Optical observations of close binary systems with a compact component
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Spin-up of the white dwarf in the intermediate polar 
BG CMi/3A0729+103

T. Augusteijn, J. van Paradijs, and H.E. Schwarz
Astronomy & Astrophysics 247, 64 (1991)

Abstract

Using 29 times of maximum light of the pulsational light curve of the intermediate polar BG CMi, we find that the white dwarf rotation period decreases on a timescale $P/\dot{P} = (0.566 \pm 0.033) \times 10^6$ yr. We have obtained a new orbital period for the system and show that it is equal to the 1-cycle alias of the previously accepted period determined from the timing of two orbital X-ray minima. Using estimates of both the magnetic dipole moment of the white dwarf and the mass accretion rate we obtain, using the disk accretion model of Lamb and Patterson (1983), a white dwarf mass of $\sim 1.0 \, M_\odot$. For the diskless accretion model of Hameury et al. (1986) we obtain $\sim 0.5 \, M_\odot$. For both models the rotation rate of the white dwarf deviates substantially from equilibrium. However, for the diskless accretion model it is not clear if the estimated mass accretion rate can be adopted.

5.1 Introduction

The V $\sim$ 14.5 magnitude star BG CMi (Kholopov et al. 1985) was identified by McHardy et al. (1982) as the optical counterpart of the X-ray source 3A0729+103 (McHardy et al. 1981). On the basis of optical photometry and spectroscopy McHardy et al. (1984) showed that this star is a cataclysmic variable of the intermediate polar (IP) subclass. The authors determined an orbital period of 3.24 hr and a 15.2 min pulse period (also detected in X-rays, see McHardy et al. 1987), which was identified with the rotation period of the white dwarf. With the exception of AE Aqr, which is in many ways an exceptional source (Chincarini and Walker, 1980; Van Paradijs et al., 1989), BG CMi is to date the only IP in which circular polarization has definitely been detected (Penning et al., 1986; West et al., 1987).

Secular changes of the white dwarf rotation period, caused by torques exerted by the accreting matter, have been found in five IP's (DQ Her: Patterson et al., 1978; EX Hya: Gilliland, 1982; Jablonski and Busko, 1985; AO Psc: Van Amerongen et al., 1985; FO Aqr: Pakull and Beuermann, 1987; Osborne and Mukai 1989; V1223Sgr: Van Amerongen et al., 1987).

Recently Vaidya et al. (1988) published additional times of maximum light in the pulsations of BG CMi, determined from data obtained in 1984 and 1987, which together with the values
5 Spin-up of the white dwarf in the intermediate polar BG CMi/3A0729+103

Table 5.1 Summary of observations

<table>
<thead>
<tr>
<th>year</th>
<th>$T_{\text{start}}$(HJD)</th>
<th>$\Delta T$ (day)</th>
<th>$T_{\text{start}}$(HJD)</th>
<th>$\Delta T$ (day)</th>
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<tr>
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</table>

published by McHardy et al. (1984, 1987) gave a 4.7 yr baseline. Vaidya et al. (1988) did not find any significant change in the white dwarf rotation period. Assuming that the errors given are 1 $\sigma$ errors, we derive a 3 $\sigma$ lower limit to the spin-up timescale of $P/\dot{P} > 0.16 \times 10^6$ yr.

In this paper we investigate the possible changes of the rotation period of the white dwarf in BG CMi/3A0729+103 using data obtained in the Walraven (VBLUW) photometric system. In Sect. 5.2 a short description of the observations and the reduction of the data is given. On the basis of these data an ephemeris is derived in Sect. 5.3 and a comparison is made with arrival times published in the literature. In Sect. 5.4 a new ephemeris is determined for the orbital period. We discuss our results in Sect. 5.5.

5.2 Observations and Reduction

We observed BG CMi on 29 nights between 6 March 1983 and 19 May 1989 using the Walraven photometer attached to the 0.91m Dutch telescope at the European Southern Observatory (ESO). A summary of the observations is given in Table 5.1.

