ROSAT All-sky survey observations of normal stars
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The ROSAT All-Sky Survey: description and data analysis

2.1 Introduction

The German, British and American X-ray satellite ROSAT (acronym for the German "Röntgensatellit") was launched on June 1, 1990, and performed an All-Sky Survey during the first 6 months of its operational period (August 1990 to January 1991). The instruments on board the satellite are an X-ray telescope (XRT), sensitive to photons with energies in the range 0.07–2.4 keV, and an extreme-ultraviolet telescope, the Wide Field Camera (WFC), sensitive between 0.025 and 0.2 keV. The WFC is not described in this chapter, because it is beyond the scope of this thesis. We refer to Barstow and Willingale (1988) for a description of the instrument; the brightest sources detected in the WFC All-Sky Survey have been catalogued by Pounds et al. (1993). The XRT has the following instruments in its focal plane: a high-resolution imager (HRI), which was not used during the All-Sky Survey, and two position-sensitive proportional counters (PSPC). In this chapter we briefly describe the All-Sky Survey and the PSPC instrument characteristics (Sect. 2.2), and we discuss the methods used to analyse the data. In Sect. 2.3 we present a method to derive count rates, upper limits to them and spectral hardness ratios, from the PSPC survey data. It is a maximum likelihood method, based on counting statistics. In Sect. 2.4 we present a method to convert the PSPC count rate into flux, using the spectral hardness and the hydrogen column density. We give an application of this method to sources whose spectra can be described by a single-temperature Mewe & Gronenschild spectrum. In Sect. 2.5 we discuss model-spectra fitting to X-ray data in general and apply it to PSPC data.

2.2 The ROSAT All-Sky Survey

2.2.1 The Survey

The ROSAT All-Sky Survey was performed between July 30, 1990 and January 25, 1991 (plus 14 days at the end of February and the beginning of August 1991, to make up for some missing data at the end of the survey). The satellite scanned the sky in the direction perpendicular to the Sun, so that the whole sky was covered in half a year. The field of view of the PSPC is a circle with a diameter of 2 deg, so that every source was observed during a period of at least 2 days (depending on ecliptic latitude). The orbital period
Figure 2.1: Theoretical exposure times during the All-Sky Survey as a function of right ascension and declination. The dashed line is the ecliptic, which has the minimum exposure time (750 seconds); contour levels (solid lines) are drawn for 800, 1000, 1200, 1600, 2000, 2600, 3200, 4000, 6000, and 8000 seconds. The shortest times are near the ecliptic, the longest are near the ecliptic poles. These exposure times have not been corrected for vignetting, for switch-off periods or for efficiency of the instruments. The effective exposure times are almost a factor two smaller.

of the satellite is 96 minutes, so every source is observed at most 32 seconds per orbit. Figure 2.1 shows a contour plot of the theoretical total exposure times as a function of right ascension and declination. The PSPC instrument was switched off when the satellite penetrated the radiation belts near the magnetic poles and in the South Atlantic Anomaly, to protect it against overexposure.

2.2.2 The PSPC

The Position Sensitive Proportional Counter is a multi-wire proportional counter in a housing filled with argon, xenon, and methane. X-ray photons are photo-electrically absorbed by these components resulting eventually in a cloud of secondary electrons, of which the number is proportional to the photon energy. The position is determined from the amplitudes of signals on the two mutually perpendicular cathode grids.

The PSPC has 256 pulse height channels, which are largely overlapping in energy range. Figure 2.2 shows the pulse height response for a few individual monochromatic emission lines. From this figure one can see that the number of independent spectral
2.2 The ROSAT All-Sky Survey

![Image](image.png)

**Figure 2.2:** The response of the pulse height channels to different energies. The dashed line in the lower panel shows the expected count rate for a theoretical input spectrum consisting of monochromatic emission lines at 0.2, 0.7, 1.2, 1.7, and 2.2 keV, as shown in the top panel. Solid lines are the individual pulse height channel response curves for the corresponding monochromatic signal from the top panel.

The energy resolution for model fitting (see also Sect. 2.5) is that ROSAT PSPC spectra can always be successfully modelled with 5 independent free parameters, (e.g., a two-temperature spectrum with absorption). The uncertainty in the photon position depends on the angle of incidence (the off-axis angle) and on the energy of the photon. During the All-Sky Survey, photons of one specific source are distributed over the whole area of the detector, along successive scan lines, as illustrated in Figure 2.3. On average, more than 90% of the photons from a point source are contained within a circular area with a radius of 400", while in on-axis pointed observations this area has a typical radius of 60". The effective area of the combination of the X-ray mirror assembly (XMA) and the PSPC strongly depends on the photon
The track of a source along the PSPC field of view during the All-Sky Survey (grey lines) for a source located on the ecliptic. The progression speed (i.e., the distance between successive scan lines) is about 4' per orbit.

energy. This dependence is shown in Figure 2.4. The sharp edge at 0.28 keV is caused by absorption of photons just above this energy by carbon, which is a main ingredient of the entrance window coating. The decrease in effective area at energies above 1.2 keV is caused by a decreasing reflection efficiency of the mirrors.

