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Dynamical moving mirrors and black holes

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Abstract

A simple quantum mechanical model of N free scalar fields interacting with a dynamical moving mirror is formulated and shown to be equivalent to two-dimensional dilaton gravity. We derive the semi-classical dynamics of this system, by including the back reaction due to the quantum radiation. We develop a hamiltonian formalism that describes the time evolution as seen by an asymptotic observer, and write a scattering equation that relates the in-falling and out-going modes at low energies. At higher incoming energy flux, however, the semi-classical model appears to become unstable and the mirror seems to accelerate forever along a trajectory that runs off to infinity. This instability provides a useful paradigm for black hole formation and introduces an analogous information paradox. Finally, we indicate a possible mechanism that may restore the stability of the system at the quantum level without destroying quantum coherence.

1. Introduction

Soon after the discovery of the black hole emission effect by Hawking [1], it was realized that this effect was an example of a wider class of phenomena in which particle creation occurs due to the observer dependence of the vacuum. A particularly useful analogy, that is often made, is with particle creation due to an accelerating mirror [2]. The formal relation between the two systems becomes most transparent when one considers the trajectory of the origin of the coordinate system in the spherically reduced black hole geometry. Mathematically, this point indeed behaves as a reflecting mirror, and, due to the distortion of space-time near the black hole, it will at late times seem to rapidly recede away from the

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asymptotic observers with a constant acceleration of $a = 1/8M$. From this perspective, the black hole emission effect arises due to the distortion of the incoming vacuum after reflection of this accelerating mirror, producing an apparently thermal out-going state with temperature $T_H = 1/8\pi M$. Moreover, the out-going radiation will appear to describe a mixed state, because signals that are sent in at late times will never seem to reach the mirror point and thus never seem to be reflected back into out-going signals.

While in general the moving mirror analogy is of course incomplete, the correspondence becomes almost exact in the context of the two-dimensional dilaton gravity models of black holes. In these toy models one typically considers massless matter fields, because this leads to the technical simplification that their propagation does not depend on the Weyl factor of the metric [3]. However, this tells us that the gravitational and matter fields interact directly only via the reflection at the boundary point, that plays the role of the origin $r = 0$ in the analogy with the s-wave reduced Einstein theory. All two-dimensional dilaton gravity models of this type [4] are therefore physically equivalent to a model of free matter fields reflecting off a dynamical moving mirror [5].

In this paper we will further develop this reformulation of two-dimensional dilaton gravity. We will describe both the classical and the semi-classical dynamics of the mirror–matter system in the large N limit, and in particular derive the explicit form of the back reaction due to the quantum radiation. We will find that there exists a range of parameters for which there exists a well-defined scattering equation relating the low energy incoming and out-going matter waves. This subcritical S -matrix is energy preserving and includes a regime in which most of the out-going radiation looks thermal but still describes a pure state. At higher incoming energy flux, however, black hole formation sets in and the semi-classical mirror trajectory degenerates. This super-critical case is of course the most interesting, because we are then dealing with the black hole evaporation phenomenon and confronted with the puzzle of information loss. In the first part of this paper, however, we will restrict our attention to the low energy regime, with the hope that a good understanding of this regime will teach us something about how to extend the model to the super-critical situation.

In particular we would like to gain insight into how one can ensure *in a natural way* that the total energy carried by the emitted radiation is equal to the total energy of the incoming matter. As a first step in this direction, we will derive a hamiltonian that generates the classical time evolution of the mirror–matter system as seen by the asymptotic observer. We will find, however, that this hamiltonian is unbounded from below and that the resulting instability leads to black hole formation at high energies. In our model, this situation is described by a forever accelerating mirror trajectory, which also seems to radiate forever. In a concluding section we then explain why we believe that this instability will be cured once all interactions between the in-falling and out-going matter are taken into account. In particular we point out that this interaction results in a nonlocal commutator algebra between the *in*- and *out*-fields, and this leads to some important modifications of the standard semi-classical description.

2. Back reaction on a moving mirror

Before we turn to the study of two-dimensional dilaton gravity, let us first discuss in general the effect of the back reaction due to the emitted radiation in a moving mirror model. This discussion will be a useful preparation for the coming sections and at the same time it will summarize some of the conclusions of the more detailed subsequent analysis.

Let us consider a two-dimensional system of a relativistic particle of mass m that acts as a reflecting mirror for a collection of N massless matter fields f_i . We can specify the motion of the mirror particle by the parametrized world line trajectory $(x^+(\tau), x^-(\tau))$, and we will assume that the matter fields are restricted to live to the right of this world line. In other words, the mirror particle represents the boundary of space, which has therefore only one asymptotic region. For simplicity, let us choose the direct reflection condition that the matter fields f_i vanish along the boundary trajectory,

$$f_i(x^+(\tau), x^-(\tau)) = 0 \quad (2.1)$$

for all τ . For a given mirror trajectory, this condition relates the left-moving in-modes and right-moving out-modes via the diffeomorphism

$$f_i^{\text{out}}(x^-) = f_i^{\text{in}}(F(x^-)), \quad (2.2)$$

with

$$x^+(\tau) = F(x^-(\tau)). \quad (2.3)$$

It will be convenient to fix the freedom in the parameter τ , by identifying it with the proper time along the world line

$$\frac{dx^+}{d\tau} \frac{dx^-}{d\tau} = 1. \quad (2.4)$$

The equation of motion of the mirror particle then takes the general form

$$\frac{1}{2}m \frac{d^2 x^\pm}{d\tau^2} = \mathcal{F}_\mp, \quad (2.5)$$

where $\mathcal{F}_\mp$ defines the relativistic force. From (2.4) we know that force $\mathcal{F}$ is always perpendicular to the world line trajectory,

$$\mathcal{F}_+ dx^+ + \mathcal{F}_- dx^- = 0. \quad (2.6)$$

with $dx^\pm = (dx^\pm/d\tau) d\tau$.

In general there can be many external forces that contribute in (2.5). These external forces could e.g. be used to generate a nonzero acceleration of the mirror particle even in the absence of (classical) matter waves. As a concrete example, that will enable us to make a direct correspondence with two-dimensional dilaton

gravity, let us assume the mirror particle has a charge (set equal to 1) and moves in a constant electric field E ,

$$\mathcal{F}_{\pm}^e = \pm E \frac{dx^{\mp}}{d\tau}. \quad (2.7)$$

The vacuum mirror trajectories in the presence of this constant Lorentz force are of the form

$$(x^+ - c^+)(x^- - c^-) = -m^2/4E^2, \quad (2.8)$$

with $c^{\pm}$ integration constants. The sign of E/m determines which branch of this hyperbola is picked out.

In nonvacuum situations, there will be an additional force on the boundary particle due to the interaction with the massless scalar fields. Each time an f -particle bounces off the mirror it will give off some of its momentum, resulting in a variable force equal to the rate of change in the momentum in the matter fields,

$$\mathcal{F}_{\pm}^m = \pm T_{\pm\pm} \frac{dx^{\pm}}{d\tau}. \quad (2.9)$$

Here

$$T_{\pm\pm} = \sum_i \frac{1}{2} (\partial_{\pm} f_i)^2 \quad (2.10)$$

denotes the traceless matter stress–energy tensor. The orthogonality condition (2.6) on the force is equivalent to the conservation of stress–energy,

$$T_{--}(dx^-)^2 = T_{++}(dx^+)^2. \quad (2.11)$$

This conservation equation is classically consistent with the reflection condition (2.1). The resulting equation of motion (2.5) in the presence of the combined force $\mathcal{F}_{\pm}^m + \mathcal{F}_{\pm}^e$ becomes, after integrating once with respect to τ ,

$$\frac{1}{2}m \frac{dx^{\pm}}{d\tau} \pm Ex^{\pm} + P_{\mp}(x^{\mp}) = \text{const.}, \quad (2.12)$$

with

$$P_{\pm}(x^{\pm}) = \pm \int_{x^{\pm}}^{\pm\infty} dx^{\pm} T_{\pm\pm}. \quad (2.13)$$

These equations express the classical conservation of total energy and momentum.

Now we would like to incorporate into this simple model the quantum mechanical effect of particle creation by a moving mirror. It is well known that a diffeomorphism of the form (2.2) will in general mix the positive and negative energy modes of the f_i -fields, and that, as a result, an incoming vacuum state will be reflected to an out-going state with a nonzero flux of particles. To account for the energy of these particles, the classical reflection equation (2.11) for the

stress–energy tensor will receive a quantum correction. For massless fields in two dimensions, this correction is given by the conformal anomaly

$$T_{--}(\mathrm{d}x^-)^2 = (T_{++} + \kappa\{x^-, x^+\})(\mathrm{d}x^+)^2, \quad (2.14)$$

where $\kappa = \frac{1}{24}N$, and $\{, \}$ denotes the schwartzian derivative

$$\{f, x\} = \frac{f'''}{f'} - \frac{3}{2} \frac{(f'')^2}{(f')^2}, \quad (2.15)$$

with $f' = \mathrm{d}f/\mathrm{d}x$. This quantum reflection equation shows that if we start with a vacuum with $T_{++} = 0$, then after reflection there is a nonzero out-going energy flux $T_{--} = \kappa\{x^+, x^-\}$.

This particle creation phenomenon will result in a modification of the equation of motion for the mirror, to account for the back reaction due to the quantum radiation. Indeed, with the new reflection equation (2.14), the classical expression (2.9) for the force is no longer consistent with the transversality condition (2.6). To correct for this, we should rewrite the new equation (2.14) in a similar form as the old one (2.11), because only then we can consistently interpret each side of the equation as the quantum corrected force to be used in (2.5). Unfortunately, however, this procedure is not entirely unique, and thus there seems to be more than one way to include a semi-classical correction to the force equation. Presumably, this ambiguity is related to the freedom one has in writing the one-loop counterterm that compensates for the conformal anomaly (see sect. 5 for a more detailed discussion of this point).