The Walraven photometer provides simultaneous measurements in five passbands (V, B, L, U and W) with effective wavelengths between 3255 and 5467 Å which are defined in Rijf et al. (1969) and Lub and Pel (1977). The source was monitored for several hours each night with a break about every half hour to measure the sky background and nearby comparison stars. For all the observations an integration time of 16 sec was used. To avoid contamination of the light from a star located $\sim 15''$ to the North of the source, an 11''5 diaphragm was used.

The photometric data on BG CMi were reduced differentially with respect to a nearby comparison star (SAO 96986, V=8.4, spectral type B9). The timing of each measurement was taken at the middle of the exposure and the heliocentric timing correction was applied.

The comparison star was checked for variations by calculating the ratio of the sky subtracted signal of this star with respect to that of a second comparison star. This ratio was constant to
within ~ 1% during each night; the average value per night was constant over the whole period of six years to within better than 0.5%.

5.3 The White Dwarf Spin Ephemeris

5.3.1 Deriving the Ephemeris

To derive the ephemeris we have limited ourselves to arrival times determined from the Walraven observations as listed in Table 5.1. In this way a uniform set of arrival times and error determinations could be obtained. To determine the white dwarf rotation period, the data were first corrected for slow brightness variations. This was done by subtracting a smoothed light curve (using a running mean with a bin size of approximately $3 \times P_{\text{spin}}$) for each night.

The corrected data were searched for periodic signals around the value of the pulse frequency given in the literature ($f_{\text{spin}} = 1.0947 \times 10^{-4}$ Hz) using the phase dispersion minimization (PDM) method of Stellingwerf (1978). With this method, one determines the dispersion of the data for a given period, with respect to the average light curve, as determined in a number of phase bins. To avoid smearing due to a possibly changing period, this was done for the groups of data within each of the observing seasons dividing the 1985 data into two parts and taking the 1988/89 data as one (see Table 5.1). In Fig. 5.1 the result is shown for the 1988/89 data set (V band) which has the longest baseline within one observing season (about six months). The result provides a nice example of a 'window' pattern, due to the specific spacing of our observations, centered on a best fit period of $P=0.0105728(2)$ d. The broad dips seen in Fig. 5.1 are the result of one day aliasing which envelopes finer structures due to the distribution of observing nights. The overall envelope is the result of the typical length (~2.0 hr) of an observing run.

The best fit period is consistent with the periods determined from cycle fitting over a longer time base, i.e. $P=0.01057278(2)$ d by Vaidya et al. (1988), and the two possible periods (differing by one cycle over two years), $P=0.010572769(2)$ d or $P=0.010572606(2)$ d given by McHardy et al. (1987).

The next step was to determine the arrival times of the pulse maxima per night. This was done by fitting a sinusoid to the data (corrected for longterm variations) with a fixed period of $P=0.0105728$ d. In the appendix we describe in detail how the arrival time (and its error) for each data set was determined. Next, a fit of the arrival times versus the cycle number was made for each season. In this way, it is possible to maintain the cycle count from one year to the next, and hence we are able to obtain a fit over the entire data set. We find that the arrival times are better fitted by a quadratic ephemeris than by a linear ephemeris at a confidence level of higher than 99.9%. A list of the arrival times with their respective errors and cycle numbers is given in Table 5.2.

From the 29 pulse arrival times obtained from Walraven VBLUW data covering a 6.1 yr baseline, we derive the following quadratic ephemeris:

$$T_{\text{max}}(HJD) = 2446642.85448(12) + 0.0105728772(10) \times N - 2.70(16) \times 10^{-13} \times N^2$$

$$\text{Cov}(T_o, P_o) = -3.7 \times 10^{-14} d^2 \quad \text{Cov}(T_o, c) = -1.5 \times 10^{-18} d^2$$

$$\text{Cov}(P_o, c) = +9.0 \times 10^{-24} d^2$$

Following Van der Klis and Bonnet-Bidaud (1989) we have included the covariance estimates. Here $\text{Cov}(x, y)$ denotes the covariance of $x$ and $y$, with $c = \frac{1}{2} P_o \dot{P}$. The $\chi^2$ of the fit is 26.47 with 26 degrees of freedom.

