Probing the plateau-insulator quantum phase transition in the quantum Hall regime

van Schaijk, R.T.F.; de Visser, A.; Oltshoorn, S.; Wei, H.P.; Pruisken, A.M.M.

Link to publication

Probing the Plateau-Insulator Quantum Phase Transition in the Quantum Hall Regime

R. T. F. van Schaijk and A. de Visser

Van der Waals-Zeeman Institute, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

S. M. Olsthoorn

High Field Magnet Laboratory, R.I.M., University of Nijmegen, Toernooiveld, 6525 ED Nijmegen, The Netherlands

H. P. Wei

Department of Physics, Swain Hall West, Indiana University, Bloomington, Indiana 47405

A. M. M. Pruisken

Institute for Theoretical Physics, University of California at Santa Barbara, Santa Barbara, California 93106-4060
and Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
(Received 2 December 1998)

We report quantum Hall experiments on the plateau-insulator transition in a low mobility $In_{0.53}Ga_{0.47}As/InP$ heterostructure. The data for the longitudinal resistance ρ_{xx} follow an exponential law and we extract a critical exponent $\kappa = 0.55 \pm 0.05$ which is slightly different from the established value $\kappa = 0.42 \pm 0.04$ for the plateau transitions. Upon correction for inhomogeneity effects, which cause the critical conductance σ_{xx}^* to depend *marginally* on temperature, our data indicate that the plateau-plateau and plateau-insulator transitions are in the same universality class.

PACS numbers: 73.40.Hm, 72.10.−d, 73.20.Dx

In the field of two dimensional electron gases the nature of the transitions between adjacent quantum Hall plateaus (*PP* transition) is an ardent topic of research. Experiments on low mobility $In_{0.53}Ga_{0.47}As/InP$ heterostructures are a remarkable demonstration of a *quantum phase transition* indicating that the quantum Hall steps become infinitely sharp as the temperature (T) approaches absolute zero [1,2]. The maximum slope in the Hall resistance (ρ_{xy}) with varying magnetic field B was shown to diverge algebraically in T, $(d\rho_{xy}/dB)_{max} \propto T^{-\kappa}$ with the critical exponent $\kappa = 0.42 \pm 0.04$. On the other hand, the half-width (ΔB) of the longitudinal resistance (ρ_{xx}) was shown to vanish like $\Delta B \propto T^{\kappa}$ with the same exponent κ.

Because of the short range random alloy potential scattering, the low mobility $In_xGa_{1-x}As/InP$ structure has proven to be exceptionally important for studying scaling phenomena. This produces a wide range in T where the transport is dominated by Anderson (de)localization effects. This is in sharp contrast to high mobility $GaAs/Al_xGa_{1-x}As$ heterostructures where the long range potential fluctuations dramatically complicate the observability of the critical phenomenon, given the limitations of the experiment [3,4].

Nevertheless, the *PP* transitions in GaAs heterostructures have been studied extensively. It is therefore not surprising that the *universality* of the plateau transitions has not been established by these investigations. In these experiments, the same value of $\kappa \approx 0.42$ was found but for a few samples only and for a small range in lowest T [4]. However, in most of the samples simple data fitting produced κ's ranging from 0.2 up to 0.9. These results are

Landau level dependent and even for a given Landau level the ρ_{xx} and ρ_{xy} data give rise to different values for κ [5].

The focus in the last few years has been on transport in the lowest Landau level. For this purpose samples of lower density were used [6–10]. The bare resistance data look quite different from those of the other Landau levels since the transition is now between a quantum Hall (plateau) phase and an *insulator* (*PI* transition). There are striking similarities between the *PI* transition at high B and the superconductor-insulator transition [11] and the metal-insulator transition in two dimensions at $B = 0$ [12].

One of the most important predictions of the renormalization theory is that the *PP* and *PI* quantum phase transitions are the same [2]. This stipulates that the same κ be observed as T approaches absolute zero and that the (electron-hole) symmetry in the σ_{xx}, σ_{xy} conductance plane be retained. These aspects of Landau level systems serve as an important guide for selecting samples, particularly since the range of available T in transport measurements is rather limited and usually not sufficient to establish a critical point. However, in the experiments of Refs. [6–10] a comparison between the *PP* and *PI* transitions within the same sample was either not possible or not drawn. Moreover, these experiments provide inconclusive results because the samples used suffer from the aforementioned complications.