The most convenient choice, that leads to the simplest equations of motion for the boundary, is to rewrite (2.14) as follows:

$$\left[T_{--} + \frac{1}{2}\kappa\partial_-^2 \log(\partial_- x^+)\right](\mathrm{d}x^-)^2 = \left[T_{++} + \frac{1}{2}\kappa\partial_+^2 \log(\partial_+ x^-)\right](\mathrm{d}x^+)^2. \quad (2.16)$$

Thus, comparing with (2.6), this choice leads to the following expression for the quantum corrected force:

$$\begin{aligned} \mathcal{F}_\pm &= \pm \left[T_{\pm\pm} + \frac{1}{2}\kappa\partial_\pm^2 \log(\partial_\pm x^\mp)\right] \frac{\mathrm{d}x^\pm}{\mathrm{d}\tau} \\ &= \frac{\mathrm{d}}{\mathrm{d}\tau} \left(P_\pm(x^\pm) \pm \kappa \frac{\mathrm{d}^2 x^\pm}{\mathrm{d}\tau^2} \right). \end{aligned} \quad (2.17)$$

After inserting this result into (2.5) we obtain a quantum corrected equation of motion. The fact that it becomes a third-order differential equation is typical for systems where the emitted radiation depends and reacts back on the motion of the source. As before, however, we can integrate the equation of motion once with respect to the proper time τ and obtain second-order differential equations

$$\frac{1}{2}m \frac{\mathrm{d}x^\pm}{\mathrm{d}\tau} \pm \kappa \frac{\mathrm{d}^2 x^\pm}{\mathrm{d}\tau^2} \pm Ex^\pm + P_\mp(x^\mp) = \text{const.}, \quad (2.18)$$

where we recall that τ is defined as the proper time along the boundary. The above equations can in principle be used to compute an explicit scattering equation describing the reflection of f -waves off the dynamical mirror.

In the following sections we will examine two-dimensional dilaton gravity and we will arrive at an essentially identical set of equations of motion for the boundary point. An unusual feature, however, will be that the mirror particle in that case has a negative rest mass. We will discuss the properties of the above equations in more detail in sect. 6.

3. Dilaton gravity as a moving mirror model

In this section we describe the reformulation of two-dimensional dilaton gravity as a simple two-dimensional model of free scalar fields interacting with a dynamical moving mirror. Our discussion here will be purely classical. The semi-classical corrections due to the conformal anomaly will be discussed in sects. 5 and 6.

3.1. Classical dilaton gravity

Two-dimensional dilaton gravity coupled to N massless scalar fields is described by the action

$$S_0 = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left(e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\lambda^2] + \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right). \quad (3.1)$$

This action shares many features with the s-wave reduction of $(3+1)$ -dimensional gravity and has been extensively studied as a toy model of two-dimensional quantum gravity which contains black hole solutions. The classical equations of motion of this action can be solved explicitly due to the property that the rescaled metric

$$\hat{g}_{ab} = e^{2\phi} g_{ab} \quad (3.2)$$

is flat everywhere. To write the general solution, it is thus natural to introduce coordinates $x^\pm$ such that

$$ds^2 = \exp[2\rho(x^+, x^-)] dx^+ dx^-, \quad (3.3)$$

$$\rho(x^+, x^-) = \phi(x^+, x^-). \quad (3.4)$$

In these coordinates the remaining equations of motion

$$\partial_+ \partial_- e^{-2\phi} = -\lambda^2, \quad (3.5)$$

$$\partial_\pm^2 e^{-2\phi} = T_{\pm\pm}, \quad (3.6)$$

where $T_{\pm\pm}$ is the traceless matter energy-momentum tensor, are trivial to integrate to obtain the most general solution

$$\begin{aligned} \exp[-2\phi(x^+, x^-)] = & M - \lambda^2 x^+ x^- - \int_{x^+}^{\infty} dy^+ \int_{y^+}^{\infty} dz^+ T_{++} \\ & - \int_{-x^-}^{\infty} dy^- \int_{-y^-}^{\infty} dz^- T_{--}. \end{aligned} \quad (3.7)$$

If we put the $T_{\pm\pm} = 0$ this reduces to the static black hole solution of mass M . The terms involving $T_{\pm\pm}$ represent the classical back reaction of the metric due to the incoming or out-going matter. The physical interpretation of this general solution as describing gravitational collapse and black hole formation in two dimensions has been discussed extensively elsewhere (for a review see ref. [6]).

Without matter present, the metric ds^2 reduces to the two-dimensional Minkowski metric

$$ds^2 = \frac{dx^+ dx^-}{x^+ x^-} \quad (3.8)$$

with the two light-cone coordinates $x^\pm$ defined on the half-lines $\pm x^\pm > 0$. In this parametrization, the four asymptotic regions of the Minkowski plane correspond to $x^\pm \rightarrow 0$ or $\pm\infty$. In the following, however, we will consider only the right regions as asymptotic regions, while the left regions $x^\pm \rightarrow 0$ will be replaced by a dynamical boundary. Namely, since the field e^ϕ is known to play the role of coupling constant, it seems a natural procedure [4,5] to define a cutoff at strong coupling by introducing a reflecting boundary located on a line on which e^ϕ takes a certain large but constant value $\exp(\phi_{\text{cr}})$. We can think of this dynamical boundary as describing the trajectory of a single particle, parametrized by two coordinate functions $x^\pm(\tau)$ of an (arbitrary) time variable τ . The condition that the dilaton is constant along the boundary,

$$\phi(x^+(\tau), x^-(\tau)) = \phi_{\text{cr}}, \quad (3.9)$$

will imply a specific equation of motion for its trajectory, provided we supplement it with an appropriate reflection condition for the matter fields. For most of the following discussion, we will keep the specific value of ϕ_{cr} as a free parameter of the model.

If we set things up in this way, there will exist a low energy regime in which this boundary remains time-like everywhere. As long as we restrict ourselves to this regime, we are allowed to impose the condition that the stress-energy must reflect at the boundary. This guarantees that energy is conserved. In this section we will work with the classical reflection formula (2.11). The quantum correction term will be included later. There will of course also be a high energy regime at which black hole formation sets in. In this case part of the boundary is replaced by a space-like singularity. The critical energy flux above which this happens depends on which critical value of ϕ we choose in (3.9), (see subsect. 3.3). We will comment on the super-critical case in the concluding sect. 7.

3.2. Classical dynamics of the boundary

The boundary trajectory $x^\pm(\tau)$ represents the only dilaton gravity degree of freedom that couples directly to the matter fields. So in principle one should be allowed to eliminate all other gravitational fields from the action and derive a simple reparametrization invariant action describing the dynamics of the boundary particle. Such a derivation, however, would be slightly ambiguous, since it would depend on the choice of boundary terms one could add to the original dilaton gravity action. We will therefore follow a more direct route.

The boundary conditions (3.9) and (2.11), together with the explicit solution (3.7) for the dilaton field, uniquely determine the boundary equations of motion. We will now show that these equations are the same as those derived from the following very simple boundary action:

$$S_b[x] = m \int d\tau \sqrt{\partial_\tau x^+ \partial_\tau x^-} - \lambda^2 \int d\tau x^+ \partial_\tau x^-, \quad (3.10)$$

where the coupling of this boundary particle to matter is simply described via the restriction that matter fields must live to the right of the boundary. So the total action of the system is

$$S = S_b[x] + S_m[f, x], \quad (3.11)$$

with $S_b[x]$ as above and

$$S_m[f, x] = \int_{\pm x^\pm < \pm x^\pm} d^2x \partial_+ f \partial_- f, \quad (3.12)$$

where the integral over $x^\pm$ in the matter action is restricted by the boundary as indicated. The action (3.10) is identical to that of a single particle moving in a constant electric field $E = \lambda^2$. Note, however, that the coordinates $x^\pm$ are not the usual Minkowski coordinates, for which the metric takes the standard form $ds^2 = -dt^2 + dr^2$, but are (in the asymptotic region) related to these via

$$r \pm t = \lambda^{-1} \log(\pm \lambda x^\pm). \quad (3.13)$$

We have not yet put any restrictions on the sign of the parameter m , and it in fact turns out that the relevant regime corresponds to a particle of *negative* mass. Namely, the free classical boundary trajectories that follow from the action (3.10) are hyperbolae of the form (2.8) with $E = \lambda^2$, and to select the correct branch requires that we take $E/m < 0$. In the following we will therefore replace m to $-m$, so that m will continue to be a positive number, equal to minus the negative mass of the mirror particle. Further, we will be using a definite prescription for fixing the integration constants $c^\pm$ in (2.8) by imposing the asymptotic condition that the past and future asymptotes of the boundary trajectory are given by $x^+ = 0$ and $x^- = 0$, respectively.

The influence on the form of the boundary trajectory due to a nonvanishing influx of energy is obtained by computing the variation of the matter action with respect to $x^\pm$. One finds

$$\delta S_m = \int d\tau (T_{++} \partial_\tau x^+ \delta x^+ + T_{--} \partial_\tau x^- \delta x^-). \quad (3.14)$$

Integrating the resulting equation of motion $\delta S_b + \delta S_m = 0$ once with respect to τ gives

$$-\frac{1}{2}m\sqrt{\partial_{\mp}x^{\pm}} \pm \lambda^2 x^{\pm} + P_{\mp}(x^{\mp}) = 0, \quad (3.15)$$

with $P_{\pm}$ as defined in (2.13). As in sect. 2, the above two equations express the conservation of total $x^{\pm}$ momentum. Note, however, that $P_{\pm}(x^{\pm})$ are not the usual total momenta when translated back to (r, t) coordinates.