The quadratic term of the ephemeris corresponds to a period derivative of

$$\dot{P} = -5.11(30) \times 10^{-11} s/s$$
and a spin-up timescale of
\[ \frac{P}{P} = 0.566(31) \times 10^6 \text{yr} \]

This value is of the same order as that derived for other IP (see e.g. Van Amerongen et al. 1987).

5.3.2 Comparison with previous work

There are six pulse arrival times given in the literature (McHardy et al. 1984, McHardy et al. 1987, Vaidya et al. 1988). Only McHardy et al. (1987) give an error estimate for that result. Our ephemeris agrees well with both arrival times published by McHardy et al. (1984, 1987), but is inconsistent with the four arrival times published by Vaidya et al. (1988). Table 5.3 lists the previous arrival times as compared with the present work.

As the three publications mentioned above do agree on the basic period, and any period corresponding to a 1 cycle shift over a time base of between one week and our entire time-base of 6.1 yr can be excluded on the basis of cycle counting within our data set, we explored the possibility of periods which differ by 1 cycle over one day with the \( P_{\text{spin}} = 0.0105728 \) d period.

We followed the same reduction procedure as described above for periods of \( P_{\text{spin}} = 0.0104620 \) d, and \( P_{\text{spin}} = 0.0106860 \) d separately and found for both the corresponding sets of arrival times that a quadratic fit is better than a linear fit on a confidence level of higher than 99.9% with a similar quadratic coefficient of \( \sim -2 \times 10^{-13} \). It should be noted that for data distributed equally before and after the time of maximum light, the maximum difference between the times
Table 5.2 Arrival times pulse maxima and associated errors

<table>
<thead>
<tr>
<th>Cycle No.</th>
<th>$T_{max}$ (HJD)</th>
<th>Error</th>
<th>Cycle No.</th>
<th>$T_{max}$ (HJD)</th>
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Figure 5.2. This plot gives the arrival times of the pulse maxima, as listed in Table 5.2, with respect to a linear fit through the arrival times. Along the X-axis the cycle number is given. The Y-axis gives the difference of the observed values with the linear fit. The quadratic fit to the arrival times as given in the text is indicated by the solid (and dashed) line. The two arrival times given by McHardy et al. (1984, 1987) are indicated by stars.

of maximum light for curves fitted with periods $P_1$ and $P_2$ is $\sim 0.5 \times (P_1 - P_2)$. For the three given periods this would correspond to a total spread in $T_{max}$ of 0.00011 d (this is a significant fraction of the typical error; see Table 5.2). This expectation is confirmed by the derived values of $T_{max}$ for each of the three periods.

The arrival times in the literature were shifted (as described in the appendix) to enable us to compare them directly with an ephemeris based on another period. In four cases the arrival time has been determined from one night of observations, in one case from two consecutive nights, and in one case from sixteen nights distributed over a 92 day period. In all of these cases the arrival times are given for a pulse cycle at the beginning of the observations. As only the start
5 Spin-up of the white dwarf in the intermediate polar BG CMi/3A0729+103

Table 5.3 Pulse maxima arrival times from the literature

<table>
<thead>
<tr>
<th>T_{max} (mid)</th>
<th>systematic error</th>
<th>O-C(P_{pulse})</th>
<th>σ_{T_{max}}</th>
<th>Reference</th>
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and the duration of the observations for each night are given, we assumed that the data were equally spaced during every observing run and that the number of integrations was proportional to the duration of each run. McHardy et al. (1987) show the whole (one night) observation which does not contain any large gaps; this gives us some confidence that the shifted maximum should at least be within one period of the average time of all the observations. This implies a possible systematic error of the arrival time when used for either of the other periods of at most 0.00011 d, or 0.01 in phase. The data shown in Vaidya et al. (1988), being only part of the data, shows many irregularly spaced gaps. As three of their arrival times are determined from one night of observations, we expect the adopted average time to be well within 0.02 d and, as the shifted arrival times are very close to the calculated average time, we expect the systematic error when used with the other periods to be less than 0.00022 d or 0.02 in phase. The remaining arrival times of Vaidya et al. (1988), determined from two consecutive nights in 1987 and McHardy et al. (1984), determined from 16 nights in 1982, were shifted to the middle of the data set closest to the average time (see appendix). As only part of the data is shown by McHardy et al., not including the data set to which the arrival time is shifted, but noting that the data shown does not contain any big gaps, we expect this arrival time also to be within 0.02 d.

