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DOI

[10.1016/0370-2693\(94\)90693-9](https://doi.org/10.1016/0370-2693(94)90693-9)

Publication date

1994

Published in

Physics Letters B

[Link to publication](#)

Citation for published version (APA):

de Bakker, B. V., & Smit, J. (1994). Volume Dependence of the phase boundary in 4D dynamical triangulation. *Physics Letters B*, 334(3-4), 304-308. [https://doi.org/10.1016/0370-2693\(94\)90693-9](https://doi.org/10.1016/0370-2693(94)90693-9)

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ELSEVIER

18 August 1994

PHYSICS LETTERS B

Physics Letters B 334 (1994) 304–308

Volume dependence of the phase boundary in 4D dynamical triangulation

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Received 20 May 1994

Editor: P.V. Landshoff

Abstract

The number of configurations of the dynamical triangulation model of 4D euclidean quantum gravity appears to grow faster than exponentially with the volume, with the implication that the system would end up in the crumpled phase for any fixed κ_2 (inverse bare Newton constant). However, a scaling region is not excluded if we allow κ_2 to go to infinity together with the volume.

1. Introduction

The dynamical triangulation model of four dimensional euclidean quantum gravity has recently been the subject of interest [1–7].

The canonical partition function of the model at some fixed volume (number of four-simplices) V may be defined as a sum over triangulations $\mathcal{T}$ of the hypersphere S^4

$$Z(V, \kappa_2) = \sum_{\mathcal{T}(N_4=V)} \exp(\kappa_2 N_2), \quad (1)$$

where N_i is the number of simplices of dimension i in the triangulation $\mathcal{T}$ with fixed edge lengths. The N_i ($i = 0 \dots 4$) satisfy three constraints which means that only two of them are independent. We have chosen N_2 and N_4 as the independent variables. For comparison with other work we remark that if N_0 is chosen instead

of N_2 then the corresponding coupling constant κ_0 is related to our κ_2 as $\kappa_0 = 2\kappa_2$.

The weight $\exp(\kappa_2 N_2)$ is part of the Regge-Einstein action

$$S = \frac{-1}{16\pi G_0} \sum_{\Delta} V_{\Delta} R_{\Delta} = \kappa_2 (\rho N_4 - N_2), \quad (2)$$

$$\kappa_2 = \frac{V_2}{8G_0}, \quad \rho = \frac{10 \arccos(1/4)}{2\pi}, \quad (3)$$

where $V_2 = (324/5)^{1/4}$ is the volume of 2-simplices (triangles Δ) in units of $V_4 = 1$. Numerical simulations have shown that the system at fixed volume can be in two phases. For $\kappa_2 > \kappa_2^c(N_4)$ (weak bare coupling G_0) the system is in an elongated phase with high $\langle R_{\Delta} \rangle$ while for $\kappa_2 < \kappa_2^c(N_4)$ (strong coupling) it is in a crumpled phase with low $\langle R_{\Delta} \rangle$. The elongated phase has a low effective dimensionality and a large average distance while the crumpled phase has a high effective dimensionality and a small average distance between the simplices. The transition at κ_2^c is found to be continuous with a susceptibility $\partial^2 \ln Z(N_4, \kappa_2) / \partial \kappa_2^2$

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that grows with V . This opens the exciting possibility that continuum behavior may be found at the phase boundary [1–6].

Recently, evidence was presented [7] that the partition function $Z(N_4, \kappa_2)$ grows faster than exponentially in N_4 at fixed κ_2 . This result was interpreted as to cast doubt on the dynamical triangulation approach to four dimensional gravity, because it implies that the grand canonical partition function

$$Z(\kappa_2, \kappa_4) = \sum_{\mathcal{T}} \exp(\kappa_2 N_2 - \kappa_4 N_4) \quad (4)$$

is ill defined (the parameter κ_4 is related to the bare cosmological constant). We like to argue however that it is useful to study the local and ultraviolet properties of the model separately from the global and cosmological properties. The nonexistence of $Z(\kappa_2, \kappa_4)$ need not invalidate the possibility of a scaling regime at large N_4 . In this letter we report on our results for the behavior of $Z(N_4, \kappa_2)$ and shall argue that a possible continuum limit involves sending also κ_2 to infinity, i.e. $G_0 \rightarrow 0$.