2.2.3 The data

The data of the All-Sky Survey have been processed at the Max Planck Institut für extraterrestrische Physik (MPE) in Garching-bei-München. All photons collected have been assigned a sky-position and an energy channel and have been archived in so-called
2.2 The ROSAT All-Sky Survey

![Figure 2.4](image)

**Figure 2.4:** The effective area of the XMA+PSPC combination as a function of the photon energy.

Photon Event Tables (PETs), together with housekeeping information on satellite and instrument status as a function of time. All this is done with the fully automated Standard Analysis Software System (SASS), developed at MPE for this purpose. Detection of X-ray sources and calculation of physical information (e.g., count rates, upper limits, spectral hardness, background level, extent) have been done in the final steps of the SASS. Here, we give a brief description of these final steps; the SASS is described in more detail by Voges et al. (1992).

The detection of sources has been done with several sliding-window algorithms with several detection cell sizes, and in three different energy bands (i.e., pulse height channel ranges).

After a source is pointed out as a possible detection, a maximum likelihood algorithm developed by Crudecc et al. (1988) quantifies the significance level of detection. It determines the position of the source, its intensity, and its angular diameter (in case of an extended source). Potential sources of uncertainty in this method come from the determination of the background level, and from the (effective) exposure time. Background maps have been created by excluding the detected sources and fitting a smooth function through the remaining photons. X-ray sources which are not bright enough to be detected by the detection algorithms, can contaminate the background maps. Furthermore, the background is smoothed over large cells, so that local sources of background are not considered. Exposure maps have been created using the housekeeping and attitude infor-
ensation of the satellite and the instruments. The large scale average exposure times from these maps have been used for determination of the count rate.

The SASS-results used in this thesis, are all from the first processing of the All-Sky Survey, made between August 1991 and April 1992. In this version of SASS a few problems have been discovered, most of them have been located and have already been corrected in later versions of the SASS. For the studies described in Chapters 3 and 4 we chose to use an independent analysis method, as described in the next section. We used the SASS-results for a few detected sources in Chapter 3, for all sources in Chapter 8, and for most of the upper limits in Chapter 3. We have corrected the SASS results with respect to some recognized problems in the following way:

1. For some of the data, the vignetting correction has been applied twice. We have multiplied the count rate by a mean vignetting factor of 0.7.

2. The North-Pole data have a large offset in sky-position. We have excluded sources in this region, if there was no additional information about them in other survey strips.

3. Some of the stars which are not detected with the SASS, should have been, because their likelihood, as calculated by the maximum likelihood algorithm, exceeds the criterium for detection. We have denoted these sources as detections and calculated their count rates by the method described in the next section.

2.3 Calculation of count rates, upper limits and hardness ratios

The subjects discussed in Chapters 3 and 4 of this thesis require large accuracy and a coherent approach, because of the statistical nature of the investigation. As discussed in Sect. 2.2.3, we have chosen another method to analyse the All-Sky Survey data used in this investigation.

A circle is selected around each of the stars, large enough to contain nearly all source counts. Two additional circles are selected to determine the background. These background circles lie along the scan direction, and are therefore observed just before and after the observation of the source. They have the same ecliptic longitude as the source, are chosen on both sides of the source, and have the same radius as the source circle, as illustrated in Fig. 2.5. In the case of 'pollution' of one of the background circles by a nearby source, only the other background circle was used in the analysis.

The extraction of photons and calculation of effective exposure times is done following Belloni et al. (1994). The important feature of this method is that photons collected during certain time periods are excluded from the analysis. These are the periods, in which the source (or better: its extent due to positional inaccuracy), was not fully in the field of view of the PSPC. For these periods, the count rate would be underestimated. At the same time this could be a disadvantage of the method, because the information contained in these photons is not used. The detector area containing the photons that are not used is approximately 20% of the total area. Considering the effective area, which is smaller at the edge of the detector, than for on-axis observations, the loss of photons is less than 10%.
2.3 Calculation of count rates, upper limits and hardness ratios

2.3.1 Detection or non-detection

After selecting a suspected source region and two background regions (or one if other sources pollute the background), we first establish whether there is an X-ray emitting source, i.e. we test the null hypothesis $H_0$ that the suspected source region contains only background. We reject $H_0$, if the probability that all counts are background counts is less than an adopted critical value $\alpha$. This probability is calculated in the following way.