For each of these arrival times we determined the O-C value with respect to the ephemeris for each of the three periods. We listed these values together with the possible systematic error when using these arrival times to compare them to an ephemeris based on an other period and the uncertainties in the ephemeris values themselves in Table 5.3. To calculate the uncertainties in the predicted arrival times, we followed Van der Klis and Bonnet-Bidaud (1989) and determined for each arrival time and ephemeris:

\[
\sigma_{T_{\text{max}}}^2 = \sigma_{T_0}^2 + \sigma_{P_0}^2 n^2 + \sigma_c^2 n^4 + 2Cov(T_0, P_0)n + 2Cov(T_0, c)n^2 + 2Cov(P_0, c)n^3
\]

where \(\sigma_X^2\) denotes the variance in X, and the covariances have the same meaning as above. As for all three ephemerides these values are practically the same and only one value is listed for each arrival time given in Table 5.3.

It can be seen from Table 5.3 that the arrival times given by McHardy et al. (1984, 1987) are best fitted with the ephemeris for a period of \(P=0.0105728\) d (the corresponding O-C values with respect to a linear fit through our arrival times are, indicated by a star, plotted in Fig. 5.2). However, the set of arrival times given by Vaidya et al. (1988) cannot be fitted by any of the three. Since we see no other possible explanation for this discrepancy we are forced to conclude that these arrival times are in error. We note that the arrival times of Vaidya et al. (1988) by themselves are best fitted by a period of \(P=0.0105728\) d. A comparison of this fit with a linear
5.4 The Orbital Ephemeris

From their optical data McHardy et al. (1984) found an orbital period $P_{\text{orb}} = 0.13480(1) \ \text{d}$. On the basis of two orbital X-ray minima, separated by 462 days, this value was refined to $P_{\text{orb}} = 0.134790(5) \ \text{d}$ (McHardy et al. 1987). Using the PDM method, described in Sect. 5.3, we searched for periodic signals around the above period. In Fig. 5.3 we show the result based on all data obtained in the Walraven V band (about 7000 measurements). The value of the orbital period given above has been indicated in Fig. 5.3; clearly, the deepest dip is not consistent with this value.

The deep dip to the right of the indicated value of the orbital period is by far the strongest for periods in the region of a few hours (including the 1-day alias of this period) in all the five pass-bands and is most likely to be the correct orbital period. The dip is located at a frequency of $\nu_{\text{orb}} = 8.5895 \times 10^{-5} \ \text{Hz}$ which is, within the uncertainty, equal to the 1-cycle alias (over 462 days) of the period determined from the X-ray minima. The dip just to the left of the indicated value, as is also the dip of similar depth to the right of the strongest dip, is consistent with being the 1 year alias of the period corresponding to the strongest dip. Following the same procedure as before, we determined an arrival time for each observing season (the 1988/89 season was divided into two parts). The uncertainty in the period is small enough that the arrival times of the maxima were only corrected to the average time of the observations; also the cycle count from one season to the next can be maintained without any problems. The arrival times and

![Figure 5.3: The "Frequency"-gram around the value of the orbital frequency given in the literature. This value ($\nu_{\text{orb}} = 8.5867 \times 10^{-5} \ \text{Hz}$) is indicated by an arrow. The axes have the same meaning as in Fig. 5.1.](image-url)
their cycle number determined in this way, together with the arrival time given by McHardy et al. (1984), similarly corrected to the average time of the observations, are listed in Table 5.4.