Recently, an interesting empirical result for the lowest Landau level ρ_{xx} has been reported [13]. For arbitrary samples at finite T, the ρ_{xx} data seems to depend linearly rather than algebraically on T, indicating that the problem is generically the same for all GaAs samples. Once

again, the experimental design has overlooked an essential requirement for studying scaling phenomena: the importance of short range random potential scattering—an essential (but certainly not sufficient [14]) prerequisite for sample choice.

In this paper we report the results of magnetotransport experiments performed on an $In_{0.53}Ga_{0.47}As/InP$ heterostructure. Our main objective is to study critical aspects of the PI transition and to compare the results to the PP transitions measured on the same sample. We benefit from the fact that our sample has been studied before [15]. In particular, the exponent κ for the PP transitions was found to be 0.42 and 0.20 for spin polarized and spin degenerate Landau levels, respectively. The mobility of the sample is $\mu = 16\,000$ cm^2/V s measured at $T = 4.2$ K. The electron density is 2.2×10^{11} cm^{-2} which means that the PI transition occurs at $B \approx 16$ T.

Our experiments were carried out in a Bitter magnet ($B < 20$ T) using a plastic dilution refrigerator (0.1–2 K) and a bath cryostat (1.5–4.2 K). The magnetotransport properties were measured using a standard ac technique with a frequency of 6 Hz and an excitation current of 5 nA. The main experimental results are presented in Fig. 1, where we have traced ρ_{xx} and ρ_{xy} as B sweeps through the PI transition. The different sets of data show a characteristic value $B \approx 16$ T separating the insulating phase at high B and the quantum Hall phase at lower B. The ρ_{xy} data are shown in the inset of Fig. 1. This quantity, at low T, is clearly not quantized through the transition.

In order to recognize the relevant structure in the combined ρ_{xx}, ρ_{xy} data we have computed the conductance components σ_{xx} and σ_{xy} in the standard fashion. The results for different T are plotted in the σ_{xx}, σ_{xy} flow diagram (Fig. 2, upper inset). The symmetry about the

line $\sigma_{xy} = 1/2$ is striking and reflects the high quality of the experimental data. The same symmetry was observed and discussed in the original work on the PP transitions [16]. Following the renormalization theory of the quantum Hall effect [2] we extract the critical value B_c from the maximum in σ_{xx} or from $\sigma_{xy} = 1/2$. The B_c ranges from 16.3 T at 0.13 K up to 16.9 T at 4.2 K, indicating a change in electron density of 3.5%. This is smaller than what is obtained from the low B Hall data, which gives a density of 2.0×10^{11} cm^{-2} at 0.13 K as compared to 2.2×10^{11} cm^{-2} at 4.2 K. The symmetry in the σ_{xx}, σ_{xy} diagram is a direct consequence of the following relations which hold at low but fixed T:

$$\sigma_{xx}(\Delta\nu) = \sigma_{xx}(-\Delta\nu);$$
$$\sigma_{xy}(\Delta\nu) = 1 - \sigma_{xy}(-\Delta\nu). \tag{1}$$

Here $\Delta\nu = \frac{1}{B} - \frac{1}{B_c} = \nu - \nu_c$. We have explicitly verified the validity of Eq. (1). This result is important since it fundamentally reflects the electron-hole symmetry in the problem. Our data do not follow the statement of "duality" [17,18] which says that $\rho_{xx}(\Delta\nu) = \rho_{xx}^{-1}(-\Delta\nu)$ and ρ_{xy} remains quantized through the PI transition. Instead, we observe that the σ_{xx} peak at $\Delta\nu = 0$ develops a maximum around 1 K. Thus only for $T < 1$ K the asymptotic scaling regime for the PI transition is attained [1]. For the PP transitions this range is larger ($T < 3$ K).