Eqs. (3.15) combined imply that

$$\partial_{\tau}x^{+}[\lambda^2x^{-} - P_{+}(x^{-})] + \partial_{\tau}x^{-}[\lambda^2x^{+} + P_{-}(x^{+})] = 0. \quad (3.16)$$

This is, as promised, the equation that the dilaton field, as given via its classical solution (3.7), is constant along the boundary. The value of this constant depends on the choice of the parameter m via

$$e^{-2\phi}(x^{+}, x^{-}) = m^2/4\lambda^2. \quad (3.17)$$

As a further comment we note that, using the energy reflection equation, one can show that also the following quantity:

$$\mathcal{M}(x^{+}, x^{-}) = M_{+}(x^{+}) + M_{-}(x^{-}) + \frac{1}{\lambda^2}P_{+}(x^{+})P_{-}(x^{-}), \quad (3.18)$$

with

$$M_{\pm}(x^{\pm}) = \int_{x^{\pm}}^{\pm\infty} dx^{\pm} x^{\pm} T_{\pm\pm}, \quad (3.19)$$

is conserved. This quantity $\mathcal{M}$ reduces in the far past and future to the total energy as measured by an asymptotic observer. The result that $\mathcal{M}$ is constant can therefore be interpreted as the statement of energy conservation.

3.3. Boundary trajectory for a shock wave

The total system of matter and boundary only describes a well-defined dynamical system if one restricts to field configurations below a certain critical energy flux. As an example, we consider the classical boundary equation when the incoming wave is a shock wave located at $x^{+} = q^{+}$, with amplitude p_{+} ,

$$T_{++}(x^{+}) = p_{+}\delta(x^{+} - q^{+}). \quad (3.20)$$

As long as the total energy $E = p_{+}q^{+}$ carried by the pulse is smaller than $m^2/4\lambda^2$, the boundary trajectory is time-like everywhere and given by

$$(\lambda^2x^{-} - p_{+})x^{+} = -m^2/4\lambda^2 \quad (3.21)$$

for $x^{+} < q^{+}$ and

$$x^{-}(\lambda^2x^{+} + p_{-}) = -m^2/4\lambda^2, \quad (3.22)$$

with

$$p_{-} = \frac{\lambda^2 p_{+} q^{+}}{m^2/4\lambda^2 q^{+} - p_{+}}, \quad (3.23)$$

Fig. 1. Schematic depiction of the classical boundary trajectory for (a) a sub-critical and (b) a super-critical shock wave.

for $x^+ > q^+$. The typical form of this boundary trajectory is depicted in Fig. 1a. Note that the mirror point indeed behaves as a particle with negative rest mass: when the shock wave hits it, it does not bounce back to the left but in the opposite direction to the right.

In case $p_+ q^+ > m^2/4\lambda^2$ then the solution to Eq. (3.15) cannot be time-like everywhere, but turns space-like for $x^+ > q^+$ (see Fig. 1b). So in this regime it is no longer classically consistent to treat the boundary as a reflecting mirror. This example suggests that the following inequality:

$$P_+(x^+) < m^2/4\lambda^2 x^+ \quad (3.24)$$

for all x^+ , with $P_+(x^+)$ defined in (2.13), is a necessary and possibly sufficient criterion for the energy flux to ensure that the classical boundary remains time-like. Note that this inequality does not imply any specific *local* upper bound on the energy flux T_{++} .

4. Classical hamiltonian formalism

To prepare for the transition to the quantum theory and to understand better why the mirror-matter dynamics conserves energy, we will now consider the reformulation of the system in hamiltonian language. Our eventual goal will be to construct this hamiltonian in such a way that it exactly describes the dynamics as seen by the asymptotic observer.

4.1. Derivation of the hamiltonian

A hamiltonian formulation always depends on a specific choice of a space and time coordinate. We will denote these by σ and τ , respectively. To start with, it

will be most convenient to choose them in such a way that the location of the boundary is always at a fixed value of σ . This will have the advantage that we will not have to deal with a dynamical restriction on the coordinates. Instead, the interaction between the matter and the boundary $x^\pm(\tau)$ will be described via an explicit term in the action. A particularly simple choice of coordinates (σ, τ) is as follows:

$$x^\pm(\sigma, \tau) = x^\pm(\tau) \pm \sigma, \quad (4.1)$$

where the space coordinate is restricted to the positive half-line $\sigma > 0$. The scalar field action then takes the following form (here and in the rest of this section we will suppress the (sum over the) i -index of the scalar fields f_i):

$$S_m = \int \frac{d\tau d\sigma}{\partial_\tau x^+ + \partial_\tau x^-} (\partial_\tau f - \partial_\tau x^+ \partial_\sigma f)(\partial_\tau f + \partial_\tau x^- \partial_\sigma f). \quad (4.2)$$

The form of this action can be considerably simplified, by introducing fields π_σ that denote the canonical conjugate of the f -fields. We can then write the action in the first-order form

$$S_m = \int d\tau \int_0^\infty d\sigma (\pi_\sigma \partial_\tau f - \partial_\tau x^+ T_{++} - \partial_\tau x^- T_{--}), \quad (4.3)$$

where

$$T_{\pm\pm} = \frac{1}{4}(\pi_\sigma \pm \partial_\sigma f)^2 \quad (4.4)$$

denote the left- and right-moving stress tensors. The original action (4.2) is recovered from (4.3) by eliminating the fields π_i via their equations of motion.

The boundary action itself can also be written in a hamiltonian form by introducing conjugate coordinates $P_\pm$ to $x^\pm$ and a Lagrange multiplier field e ,

$$S_b = \int d\tau [P_+ \partial_\tau x^+ + P_- \partial_\tau x^- + \lambda^2 x^- \partial_\tau x^+ + e(P_+ P_- + m^2)]. \quad (4.5)$$

Again, this is the familiar form of the action of a Klein–Gordon particle moving in a constant electric field in two dimensions^{*}.

Both the scalar field action and the boundary action are invariant under arbitrary reparametrizations of the time variable τ . This local gauge invariance means that the hamiltonian, when constructed in the standard way, is identically zero. Indeed, by performing a redefinition of the momentum variables $P_\pm$, the combined lagrangian can be written in the standard first-order form

$$S = \int d\tau \int_0^\infty d\sigma \pi_\sigma \partial_\tau f + \int d\tau (P_+ \partial_\tau x^+ + P_- \partial_\tau x^-) + \int d\tau e[(P_+ + \frac{1}{2}\lambda^2 x^- - P_-)(P_- - \frac{1}{2}\lambda^2 x^+ - P_+) + m^2], \quad (4.6)$$

^{*} This point particle action was also considered in relation with two-dimensional dilaton gravity in ref. [7].

with

$$P_{\pm} = \int_0^{\infty} d\sigma \, T_{\pm\pm}. \quad (4.7)$$

It is now manifest that we are dealing with a theory with a Hamilton constraint rather than a hamiltonian. This may look somewhat surprising, since we started with a theory of free scalar fields that contains states of nonzero energy. The reason is of course that the dependence of the physical coordinates $x^{\pm}$ on the fiducial time τ is defined in (4.1) via the arbitrarily parametrized boundary trajectory $x^{\pm}(\tau)$.

However, we want to construct a nontrivial time evolution and a corresponding nonzero hamiltonian. To this end we will have to choose a specific physical time τ and fix the reparametrization invariance in τ by setting $\tau = \tau$. We would like this time coordinate to coincide with the physical time as seen by an asymptotic observer. This leads us to the following definition of τ :

$$2\lambda\tau(\tau) = \log(-x^+/x^-). \quad (4.8)$$

We also define

$$2\lambda R(\tau) = \log(-\lambda^2 x^+ x^-). \quad (4.9)$$

Furthermore, instead of (4.1), we will choose a parametrization of the original light-cone coordinates $x^{\pm}$ that is appropriate for this choice of time

$$\begin{aligned} x^{\pm}(\sigma, \tau) &= x^{\pm}(\tau) e^{\lambda\sigma} \\ &= \pm \lambda^{-1} \exp\{\lambda[\pm\tau(\tau) + R(\tau) + \sigma]\}, \end{aligned} \quad (4.10)$$

with σ again defined on the positive half-line. For later reference, we note that the physical space coordinate r is equal to $R(\tau) + \sigma$. Now, after following the identical steps as above, we find that in terms of these new variables the action takes the form

$$\begin{aligned} S &= \int d\tau \int_0^{\infty} d\sigma \, \pi_{\sigma} \partial_{\tau} f + \int d\tau \, (P_T \partial_{\tau} T + P_R \partial_{\tau} R) \\ &\quad + \int d\tau \, e \left[-(P_T - \lambda e^{2\lambda R} - P_R)^2 + m^2 e^{2\lambda R} \right], \end{aligned} \quad (4.11)$$

with

$$P_T = \int_0^{\infty} \frac{1}{2} d\sigma \left[\pi_{\sigma}^2 + (\partial_{\sigma} f)^2 \right], \quad (4.12)$$

$$P_R = \int_0^{\infty} d\sigma \, \pi_{\sigma} \partial_{\sigma} f. \quad (4.13)$$

This action is completely equivalent to the previous one, and is again reparametrization invariant in the fiducial time τ . However, now we can identify a natural hamiltonian H of the model, namely as the operator that generates the

time translations in the *physical* time τ . From the form (4.11) of the action we read off that

$$H = p_\tau, \quad (4.14)$$

which, using the constraint imposed by e , can be solved:

$$H = -\sqrt{(P_R - P_r)^2 + m^2 e^{2\lambda R}} + \lambda e^{2\lambda R} + P_\tau. \quad (4.15)$$

This hamiltonian acts in the phase space of the free fields (π, f) and of the boundary coordinates (P_R, R) .

