, Lecture Notes in Mathematics, vol. 344, Springer, Berlin.
- [1982] A. S. TROELSTRA and D. VAN DALEN (EDITORS), *The L. E. J. Brouwer centenary symposium*, North-Holland, Amsterdam.

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