Since the σ_{xx}, σ_{xy} data show the characteristic Landau level independent behavior, we can now extract the
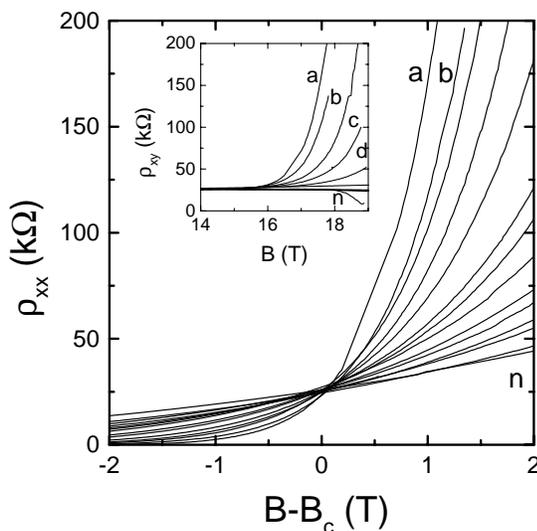

FIG. 1. ρ_{xx} and ρ_{xy} data with varying B. The curves are labeled $a, b, \ldots, n$ and the corresponding T's are 0.13, 0.21, 0.26, 0.35, 0.47, 0.59, 0.83, 1.04, 1.4, 1.5, 1.9, 2.2, 3.1, and 4.2 K. B_c is the critical B (see text).
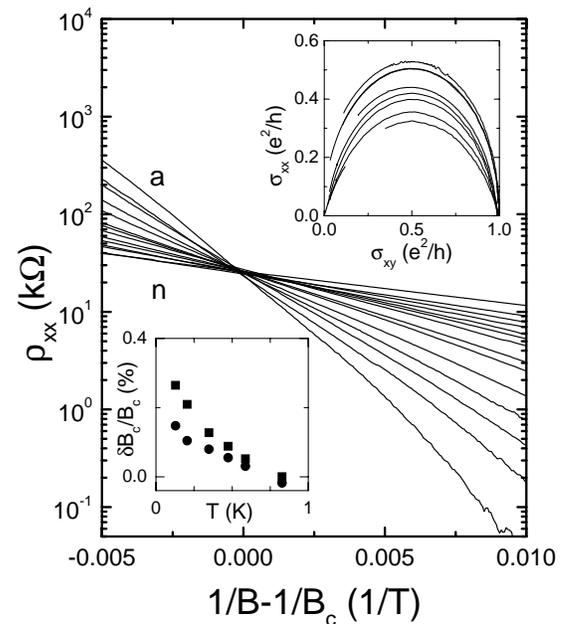

FIG. 2. ρ_{xx} data on a log scale with varying $1/B$. The labels and T's are the same as in Fig. 1. Upper inset: σ_{xx} vs σ_{xy} at various T in the range 1 K–130 mK. Lower inset: $\delta B_c/B_c$ vs T. The squares are the data derived from ρ^*, Eqs. (2) and (4), and the circles are the data obtained from the different definitions of B_c (see text).

critical exponents in a similar fashion as done previously with the ρ_{xx} and ρ_{xy} data from the higher Landau levels. For example, from σ_{xx} with varying B we obtain the half-width $\Delta B \propto T^{\kappa}$ with an exponent $\kappa = 0.46 \pm 0.05$. On the other hand, we find $(\frac{d\sigma_{xy}}{dB})_{min} \propto T^{-\kappa}$ with $\kappa = 0.43 \pm 0.05$. We attribute the small difference between the exponents to the uncertainties caused by mixing the ρ_{xx}, ρ_{xy} data in the computation of the conductances [16]. In Fig. 3 we plot the $(\Delta B)^{-1}$ vs T for both the PI and the $2 \rightarrow 1$ plateau transition. The latter was derived from the half-width in ρ_{xx}. For later purposes we have also plotted the low T data for $(\frac{d\sigma_{xy}}{dB})_{min}$ vs T in the lower inset of Fig. 4. The resulting exponents $\kappa = 0.46 \pm 0.05$, 0.42 ± 0.05, and 0.43 ± 0.05 are all the same, within the experimental error, indicating that the PP and the PI transitions are in the same universality class.