4.2. Equations of motion and energy conservation

It is instructive to analyze the classical mirror–matter dynamics in this coordinate system. From the Poisson brackets on the phase space,

$$\{\pi_\sigma(\sigma_1), f(\sigma_2)\} = \delta(\sigma_{12}), \quad (4.16)$$

$$\{P_R, R\} = 1 \quad (4.17)$$

and the form (4.15) of the hamiltonian, we deduce the following equations of motion for the boundary variables R and P_R :

$$\dot{R} = -\frac{P_R - P_r}{\sqrt{(P_R - P_r)^2 + m^2 e^{2\lambda R}}}, \quad (4.18)$$

$$\dot{P}_R = \frac{\lambda m^2 e^{2\lambda R}}{\sqrt{(P_R - P_r)^2 + m^2 e^{2\lambda R}}} - 2\lambda^2 e^{2\lambda R}, \quad (4.19)$$

with $\dot{R} = dR/d\tau$, etc.. The equation of motion for the f -fields can be written as

$$\dot{f} = \pi_\sigma + \dot{R} \partial_\sigma f, \quad (4.20)$$

$$\dot{\pi}_\sigma = \partial_\sigma^2 f + \dot{R} \partial_\sigma \pi_\sigma. \quad (4.21)$$

This can be integrated to the standard form of a sum of a left- and right-moving wave *

$$f = f_{\text{in}}(R(\tau) + \sigma - \tau) + f_{\text{out}}(R(\tau) + \sigma + \tau) \quad (4.22)$$

and similar for π_σ . This shows that $R(\tau)$ indeed parametrizes the physical location in (r, t) space of the mirror trajectory at $\sigma = 0$.

We can use Eq. (4.18) to eliminate P_R from the expression (4.15) for the hamiltonian. We find

$$H = -\gamma m e^{\lambda R} + \lambda e^{2\lambda R} + P_\tau, \quad (4.23)$$

* The suffices in and out here refer to the respective asymptotic regions.

with

$$\gamma = 1/\sqrt{1 - \dot{R}^2}. \quad (4.24)$$

The combination on the right-hand side of (4.23) is constant in time and can be identified with the total energy E of the mirror-matter system. Thus we find that the mirror behaves in this coordinate system as a particle with a position-dependent negative mass $m(R) = -m e^{\lambda R}$ moving in an electric field $E(R) = 2\lambda^2 e^{2\lambda R}$.

To understand qualitatively the motion of this particle, it is useful to write the force equation in this language. The force due to the reflecting matter is determined via the rate of change in the total matter energy P_r , given by

$$\dot{P}_r = -(1 + \dot{R})T_{\text{in}} + (1 - \dot{R})T_{\text{out}}. \quad (4.25)$$

Here T_{in} and T_{out} denote the left-moving *in*-flux and right-moving *out*-flux of stress-energy at the location of the boundary. The two energy fluxes are related via the classical reflection equation, which takes the form

$$(1 + \dot{R})^2 T_{\text{in}} = (1 - \dot{R})^2 T_{\text{out}}. \quad (4.26)$$

Thus we can express $\dot{P}_r$ solely in terms of the incoming flux T_{in} as

$$\dot{P}_r = \dot{R} \mathcal{F}_m, \quad \mathcal{F}_m = -2 \frac{1 - \dot{R}}{1 + \dot{R}} T_{\text{in}}. \quad (4.27)$$

The quantity $\mathcal{F}_m$ represents the force due to the matter bouncing off the mirror.

The equation of motion of the mirror particle can now be written in the following form:

$$-\gamma^3 m e^{\lambda R} \ddot{R} = -\frac{\partial V}{\partial R} + \mathcal{F}_m, \quad (4.28)$$

with

$$V(R) = -\gamma m e^{\lambda R} + \lambda e^{2\lambda R} \quad (4.29)$$

The form of this potential energy is indicated in Fig. 2.

Fig. 2. The form of the effective potential $V(r)$, showing the equilibrium position $r = R_0$. This equilibrium is unstable because the effective mass of the mirror particle is negative.

Fig. 3. The form of the mirror trajectory in (r, t) coordinates for (a) a vacuum runaway solution and (b) the solution with a subcritical matter shock wave.

4.3. Classical mirror trajectories

Before we describe what the motion of the mirror interacting with a matter wave looks like, let us first discuss the vacuum solutions to these equations, i.e. with no matter present. From the shape of the effective potential we then see that there is one isolated solution for which R is constant,

$$e^{2\lambda R_0} = m^2 / 4\lambda^2. \quad (4.30)$$

This is the unique physical vacuum solution. Note, however, that this trajectory is in fact unstable, due to the negative effective mass of the boundary particle. The vacuum equations in principle also allow for a more general class of solutions of the form

$$2 \exp\{\lambda[R(T) - R_0]\} = \sqrt{4 + \exp[2\lambda(T - T_0)]} - \exp[\lambda(T - T_0)], \quad (4.31)$$

with T_0 a constant of integration. They describe (see Fig. 3a) a mirror that begins at $R = R_0$ and then starts to accelerate to the left until it reaches a constant acceleration at late times and approaches the light-like asymptote

$$R - R_0 = -(T - T_0). \quad (4.32)$$

The time reverse of this runaway solution is also a vacuum solution*.

The typical motion of the mirror interacting with a sub-critical matter wave is now described as follows (cf. subsect. 3.3). A short time before the matter wave

* We should note that the vacuum equations in fact also allow solutions that run off to $R = +\infty$ in a finite time. This motion, however, is clearly unphysical and is eliminated after choosing appropriate initial conditions.

reaches the mirror, it starts to pre-accelerate to the left and follows one of the vacuum trajectories in Fig. 3 of the form (4.31). The parameter τ_0 of this motion is not free but determined in terms of the total x^+ momentum $P_+ = \int dx^+ T_{++}$ carried by the incoming f -wave via

$$P_+ = \lambda \exp[\lambda(R_0 - \tau_0)]. \quad (4.33)$$

If we consider the specific example of a matter pulse concentrated into a shock wave along the line $r + t = 0$, with total energy E , then this relation reads

$$E = \lambda \exp[\lambda(R_0 - \tau_0)]. \quad (4.34)$$

Provided the matter wave is subcritical, it will reflect off the mirror in a finite time, after which the mirror will eventually return to the (unstable) vacuum position r_0 via a trajectory described by the time reverse of (4.31). The form of this trajectory is indicated in Fig. 3b.

It is clear what will happen in the super-critical regime. For the shock wave example we deduce from (4.34) that, if the total energy exceeds the critical energy

$$E_c = \lambda \exp(2\lambda R_0), \quad (4.35)$$

the asymptote will be located at $R_0 + \tau_0 < 0$. Since the wave itself travels along the line $r + t = 0$, the initial data in this case prevent the incoming signal from ever reaching the mirror. This is the manifestation in this coordinate system of black hole formation. Classically, there will be information loss, in the sense that the incoming wave will never be reflected into a right-moving signal. The question we wish to address in the following sections is whether this conclusion will continue to hold when we include the quantum effect of particle creation and the corresponding back reaction on the mirror trajectory.

Let us make one final important comment about the classical model. From the above description of the (sub-critical) motion of the boundary point as seen by an asymptotic observer, it may appear that we had to make an acausal fine-tuning (4.33) of the initial data. This fine-tuning is necessary to ensure that after the reflection, the mirror will eventually return to the original equilibrium point. One should keep in mind, however, that it is not the mirror trajectory but the (r, t) coordinate system *itself* that is fine-tuned: it is determined (causally) in terms of the mirror trajectory, via the condition that the future asymptote corresponds to $\kappa(\tau) = R_0$. In other words, the apparent acausality arises because we describe the entire motion of the system in terms of asymptotic coordinates that depend on events taking place at late times. As we will discuss in sect. 7, this fact will have important consequences at the quantum level.

5. Semi-classical analysis

We now wish to discuss the two-dimensional dilaton gravity model at the semi-classical level. As pointed out in ref. [3], one then needs to include a one-loop term in the action. This counterterm takes care of the conformal anomaly and also

effectively represents the back reaction due to the quantum radiation emitted via the Hawking process. The purpose of this section is to compute the effect of this counterterm on the equation of motion of the boundary. The final result for this correction term will be the same as that described in sect. 2.

5.1. Semi-classical dilaton gravity

It would greatly simplify the theory if we can choose the counterterm in a way that respects all the symmetries of the classical theory. In particular, we would like to preserve the property that the classical equations of motion imply that the rescaled metric is flat everywhere, even in the presence of matter. In addition, the counterterm is subject to the requirement that asymptotically on $\mathcal{I}^+$ it must represent the correct energy flux of the physical Hawking particles. These considerations lead us to choose the following one-loop correction to the effective action:

$$S_1 = -\frac{1}{8\pi} \int d^2x \left(\frac{N-24}{12} \hat{R} \frac{1}{\nabla^2} \hat{R} - \frac{N}{12} (2\phi \hat{R}) \right), \quad (5.1)$$

with $\hat{R}$ being the Ricci scalar of the rescaled metric $\hat{g}$ defined in (3.2). This choice of correction term combines the approaches proposed in refs. [4,8]. The first term above is the usual nonlocal quantum anomaly term that compensates for the conformal anomaly, while the second term is a local counterterm that ensures that in the far future the physical metric g_{ab} couples to the correct Hawking flux carried by the physical particles.