Let $n_i$ be the observed number of counts in region $i$, region 1 being the suspected source region, regions 2 and 3 the background regions. We assume that the average background count rate $r_{bg}$ (cts sec$^{-1}$ cm$^{-2}$) is the same for all selected regions and that the background counts in all three regions follow a Poisson distribution: the probability $p(n_i|b_i)$ of finding $n_i$ background counts in region $i$, for a background level $b_i$, is:

$$p(n_i|b_i) = \frac{e^{-b_i}b_i^{n_i}}{n_i!}$$  \hspace{1cm} (2.1)

where $b_i = c_i r_{bg} \equiv t_{\text{exp},i} A_i G_{V,i} r_{bg}$ is the expected number of background counts in a region.
with area $A_i$, effective exposure time $t_{\text{exp},i}$ and mean vignetting factor $C_{V,i}$ (calculated from the off-axis angles and energies from the observed counts in region $i$; Belloni et al. 1994).

The probability $P_0(n_b = n_1)$ that the observed $n_1$ counts in region 1 are all background counts, i.e., the probability that none of the observed counts are source counts, is for a known background level $b$:

$$P_0(n_b = n_1) = \frac{p(n_1|b)}{\sum_{n=0}^{n_1} p(n|b)}$$  \hspace{1cm} (2.2)

We do not know the exact value of $b$, but we estimate its probability distribution from the observed number of background counts in regions 2 and 3 in the following way. We can calculate a likelihood function $q(b|n_2, n_3)$, which is proportional to the probability $p(n_2, n_3|b)$ of finding $n_2$ counts in region 2 and $n_3$ counts in region 3, for a given $b$, and which is normalized so that $\int q(b|n_2, n_3) \, db = 1$:

$$q(b|n_2, n_3) = \frac{p(n_2, n_3|b)}{\int p(n_2, n_3|b) \, db}$$ \hspace{1cm} (2.3)

Under the a priori assumption that every value for the background count rate is equally likely to occur, $q(b|n_2, n_3)$ is the probability distribution of $b$, given $n_2$ and $n_3$.

Now we can calculate the probability $P(n_b = n_1)$ that all counts in region 1 are background counts (given the observed $(n_1, n_2, n_3)$), by integrating Eq. (2.2) over all possible background levels $b$, weighted with the probability distribution $q(b|n_2, n_3)$:

$$P(n_b = n_1) = \int P_0(n_b = n_1) q(b|n_2, n_3) \, db$$

$$= \int \frac{e^{-x}x^N}{N!\sum_{n=0}^{n_1} \frac{n_1!}{n!}(cx)^n} dx$$ \hspace{1cm} (2.4)

where $c = c_1/(c_2 + c_3)$ ($c_1$ as defined after Eq. (2.1)), $N = n_2 + n_3$ and the substitution $b = cx$ has been made.

If $P(n_b = n_1) < \alpha$ we say that there is an X-ray emitting source in region 1. With this method, the expected number of false detections in the whole sample $f(\alpha)$, given a threshold value $\alpha$, will be:

$$f(\alpha) = \sum_{\text{detections}} P(n_b = n_1)$$ \hspace{1cm} (2.5)

where the summation is taken over all stars for which $P(n_b = n_1) < \alpha$ (i.e. over all detections).

We set $\alpha$ such that we expect 0.5 false detections for the total sample, i.e. $f(\alpha) = 0.5$. We find that this condition is fulfilled when we choose $\alpha = 0.035$, for the sample in Chapter 3, and $\alpha = 0.025$, for the sample in Chapter 4.

### 2.3.2 Count rate

If a source is detected, we estimate the source count rate $r_s$ and a region in which $r_s$ can be found within a certain level of confidence, as follows.
We assume that \( r_s \) (cts sec\(^{-1}\)) is constant during the time of the observation. The probability \( p(n_1, n_2, n_3|b, s) \) of finding the observed number of counts in each region, given an average background level \( b \) and a source level \( s \) is:

\[
p(n_1, n_2, n_3|b, s) = \sum_{n_3=0}^{n_1} \left\{ \frac{e^{-s} s^n b_{n_1-n}}{n! (n_1-n)!} \right\} \left\{ \frac{e^{-b_2} b_{n_2}^n}{n_2!} \right\} \left\{ \frac{e^{-b_3} b_{n_3}^n}{n_3!} \right\}
\]

(2.6)

where \( s = c^2 r_s = t \exp(C_{V,1} r_s) \) is the expected number of source counts, \( b_i = b c_i/c_1 \) is the expected number of background counts in region \( i \) (see Eq. 2.1) and the summation is over every possible number of source counts.

