Top physics at the LHC

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Abstract

We review the present situation in top quark physics, in these early days of Run II of the LHC. We take mostly a Standard Model perspective, showing recent results, and review the key concepts and results of the associated theoretical predictions. The issues we discuss are the top quark mass, top quark pair and single top production, production in association with other particles, charge asymmetry and top quark decay.

1 Introduction

Ever since its discovery \cite{1,2} in 1995 by the CDF and D0 experiments at the Tevatron, the top quark has been in or near the center of attention in high-energy physics. Its remarkably large mass, still the largest of any known elementary particle, implies that it couples strongly to the agents of electroweak symmetry breaking, making it both an object of interest itself, and a tool to investigate that mechanism in detail.
The history of heavy flavours anyway is such each of them has taught us much about Nature. From the charm quark we learned that the Standard Model is consistent, through the GIM \[3\] mechanism. Moreover, its discovery cemented the belief in QCD as the quantum theory of the strong interactions. From the bottom quark we learned that a complete third family was there to find, in turn allowing for weak CP violation \[4\] to be part of the Standard Model. However, although already discovered 20 years ago, the top quark has not yet taught us fundamentally new insights. The top may well do this in the coming decade after all, a belief that rests on top’s attributes.

The top quark couples to other particles through various (chiral, vector, scalar) structures according to the Standard Model Lagrangian. In our search for physics beyond the Standard Model, all of these bear scrutiny for deviations, and there is therefore much to test. Such precise scrutiny is feasible because the large top mass implies that hadronization effects do not occur and spin information is preserved.

With the Tevatron having made the first precious thousands top quarks, leading to its discovery and tests of some of its properties, the LHC is a genuine top quark factory, in particular in Run II, which is now underway. The data gathered already and especially upcoming data will allow us to study the top quark and its behavior in LHC collisions in great detail, if also the theoretical descriptions and simulations are of commensurate quality.

Here we provide a compact review of some of the key aspects of top quark physics, largely from a Standard Model point of view. We highlight key issues from a mostly conceptual standpoint, and list the present state of affairs in terms of calculations and corresponding experimental analyses. We refer to other excellent reviews \[5–9\] for more extensive explanations. In section \[2\] we discuss mostly issues regarding properties of the top itself, and characteristics of its decay. In section \[3\] we mostly discuss its production, either in pairs, singly, or in association. We end with a brief conclusion.

2 Top properties and decays

In this first section we give a brief description of how the top quark is embedded in the Standard Model, and motivates physics beyond it. We also discuss aspects of its decay and properties such as mass and spin.

2.1 Top in the Standard Model

We recall the various interactions of the top quark field \(t(x)\) in the Standard Model Lagrangian. The interaction with gluons is a vectorlike coupling involving an \(SU(3)\) generator in the fundamental representation

\[
g_s \bar{t}_i(x) \gamma^\mu [T^a]_{ij} t_j(x) G^a_\mu(x),
\]

where \(i, j\) label QCD colour charge. The interaction with photons is also simply vectorlike and proportional to the top quark electric charge

\[
\frac{2}{3} e \bar{t}(x) \gamma^\mu t(x) A_\mu(x).
\]
Its charged weak interaction is left-handed and flavour-changing

\[ \frac{g_w}{2\sqrt{2}} V_{lf} \bar{t}(x) \gamma^\mu (1 - \gamma_5) f(x) W_\mu(x), \quad f = d, s, b, \]  

while its neutral weak interaction is flavour-conserving and parity violating

\[ \frac{g_w}{4 \cos \theta_W} \bar{t}(x) \gamma^\mu \left( 1 - \frac{8}{3} \sin^2 \theta_W - \gamma_5 \right) t(x) Z_\mu(x). \]  

Finally, the interaction of the top with the Higgs boson is of the Yukawa type

\[ y_t \ h(x) \bar{t}(x) t(x), \]  

with a coupling constant \( y_t = \sqrt{2} m_t / v \) directly proportional to the top quark mass \( m_t \), and \( v \) is the Higgs vacuum expectation value.

Beyond these, effective interactions such as for flavour-changing neutral currents, occur due to loop corrections. They can be calculated and are generally very small compared to the ones above. All these interactions, either elementary or effective, could be modified in structure and strength by virtual effects due to new interactions associated with physics beyond the Standard Model. This is a particularly interesting line of investigation for the top quark, if only because it evidently has a large coupling to the electroweak symmetry breaking sector (the Yukawa coupling \( y_t \) in Eq. (5) is almost exactly 1 in the Standard Model). It is then important to test these structures in detail, and indeed this is the thrust behind the field of top physics.