Setting the conformal gauge,

$$g_{uu} = g_{vv} = 0, \quad g_{uv} = g_{vu} = -\frac{1}{2} e^{2\rho}, \quad (5.2)$$

the semi-classical effective action becomes

$$S = \frac{1}{\pi} \int d^2x \left(e^{-2\phi} (2\partial_u \partial_v \rho - 4\partial_u \phi \partial_v \phi + \lambda^2 e^{2\rho}) - \frac{1}{2} \sum_{i=1}^N \partial_u f_i \partial_v f_i \right. \\ \left. + 2\kappa \phi \partial_u \partial_v (\rho - \phi) - 2(\kappa - 1) \partial_u (\rho - \phi) \partial_v (\rho - \phi) \right), \quad (5.3)$$

with $\kappa = \frac{1}{24}N$. This one-loop effective action can be brought into a form which is essentially identical to the original classical action, by performing the change of variables

$$\hat{\rho} = \rho - \phi, \quad (5.4)$$

$$\Omega = e^{-2\phi} + \kappa \phi. \quad (5.5)$$

The action then becomes

$$S = \frac{1}{\pi} \int d^2x \left(\lambda^2 e^{2\hat{\rho}} + 2(\Omega - \kappa \hat{\rho}) \partial_u \partial_v \hat{\rho} - \frac{1}{2} \sum_{i=1}^N \partial_u f_i \partial_v f_i \right), \quad (5.6)$$

with $\hat{\kappa} = \kappa - 1$. The corresponding equations of motion of Ω and $\hat{\rho}$,

$$\partial_u \partial_v \hat{\rho} = 0, \quad (5.7)$$

$$\partial_u \partial_v (\Omega - \hat{\kappa} \hat{\rho}) + \lambda^2 e^{2\hat{\rho}} = 0, \quad (5.8)$$

indeed take the same form as the classical equations (namely via the substitution $\Omega - \hat{\kappa} \hat{\rho} \rightarrow e^{-2\phi}$, $\hat{\rho} = \rho - \phi$). The difference, however, between the one-loop corrected and the classical theory resides in the form of the stress–energy tensor, which reads

$$T_{uu} = -2\partial_u \Omega \partial_u \hat{\rho} + \partial_u^2 \Omega + 2\hat{\kappa}(\partial_u \hat{\rho} \partial_u \hat{\rho} - \partial_u^2 \hat{\rho}) \quad (5.9)$$

and similar for T_{vv} . Under the conformal transformations generated by these stress tensors, Ω transforms as a scalar, while $e^{2\hat{\rho}}$ defines a (1, 1) form.

The one-loop corrected dilaton gravity theory is still essentially a free field theory. To see this, we notice that the rescaled metric $\hat{ds}^2 = e^{2\hat{\rho}} du dv$ is still flat everywhere. As in the previous section, we can in principle use this fact to perform a further gauge fixing, and eliminate the conformal invariance by imposing the condition $\hat{\rho} = 0$. Instead, however, let us for the moment keep the conformal symmetry, and use the flatness of $\hat{ds}^2$ to introduce two chiral fields $X^\pm$ via

$$\hat{\rho}(u, v) = \frac{1}{2} \log[\partial_u X^+(u) \partial_v X^-(v)]. \quad (5.10)$$

These fields parametrize the coordinate transformation from the (u, v) system to the coordinates $x^\pm$ in which $\hat{\rho} = 0$. The general solution to the equation of motion of Ω can now be written as

$$\Omega(u, v) = -\lambda^2 X^+(u) X^-(v) + \omega^+(u) + \omega^-(v). \quad (5.11)$$

The chiral fields $\hat{\rho}$ (or equivalently $X^\pm$) and $\omega_\pm$ are the free field variables. The classical Poisson brackets are

$$\{\omega^+(u_1), \partial_u \hat{\rho}(u_2)\} = \frac{1}{2} \delta(u_{12}), \quad (5.12)$$

$$\{\omega^+(u_1), \partial_u \omega^+(u_2)\} = \hat{\kappa} \delta(u_{12}). \quad (5.13)$$

It is also useful to introduce variables $P_\pm$ via

$$\partial_u \omega^+ = P_+ \partial_u X^+ + \frac{1}{2} \hat{\kappa} \partial_u \log(\partial_u X^+), \quad (5.14)$$

$$\partial_v \omega^- = -P_- \partial_v X^- + \frac{1}{2} \hat{\kappa} \partial_v \log(\partial_v X^-). \quad (5.15)$$

These variables $P^\pm$ are canonically conjugate to the coordinate fields $X^\pm$,

$$\{P_\pm(u_1), \partial X^\pm(u_2)\} = \delta(u_{12}), \quad (5.16)$$

while the $P_\pm$ -fields commute among themselves. This last requirement fixes the form of the second term in (5.14) and (5.15). The stress tensors in the new variables read

$$T_{uu} = \partial_u X^+ \partial_u P_+ - \frac{1}{2} \hat{\kappa} \partial_u^2 \log(\partial_u X^+), \quad (5.17)$$

$$T_{vv} = -\partial_v X^- \partial_v P_- - \frac{1}{2} \hat{\kappa} \partial_v^2 \log(\partial_v X^-). \quad (5.18)$$

These satisfy the Poisson bracket of a Virasoro algebra with central charge $\hat{\kappa}$. Note that the $P_{\pm}$ -fields do not have the simple scalar transformation law under the conformal symmetry generated by this stress tensor, but transform such that the combination on the right-hand side of (5.14) and (5.15) behave as proper conformal fields of dimension one.

In the remainder of this section we will restrict our attention to the semi-classical physics of this system. In principle, this can be justified only if we take the limit of large N . For this reason we will no longer make any distinction between $\hat{\kappa}$ and κ , as they become identical in this limit.

5.2. Boundary conditions

Let us return to the discussion of the boundary conditions. We choose the (u, v) -coordinate system in such a way that the boundary becomes identified with the line $u = v$, and denote the parameter along this boundary by s . As suggested above, we first require that the dilaton field is constant along the boundary

$$\partial_s \Omega = 0, \quad (5.19)$$

where $s = u = v$ is the coordinate along the boundary. In terms of the X - and P -fields this condition reads

$$\partial_s X^+ (\lambda^2 X^- - P^+) + \partial_s X^- (\lambda^2 X^+ + P^-) + \frac{1}{2} \kappa \partial_s \log(\partial_s X^+ \partial_s X^-) = 0. \quad (5.20)$$

The above boundary condition is coordinate invariant, which allows us to impose the additional condition that the gravitational and matter components of the energy-momentum flux each separately get directly reflected off the boundary. So Eq. (5.20) is supplemented with the condition that

$$T_{uu}^g = T_{vv}^g, \quad (5.21)$$

with T_{uu}^g and T_{vv}^g as given in (5.17) and (5.18). The two equations (5.20) and (5.21) combined specify the precise reflection condition that relates the incoming canonical variables (X^+, P_+) to the out-going canonical variables (X^-, P_-) .

We would like to make manifest that this relation defines a canonical transformation. To this end, we should write a generating function $S[X^+, X^-]$ of the coordinate fields such that the momenta $P_{\pm}$ defined by

$$P_{\pm} = \frac{\delta S[X]}{\delta \partial_s X^{\pm}} \quad (5.22)$$

identically solve the boundary equations (5.20) and (5.21). This results in a set of functional equations for $S[X]$ that can be solved explicitly. The form of the solution is unique, once we fix the constant value of Ω along the boundary. If we set $\Omega(x^+, x^-) = m^2/4\lambda^2$, then the generating functional $S[X]$ takes the following form:

$$S[X] = m \int ds \sqrt{\partial_s X^+ \partial_s X^-} - \lambda^2 \int ds X^+ \partial_s X^- + \frac{1}{2} \kappa \int ds \log(\partial_s X^+) \partial_s \log(\partial_s X^-). \quad (5.23)$$

In this result we recognize the classical boundary action discussed in sect. 3, and the above formula can be interpreted as the quantum corrected version of it. The formulas (5.22) defining the momenta $P_{\pm}$ become

$$\pm \frac{1}{2} m \sqrt{\partial_s X^+ \partial_s X^-} + \partial_s X^{\pm} (\lambda^2 X^{\mp} \mp P_{\pm}) + \frac{1}{2} \kappa \partial_s \log(\partial_s X^{\mp}) = 0. \quad (5.24)$$

In the next subsection we will see that this relation between the in- and out-dilaton gravity fields can be reinterpreted as the semi-classical equations of motion that determine the boundary trajectory for given in and out energy flux.

The action (5.23) and the relations (5.24) are well defined as long as both $X^+(s)$ and $X^-(s)$ are invertible functions of s . This, however, puts a nontrivial restriction on the possible values of the canonical momenta $P_{\pm}$, which, as we will see shortly, is related to the inequality (4.35) on the incoming matter energy flux. For the time being, we will restrict our attention to this sub-critical regime.