We define the likelihood-function \( q(b, s|n_1, n_2, n_3) \):

\[
q(b, s|n_1, n_2, n_3) = \frac{p(n_1, n_2, n_3|b, s)}{\int \int p(n_1, n_2, n_3|b, s) \, db \, ds}
\]

(2.7)

which is equal to the probability distribution of the pair \((b, s)\), assuming that every pair \((b, s)\) is in principle equally likely to occur.

Then the likelihood function \( q(s|n_1, n_2, n_3) \) of \( s \) can be defined as:

\[
q(s|n_1, n_2, n_3) = \int q(b, s|n_1, n_2, n_3) \, db
\]

(2.8)

As an estimate for the source count rate \( r_s \) we take the value for which the likelihood function \( q(r_s | c^2_1|n_1, n_2, n_3) \) is at its maximum.

The uncertainties (\( \Delta_+ \) and \( \Delta_- \)) in \( r_s \) are calculated by requiring that the probability \( P(r_s - \Delta_- < r_s < r_s + \Delta_+) \) of \( r_s \) lying between \( r_s - \Delta_- \) and \( r_s + \Delta_+ \) is 68\% (corresponding to the Gaussian one \( \sigma \) confidence level), and that the values of the likelihood function at \( r_s - \Delta_- \) and \( r_s + \Delta_+ \) are equal. In this calculation we assume that every source count rate and every background level is equally likely to occur, so that \( q(s|n_1, n_2, n_3) \) is the probability distribution of \( s \), and

\[
P(r_s - \Delta_- < r_s < r_s + \Delta_+) = \int_{c^2_1(r_s+\Delta_+)}^{c^2_1(r_s-\Delta_-)} q(s|n_1, n_2, n_3) \, ds = 0.68
\]

\[
q(c^2_1(r_s - \Delta_-)|n_1, n_2, n_3) = q(c^2_1(r_s + \Delta_+)|n_1, n_2, n_3)
\]

(2.9)

2.3.3 Upper limit

If a suspected source is not detected, i.e. when the probability is larger than \( \alpha \) (see Sect. 2.3.1) that the counts in the source region are all caused by background photons, an upper limit is calculated. We define the upper limit count rate \( r_{ul} \) by requiring that the probability \( P(r_s < r_{ul}) \) of \( r_s \) being smaller than \( r_{ul} \) is 99.7\%, which corresponds to a Gaussian 3\( \sigma \) confidence level:

\[
P(r_s < r_{ul}) = \int_0^{c^2_1 r_{ul}} q(s|n_1, n_2, n_3) \, ds = 0.997
\]

(2.10)
2.3.4 Hardness ratio

The hardness ratio $h$ is defined as follows:

$$h = \frac{r_b}{r_a + r_b}$$  \hspace{1cm} (2.11)

where $r_b$ is the source count rate in the high-energy band (b-band: channels 41 to 240, $\sim 0.4-2.4$ keV) and $r_a$ is the source count rate in the low-energy band (a-band: channels 7 to 40, $\sim 0.1-0.4$ keV). Let $n_{a,i}$ and $n_{b,i}$ be the observed number of counts in region $i$ in the a-band and the b-band respectively, let $q(a;n_{a,i},i=1,3)$ and $q(b;n_{b,i},i=1,3)$ be the likelihood functions of the source level in the a-band, $s_a$, and in the b-band, $s_b$, respectively, given the observed number of counts in the appropriate energy band (derived following Eqs. (2.6)-(2.8)). To calculate the most probable value of this ratio, we first calculate the probability $P(h > h_0)$ of $h$ being larger than a certain value $h_0$ (assuming that every pair $(s_a, s_b)$ is in principle equally likely to occur):

$$P(h > h_0) = \int_{h_0}^{\infty} q(a;n_{a,i},i=1,3) \int_{h_0}^{\infty} q(b;n_{b,i},i=1,3) \, ds_b \, ds_a$$  \hspace{1cm} (2.12)

where $x = h_0/(1 - h_0)$. The probability distribution $q(h)$ of the hardness ratio then is:

$$q(h) = -\frac{d P(h' > h)}{d h} = \frac{\sum_{n=0}^{n_{a,i}} \sum_{n'=0}^{n_{b,i}} I_1(n,n') I_2(n,n') h^{n_{a,i}+n_{b,i}-n'(1-h)^{n_{a,i}+n_{b,i}-1}}}{\sum_{m=0}^{n_{a,i}} \sum_{m'=0}^{n_{b,i}} I_1(m,m')}$$  \hspace{1cm} (2.13)

Here $I_1(n,n') = C^a C^b ((N_a + n)!(N_b + n')!/(n!n'!))$, with $N_a = n_{2,a} + n_{3,a}, N_b = n_{2,b} + n_{3,b}, C_a = c_{1,a}/(c_{1,a} + c_{2,a} + c_{3,a})$ and $C_b = c_{1,b}/(c_{1,b} + c_{2,b} + c_{3,b})$, and $I_2(n,n') = (n_{1,a} - n + n_{b} - n')!/((n_{1,a} - n)!(n_{1,b} - n')!)$. The most probable value $\tilde{h}$ of the hardness ratio is the value for which $q(h)$ is at its maximum.

