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Gauge dependence of the AdS instability problemFotios V. Dimitrakopoulos,^{1,*} Ben Freivogel,^{1,†} Juan F. Pedraza,^{1,‡} and I-Sheng Yang^{2,§}¹*GRAPPA and ITFA, Institute of Physics, Universiteit van Amsterdam,
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Previous work on the anti-de Sitter (AdS) instability problem within the two-time framework (TTF) has found an “oscillating singularity” whose presence depends on the gauge choice. We give a physical interpretation of this singularity as a diverging redshift between the boundary and the center of AdS. This signals a genuine breakdown of the linearized gravity. One can also identify the diverging redshift through a backreaction calculation purely in the boundary gauge, where the TTF result stays regular.

DOI: [10.1103/PhysRevD.94.124008](https://doi.org/10.1103/PhysRevD.94.124008)**I. INTRODUCTION**

The question of whether global anti-de Sitter (AdS) spacetime is generically stable or unstable under small perturbations is a very interesting problem. A conclusive resolution still eludes us despite the combined efforts of many people [1–17]. The two-time framework (TTF) is a well-established tool that reduces the full gravitational dynamics into the “slow-time” evolution of complex amplitudes of approximate eigenstates. It operates on two approximations:

- (i) The deviation from empty AdS metric is small, so we can keep only the leading order gravitational backreaction.
- (ii) The evolution can be averaged over a “fast” time scale set by the AdS radius, reducing to the dynamics in a “slow” time scale.

One can simply follow the two-time evolution and observe whether the first approximation breaks down. If it does not, then the metric stays near empty AdS and an instability is not triggered. If it does break down, then it implies an order one deviation from empty AdS, thus triggering an instability.

In [1], numerical results suggested that gravitational instability seems to coincide with a breakdown of TTF from an oscillating singularity—the complex amplitudes all start to acquire infinite phases. However, a direct logical link between the two was missing, because the physical interpretation of the oscillating singularity remained unclear. That is because a breakdown of TTF could be due to failure of either one of the two approximations, but only the breakdown of the first approximation has direct

implications for the instability.¹ Later, in [3], was suggested that TTF might not suffer from an oscillating singularity if one chooses a different gauge, a fact that was subsequently verified numerically in [2]. Those results appeared to add more confusion.

In this short article, we point out that the combination of [1] and [2,3] actually eliminates the confusion. A diverging difference² between the results in two different gauges implies a diverging redshift between two different locations in AdS, which in turn implies a diverging deviation in the metric. Alternatively, one could have used only the result in the boundary gauge where the TTF solutions stays finite [2]. Explicitly calculating its geometric back-reaction demonstrates the same divergence [14].

Note that the actual geometric backreaction is the TTF result multiplied by the amplitude squared of the initial perturbation. A diverging TTF redshift means that linearized gravity breaks down for arbitrarily small initial amplitude, which triggers a genuine instability of global AdS.

In Sec. II, we will briefly review the model of spherically symmetric scalar perturbations in AdS and the perturbation theory leading to the two-time framework. In Sec. III, we will compare the result between two different gauges. We will show that a diverging difference of the phases in these two different gauges is equivalent to a diverging “averaged” redshift calculated from backreaction. When the averaged redshift diverges, the actual redshift must diverge at some moment, guaranteeing a large deviation from the empty AdS metric.

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¹Some may have the intuition that the breakdown of the second approximation also can only come from large deviations from the AdS metric, but such a statement is never proven explicitly.²We will specify what this means in the subsequent sections.

II. REVIEW OF THE MODEL

The model that is mainly used is a perturbation in the form of a spherically symmetric massless scalar field which propagates under its own self-gravitation in the AdS background. For the metric of asymptotically AdS space-times we use the following ansatz³:

$$ds^2 = \frac{1}{\cos^2 x} (-Ae^{-2\delta} dt^2 + A^{-1} dx^2 + \sin^2 x d\Omega^2), \quad (1)$$

where the functions A and δ depend only on time t and the radial coordinate $x \in [0, \frac{\pi}{2}]$. The metric (1) is not entirely gauge fixed. Two gauge fixing conditions that are common in the literature are $\delta(t, 0) = 0$ and $\delta(t, \pi/2) = 0$. The first one constitutes the so-called *central gauge*, and t corresponds to the proper time in the center of AdS, while the second choice constitutes the *boundary gauge*, and t corresponds to the proper time at the boundary.