A linear fit to the arrival times of the Walraven data gives the following ephemeris:

\[ T_{\text{max}}(\text{HJD}) = 6694.2888(17) + 0.13474853(29) \times N \]

\[ \text{Cov}(T_{\text{max}}, P_{\text{max}}) = -8.4 \times 10^{-11} \text{ } \text{d}^2 \]

with \( \chi^2 = 7.30 \) with 4 degrees of freedom. The error and covariance estimated are based on the arrival time errors scaled to give \( \chi^2_{\text{red}} = 1.0 \).

## 5.5 Discussion

Using the measurement of \( \dot{P} \), the magnetic moment of the white dwarf can be derived from accretion models. The magnetic moment of the white dwarf in IP's are of special interest when comparing them with those of the white dwarfs in polars (see for a recent review Cropper 1989). In contrast to IP's, the white dwarf in polars is corotating with the orbital frequency. This is thought to be due to magnetic interaction of the white dwarf with the secondary. By comparing the magnetic moments of white dwarfs in polars and IP's it can be determined if these types of systems form two distinct groups with intrinsically different distributions of magnetic moments, or form together one group of objects. In the later case the difference between polars and IP's is the result of a different evolutionary status of the systems, combined with a single distribution of the magnetic moments of the white dwarfs. Also an intrinsic difference of the mass of the white dwarfs in the different types of systems could play a roll (see below).

### Table 5.4 Arrival times orbital maxima and associated errors

<table>
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<th>Cycle No</th>
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<th>Error</th>
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</tr>
<tr>
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</table>

^aMcHardy et al. 1984.
The magnetic moments derived from the accretion models is based on the assumption that the white dwarf is spinning close to its equilibrium period (Lamb and Patterson 1983). It has already been shown by Van Amerongen et al. (1987) that this assumption is not generally true, and these authors determined the magnetic moments from the accretion models using estimates of the mass accretion rates.

On the basis of the rise of the degree of polarization towards the infrared, and assuming that the origin of the wavelength dependence is similar to that observed in polars but at longer wavelength, West et al. (1987) concluded that the magnetic field of the white dwarf in BG CMi is in the range 5-10 MG (MG = 10^6 Gauss). BG CMi is the first IP in which the magnetic field and the period derivative have been determined separately. This allows for the first time to see if the accretion models can, for reasonable values of the system parameters, give a satisfactory fit to the observed values and, vice versa, could allow to derive constraints on these system parameters.

5.5.1 Accretion models

Generally the period changes found in IPs are thought to be due to accretion torques. In the framework of models for accreting magnetic stars (see e.g. Ghosh and Lamb 1979), the rate of change of the spin rate $\Omega$ can be written as:

$$I\ddot{\Omega} = \dot{M}(GM_{\text{r}})^{1/2}n(\omega_s)$$

Here $I$ and $M$ are the moment of inertia and the mass of the accreting star respectively, $\dot{M}$ is the mass accretion rate; $r_m$ is equal to a (model-dependent) fraction $f$ of the magnetospheric radius $r_{\mu}$:

$$r_m = fr_{\mu} = (2.660 \times 10^{10}\text{cm})f\mu_3^{4/7}\tilde{M}_{16}^{-2/7}m^{-1/7}$$

$\mu_3, \tilde{M}_{16},$ and $m$ are the magnetic moment of the white dwarf, the mass accretion rate, and the white dwarf mass in units of $10^{33} G \text{cm}^3, 10^{16} \text{gs}^{-1},$ and $M_\odot$ respectively; $n(\omega_s)$ describes the interaction between the magnetic white dwarf and the accretion disk, and is a dimensionless function of the ratio, $\omega_s$, of the white dwarf spin rate $\Omega$ to the Kepler frequency, $\Omega_K(r_m)$, at the radial distance $r_m$:

$$\omega_s = \Omega/\Omega_K(r_m)$$

If the white dwarf magnetic field, and the mass accretion rate are constant, the spin rate will eventually reach an equilibrium value $\Omega_{eq}$, given by a critical value $\omega_c$ of $\omega_s$, for which $n(\omega_s)=0$:

$$\Omega_{eq} = \omega_c\Omega_K(r_m)$$

Here we follow the discussion given by Van Amerongen et al. (1987). These authors use an approximating description of the dimensionless function $n(\omega_s)$ which appears in accretion torque models for magnetic accreting compact objects (Ghosh and Lamb, 1979), to derive a (model dependent) relation between $\dot{M}$ and $\mu$ as function of $Q = \Omega_{\text{obs}}/\Omega_{eq}$ ($= \omega_s/\omega_c$). Taking a linear approximation of the dimensionless function $n(\omega_s) = a\omega_s + b$, they derive:

$$\mu_3 = 1.157\tau_{\text{obs}}(10^6 yr)^{-1/2}\Omega_{\text{obs}}(10^{-2} s^{-1})^{-1/2}(1 - Q)^{-1/2}(f/0.5)^{-7/4}b^{1/2}(a^2)^{-1/2}m^{0.2} \quad (5.1)$$

$$\tilde{M}_{16} = 2.610\Omega_{\text{obs}}(10^{-2} s^{-1})^{4/3}\tau_{\text{obs}}(10^6 yr)^{-1}Q^{-1/3}(1 - Q)^{-1}(-a)^{1/3}b^{-4/3}m^{-1.267} \quad (5.2)$$

with $\tau_{\text{obs}} = \Omega/\dot{\Omega}_{\text{obs}} = -P_{\text{rot}}/\dot{P}_{\text{rot}}$. It should be noted that these relationships are incorrectly reproduced in the article by Van Amerongen et al. (1987), although the correct relationships were used in their analysis and final results.
Figure 5.5. a–f These plots give the relation between the magnetic moment $\mu$ ($G \ cm^3$) and the mass accretion rate $\dot{M}$ ($M_\odot \ yr^{-1}$) for: a the disk accretion model of Lamb and Patterson (1983) and a white dwarf mass of $m_{\text{wd}} = 1.0 \ (M_\odot)$; b for the same model and $m_{\text{wd}} = 0.5$; c for the same model and $m_{\text{wd}} = 1.4$; d for the diskless accretion model of Hameury et al. (1986) and $m_{\text{wd}} = 1.0$; e $m_{\text{wd}} = 0.5$; and f $m_{\text{wd}} = 1.4$. In each plot the appropriate ranges of $\mu$ and $\dot{M}$, as calculated for the assumed mass of the white dwarf, are indicated by dashed lines.

5.5.2 Comparison with the observed values of $\dot{P}$ and $\mu$

The relationships (5.1) and (5.2) are plotted in Fig. 5.5a for the values of $\Omega$ and $\dot{\Omega}$, observed for BG CMi. Values of $f = 0.63$, $a = -4$, and $b = 1.4$ (appropriate for the disk accretion model of Lamb and Patterson, 1983), and of $Q$ ranging between $\sim 0$ and $\sim 10$ were used.

Intermediate polars with white dwarfs spinning very close to equilibrium are located on the branch in the upper right corner of the $\mu - \dot{M}$ diagram. Systems which deviate substantially from equilibrium, with white dwarfs spinning up, appear on the upper branch on the left-hand side of the diagram those with white dwarfs spinning down on the lower branch on the right-hand side. For the diagram shown, a white dwarf mass of $m_{\text{wd}} = 1(M_\odot)$ was used. For a white dwarf mass of $0.5M_\odot$ the log $\mu$ scale changes by $-0.06$ and the log $\dot{M}$ by $+0.38$.

Also shown in Fig. 5.5a is an estimate of $\dot{M}$ ($\log \dot{M} = -8.17$). In deriving this value we followed the calculations of Mouchet (1983), and used a disk model spectrum fitting published by Falomo et al. (1985). An estimated uncertainty of $\pm 0.4$ in log $\dot{M}$ (Patterson 1984) was taken into account. This way we can constrain the value of the magnetic field of the white dwarf, and the value of $Q$ which describes the deviation from equilibrium.