5.3. The physical mirror trajectory

We are now finally in a position to derive the semi-classical correction to the classical mirror-matter dynamics described in sect. 3 and make contact with the discussion in sect. 2. To do this we must eliminate the redundancy due to the conformal invariance and translate the above description of the scattering off the boundary into physical variables that commute with the physical constraint

$$T_{uu}^g + T_{uu}^m = 0. \quad (5.25)$$

The simplest way to do this is to use the physical fields $X^{\pm}$ as the reference coordinate system. This procedure amounts to choosing the gauge $\rho = \phi$. Thus we are instructed to adopt the following definition of the physical left-moving field $f_i(x^+)$:

$$f_i(x^+) = \int du \partial_u X^+ \delta(X^+(u) - x^+) f_i(u), \quad (5.26)$$

where x^+ denotes a c-number coordinate. The same construction can be used to define the physical fields $f_i(x^-)$ in the left-moving sector. Furthermore, we can also introduce physical variables $\hat{P}_{\pm}(x^{\pm})$ that are made from the gravitational fields via

$$\hat{P}_+(x^+) = \int du [P_+ \partial_u X^+ + \frac{1}{2} \kappa \partial_u \log(\partial_u X^+)] \delta(X^+(u) - x^+). \quad (5.27)$$

Here the combination on the right-hand side has been chosen such that it commutes with the gravitational stress tensor T_{uu}^g .*

* Note that, although we assume that $X^+(u)$ is invertible, the above definitions of the physical variables would in principle also make sense without this restriction.

The physical fields $\hat{P}_{\pm}(x^{\pm})$ are not independent from the matter fields, but are related to them via the Virasoro constraint equations (5.25) and identified with the integrals with

$$\hat{P}_{\pm}(x^{\pm}) = \pm \int_{x^{\pm}}^{\pm\infty} dx^{\pm} T_{\pm\pm} \quad (5.28)$$

of the respective components of the matter stress tensor. Thus we can now finally determine the physical trajectory $x^-(x^+)$ of the boundary by rewriting relations (5.24) in terms of the physical variables $\hat{P}_{\pm}(x^{\pm})$. These relations give a semi-classical equation of motion, which takes the form

$$-\frac{1}{2}m\sqrt{\partial_{\pm}x^{\pm}} \pm \lambda^2 x^{\pm} + \hat{P}_{\mp}(x^{\mp}) \pm \frac{1}{2}\kappa\partial_{\mp}\log(\partial_{\pm}x^{\pm}) = 0. \quad (5.29)$$

These are the quantum corrected conservation laws of total $x^{\pm}$ momentum. These differential equations are (upto the sign of m) identical to the ones we arrived at in sect. 2. There we wrote them in terms of the proper time coordinate τ along the boundary, as defined in (2.4), as

$$-\frac{1}{2}m\partial_{\tau}x^{\pm} \pm \kappa\partial_{\tau}^2x^{\pm} \pm \lambda^2 x^{\pm} + \hat{P}_{\mp}(x^{\mp}) = 0. \quad (5.30)$$

As was shown before, these semi-classical momentum conservation laws are compatible with the quantum reflection equation (2.16) for the stress–energy tensor. In the following subsection we will investigate some of the physical properties of these equations.

First let us determine the range of parameters for which there still exist vacuum trajectories of the form

$$-\lambda^2 x^+ x^- = \exp(2\lambda R_0). \quad (5.31)$$

For this we must take into account that the stress–energy flux $T_{\pm\pm}$ is in fact not identically zero in the vacuum. The physical vacuum is defined with respect to the physical asymptotic time t , while $T_{\pm\pm}$ is normal ordered with respect to the $x^{\pm}$ -coordinate system. Due to conformal anomaly, this implies that $T_{\pm\pm}$ receives a negative contribution equal to $-\kappa/2(x^+)^2$. Thus we find that the vacuum trajectory (5.31) solves (5.29) when R_0 is a solution to

$$2\lambda \exp(2\lambda R_0) - m \exp(\lambda R_0) + \kappa\lambda = 0. \quad (5.32)$$

In order for the solution to be real, the parameter m has to be chosen such that

$$m^2 > 8\kappa\lambda^2. \quad (5.33)$$

Hence, to have a time-like vacuum boundary trajectory, we can no longer take m to be arbitrarily small. Note further that, if (5.33) is satisfied, eq. (5.32) has in fact *two* real solutions.

6. Semi-classical hamiltonian dynamics

We now wish to return to the issue of energy conservation in this model, while including the quantum radiation and the effect of the back reaction. Let us

therefore again translate the above equations of motion into the (R, T) -coordinate system. In sect. 4 we did this via the hamiltonian formulation. This method, however, is now not so easily available, because of the higher-order derivatives present in the quantum corrections. Still, the timetranslation symmetry guarantees that there exists an expression for the conserved energy, that generalizes the classical result (4.23).

6.1. Semi-classical energy conservation

The quantum corrected energy equation can be derived directly from the two boundary equations (5.30) as follows. The total energy P_t and the total $x^\pm$ momenta $P_\pm$ are related by

$$P_t(\tau) = \lambda \int^\tau d\tau \left[x^+ \partial_\tau P_+ - x^- \partial_\tau P_- + \frac{1}{2} \kappa \partial_\tau \log(-\lambda^2 x^+ x^-) \right]. \quad (6.1)$$

The term proportional to κ arises due to the anomalous transformation law of the stress–energy tensor. Now we use the momentum conservation equations (5.30) to eliminate the matter momenta on the right-hand side of (6.1) in favor of the (τ) -derivatives of the boundary coordinates $x^\pm$, which we in turn replace by R and T . After some algebra, this procedure leads to the following equation:

$$-m e^{2\lambda R} \partial_\tau T + 2\kappa e^{2\lambda R} \left[\partial_\tau^2 R + 2\lambda (\partial_\tau R)^2 \right] + \lambda e^{2\lambda R} + \kappa \lambda^2 R + P_\tau = \text{const.} \quad (6.2)$$

The constant on the right-hand side is the total conserved energy.

We can eliminate the time variable τ in favor of the physical time T , by writing eq. (2.4),

$$\partial_\tau T = \gamma e^{-\lambda R}, \quad \gamma = 1/\sqrt{1 - \dot{R}^2}, \quad (6.3)$$

with $\dot{R} = dR/dT$. This allows us to remove τ from the energy equation (6.2), leading to the following semi-classical result for the total energy:

$$H = -\gamma m e^{\lambda R} + 2\kappa \lambda \gamma^2 + 2\kappa \gamma^4 \ddot{R} + \lambda e^{2\lambda R} + \kappa \lambda^2 R + P_\tau. \quad (6.4)$$

We want to use this expression to examine the energy balance and (in)stability of the mirror–matter dynamics.

As before, we can try to obtain a qualitative understanding of the boundary dynamics by considering the force equation of motion. This equation is obtained by computing the time derivative of (6.4) and putting the result equal to zero after dividing by an overall factor of $\dot{R}$. The time derivative $\dot{P}_\tau$ of the total matter energy is still given by the classical expression (4.25) in terms of the left- and right-moving energy flux. The reflection equation (4.26), however, now receives an extra quantum contribution

$$(1 + \dot{R})^2 T_{\text{in}} = (1 - \dot{R})^2 T_{\text{out}} + 2\kappa \gamma^2 (R^{(3)} + 3\gamma^2 \dot{R} \ddot{R}), \quad (6.5)$$

with $\dot{r}^{(3)} = d^3R/dt^3$. Thus the change in the total matter energy in terms of the incoming flux now becomes

$$\dot{P}_T = 2\dot{R} \frac{1 + \dot{R}}{1 - \dot{R}} T_{\text{in}} - 2\kappa\gamma^4(1 + \dot{R})(R^{(3)} + 3\gamma^2\dot{R}\ddot{R}^2). \quad (6.6)$$

The second term represents the change in the total matter energy due to the quantum radiation by the accelerating mirror.

The final result for the quantum equation of motion of the mirror reads

$$\gamma^3 m_q(R) \ddot{R} = -\frac{\partial V_q}{\partial R} + \mathcal{F}_m + \mathcal{F}_q, \quad (6.7)$$

where

$$m_q(R) = -m e^{\lambda R} + 4\kappa\lambda^2\gamma,$$

$$V_q(R) = -\gamma m e^{\lambda R} + \lambda e^{2\lambda R} + \kappa\lambda R,$$

$$\mathcal{F}_m = -2\frac{1 + \dot{R}}{1 - \dot{R}} T_{\text{in}},$$

$$\mathcal{F}_q = 2\kappa\gamma^4 R^{(3)} - 2\kappa\gamma^6(1 - 3\dot{R})\ddot{R}^2. \quad (6.8)$$

Here $m_q(R)$ and $V_q(R)$ are the quantum corrected effective mass and potential energy of the mirror particle. The shape of the potential $V_q(R)$ is plotted in Fig. 4. The term $\mathcal{F}_m$ is recognized as the classical force due to the direct reflection of the matter waves and $\mathcal{F}_q$ is the remaining part of the force induced by the quantum anomaly. The sum of all the terms proportional to κ represent the total back reaction due to the quantum radiation.

Fig. 4. The form of the quantum effective potential $V_q(r)$, showing the equilibrium position $r = R_0$ and a second extremum. For the runaway solutions described in subsect. 6.2, the difference between the quantum and classical potential energy is exactly cancelled by energy contained in the emitted quantum radiation.

6.2. Instability of the semi-classical equations

What do the semi-classical boundary trajectories look like? First, we note that there still exists a vacuum trajectory of the form $R = R_0$. Here R_0 is found by solving $\partial V_q / \partial R = 0$, which is Eq. (5.32). The interesting question, however, is whether the semi-classical vacuum equations of motion still allow for runaway solutions of the form

$$2 \exp\{\lambda[R(T) - R_0]\} = \sqrt{4 + \exp[2\lambda(T - T_0)]} - \exp[\lambda(T - T_0)], \quad (6.9)$$

that describe a mirror that keeps accelerating to the left (see Fig. 3a). This is not immediately obvious, since the mirror will now radiate and produce a constant flux of energy. From (6.6) (with T_{in} put to zero) we can explicitly evaluate the energy as a function of the mirror position $R(T)$ for the trajectory (6.9),

$$P_T(T) = \kappa\lambda^2[R_0 - R(T)] - \frac{1}{2}\kappa\lambda \exp\{-2\lambda[R_0 - R(T)]\}. \quad (6.10)$$

This expression indeed grows linearly for large T .