Next we compare our data to the exponential result [13] $\rho_{xx}(\nu, T) \propto \exp[-\Delta\nu/\nu_0(T)]$. In Fig. 2 we have traced ρ_{xx} on a log scale as a function of the difference $\Delta\nu$. This gives an adequate description of the data. The slope (ν_0^{-1}) of the straight lines at the transition regime can be accurately determined at each T. In Fig. 3 we plot ν_0^{-1} vs T on a log-log scale. The data nicely follow the algebraic behavior $\nu_0^{-1} \propto T^{-\kappa'}$ with $\kappa' = 0.55 \pm 0.05$. This value differs from the expected value $\kappa = 0.42 \pm 0.05$ by more than the experimental error.

It is important to note that the data do not follow the linear behavior $\nu_0 = \alpha T + \beta$ as proposed and advocated in Ref. [13]. Such linear dependence on T clearly does not describe the asymptotics of the quantum phase transition at T equal to zero. Instead, it is semiclassical in nature and typically observed at finite T and on samples with predominantly slowly varying potential fluctuations [3].
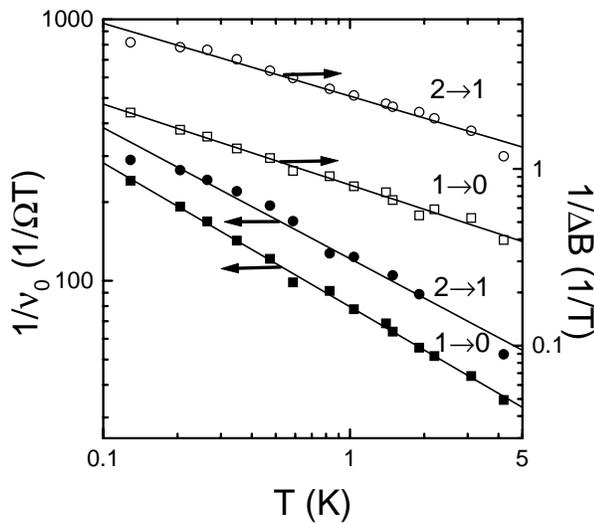
In order to show that the value $\kappa' = 0.55 \pm 0.05$ is not a specific property of the PI transition, we have mapped the $2 \rightarrow 1$ plateau transition onto the lowest Landau level following the steps $\rho_{xx}, \rho_{xy} \rightarrow \sigma_{xx}, \sigma_{xy} \rightarrow \sigma_{xx}, \sigma_{xy} - 1 \rightarrow \rho_{xx}, \rho_{xy}$. The ρ_{xx} data thus obtained were fitted to the exponential expression leading to a value $\kappa' = 0.51 \pm 0.05$ (see Fig. 3). Transformations like this generally lead to less quality data. Nevertheless, the results in Fig. 3 indicate that different exponents can be extracted from the same experimental data.

We next address the origin of this difference. First we point out that the transport data of the PI transition are accurately described by writing

$$\rho_{xx}(\nu, T) = \rho^*(T) \exp(-T^{-\kappa'}\Delta\nu). \qquad (2)$$

Here ρ^* denotes the *critical* resistance. It can be written as $\rho^* = \sigma_{xx}^*/[(\sigma_{xx}^*)^2 + 1/4]$ where σ_{xx}^* stands for the peak in σ_{xx}. Both quantities are weakly dependent on T and, quite surprisingly, this T dependence is not simply *irrelevant* as thought previously. It is, in fact, *marginal*, and we next show that it accounts for the difference in the observed exponents.

Following Eq. (1) we can relate $\rho_{xx}(\Delta\nu)$ to $\rho_{xx}(-\Delta\nu)$ such that we can write the ratio as $\rho_{xx}(\Delta\nu)/\rho_{xx}(-\Delta\nu) = \exp(-2 T^{-\kappa'}\Delta\nu)$. As a good check upon the validity of this result we have fitted the exponential on the right-hand side to the experimental data which were inserted into the left-hand side. The same numerical value $\kappa' = 0.55 \pm 0.05$ was obtained indicating once more that Eq. (1) represents the fundamental symmetry in the problem. From the ratio we obtain the following renormalization group
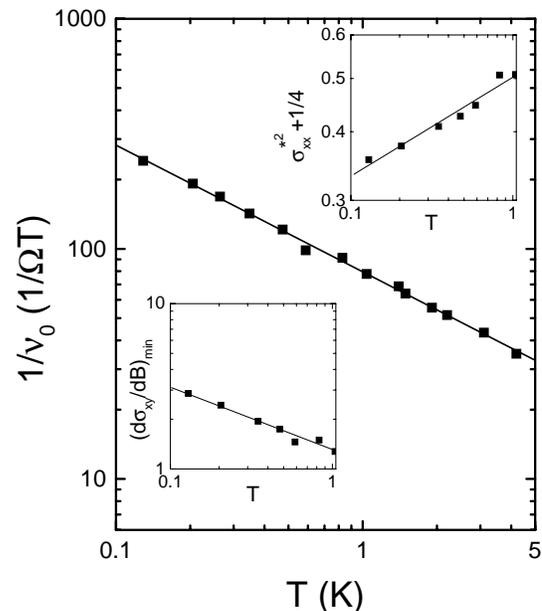