The uncertainties in the hardness ratio are calculated from $q(h)$ and $\tilde{h}$ in the same way as the uncertainties in the count rate were calculated from $q(s|n_{1},n_{2},n_{3})$ and $\bar{r}_s$ (see Sect. 2.3.2, Eq. (2.9)).

2.4 Conversion of count rate to flux

To derive the stellar flux density, the count rate is divided by an energy conversion factor $C_{cf}$, which depends on the spectral characteristics of the star and on the amount of interstellar matter along the line of sight. If the energy distribution of a source is known, the energy conversion factor $C_{cf}$ can easily be calculated, using the instrument characteristics, as described in the following subsection. We have calculated theoretical energy conversion factors for the specific case of one-temperature Mewe & Gronenschild spectra (Sect. 2.4.1). The observed spectral hardness (Sect. 2.4.2) gives information about the energy distribution, and therefore provides us with a tool to estimate the energy conversion factors more accurately. This tool is presented in Sect. 2.4.3.
2.4 Conversion of count rate to flux

The energy conversion $C_{cf}$ as a function of temperature. Different lines denote different hydrogen column densities: the top curve is for $n_H = 10^{18}$ cm$^{-2}$, the increase of $n_H$ between two successive curves is a factor $\sqrt{10}$, the bottom curve is for $n_H = 10^{22}$ cm$^{-2}$.

2.4.1 Theoretical energy conversion factor

For the calculation of the energy conversion factor $C_{cf}(T, n_H)$, we have used the PSPC detector-response matrix released by the ROSAT science data center in November 1992 and the table of effective areas (for data before January 25, 1991) released in October 1992. The detector-response matrix gives the response per channel per energy bin. In Fig. 2.2 we show response curves for photons with energies between 0.2 and 2.2 keV. The on-axis effective area is shown in Fig. 2.4 as a function of energy. For an X-ray source with a known spectrum, we can calculate the total count rate, $r_s(k)$ (cts/sec), in a certain PSPC channel, $k$, by multiplying the photon flux at every energy bin $E_i$ with the response $R_{PSPC}(E_i, k)$ for that energy in channel $k$ and the on-axis effective area $A_{eff}(E_i)$ for that energy.

We calculate the expected count rate using theoretical spectra, $F(E, T)$ (photons sec$^{-1}$cm$^{-2}$), for optically thin plasmas in thermal equilibrium and with solar abundances (Mewe et al. 1985), for temperatures $T$ between $10^5$ K and $10^8$ K, and for hydrogen column densities $n_H$ between $10^{18}$ cm$^{-2}$ and $10^{22}$ cm$^{-2}$, using interstellar absorption cross sections of Morrison & McCammon (1983):

$$r_s(k) = \sum_{i=1}^{N} R_{PSPC}(E_i, k)A_{eff}(E_i)e^{-n_H X(E_i)}F(E_i, T)$$  \hspace{1cm} (2.14)
where $N$ is the number of energy bins and $X(E_i)$ is the absorption cross section for energy $E_i$.

The factor $C_{cf}$ is derived by dividing the expected total count rate by the associated net flux in the ROSAT energy band 0.1–2.4 keV (in erg sec$^{-1}$ cm$^{-2}$):

$$C_{cf}(T, n_H) = \frac{\sum_{k=1}^{256} r_s(k)}{K + \sum_{i=1}^{n_H} F(E_i, T)E_i}$$

(2.15)

where $i_1$ and $i_2$ have been chosen so that energy bins $i_1$ and $i_2$ are the first and the last complete bins within the 0.1–2.4 keV range, and $K = \frac{(E_{i_1} - 0.1)}{\Delta E_{i_1}} F(E_{i_1-1}, T)E_{i_1-1} + \frac{(E_{i_2} - E_{i_2+1})}{\Delta E_{i_2+1}} F(E_{i_2+1}, T)E_{i_2+1}$ is the contribution to the flux from the two outer energy bins, corresponding to 0.1 and 2.4 keV.

The computed $C_{cf}$ is shown in Fig. 2.6, as a function of the temperature of the X-ray emitting plasma, for a range of different hydrogen column densities.