Where does this energy come from? The only possibility appears to be that the energy carried by the mirror itself grows negative at the same rate. The semi-classical hamiltonian (4.23) indeed contains several terms that can become negative, in particular the term $\kappa\lambda^2 R$ in the effective potential energy $V_q(R)$. As indicated in Fig. 4, the linear growth (6.10) in the energy P_T cancels against this linear term in $V_q(R)$ and effectively restores the classical form of the potential. As a result of this cancellation, we find that the total energy for the runaway solution (6.9) is indeed constant, provided R_0 solves (5.32). Thus we conclude that (6.9) is still an allowed solution of the semi-classical vacuum equations of motion.

Let us now consider the semi-classical modifications to classical mirror-matter dynamics described in subsect. 3.3. At early times, there is not too much difference. Some time before the matter wave arrives, the mirror will start to pre-accelerate to left along the trajectory (6.9), where the parameter T_0 is determined in the same way as before in terms of the total x^+ momentum of the incoming matter wave via

$$P_+ = \lambda \exp[\lambda(R_0 - T_0)], \quad (6.11)$$

except now with the new value for R_0 . Here

$$P_+ = \int_{-\infty}^{\infty} dt e^{-\lambda(r+t)} T_{\text{in}}(r+t). \quad (6.12)$$

So there will still exist a sub-critical regime in which the boundary trajectories will look roughly as in Fig. 3b, and for which the above equations in principle provide an explicit and well-defined relation between the in- and out-going waves. This low energy scattering equation is energy and information preserving, even though it is possible that a sizable fraction of the out-going matter comes out in the form of thermal radiation emitted during the period of pre-acceleration.

In the super-critical regime, on the other hand, we seem to find that, in spite of the presence of the quantum radiation, the semi-classical model has the same instability as the classical theory. It still seems true that when the matter shock wave carries more than the critical energy $E_c = \lambda e^{2R_0}$, the mirror point will keep accelerating forever and the matter pulse will never catch up with it. So there will still be information loss, since eventually the incoming wave will end up in the left asymptotic region and disappear into the black hole.

However, unlike the classical case, there is now something very unphysical about this situation, because if the mirror never stops accelerating, it also never stops emitting radiation. Clearly, in any physically realistic model, a black hole should stop radiating and disappear after it has emitted all of its mass. In our model, on the other hand, it seems that the energy of the mirror can become arbitrarily negative, so no mechanism seems available that stops it from accelerating after all incoming energy has been emitted. The question is how to deal with this (apparent) instability?

7. The super-critical case

The situation we have arrived at is similar to the instability that at some point seemed to plague other semi-classical models of two-dimensional dilaton gravity (see e.g. ref. [9]). A particular prescription for stabilizing the system has been proposed in ref. [4]. When translated into the language of our moving mirror model, this proposal effectively amounts to imposing the boundary condition that, when all the energy has been emitted, the mirror jumps back to the original equilibrium point $R = R_0$, after which it stops radiating. This prescription mimics the physical effect of the black hole completely disappearing into nothing after it has evaporated. By imposing this boundary condition, it seems one has indeed restored the stability of the system, but only at a very high cost. At the instant the mirror jumps back, all part of space to the left of $R = R_0$ suddenly becomes forever invisible to the outside world and the corresponding information cannot be retrieved. Thus, based on this physical picture, information loss seems inevitable, at least at the level of this semi-classical approximation.

We believe, however, that this conclusion is drawn too quickly. The reason is that, as we will now argue, the instability described in subsect. 6.2 may well be an artifact of the approximations that have been made. Namely, an important point that has not yet been properly taken into account is that the parameter t_0 in the runaway solution (6.9) is not a c-number, but an operator that depends on the incoming x^+ momentum via (6.11). This equation is directly related to the fact that the (r, t) coordinates are determined dynamically in terms of the incoming matter flux. Because this relation is nonlocal, it leads to some surprising consequences at the quantum level. In particular, we will argue that it opens up the possibility for an alternative physical mechanism for shutting down the radiation by the mirror, that does not destroy quantum coherence.

7.1. The algebra of in- and out-fields

The most direct and important consequence of the nonlocal initial condition (6.11) of the mirror trajectory in (r, t) space is that the left- and right-moving field operators $f_{\text{in}}(r_1, t_1)$ and $f_{\text{out}}(r_2, t_2)$ will *not* commute with each other. Instead they will satisfy a nontrivial commutator algebra, even when they are space-like separated. The physical interpretation of this nonlocal algebra is that it represents a gravitational shock-wave interaction between the incoming and out-going matter waves. As has been emphasized by 't Hooft in the context of $(3+1)$ -dimensional black holes [10], whenever one sends in a particle into a black hole, it will produce a gravitational shock wave near the horizon, that results in an exponentially growing shift in the position of all out-going particles. In the present model, this gravitational shock wave is generated via the small shift in the mirror trajectory as a result of the small change in the parameter t_0 due to the x^+ momentum of the in-falling particle.

To compute the resulting algebra, let us consider the semi-classical runaway solution (6.9), and use it to express the out-fields f_{out} in terms of the in-fields f_{in} . Thus we imagine sending some test wave back in time and letting it reflect off the space-time trajectory of the mirror point. This leads to the relation

$$f_{\text{out}}(t-r) = f_{\text{in}}\left(R_0 + T_0 - \frac{1}{\lambda} \log\{1 + \exp[\lambda(r - R_0 - t + T_0)]\}\right). \quad (7.1)$$

Although in the full quantum theory this equation is not entirely exact (since it does not include all effects of the back reaction due to the test wave itself) it should be accurate for in- and out-waves of reasonable energies and it becomes exact in the semi-classical limit when $\hbar \rightarrow 0$.

Once we adopt this approximation, then we immediately note that the above relation looks noninvertible, since the argument of f_{in} never exceeds the limiting value $R_0 + T_0$. This corresponds to the classical fact that f_{in} -waves that depart at any later time will never be reflected to out-waves. Now, if we would treat T_0 just as a c-number quantity, we would conclude from this that the commutator

$$[f_{\text{out}}(r_2, t_2), f_{\text{in}}(r_1, t_1)] \quad (7.2)$$

will vanish in the region

$$r_1 + t_1 > R_0 + T_0. \quad (7.3)$$

If this were indeed true, then this would prove that the in-Hilbert space is larger than the out-Hilbert space, and this would imply that information loss is inevitable. Instead, however, since T_0 is expressed in terms of the total incoming x^+ momentum via the relation (6.11), we must take into account that there is a nonzero commutator between T_0 and $f_{\text{in}}(r_1, t_1)$. A simple computation gives

$$[T_0, f_{\text{in}}(r_1, t_1)] = i\lambda^{-2} \exp[-\lambda(r_1 + t_1 + R_0 - T_0)] \partial_{r_1} f_{\text{in}}(r_1, t_1). \quad (7.4)$$

Fig. 5. Due to the nonlocal definition of the (r, t) coordinate system, the left- and right-moving fields do *not* commute with each other, even when they are space-like separated.

Although the right-hand side becomes exponentially small at late times t_1 , its effect in (7.2) can become very large. After combining (7.4) with (7.1), we can now compute the commutator (7.2), with the result (Fig. 5)

$$[f_{\text{out}}(2), f_{\text{in}}(1)] = i\lambda^{-2} \exp[\lambda(-r_1 - r_2 + t_2 - t_1)] \partial_2 f_{\text{out}}(2) \partial_1 f_{\text{in}}(1). \quad (7.5)$$

The above expression for the algebra is valid in the semi-classical limit $\hbar \rightarrow 0$ and then only in the regime (7.3), which is for those incoming waves f_{in} that would classically never reflect off the mirror trajectory (6.9). For $r_1 + t_1 < R_0 + T_0$ there will be other contributions since the left- and right-movers can then interact directly via reflection off the mirror. Note further that this algebra is *symmetric* between f_{in} and f_{out} , although its derivation looks asymmetric.

For correctness we should note that, since the right-hand side of (7.5) grows exponentially with the time difference $t_2 - t_1$ and also for negative $r_1 + r_2$, it is after some point no longer a good approximation to work to first order in $\hbar$. For large $t_2 - t_1$ this semi-classical approximation breaks down and we must start to take into account multiple commutators between τ_0 and f_{in} . The net result of this is to replace the classical algebra (7.5) by a quantum exchange algebra of the form

$$f_{\text{out}}(2)f_{\text{in}}(1) = \exp\{i\lambda^{-2} \exp[\lambda(-r_1 - r_2 + t_2 - t_1)] \partial_1 \partial_2\} f_{\text{in}}(1)f_{\text{out}}(2). \quad (7.6)$$

This exchange algebra reduces to the semi-classical formula in the $\hbar \rightarrow 0$ limit. It explicitly reveals the shock-wave interaction between the left- and right-movers: it shows that when an in- and out-wave cross, each will undergo an exponentially growing displacement proportional to the $x^\pm$ momentum carried by the other wave (see ref. [11]).

In the following, however, we will continue to work in the semi-classical limit. It is clear, however, that even in this case the nonlocal algebra (7.5) has nontrivial consequences. In particular, it tells us that, due to the quantum uncertainty principle, we should be very careful in making simultaneous statements about the left- and right-moving fields (that is, as long as we work in the (r, t) coordinate system).