FIG. 3. Left axis: $1/\nu_0$ vs T for the $2 \rightarrow 1$ plateau transition (full circles, $\kappa' = 0.51$) and the PI transition (full squares, $\kappa' = 0.55$). Right axis: $1/\Delta B$ vs T for the $2 \rightarrow 1$ plateau transition (open circles, $\kappa = 0.42$) and the PI transition (open squares, $\kappa = 0.46$).

FIG. 4. $1/\nu_0$ vs T for the PI transition (full squares, $\kappa' = 0.55$). Upper inset: $\sigma_{xx}^{*2} + 1/4$ vs T. The slope of the straight line equals 0.15. Lower inset: $(\frac{d\sigma_{xy}}{dB})_{min}$ vs T. The slope of the straight line equals 0.43.

equation for small $\theta = \sigma_{xy} - 1/2$:

$$\frac{d\theta}{d\ln T} = -\kappa\theta; \qquad \kappa = \kappa' - \frac{d\ln(\sigma_{xx}^{*2} + 1/4)}{d\ln T}. \quad (3)$$

Equation (3) shows how a relatively weak T dependence in σ_{xx}^* can lead to different exponents extracted from different quantities. In Fig. 4 we replot $1/\nu_0$ vs T on a log-log scale. The solid line gives $\kappa' = 0.55$. In the upper inset we plot the low T data for $\ln[(\sigma_{xx}^*)^2 + 1/4]$ vs $\ln T$ and obtain a slope of 0.15 ± 0.03. According to Eq. (3) we have $\kappa \approx 0.40$. This value should be compared with $\kappa = 0.43$ extracted from the low T dependence of $(d\sigma_{xy}/dB)_{\min}$ (Fig. 4).

A naive interpretation of the *marginal* dependence of σ_{xx}^* on T is that the electron gas has not yet fully developed criticality. This would mean that a much lower T is necessary before the critical fixed point is truly reached. However, it is important to stress that the small changes in σ_{xx}^* observed at low T are most likely the result of macroscopic inhomogeneities in the sample. One way of showing this is by writing Eq. (2) as

$$\rho_{xx}(\Delta\nu, T) = e^{-[\Delta\nu - \delta\nu_c(T)]/\nu_0(T)}. \quad (4)$$

Here the shift in the critical filling fraction ($\delta\nu_c$) and the critical resistance (ρ^*) are related through $\rho^*(T) = e^{\delta\nu_c/\nu_0}$. This shift is next to be compared to the difference ($\delta\nu_c$) as it is obtained from the definitions $\sigma_{xy} = 1/2$ and $d\sigma_{xx}/dB = 0$. In the lower inset of Fig. 2 we plot the $\delta\nu_c/\nu_c$ or, equivalently, $\delta B_c/B_c$ with varying T for both cases. The two effects are comparable.

Notice that the uncertainty $\delta\nu_c/\nu_c$ in the definition of ν_c clearly shows the effect of macroscopic inhomogeneities (in electron density) which cause ν_c to be slightly different in the different regions of the sample where the ρ_{xx} and ρ_{xy} are being probed. Figure 2 therefore indicates that the weak or *marginal* T dependence of σ_{xx}^* is, in fact, an *inhomogeneity* effect. This lack of universality in σ_{xx}^* also shows up in the different data sets taken at different experimental runs. After heating the system up to room T and then cooling down again one usually finds that the B_c has shifted (indicating that the electron density has changed) along with a shift in σ_{xx}^* (indicating that the inhomogeneity profile of the density has changed).