2. Method

Unfortunately, no set of local moves to simulate the canonical ensemble is known and probably no such set exists [8]. Therefore, we rewrite the partition function (1) as

$$Z(V, \kappa_2) = \exp(\Delta S(V)) \times \sum_{\mathcal{T}} \exp(\kappa_2 N_2 - \Delta S(N_4)) \delta_{N_4, V}, \quad (5)$$

where the sum is now over all triangulations $\mathcal{T}$ with S^4 topology. In a simulation we can then generate configurations with Boltzmann weight

$$\exp(\kappa_2 N_2 - \Delta S(N_4)) \quad (6)$$

and select only those configurations with $N_4 = V$. The precise form of ΔS is unimportant. We have taken followed [1,2] and used

$$\Delta S(N_4) = -\kappa_4 N_4 - \gamma(N_4 - V)^2, \quad (7)$$

with $\gamma = 5 \cdot 10^{-4}$. This is smaller than the value used in [6,7], resulting in somewhat larger volume fluctuations. As the number of triangulations cannot grow

faster than $(5N_4)! \sim \exp(5N_4 \ln N_4)$ such a term ensures that the volume does not blow up in the simulation. The interesting question is now how $Z(V, \kappa_2)$ behaves as a function of the volume and the coupling constant κ_2 .

The simulation of the system

$$Z' = \sum_{\mathcal{T}} \exp(\kappa_2 N_2 - \kappa_4 N_4 - \gamma(N_4 - V)^2) \quad (8)$$

$$= \sum_{N_4} \exp(\ln Z(N_4, \kappa_2) - \kappa_4 N_4 - \gamma(N_4 - V)^2) \quad (9)$$

allows the measurement of $\partial \ln Z / \partial N_4$ through the equation

$$\begin{aligned} \kappa_4^c(V, \kappa_2) &\equiv \frac{\partial \ln Z(V, \kappa_2)}{\partial V} \\ &= \kappa_4 + 2\gamma(\langle N_4 \rangle - V) + O\left(\frac{1}{V}\right), \end{aligned} \quad (10)$$

which follows from a saddle point approximation for $\langle N_4 \rangle$ in this ensemble. At the same time this simulation gives us configurations with the weights (6) to measure properties of (5).

We will not expand here upon our method to generate simplicial complexes, but for definiteness let us mention that we allow all those complexes where no two d -dimensional (sub)simplices share exactly the same set of $(d+1)$ points. (In [9] our criteria were somewhat different, our present criteria lead to numerical results in agreement with [2,3,5–7].)

3. Results

Fig. 1 shows the value of κ_4^c as a function of the volume at $\kappa_2 = 0$, where the configurations at any particular volume all have the same weight. The points at the highest volumes are somewhat correlated as the autocorrelation time became of the order of the time between volume increments. The line is a fit to

$$\kappa_4^c = a + b \ln \langle N_4 \rangle, \quad (11)$$

where the fitting parameters are

$$a = 0.82(1), \quad b = 0.0323(9), \quad (12)$$

Fig. 1 The effective critical κ_4^c for various volumes at $\kappa_2 = 0$.

with $\chi^2 = 24$ at 46 d.o.f. We have also tried fitting the data to the converging functions

$$\kappa_4^c = \sum_{n=0}^k a_n \langle N_4 \rangle^{-n} \quad (13)$$

for various small k and

$$\kappa_4^c = a + b \langle N_4 \rangle^{-c}. \quad (14)$$

The function (13) simply does not yield any reasonable fit. On the other hand, the small converging power (14) fits with $a = 1.7(9)$ and $c = 0.06(8)$, which is consistent with zero. This also points to a logarithm. The large error in a is due to its large correlation with the power c . For fixed c the error in a would be much smaller. Both the qualitative picture and the value of b (12) agree with data reported in [7], produced via completely independent code.

The forms (11) and (14) are virtually indistinguishable for small powers. Thus, if it is indeed a logarithm, a converging small power can never be excluded and will even be favored if the low volume points have somewhat lower κ_4^c than that of a pure logarithm.

At the other side of the transition, at $\kappa_2 = 2.0$, the same plot (not shown) is just a horizontal line for all volumes we have used, which went up to 90,000. The

fitting parameters for the logarithmic function (11) are in this case

$$a = 5.662(5), \quad b = 1(5) \times 10^{-4}, \quad (15)$$

with $\chi^2 = 7.7$ at 86 degrees of freedom, which is consistent with a constant (i.e. b is consistent with 0). So in this phase the number of configurations making an important contribution to the partition function does rise only exponentially with the volume.

The values of κ_4^c at various volumes and κ_2 values in the region of the transition can be seen in Fig. 2. We have subtracted $2.3\kappa_2$ from the data for κ_4^c to expand the vertical scale. The diamonds indicate the value of κ_2^c at each volume. For the lowest three volumes these are the points obtained in [6], while those of 8000 and 16000 are our own.