However, as mentioned above there is an independent estimate of the white dwarf magnetic
field (West et al., 1987). The magnetic moment $\mu$ is related to the magnetic field by:

$$\mu = B_0 R_{wd}^3$$

(5.3)

We approximate the mass-radius relation of white dwarfs (Hamada and Salpeter, 1961) by

$$R_9 = 0.5 \ m_{wd}^{-0.8}$$

(5.4)

Here $R_9$ is the white dwarf radius in units of $10^9$ cm. Combining (4.3) and (4.4) yields;

$$\mu = (1.25 \ 10^{32} \ G cm^3) \ B_0 m_{wd}^{-2.4}$$

(5.5)

Here $B_0$ is the magnetic field strength in units of $10^6$ G. Relation (5.5) shows clearly that $\mu$, which is an important parameter in the models for accreting magnetic stars and the evolutionary scenarios of the systems containing these stars (see e.g. Lamb and Melia 1986), is a strong function of the white dwarf mass. For plausible white dwarf masses of 0.4 and 1.0 $M_\odot$ and a given (observed) magnetic field strength, this relation implies magnetic moments which differ by a full order of magnitude.

Using relation (5.5), and substituting the range of values given by West et al. (1987), we derive, for a value of $m_{wd} = 1$, a range in $\mu$ which is also shown in Fig. 5.5a. This can alternatively be used to constrain the value of the mass accretion rate $\dot{M}$, and $Q$.

It is striking how well the model agrees with the range allowed by the two estimates of $\dot{M}$ and $\mu$. However, both the model (through Eq. (5.1)), and the estimated values of $\mu$ (5) and $\dot{M}$ (via the modelling of the spectrum) are dependent on the white dwarf mass.

To investigate the effect of varying the white dwarf mass we repeated the model calculation for values of the white dwarf mass of $M_{wd} = 0.5$ and $1.4 M_\odot$, and estimated ranges for both $\mu$ and $\dot{M}$ corresponding to these masses. As can be easily seen from (5) the range of $\mu$ will be shifted to higher values for a smaller mass, and to lower values for larger mass. As mentioned above, the model curve will move to higher values in $\dot{M}$ and slightly smaller values in $\mu$ for smaller values of $M_{wd}$, and vice versa for bigger values of $M_{wd}$. The effect on the estimate of $\dot{M}$ by varying $M_{wd}$ is very small. It can be seen from Fig. 5.5b,c that for both values of $M_{wd} = 0.5$ and $1.4 M_\odot$, the mutual ranges in $\mu$ and $\dot{M}$ do not agree, and cover quite different parts of the model curve. The value of the rotation rate of the white dwarf for the "best-fit" model (Table 5.5) for a white dwarf mass of $M_{wd} = 1.0 M_\odot$, deviates substantially from equilibrium.

To study the effect of the accretion model used, we performed the calculations for the same values of $M_{wd}$ using values of $f=0.37$, $a=-1$, and $b=1$ (appropriate for the diskless accretion model of Hameury et al., 1986). We note, however, that it is not clear if the estimate of $\dot{M}$, based on a disk model spectrum fitting, can be applied for this specific diskless model. The major effect of the different choice for the accretion model is that the values of log $\mu$ are increased by $\sim 1.0$ in log. The values of log $\dot{M}$ are essentially the same. As a result the model of Hameury et al. (1986) agrees best with the estimated values of $\mu$ and $\dot{M}$ for a white dwarf mass of $0.5 M_\odot$, as can be seen in Fig. 5.5d-e.

As in the previous accretion model, for the parameters that agree best with the observational estimates of $\mu$ and $\dot{M}$, the rotation period of the white dwarf deviates substantially from its equilibrium value. The respective ranges in $\mu$, $\dot{M}$ and $Q$, of the "best-fit" model, are listed in Table 5.5.