7.2. The super-critical energy balance

A second important consequence of the relation (6.11) is that it turns (7.1) into a nonlinear, *energy preserving* relation between the in- and out-fields. Namely, we can use the conserved quantum hamiltonian H , that generates the time evolution of the in- and out-fields via

$$-i\partial_t f_{\text{in,out}}(t \pm r) = [H, f_{\text{in,out}}(t \pm r)], \quad (7.7)$$

to compare the energy carried by the out-wave (the left-hand side of (7.1)) to that carried by the same wave before reflection off the mirror (the right-hand side of (7.1)). If we would treat τ_0 as a c-number, these energies would clearly be different. Indeed, since the mirror recedes fast to the left, the reflected out-signal has a much lower frequency than the in-signal. In our dynamical mirror model, on the other hand, any energy change must be compensated by an energy change somewhere else. To make this energy balance work, we must again take into account (6.11). From this relation we find that, as an operator in the in-Hilbert space, τ_0 satisfies

$$[H, \tau_0] = i. \quad (7.8)$$

Here we used that $[H, P_+] = i\lambda P_+$, which holds formally. It is now a simple calculation to show that the commutators of H with the left- respectively right-hand side of (7.1) are indeed equal. Thus, as quantum operators, the expressions on both sides carry the same energy.

Physically this means that, when we send back a signal from the out-region and let it reflect to an in-signal, the increase in energy due to the blue-shift will be exactly compensated by a decrease in the energy carried by the matter forming the black hole. The interaction responsible for this nonlocal transfer of energy is the just described commutator algebra (7.5). Indeed, another way of seeing that the super-critical scattering equation (7.1) preserves energy is that the resulting algebra (7.5) is time-translation invariant, and thus also energy preserving.

This observation again teaches us a useful lesson. Namely, it tells us that it is essentially *impossible* that more energy is contained in the out-going radiation than went in, and thus that the instability that seems to plague the semi-classical model should disappear in a proper quantum treatment of the mirror–matter dynamics. We will now use this important insight, in combination with nonlocal algebra (7.5), to propose a new physical mechanism that will indeed stop the acceleration of the mirror by the time the total in- and out-energy are equal. This should then automatically restore the stability of the model.

7.3. Effective time evolution of the in-falling matter

Let us now reconsider the time evolution of the mirror–matter system in the light of these new insights. To this end, let us imagine setting up an experiment along a time slice $t = \text{const.}$ in which we try to detect all the in- and out-going particles and measure their positions and energies *. Due the nonlocal commutator between both kinds of particles, we can do this experiment only upto a certain accuracy. Mathematically, the Hilbert space of the f -fields at a given time t does not decompose into a simple tensor product of a left- and right-moving Hilbert space, since it makes a difference if we put, say, all out-going operator f_{out} to the left or to the right of the in-falling fields f_{in} . It is therefore an ill-defined question how much energy at a given time is contained in the in-falling or the out-going matter. The answer depends on the specific ordering prescription. Only the total (left plus right) energy remains well defined.

It is now clear that the description we gave in sect. 6 needs to be modified. There we had assumed that we could assign simultaneous physical meaning to the energy carried by the left- and right-moving fields. This allowed us to split the total hamiltonian H into two parts, a part H_R that measures the energy contained the out-going quantum radiation, given by (6.10), and a part H_L that measures all the remaining energy, given by

$$H_L(t) = H - \kappa\lambda^2 [R_0 - R(T)] + \frac{1}{2}\kappa\lambda \exp\{-2\lambda[R_0 - R(T)]\}. \quad (7.9)$$

In sect. 6 we interpreted this expression as the constant total energy H of the left-moving matter plus the negative (quantum) effective potential of the mirror. Our new proposal, however, is that this negative potential energy does in fact not really exist, but has its origin in the uncertainty relation between the left- and right-moving energies. It is namely far from clear that the ordering prescription that gives rise to the result (6.10) for the out-going energy is the same as the ordering in which the in-going energy remains constants. In fact, intuitively one expects these orderings to be opposite to each other, since to measure the out-going energy one needs to bring all *out*-fields to the left of the *in*-fields and the other way around if one wants to measure the total in-going energy. Thus, if this intuition is correct, it means that, in assigning simultaneous physical reality to the energy H_R of the Hawking radiation and the constant energy H of the in-falling matter, we have in fact overestimated the total amount of energy carried by the matter. The super-critical energy balance described above suggests that correcting the mistake should in fact precisely cancel the negative quantum contribution to the potential energy of the mirror.

Thus we arrive at a new physical picture in which there is a certain *complementarity* between the physical realities as seen by an asymptotic observer and that seen by an in-falling observer [10,12]. The reason for this complementarity is that

* Warning: This thought experiment is in fact highly hypothetical, since r and t have only real physical meaning as an asymptotic coordinate system.

each of these observers will use a different ordering prescription to assign physical meaning to the same quantum state. For the in-falling observer, the in-falling matter will simply propagate freely without any perturbation, but he will not see the out-going radiation. For the asymptotic observer, on the other hand, the Hawking radiation is physically real, and, due to the nonlocal interaction between in- and out-fields, the in-falling matter will therefore satisfy a nontrivial time-evolution equation. To make this idea a little bit more concrete, we will in the last part of this section describe a simple proposal for a possible effective description of the time evolution of the in-falling matter as seen by an asymptotic observer. The description will be far from complete, but it will give a clear indication of what kind of modifications from the standard picture can be expected when one takes this new quantum effect into account.

To simplify the following discussion, let us concentrate on the time evolution at late times $\lambda(t - t_0) \gg 1$, in which case (7.9) reduces to

$$H_L(t) = H - \kappa\lambda^2(t - t_0) \quad (7.10)$$

plus exponentially small corrections. In the κ -term we recognize the linear decrease in the left-moving energy as a consequence of the constant Hawking emission process. The idea is now to define an effective time evolution in such a way that this expression (7.10) represents the energy contained in just the left-moving matter. It turns out that this can be achieved in a very simple and natural way via the *ansatz* that the hamiltonian H_L not only *measures* the total remaining energy, but also *generates* the effective time evolution of the in-falling fields. Thus we define new left-moving matter fields $f_L(r, t)$ that satisfy the time evolution

$$-i\partial_t f_L(r, t) = [H_L(t), f_L(r, t)]. \quad (7.11)$$

Using (7.4) we find that this leads to a modified free field equation of motion

$$(\partial_t - \partial_r)f_L(r, t) = \frac{1}{2}\kappa \exp[-\lambda(r + t + R_0 - T_0)](\partial_t + \partial_r)f_L(r, t). \quad (7.12)$$

This equation, which is valid for in-falling waves at super-critical trajectories $r + t \gg R_0 + T_0$, shows that the effective fields $f_L(r, t)$ travel with a velocity that is slightly bigger than the speed of light! It can be integrated to a complete trajectory for the left-moving signals, which takes the form *

$$f_L(r, t) = f_{\text{in}}\left(\frac{1}{\lambda} \log\{\exp[\lambda(r + t + R_0 - T_0)] + \frac{1}{2}\kappa\lambda(r - t - R_0 + T_0)\}\right). \quad (7.13)$$

This equation shows that the f_L -waves will travel for a long time along an approximately light-like trajectory, while they slowly move towards the asymptotic mirror trajectory $r + t = R_0 + T_0$.

* The integration constant is chosen such that $f_L(r, t) = f_{\text{in}}(r + t)$ at $r - t = R_0 - T_0$.

The special property of the effective time evolution is that the total left-moving wave constantly loses energy at precisely the rate of the Hawking energy flux. A simple calculation gives that

$$\begin{aligned} & \partial_t \left(\frac{1}{2} \int [(\partial_t + \partial_r) f_L]^2 \right) \\ &= -\frac{1}{2} \kappa \lambda \int \exp[-\lambda(r + t + R_0 - T_0)] [(\partial_t + \partial_r) f_L]^2 = -\kappa \lambda^2, \end{aligned} \quad (7.14)$$

where the integrals run over $r - t = \text{const.}$ and in the second step we again used (6.11). This demonstrates the consistency of the interpretation of H_L as the total energy carried by the effective fields f_L .

Another interesting feature of (7.11) is that the total x^+ momentum P_+ defined in (6.12), and therefore also the parameter t_0 that determines the asymptotic mirror trajectory, in fact remain constant. This tells us that the in-falling fields f_L are capable of catching up with the mirror in a finite time. When we consider the specific example of an incoming shock wave of energy E_{in} that initially falls in along the light-like trajectory $r_1 + t_1 > R_0 + T_0$, then we find from (7.13) that the wave will have reached the asymptotic mirror trajectory when

$$\begin{aligned} t - r + R_0 - T_0 &= \frac{2}{\kappa \lambda} \{ \exp[\lambda(r_1 + t_1 + R_0 - T_0)] - \exp(2\lambda r_0) \} \\ &= \frac{2}{\kappa \lambda^2} (E_{\text{in}} - E_c), \end{aligned} \quad (7.15)$$

with E_c the critical energy (4.34). This is precisely the black hole evaporation time $T_{\text{evap}} = (E_{\text{in}} - E_c)/\kappa \lambda^2$ after which the evaporating black hole has reached a sub-critical energy.

Although we have introduced them in a somewhat ad hoc way, all this suggests to us that these fields $f_L(r, t)$ may indeed give a reasonable effective description of the in-falling matter as seen by an asymptotic observer, the idea being that their somewhat unusual time evolution arises because of the interaction with the out-going radiation. In any case, the above description indicates that taking this interaction into account may indeed lead to a new possible mechanism for stopping the acceleration of the mirror after all energy has been emitted. It is clear, however, that much work has to be done to see if an effective description of this kind can be developed into a complete and consistent semi-classical treatment of the model at super-critical energies.

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