Going from right to left the curves are close together and then break away from each other as their slope decreases. The break away point moves to the right as the volume increases. Assuming that it is correct to extrapolate the pattern in this figure and Eqs. (12,15) to large volumes, we conclude that κ_4^c first stays constant as the volume increases and then, depending on the value of κ_2 , starts to diverge logarithmically. We also see that the transition between the crumpled and elongated phase moves to larger κ_2 as the volume in-

Fig. 2. The effective critical κ_4^c as a function of κ_2 at various volumes. To expand the vertical scale we have subtracted a straight line $-2.3\kappa_2$ from the data. The values of $\kappa_2^c(V)$ are indicated with diamonds.

creases, staying near the break away points.

There is a close relation between these curves and the average curvature, due to the relation

$$\frac{\partial \langle N_2 \rangle}{\partial V} = \frac{\partial^2 \ln Z(V, \kappa_2)}{\partial \kappa_2 \partial V} = \frac{\partial \kappa_4^c}{\partial \kappa_2}. \quad (16)$$

This implies that the largest change in the slope of these curves will coincide with the susceptibility peak and that as that peak becomes narrower the bend in the $\kappa_4^c(\kappa_2)$ curve becomes sharper.

4. Discussion

These results indicate that the number of crumpled configurations grows factorially with the volume, while the number of elongated configurations only grows exponentially. This would mean that at any fixed κ_2 the crumpled configurations will always dominate for large enough volumes.

From this it already follows that the κ_2^c of the transition between the phases must diverge with the volume. In Ref. [6] a converging scenario is favored but a diverging one not excluded. In fact, it is mentioned [6] that leaving out data points with $V = 500, 1000$

leads to fit with κ_2^c diverging logarithmically with volume. The large κ_2 region in Fig. 2 seems indeed to suggest that $V = 1000$ is somewhat too low to see the asymptotic trend.

It is an interesting question whether the bare free energy

$$F(V, \kappa_2) = -\ln \sum_{T(N_4=V)} \exp(-S) \quad (17)$$

is extensive at the phase boundary, $\partial F(V, \kappa_2^c(V))/\partial V = \text{const.}$ From (2) we see that this would be the case if

$$\begin{aligned} &\kappa_4^c(V, \kappa_2^c(V)) - \rho \kappa_2^c(V) \\ &- \rho V \frac{\partial \kappa_2^c(V)}{\partial V} + \langle N_2 \rangle \frac{\partial \kappa_2^c(V)}{\partial V} \end{aligned} \quad (18)$$

is independent of V for large V . Assuming a logarithmic dependence of κ_2^c on V the last two terms are constant so the question is whether the difference of the first two is V independent. We see from Fig. 2 that this unlikely: in the range $V = 2000 - 16000$, $\kappa_4^c \approx 2.4 \kappa_2^c(V)$ which rises faster with than $\rho \kappa_2 \approx 2.098 \kappa_2$.

Expecting extensivity with the bare Einstein-Regge action is questionable. It consists of two very different terms $\propto N_2$ and $\propto N_4$ which will require different renormalization, such that the particular linear combination $\kappa_2(N_2 - \rho N_4)$ has to be modified. In other words, the curvature term will mix with the volume N_4 under renormalization. A natural candidate is $\kappa_2 N_2 - \kappa_4^c(N_4, \kappa_2) N_4 + \text{const } N_4$, where the *const* may be fixed by some normalization condition.

We are therefore not disturbed by the fact that the average bare curvature is rather different from zero at the transition [3]. One may also contemplate obtaining a physical curvature in terms of a physically defined metric, but there are other perhaps more easily accessible quantities. Elsewhere [9] we have proposed a way of measuring the renormalized Newton constant G in terms of the binding energy of test particles. It is however not even clear to us that V should be interpreted as the physical volume as measured in terms of a physical metric at physical scales, because V is sensitive to the proliferation of baby universes [10].

We have presented results suggesting that the bare gravitational coupling G_0 has to go to zero in a possible scaling limit. It is interesting that also in matrix models of 2D gravity with unrestricted topology the number of configurations rises factorially with the volume and a sensible continuum limit can only be taken by letting the bare G_0 go to zero in the so called double scaling limit [11–13]. Perhaps 2D gravity in its dynamical triangulation formulation (without restriction on topology) has also a strong coupling phase.

Similarly, the scenario with G_0 going to zero might also be applicable to the case of four-dimensional simplicial gravity with unrestricted topology. In this model it is certain that there is no exponential bound on the number of configurations.

Acknowledgement

This work is supported in part by the Stichting voor Fundamenteel Onderzoek der Materie (FOM). The numerical simulations were partially carried out on the IBM SP1 at SARA.

Note

Just as we finished this paper we received a preprint [14] presenting data that favors the scenario with κ_4^c converging according to (14) with an exponent c of $1/4$.

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