The equations that govern the evolution of the system are the wave equation for a massless scalar field and the Einstein equations with a stress-energy tensor due to ϕ . Using the variables $\Phi = \phi'$ and $\Pi = A^{-1} e^{\delta} \dot{\phi}$, the equations of motion can be written as

$$\dot{\Phi} = (Ae^{-\delta}\Pi)', \quad \dot{\Pi} = \frac{1}{\tan^x} (\tan^2 x A e^{-\delta} \Phi)', \quad (2)$$

while the Einstein equations reduce to the constraints

$$A' = \frac{1 + 2\sin^2 x}{\sin x \cos x} (1 - A) - \sin x \cos x A (\Phi^2 + \Pi^2)$$

$$\delta' = -\sin x \cos x (\Phi^2 + \Pi^2). \quad (3)$$

We usually turn to perturbation theory to solve this system of equations. We start with some initial data of the form $(\phi, \dot{\phi})_{t=0} = (\epsilon f(x), \epsilon g(x))$ and we look for an approximate solution as a perturbative, in the amplitude ϵ , expansion:

$$\phi(t, x) = \sum_{k=0}^{\infty} \phi_{2k+1}(t, x) e^{2k+1},$$

$$A(t, x) = 1 + \sum_{k=1}^{\infty} A_{2k}(t, x) e^{2k},$$

$$\delta(t, x) = \sum_{k=1}^{\infty} \delta_{2k}(t, x) e^{2k}. \quad (4)$$

Inserting this ansatz into the equations of motion and collecting terms of the same order of ϵ we obtain a set of linear equations which can be solved order by order.

To first order, we merely have a scalar field propagating in the AdS background

$$\ddot{\phi}_1 + L\phi_1 = 0. \quad (5)$$

Here, $L = -\frac{1}{\tan^{d-1} x} \partial_x (\tan^{d-1} x \partial_x)$ is the Laplacian of AdS _{$d+1$} with eigenvalues $\omega_j^2 = (2j + d)^2$ and eigenfunctions

$$e_j = d_j \cos^d x P_j^{(\frac{d-1}{2}, \frac{d}{2})}(\cos(2x)),$$

$$d_j = \frac{2\sqrt{j!(j+d-1)!}}{\Gamma(j+\frac{d}{2})}. \quad (6)$$

Solving Eq. (5) one simply gets

$$\phi_1(t, x) = \sum_j c_j^{(1)}(t) e_j(x)$$

$$= \sum_j (\alpha_j e^{i\omega_j t} + \bar{\alpha}_j e^{-i\omega_j t}) e_j(x). \quad (7)$$

To second order we have the backreaction in the metric described by A_2 and δ_2 . The solutions are

$$A_2(t, x) = -\nu(x) \int_0^x (\dot{\phi}_1(t, y)^2 + \phi_1'(t, y)^2) \mu(y) dy, \quad (8)$$

$$\delta_2(t, x) = \begin{cases} -\int_0^x (\dot{\phi}_1(t, y)^2 + \phi_1'(t, y)^2) \nu(y) \mu(y) dy, & \text{for } \delta(t, 0) = 0 \\ \int_x^{\pi/2} (\dot{\phi}_1(t, y)^2 + \phi_1'(t, y)^2) \nu(y) \mu(y) dy, & \text{for } \delta(t, \pi/2) = 0. \end{cases} \quad (9)$$

Here $\mu(x) = \tan(x)^{d-1}$ and $\nu(x) = \frac{\sin(x)\cos(x)}{\tan(x)^{d-1}}$.