### 2.4.2 The spectral hardness ratio

The spectral hardness ratio, $h$, is defined as the ratio of the number of counts $N_h$ in the high-energy band (channels 41–240) and the number of counts $N_s + N_h$ in the ‘total’ band.

**Figure 2.7:** The hardness ratio $h$ as a function of temperature. Different lines denote different hydrogen column densities: the bottom curve is for $n_H = 10^{18}$ cm$^{-2}$, the increase of $n_H$ between two successive curves is a factor $\sqrt{10}$, the top curve is for $n_H = 10^{22}$ cm$^{-2}$.
2.4 Conversion of count rate to flux

\begin{align*}
\text{energy (keV)} & \\
T = 0.3 \text{ MK} & h = 0.00 \\
T = 1 \text{ MK} & h = 0.03 \\
T = 3 \text{ MK} & h = 0.56 \\
T = 10 \text{ MK} & h = 0.59 \\
\end{align*}

Figure 2.8: Theoretical one temperature Mewe & Gronenschild models (thick solid lines) for different temperatures, the scales for these models are at the top and on the right. In each panel the resulting PSPC pulse height channel is shown as a thin solid line. Scales for the pulse height spectra are at the bottom and on the left.

(Channels 7-40 plus 41-240):

\[ h \equiv \frac{N_h}{N_h + N_s} = \frac{\sum_{k=41}^{240} r_s(k)}{\sum_{k=7}^{240} r_s(k)} \]  \hfill (2.16)

We calculated theoretical hardness ratios for the Mewe & Gronenschild spectra. The results are shown in Fig. 2.7 for different values of the hydrogen column density.

The hardness ratio is very sensitive to the temperature of the radiating plasma, in a small range of temperatures. The hardness ratio increases with increasing temperature up to 5 MK, because the emission lines in the region around 1 keV increase in strength from 2 MK to 5 MK, and decrease for higher temperatures (Fig. 2.8). These emission lines show up in the PSPC pulse height spectrum as the hard 'peak' above channel 40. For higher temperatures the emission lines disappear and the theoretical spectra are relatively flat, resulting in the almost constant hardness ratio for temperatures larger than about 20 MK.

The hardness ratio is also very sensitive to galactic absorption. In Fig. 2.9 the effect is shown of increasing hydrogen column densities on a Mewe & Gronenschild spectrum with a temperature of 3 MK. As the column density increases, the low-energy peak is suppressed, and consequently the hardness ratio becomes higher.
Figure 2.9: Theoretical one-temperature Mewe & Gronenschild model (thick solid lines) for 3 MK, for different \( n_H \); the scales for these models are at the top and on the right. In each panel the resulting PSPC pulse height channel is shown as a thin solid line. Scales for the pulse height spectra are at the bottom and on the left.

In Fig. 2.10 we show (for the same models as in Fig. 2.6) a plot of the energy conversion factor, \( C_{cf} \) vs. the hardness ratio, \( h \). From this plot one can see that it is possible to derive an estimate for the energy conversion factor from the observed hardness ratio, if we assume that the source spectrum is described by a one-temperature Mewe & Gronenschild spectrum, and that \( n_H \) is known.

2.4.3 Determination of \( C_{cf} \) and its uncertainty, using the observed \( h \) and its probability distribution

The number of counts \( N_h \) in the high-energy band (channels 41–240) and \( N_s \) in the low-energy band (channels 7–40), define the hardness ratio (Eq. 2.16) and the uncertainty in the hardness ratio (assuming \( N_h \) and \( N_s \) follow Poisson distributions):

\[
e_{h} = \sqrt{\left( \frac{\partial h}{\partial N_h} \Delta N_h \right)^2 + \left( \frac{\partial h}{\partial N_s} \Delta N_s \right)^2} = \sqrt{\frac{N_h N_s}{(N_h + N_s)^3}}
\]

(2.17)

where \( \Delta N_h = \sqrt{N_h} \) and \( \Delta N_s = \sqrt{N_s} \).
From Fig. 2.10 one can derive the range in $C_{cf}$ values that is consistent with observed hardness ratios between $h - \epsilon_h$ and $h + \epsilon_h$, and for hydrogen column densities between $n_H - \epsilon_{H}$ and $n_H + \epsilon_{H}$, from which it follows that the $C_{cf}$ value for a particular source lies in this range within a 1σ confidence level. For a quick estimate of the possible range of $C_{cf}$ values, this is a sufficient method if the observed number of counts is large enough ($N_h$ and $N_s$ both larger than 20), and if the hardness ratio interval is in the region where $C_{cf}$ varies gradually with the hardness ratio ($[h - \epsilon_h, h + \epsilon_h] \in [0.05, 0.45]$).