In summary, we can say that the *PP* and *PI* transitions are the same. This is in complete agreement with the predictions of the renormalization theory [2]. We have shown that the critical conductance σ_{xx}^* as well as the exponent of the *PI* transition are weakly affected by the (weak) macroscopic inhomogeneities in the sample. Our data retain fundamental aspects such as the electron-hole symmetry in the σ_{xx}, σ_{xy} diagram. It is important that this symmetry is not confused with the statement of duality [17] which is, in fact, not verified by our experiments.

The marginal T dependence in σ_{xx}^* is common to both the *PP* and *PI* transitions in our samples. This was previously also observed [1]. It is important to note that Eq. (3), upon modification, is applicable to the *PP* tran-

sitions as well. For example, for the $2 \to 1$ transition Eq. (3) is modified according to $\kappa' - \kappa = d\ln(\sigma_{xx}^{*2} + 9/4)/d\ln T$. By inserting the σ_{xx}^* data we find in this case $\kappa' - \kappa < 0.01$ which is well within the experimental error. This result explains why a single exponent $\kappa = \kappa' = 0.42 \pm 0.04$ was previously extracted from the $2 \to 1$ transition as well as from the higher Landau level experimental data over a wide range in T (4 K–20 mK). By combining the results for the *PP* and *PI* transitions we conclude that $\kappa = 0.42$ stands for the *universal* critical exponent of the quantum phase transition. The numerical value $\kappa' = 0.55$, on the other hand, is the result of macroscopic inhomogeneities. Following Eq. (3) it represents an *effective* exponent. A deeper understanding of the inhomogeneity effects demands a commitment to research specifically grown InGaAs/InP heterostructures.

[1] H. P. Wei *et al.,* Phys. Rev. Lett. **61**, 1294 (1988); H. P. Wei, D. C. Tsui, and A. M. M. Pruisken, in *Localization and Confinement of Electrons in Semiconductors,* edited by F. Kuchar, H. Heinrich, and G. Bauer (Springer-Verlag, Berlin, Heidelberg, 1990).

[2] A. M. M. Pruisken, Phys. Rev. Lett. **61**, 1297 (1988); A. M. M. Pruisken and M. A. Baranov, Europhys. Lett. **31**, 543 (1995).

[3] A. M. M. Pruisken, B. Škorić, and M. A. Baranov, Phys. Rev. B **60**, 16838 (1999).

[4] H. P. Wei *et al.,* Phys. Rev. B **45**, 3926 (1992).

[5] S. Koch *et al.,* Phys. Rev. B **43**, 6828 (1991); **46**, 1596 (1992).

[6] H. W. Jiang *et al.,* Phys. Rev. Lett. **71**, 1439 (1993).

[7] T. Wang *et al.,* Phys. Rev. Lett. **72**, 709 (1994).

[8] R. J. F. Hughes *et al.,* J. Phys. Condens. Matter **6**, 4763 (1994).

[9] L. W. Wong *et al.,* Phys. Rev. B **51**, 18033 (1995).

[10] W. Pan *et al.,* Phys. Rev. B **55**, 15431 (1997).

[11] A. F. Hebard and M. A. Paalanen, Phys. Rev. Lett. **65**, 927 (1990).

[12] S. V. Kravchenko *et al.,* Phys. Rev. B **50**, 8039 (1994).

[13] D. Shahar *et al.,* Solid State Commun. **107**, 19 (1998).

[14] It is quite obvious that sample problems may also occur due to drastic changes in the experimental conditions and parameters. For example, the single InGaAs/InP sample used in Ref. [13] has a density (see cond-mat/9706045) which is 1 order of magnitude smaller than those studied in Refs. [1], [15], and [16]. This dramatic difference was not even mentioned in Ref. [13].

[15] S. W. Hwang, Phys. Rev. B **48**, 11416 (1993).

[16] H. P. Wei, D. C. Tsui, and A. M. M. Pruisken, Phys. Rev. B **33**, 1488 (1986).

[17] S. Kivelson, D. H. Lee, and S. C. Zhang, Phys. Rev. B **46**, 2223 (1992).

[18] D. Shahar *et al.,* Solid State Commun. **102**, 817 (1997).