The first nontrivial dynamics appear at the $\mathcal{O}(\epsilon^3)$ order, namely in the equation for ϕ_3 in the *backreacted* background. Here we will omit the details of the derivation of this equation and we will refer the reader to the numerous works where has already been presented [4,7,16]. We will only mention, that to this order the field is expanded as

³For simplicity we have set the AdS radius to 1.

$$\phi_3(t, x) = \sum_j c_j^{(3)}(t) e_j(x), \quad (10)$$

and the equation of motion for ϕ_3 results to an infinite set of decoupled driven harmonic oscillators for the coefficients $c_j^{(3)}(t)$. However, due to the highly commensurate spectrum of AdS_{d+1} , numerous resonances appear resulting in a secular growth of these coefficients at the time scale $t \sim \epsilon^{-2}$ rendering this *naive* perturbation expansion invalid. A refined perturbation theory, known as the *two-time framework* [7,16] consists of defining a *slow time*⁴ $\tau = \epsilon^2 t$ and allow the fields in Eq. (4) to depend on τ as well. The expansion would now be

$$\begin{aligned} \phi &= \sum_{k=0}^{\infty} \phi_{2k+1}(t, \tau, x) \epsilon^{2k+1}, \\ A &= 1 + \sum_{k=1}^{\infty} A_{2k}(t, \tau, x) \epsilon^{2k}, \\ \delta &= \sum_{k=1}^{\infty} \delta_{2k}(t, \tau, x) \epsilon^{2k}. \end{aligned} \quad (11)$$

Now the resonances are entirely captured by the *slow-time* evolution of the coefficients⁵ in the expansion of ϕ_1 , Eq. (7):

$$\begin{aligned} 2i\omega_j \frac{d\alpha_j}{d\tau} &= T_j |\alpha_j|^2 \alpha_j + \sum_{i \neq j} R_{ij} |\alpha_i|^2 \alpha_j \\ &+ \sum_{\substack{j+k=l+m \\ j \neq l, j \neq m}} S_{jklm} \alpha_k \alpha_l \alpha_m. \end{aligned} \quad (12)$$

Using the *amplitude-phase* representation $\alpha_j(\tau) = A_j(\tau) e^{iB_j(\tau)}$ we can rewrite the above equation as

$$2\omega_j \frac{dA_j}{d\tau} = \sum_{\substack{j+k=l+m \\ j \neq l, j \neq m}} S_{jklm} A_k A_l A_m \sin(B_j + B_k - B_l - B_m) \quad (13)$$

$$\begin{aligned} 2\omega_j \frac{dB_j}{d\tau} &= T_j A_j^2 + \sum_{i \neq j} R_{ij} A_i^2 + A_j^{-1} \sum_{\substack{j+k=l+m \\ j \neq l, j \neq m}} S_{jklm} A_k A_l A_m \\ &\times \cos(B_j + B_k - B_l - B_m). \end{aligned} \quad (14)$$

⁴The slow-time variable $\tau = \epsilon^2 t$ characterizes the time scale of the energy transfer between the normal modes while t characterizes the oscillations of these normal modes.

⁵For a detailed treatment of how these equations are obtained and for explicit expressions of the interaction coefficients, we refer the reader to the original papers [6,7,16,17].

III. COMPARING THE TWO GAUGES

In this section we compare the results in the two gauges. We will show that within the validity of TTF, they indeed describe the same physical evolution. The relation between the two gauges has also been studied in [17] and some of the results can be found there as well. We will follow similar notations, but our attention lies on oscillating singularities that occur in one gauge and not the other. With some extra care we show what goes wrong as TTF breaks down when such a singularity develops in the central gauge.

The gauge choice should not affect any physical quantities. However, the two different gauges do lead to two different sets of differential equations, which were numerically evaluated to very different results. In [1] the case of the two-mode equal energy data in AdS_5 was studied and an oscillating singularity was reported. Namely, the derivatives of the phases blow up. In [2] it was shown that this singularity does not appear in the boundary gauge and therefore the singular behavior of the system might be only an artifact of the gauge choice.

On top of just numerical results, one can also see this difference from the asymptotic scaling of the R_{ij} coefficients as was first suggested in [3]. It was shown that for AdS_5 the R_{ij} coefficients scale in the central gauge as $R_{ij}^{CG} \sim i^3 j^2$ and therefore, for a *power-law* spectrum $A_n \sim n^{-2}$ as observed in [1], the sum in the second term of Eq. (14) diverges logarithmically. On the other hand, the asymptotic scaling of these coefficients in the boundary gauge was shown to be $R_{ij}^{BG} \sim i^2 j^2$, thus although the evolution leads to the same power-law spectrum the same sum converges. One can check that the rest of the sums do not diverge.

Despite this apparent difference, these results do not contradict each other. The oscillating singularity observed in [1], combined with the absence of that in [2], has an obvious physical meaning. It implies an infinite gravitational redshift between the boundary and the center of the spacetime.