One of the assumptions made in the above models is that the accretion rate through the disk onto the white dwarf is constant. However, in at least two IP's outburst events were observed (Szkody and Mateo 1984, Schwarz et al. 1988, Van Amerongen and Van Paradijs 1989), indicating short term variations in $\dot{M}$. Also in one IP a changing period derivative has been found (Osborne and Mukai 1989). The observed value of $\ddot{P}$ implied a change from an
increasing white dwarf rotation period to a decreasing rotation period. The most likely cause for this change in period derivative is a change in $\dot{M}$. It must be clear from the above that the models for accreting magnetic stars are not sophisticated enough to model the accretion process in any detail. However, the very smooth period changes observed in most IP’s do indicate that $\dot{M}$ can be considered constant over at least several years in which case these models can be applied.

### 5.6 Conclusions

On the basis of Walraven photometry of BG CMi we have concluded that the orbital period of this system is $P_{\text{orb}} = 0.13474853(29)$ d. The major result of this paper is the conclusion that the rotation period of the white dwarf in BG CMi is decreasing on a timescale of $(0.566 \pm 0.033) \times 10^6$ yr. The observed $\dot{P}$ makes BG CMi the sixth of the dozen known IP’s for which a changing white dwarf rotation period has been measured. In four of these systems the rotation period of the white dwarf is found to decrease, in one the rotation period is found to increase and in one a changing period derivative has been found. The observed timescales for the change of the rotation period of the white dwarf in these systems are all of the same order of $\sim 10^6$ yr which suggest that one physical timescale drives these period changes.

On the basis of both estimates of the mass accretion rate and the magnetic field strength of the white dwarf in BG CMi a comparison was made with accretion models. We find that for the disk accretion model of Lamb and Patterson (1983) a relatively massive ($\sim 1.0 M_\odot$) white dwarf is needed and that the rotation rate of the white dwarf deviates substantially from equilibrium. For the diskless accretion model of Hameury et al. (1986) a much lighter white dwarf ($\sim 0.5 M_\odot$) is favoured, but also for this model the rotation rate of the white dwarf deviates substantially from equilibrium. However, it is not clear if the estimated mass accretion rate may be adopted for this model.

The models for accreting magnetic stars when applied to IP’s do fit the observations quite well, but are clearly not capable of describing the detailed behavior of these systems. The magnetic moment, $\mu$, of the white dwarf used in these models is, for a given field strength, a strong function of the white dwarf mass. This implies that an independent mass estimate for the white dwarf in BG CMi could further constrain the accretion models and consequently the evolutionary scenarios for this system, and possibly that of the subclass of IP’s (see e.g. Lamb and Melia 1986).

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Added note: After this paper was submitted we received a preprint of a paper by J. Patterson (to appear in "Proceedings of the 12th North American Workshop on Cataclysmic Variables" on the analysis of pulse arrival times in BG CMi. The period derivative determined in this paper confirms our result and the individual timings of the pulse maxima are consistent with the ephemeris given above.

Appendix

The error in the arrival time was determined using the following formula:

\[
\Delta T = \frac{P}{2\pi} \cos^{-1}(1 - \frac{\sigma_{fit}/\sqrt{N}}{SA})
\]

where \(\sigma_{fit}\) is the dispersion of the data around the fit, \(N\) the number of data points, \(P\) the pulse period, and \(SA\) the semiamplitude of the fitted sinusoid. This formula is derived by solving \(I + \Delta I = SA \sin(\phi_0 + 2\pi(T + \Delta T)/P)\) for the maximum in the fit, and taking \(\sigma_{fit}/\sqrt{N}\) as the error in \(I\). The derived \(\Delta T\) can be understood as the time \(T_{max} + \sigma\) or \(\Delta T\) for which the fitted value can be, within the error \((\Delta I)\), as high as the derived maximum. The arrival time was determined as the error weighted average over the five colours. The derived time of maximum was taken as the maximum closest to the average time of the observations used. This should be common practice as this arrival time is best determined at this point and still could be used in the case of an erroneous period having been used to perform the fit. Of course this only holds for "continuous" data in the sense that gaps between data sets should not be much bigger than the length of individual data sets (a typical value would be less than three times). In the case that this does not hold the point should be chosen in the middle of a data set closest to the average time; close to the average time for small errors in the fit period, and in the middle of a data set for large errors in the fit period.

References