To get a more accurate estimate of the most likely $C_{cf}$ value with its uncertainty interval, we use the probability distribution $f(h)$ of $h$ and $f(\log n_H)$ of $\log n_H$. We assume that the background subtracted numbers of hard and soft source photons, as given by the ROSAT Standard Analysis Software System (SASS), follow Poisson distributions: the probabilities $p(N_h|x_h)$ and $p(N_s|x_s)$ of observing $N_h$ counts in the hard band and $N_s$ counts in the soft band, given the expected number of hard counts $x_h$ and the expected number of soft counts $x_s$, are:

$$p(N_h|x_h) = \frac{e^{-x_h} x_h^{N_h}}{N_h!}$$

$$p(N_s|x_s) = \frac{e^{-x_s} x_s^{N_s}}{N_s!}$$

(2.18)
Assuming that all values for \( x_h \) and \( x_s \) are in principle equally likely to occur, the probability distributions \( f_{N_h}(x_h) \) for \( x_h \) and \( f_{N_s}(x_s) \) for \( x_s \), given the observed \( N_h \) and \( N_s \), are:

\[
\begin{align*}
  f_{N_h}(x_h)dx_h &= p(N_h|x_h)dx_h \\
  f_{N_s}(x_s)dx_s &= p(N_s|x_s)dx_s 
\end{align*}
\]  

(2.19)

The cumulative probability distribution \( F(h) \) of the hardness ratio (i.e. the probability that the hardness ratio is smaller than \( h \)) then is:

\[
F(h) = \int_0^h f_{N_h}(x_h) \int_0^{1-h} f_{N_s}(x_s) dx_h \, dx_s
\]  

(2.20)

and the probability distribution of the hardness ratio is:

\[
f(h)dh = \frac{\partial F(h)}{\partial h}dh = \int f_{N_s}(x_s) f_{N_h}(x_h) \frac{x_s}{(1-h)^2} \, dx_s \, dh = \frac{(N_h + N_s + 1)!}{N_h!N_s!} h^{N_h}(1-h)^{N_s} dh
\]  

(2.21)

The probability distribution of \( \log n_H \) is taken to be a normal distribution with mean \( \log n_{H,0} \) and standard deviation \( \sigma = 0.434 \cdot \epsilon_{n_H}/n_H \).

\[
f(\log n_H) d\log n_H = \exp \left( -\frac{(\log n_H - \log n_{H,0})^2}{2\sigma^2} \right) d\log n_H
\]  

(2.22)

We can now determine the probability distribution of \( C_{cf} \) from the distributions of the hardness ratio (Eq. 2.21) and the hydrogen column density (Eq. 2.22), by summing the probabilities of all possible combinations of \( h \) and \( n_H \) that result in a certain \( C_{cf} \). To do this, we divide the range of \( h \in [0,1] \), \( \log n_H \in [18,22] \) and \( C_{cf} \in [0,3 \cdot 10^1] \) in equal bins of width \( \Delta h \), \( \Delta \log n_H \) and \( \Delta C_{cf} \) respectively. Then for each \( h_i = (i - \frac{1}{2})\Delta h \) and \( \log n_{H,j} = 18+(j-\frac{1}{2})\Delta \log n_H \) we derive \( C_{cf}(h_i,n_{H,j}) \) from Fig. 2.10. For some combinations of \( h \) and \( n_H \), more than one value of \( C_{cf} \) exist. In this case we take the mean value, assuming that all plasma temperatures \( T \) are in principle equally likely to occur. The derived value \( C_{cf}(h_i,n_{H,j}) \) lies in the \( k \)-th \( C_{cf} \) bin if \( (k - 1)\Delta C_{cf} \leq C_{cf}(h_i,n_{H,j}) < k\Delta C_{cf} \).

The probability \( p(h_i,n_{H,j}) \) that \( h_i \) and \( n_{H,j} \) occur is:

\[
p(h_i,n_{H,j}) = f(h_i)\Delta h \int p(\log n_H) d\log n_H
\]  

(2.23)

The probability \( p(C_{cf,k}) \) that \( C_{cf} \) lies in the \( k \)-th bin equals the sum of \( p(h_i,n_{H,j}) \) over all \( i \) and \( j \) for which \( C_{cf}(h_i,n_{H,j}) \) lies in the \( k \)-th bin:

\[
p(C_{cf,k}) = \sum_{i,j:\,(k-1)<\frac{C_{cf}(h_i,n_{H,j})}{\Delta C_{cf}}\leq k} p(h_i,n_{H,j})
\]  

(2.24)