From the metric (1), one can see that the two gauge choices are related as

$$dt_{BG} = e^{-\delta(t_{CG}; \frac{\pi}{2})} dt_{CG}. \quad (15)$$

Integrating and keeping terms only up to order $\mathcal{O}(\epsilon^2)$ we get

$$t_{BG} = t_{CG} - \epsilon^2 \int_0^{t_{CG}} dt \delta_2(t, \tau, 0) + \mathcal{O}(\epsilon^4). \quad (16)$$

Neglecting terms that oscillate in the fast time scale t , we can approximate $\delta_2(t, \tau, 0)$ by the *time averaged* quantity $\delta_2(\tau, 0)$. For completeness we will present the computation of this quantity in Sec. III A. We then get

$$\begin{aligned}
t_{BG} &\approx t_{CG} - \epsilon^2 \int_0^{t_{CG}} dt \delta_2(\tau, 0) + \mathcal{O}(\epsilon^4) \\
&= t_{CG} + 2\epsilon^2 \int_0^{t_{CG}} dt \sum_j (A_{jj} + \omega_j^2 V_{jj}) A_j^2 + \mathcal{O}(\epsilon^4).
\end{aligned} \tag{17}$$

Now, using the fact that the field $\phi(t, \tau, x)$ transforms as a scalar under such a *gauge transformation* one can derive the relation for the complex coefficients $\alpha_j(\tau)$ in the two gauges from Eq. (7):

$$\begin{aligned}
\phi^{CG}(t_{CG}) &= \phi^{BG}(t_{BG}) \Rightarrow \\
\alpha_j^{CG}(\tau_{CG}) e^{i\omega_j t_{CG}} &= \alpha_j^{BG}(\tau_{BG}) e^{i\omega_j t_{BG}}.
\end{aligned} \tag{18}$$

The relation of the slow time in the gauges is obtained simply by multiplying Eq. (17) by ϵ^2 to obtain

$$\tau_{BG} = \tau_{CG} + 2\epsilon^2 \int_0^{\tau_{CG}} d\tau \sum_j (A_{jj} + \omega_j^2 V_{jj}) A_j^2 + \mathcal{O}(\epsilon^4). \tag{19}$$

Substituting in the right-hand side of the above equation, Taylor expanding and neglecting terms that are of order $\mathcal{O}(\epsilon^2)$ we obtain

$$\begin{aligned}
\alpha_j^{CG}(\tau_{CG}) e^{i\omega_j t_{CG}} &\approx \left[\alpha_j^{BG}(\tau_{CG}) + \epsilon^2 \dot{\alpha}_j^{BG}(\tau_{CG}) \int_0^{\tau_{CG}} \delta_2 d\tau \right] \\
&\times \exp \left(i\omega_j t_{CG} + i\omega_j \int_0^{\tau_{CG}} \delta_2 d\tau \right).
\end{aligned} \tag{20}$$

Therefore, we find that the complex coefficients in the two gauges are related by

$$\alpha_j^{CG}(\tau) = \alpha_j^{BG}(\tau) \exp \left(i\omega_j \int_0^\tau \delta_2(\tau', 0) d\tau' \right) + \mathcal{O}(\epsilon^2) \tag{21}$$

as is also explained in [17]. This result can also be expressed in the amplitude-phase representation, yielding

$$A_j^{CG}(\tau) = A_j^{BG}(\tau) \tag{22}$$

$$B_j^{CG}(\tau) = B_j^{BG}(\tau) - \omega_j \int_0^\tau d\tau' \sum_i (A_{ii} + \omega_i^2 V_{ii}) A_i^2(\tau'). \tag{23}$$

That the amplitudes and the phases are related as above can be directly checked by applying Eq. (21) to the corresponding evolution equation in the two gauges, Eq. (12), and recalling that the difference is entirely contained in the coefficients [17]:

$$T_j^{BG} = T_j^{CG} + \omega_j^2 (A_{jj} + \omega_j^2 V_{jj}), \tag{24}$$

$$R_{ij}^{BG} = R_{ij}^{CG} + \omega_j^2 (A_{ii} + \omega_i^2 V_{ii}). \tag{25}$$

In [14] it was shown that a large geometric backreaction is related to the amplitude spectra and the coherence of the phases, where a phase-coherent cascade is defined by a spectrum of phases that (for large j) is linear in the mode number j :

$$B_j(\tau) = \gamma(\tau)j + \delta(\tau) + \dots \tag{26}$$

This is an *asymptotic* statement and the ellipsis represent terms that are subleading in j . The reader should be aware here that the function $\delta(\tau)$ in the above equation is not the same function appearing in Eq. (1). From Eq. (22) we see that the evolution of the amplitudes is not affected by the choice of the gauge so what remains is to show that phase coherence is also unaffected and hence the physical conclusions will be independent of the choice of the gauge. Starting from Eq. (26) for the central gauge we have

$$B_j^{CG}(\tau) \approx \gamma^{CG}(\tau)j + \delta^{CG}(\tau), \tag{27}$$

and applying Eq. (23) we can obtain the corresponding expression for the boundary gauge. This reads

$$\begin{aligned}
B_j^{BG}(\tau) - \omega_j \int_0^\tau d\tau' \sum_i (A_{ii} + \omega_i^2 V_{ii}) A_i^2(\tau') &\approx \gamma^{CG}(\tau)j \\
&+ \delta^{CG}(\tau) \Rightarrow \\
B_j^{BG}(\tau) &\approx \left(\gamma^{CG}(\tau) + \int_0^\tau d\tau' \sum_i (A_{ii} + \omega_i^2 V_{ii}) A_i^2(\tau') \right) j \\
&+ \delta^{CG}(\tau).
\end{aligned} \tag{28}$$

We see that the phase spectrum in the boundary gauge takes the form of Eq. (26):

$$B_j^{BG}(\tau) \approx \gamma^{BG}(\tau)j + \delta^{BG}(\tau), \tag{29}$$

with the functions $\gamma(\tau)$ and $\delta(\tau)$ in the two gauges being related as

$$\gamma^{BG}(\tau) = \gamma^{CG}(\tau) + \int_0^\tau d\tau' \sum_i (A_{ii} + \omega_i^2 V_{ii}) A_i^2(\tau') \tag{30}$$

$$\delta^{BG}(\tau) = \delta^{CG}(\tau). \tag{31}$$

A. The oscillating singularity as an infinite gravitational redshift

Having clarified that physical conclusions cannot be affected by the choice of the gauge, the next step is to reconcile the two different numerical results in the two gauges. In this section we will argue that the fact that \dot{B}_j diverges in the one gauge and not in the other can be

interpreted as an *infinite gravitational redshift* between the boundary and the center of the spacetime. Recall that the gravitational redshift between a source and an observer is given by the formula:

$$1 + z = \sqrt{\frac{g_{tt}(\text{obs})}{g_{tt}(\text{source})}}. \quad (32)$$

We can compute this quantity in one of the two gauges. Let us choose the nonsingular boundary gauge and compute the redshift between the boundary ($x = \pi/2$) and the center ($x = 0$) of the spacetime. Using the metric (1), the normalization $\delta(t, \pi/2) = 0$ and keeping terms only up to the order of $\mathcal{O}(\epsilon^2)$, the quantity under the square root reads

$$\frac{g_{tt}(t, 0)}{g_{tt}(t, \pi/2)} \sim 1 - \epsilon^2 \delta_2(t, \tau, 0) + \mathcal{O}(\epsilon^4). \quad (33)$$

The expression for $\delta_2(t, \tau, 0)$, Eq. (9), yields⁶

$$\begin{aligned} \delta_2(t, \tau, 0) &= \int_0^{\pi/2} (\dot{\phi}_1(t, x)^2 + \phi_1'(t, x)^2) \mu(x) \nu(x) dx \\ &= \int_0^{\pi/2} \sum_{ij} (\dot{c}_i^{(1)}(t) \dot{c}_j^{(1)}(t) e_i(x) e_j(x) \\ &\quad + c_i^{(1)}(t) c_j^{(1)}(t) e_i(x) e_j(x)) \mu(x) \nu(x) dx \\ &= \sum_{ij} (\dot{c}_i^{(1)} \dot{c}_j^{(1)} V_{ij} + c_i^{(1)} c_j^{(1)} A_{ij}). \end{aligned} \quad (34)$$