From this probability distribution we derive an estimate \( \overline{C}_{cf} \) for the energy conversion factor and its uncertainty interval. For \( \overline{C}_{cf} \) we take the most likely value of \( C_{cf,k} \) (i.e. the value for which \( p(C_{cf,k}) \) is largest). We define the interval \( I(p_0) = \)
Figure 2.11: The pulse height spectrum of HR 373. Error-bars indicate Poisson noise. The dashed line is the best fit resulting from minimizing $\chi^2$, the solid line is the best fit resulting from minimizing $\chi_p$.


\[ [C_{cf, k- (p_0)} - \frac{1}{2} \Delta C_{cf, k- (p_0)} + \frac{1}{2} \Delta C_{cf, k+ (p_0)} \text{ consisting of all } C_{cf, k} \text{ bins } k \in [k-(p_0), k+(p_0)] \text{ for which the probability } p(C_{cf, k}) \text{ is larger than a threshold value } p_0. \text{ The value of } p_0 \text{ determines the probability } p(C_{cf} \in I(p_0)) \text{ of } C_{cf} \text{ lying in the interval } I(p_0). \]

\[
p(C_{cf} \in I(p_0)) = \sum_{k=k-(p_0)}^{k+(p_0)} p(C_{cf, k})
\]

We choose $p_0$ to be the largest possible value for which $p(C_{cf} \in I(p_0))$ is larger than 0.68, so that $I(p_0)$ corresponds to a (Gaussian) $1\sigma$ uncertainty interval.

2.5 Model fitting of pulse height spectra

Model fitting to observed X-ray spectra can provide valuable information about the physical parameters of the X-ray emitting plasma, e.g., about its temperature and emissivity. We have derived coronal temperatures of a few late-type stars by fitting their PSPC pulse height spectra to one- and two-temperature Mewe & Gronenschild models (Mewe et al. 1985). The method we used to fit these spectra is fundamentally different from the usual methods in spectral fitting programs, which are based on minimization of $\chi^2$. We argue that minimization of $\chi^2$ does not result in the most likely parameter representation of the pulse height spectrum, because it assumes a normal distribution of the errors in
each pulse height channel. However, a Poisson distribution is more realistic, because the spectra consist of individual photon counts and the count rates are usually not very high.

We fitted the spectra using a maximum likelihood method for Poisson distributed errors. The probability \( p(n(i)|f(i)) \) of observing \( n(i) \) photons in pulse height channel \( i \) \((i \in [1, N])\), if the model spectrum predicts \( f(i) \) photons, then is:

\[
p(n(i)|f(i)) = \frac{e^{-f(i)}f(i)^{n(i)}}{n(i)!}.
\]

Hence, the probability \( P \) of the observed spectrum to occur is:

\[
P = \prod_{i=1}^{N} p(n(i)|f(i)) = \prod_{i=1}^{N} \frac{e^{-f(i)}f(i)^{n(i)}}{n(i)!}.
\]

Maximizing the probability \( P \) gives the model parameters for which the observed data is most likely to occur. Maximizing \( P \) is equivalent to minimizing the negative logarithm of \( P \):

\[
-\ln P = \sum_{i=1}^{N} \{f(i) - n(i)\ln(f(i)) + \ln(n(i)!)}
\]

and, because \( n(i) \) is constant in this process, this is equivalent to minimizing the expression

\[
\chi_p = \sum_{i=1}^{N} \{f(i) - n(i)\ln(f(i))}\.
\]

From the derivative of \( \chi_p \) with respect to \( f(i) \) for each pulse height channel \( i \) it follows that \( \chi_p \) reaches a minimum if the model spectrum \( f(i)_{i=1,N} \) exactly equals the observed spectrum \( n(i)_{i=1,N} \), as expected.

After finding the model parameters which result in a minimum \( \chi_p \) for the observed spectrum, we have to define a criterium for accepting this ‘best fit’ as a good fit. We have used a Monte Carlo simulation to calculate the distribution function of \( \chi_p \) for this particular best-fit model, where the simulated data have the same number of counts as the observed data. The fit is rejected, when more than 90% of the integrated distribution function is smaller than the actually derived \( \chi_p \).

An example of the difference between the conventional method of minimizing \( \chi^2 \) and the maximum likelihood method described here is shown in Fig. 2.11. The dashed line is the best fit using the \( \chi^2 \)-minimalization, the solid line gives the best fit derived by minimizing \( \chi_p \). The difference exists, because the conventional \( \chi^2 \) (which weighs every channel \( i \) with \( 1/\sigma_i^2 = 1/n(i) \)) is based on a normal distribution of counts, while the here used \( \chi_p \) (Eq. 2.29) is based on a Poisson distribution.

References

References

Voges W., Gruber R., Paul J., et al., 1992, ESA ISY-3, p. 223