To go to the second line, we simply used the expansion in eigenmodes $\phi_1(t, x) = \sum_j c_j^{(1)}(t) e_j(x)$ and in the third line we defined the interaction coefficients:

$$A_{ij} \equiv \int_0^{\pi/2} e_i'(x) e_j'(x) \mu(x) \nu(x) dx \quad (35)$$

$$V_{ij} \equiv \int_0^{\pi/2} e_i(x) e_j(x) \mu(x) \nu(x) dx. \quad (36)$$

The expansion coefficients $c_j^{(1)}$ are related to the complex coefficients α_j as⁷

$$c_j^{(1)} = \alpha_j e^{i\omega_j t} + \bar{\alpha}_j e^{-i\omega_j t} \quad (37)$$

$$\frac{dc_j^{(1)}}{dt} = i\omega_j (\alpha_j e^{i\omega_j t} - \bar{\alpha}_j e^{-i\omega_j t}). \quad (38)$$

⁶For ease of notation we have omitted to write explicitly the slow-time dependence in some cases, but it is implicitly assumed.

⁷As we have stated below Eq. (12) these are also related to A_j and B_j coefficients as $A_j = |\alpha_j|$ and $B_j = \text{Arg}(\alpha_j)$.

Substituting Eq. (37) in the above expression for $\delta_2(t, \tau, 0)$ we will get several terms of the form $e^{i\Omega t}$, where $\Omega = \omega_i \pm \omega_j$. Keeping only terms with $\Omega = 0$, the so called *resonant terms*,⁸ we finally obtain the following expression:

$$\delta_2(t, \tau, 0) \approx 2 \sum_i (A_{ii} + \omega_i^2 V_{ii}) A_i^2(\tau) \equiv \delta_2(\tau, 0). \quad (39)$$

By differentiating Eq. (23), we can see that this quantity, the time-averaged δ_2 , which was first mentioned in Eq. (17), is precisely the difference of the slow-time derivatives of the phases in the two gauges. Therefore, by comparing the results in the boundary and the central time gauge we can draw conclusions about geometric quantities, and in particular the gravitational redshift. In the case of interest, where the derivatives of the phases diverge in one gauge but not in the other, one concludes that $\delta_2(\tau, 0)$ diverges, and so does the redshift, Eq. (33). This large backreaction in turn implies the breakdown of linearized gravity. On the other hand if the derivatives are finite in both gauges there is no divergence, while the case is not clear if an oscillating singularity appears both in the boundary as well as in the central gauge. In that case $\delta_2(\tau, 0)$ could be either finite or infinite.

IV. CONCLUSIONS

In this manuscript we presented an explicit derivation on the anticipated fact that physical results can not be affected by the different gauge choices. We demonstrated that gauge-invariant quantities are related to the amplitude spectrum and the coherence of the phases in the TTF solution, and both properties are unaffected by the gauge choice. This result holds even when the difference between the two gauges diverges. Furthermore we established that the oscillating singularity observed in [1] is indeed a physical singularity, by showing that is related to an infinite redshift between the boundary and the center of the spacetime.

This means that the breakdown of the TTF observed in [1] is due to large gravitational effects which lead to the breakdown of the weak gravity approximation. Such a conclusion cannot be deduced by the observed singularity in the central time gauge alone. In that case is not clear whether the breakdown of the perturbation theory is caused by strong gravity or by the breakdown of other approximations. Therefore, with our analysis we establish that the singular solution is a genuine singular solution of the gravitational problem. Due to the scaling symmetry of the TTF system the solution will survive in the $\epsilon \rightarrow 0$ limit, and thus provide a way to address the phase space of initial conditions in this limit.

⁸These are the terms that are proportional to $e^{\pm i(\omega_i - \omega_j)t}$. This procedure is equivalent to time averaging over the *fast time* t .

An interesting thing to point out here is that for this conclusion we need to compare the derivatives of the phases in the two gauges. Therefore, the fact that in higher dimensions a discrepancy between the two gauges has not been observed [2] is rather intriguing. However since in both gauges an oscillating singularity was observed, and actually in the central time gauge this divergence was more prominent than in the boundary time gauge, it might still signal a diverging redshift, since these results are compatible with a diverging $\delta_2(\tau, 0)$, as we explained in Sec. III A.

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