### 2.2 Top beyond the Standard Model

Driving most motivations for physics beyond the Standard Model is the fact that the Higgs mass seems unnaturally small. The top quark features prominently in this argument as the main culprit for creating this situation. When considering Standard Model one-loop corrections to the inverse Higgs boson propagator there are contributions from the \( W \) and \( Z \) bosons, the Higgs boson itself, and, most importantly, the top quark. Using an ultraviolet cut-off regulator \( \Lambda \) they can be added to the bare Higgs mass squared \( m_{H,B}^2 \) to form the renormalized Higgs mass \( m_H \)

\[ m_H^2 = m_{H,B}^2 + \left( -\frac{3}{8\pi^2} g_t^2 \right) \Lambda^2 \text{ [top]} + \left( \frac{9}{64\pi^2} g^2 \right) \Lambda^2 \text{ [vector bosons]} \]

\[ + \left( \frac{1}{64\pi^2} \lambda^2 \right) \Lambda^2 \text{ [Higgs]} . \]  

Because symmetry is not enhanced by setting the Higgs mass to zero, renormalization is not necessarily multiplicative \[10\], and the divergent corrections are in fact quadratic in \( \Lambda \). Eq. (6) shows that when \( \Lambda \) is of order, say, the GUT scale, cancellations to many digits are required among these contributions, which seems a very fine-tuned setup.
Being the main troublemaker, the top may in fact also point to possible new physics in which this finetuning is avoided. A popular model is supersymmetry where stop quark loops naturally provide the cancellations that finetuning does in the Standard Model. But also in supersymmetry phenomenology the top quark plays an important role: if it weren’t for the top quark (and stop squark) corrections to the lightest Higgs boson mass, the Minimal Supersymmetric Standard Model (MSSM) would predict the latter to be lighter than the Z boson, and the MSSM would have been ruled out already. The maximum viable mass for the Higgs mass is thus about 140 GeV, comfortably above the measured value of 125 GeV.

Top could play an even more central role in the Higgs mechanism, in that its dominant contribution to the running of a Higgs potential parameter down from the GUT scale in fact leads to a negative eigenvalue for the Higgs mass matrix, thereby even explaining electroweak symmetry breaking [11].

We also note that in the last few years the precise value of the top mass has been moving further into the spotlight due to its role in regards to the stability of the electroweak vacuum [12]: the current value suggests that the vacuum is meta-stable [13–16].

In short, there is good reason to study the top quark in detail, what its properties are, how it is produced, and how it decays. We begin with the latter.

2.3 Top quark decay

The top quark decay characteristics play, directly or indirectly, an important role in studying the top quark at colliders. The top quark width is largely due to decays to a W-boson and a bottom quark. But because the top quark mass is much larger than the sum of the W and b masses, the width is sufficiently large to pre-empt top quark hadronization. The rapid decay of the top quark moreover enables transmission of top quark spin information to final states, giving us an important tool to test the role of top quark spin. At the same time, the width-to-mass ratio $\Gamma/m$ of the top quark is small enough that, for many purposes, the notion of top quark as a stable particle makes sense. This is effectively implemented through the narrow width approximation (NWA), which factorizes the production and decay processes. But, although the NWA works well for many, especially inclusive observables, it is still necessary to test its quality well, given how carefully we aim to study the top quark’s behavior.

The top width itself is very difficult to determine in a hadron collider, though a recent experimental inference of the width in the context of single top $t$-channel production was performed by D0 [17] finding $\Gamma = 2.00^{+0.47}_{-0.43}$ GeV, and CDF [18], finding $1.10 < \Gamma < 4.05$ GeV at the 68% confidence level. An optimal determination would require a threshold scan for pair production at a $e^+e^-$ collider.

The NWA full separation of production and decay is indeed an approximation, and there are corrections to it. Besides the intrinsic uncertainty of order $\Gamma/m$, there are also non-factorizable corrections from virtual partons that connect production and decay amplitudes. Another irreducible class of corrections is from diagrams with the same final state but having no intermediate top quark.
Let us briefly describe how the NWA works for the decay process of the top quark
\[ t(p) \rightarrow W^+(q) + b(r) \rightarrow t^+(k_1) + \nu(k_2) + b(r), \] (7)
where the top has been produced in the production process
\[ a(P_1) + b(P_2) \rightarrow t(p) + X(x). \] (8)

We shall also see how spin correlations can be included in the NWA. The squared amplitude for the combined process reads
\[ |A|^2 = \frac{g_\omega^4 |V_{tb}|^2}{64} \frac{1}{(p^2 - m^2)^2 + (m\Gamma)^2} \frac{1}{(q^2 - m_W^2)^2 + (m_W\Gamma_W)^2} \]
\[ \times \left[ \bar{u}(r)\gamma^\mu(1 - \gamma_5)\not{p} + m \right] M M^* \gamma^0 \not{p} + m \left( 1 + \gamma_5 \right) \gamma^\rho u(r) \]
\[ \times \left[ \bar{u}(k_2)\gamma_\mu(1 - \gamma_5)v(k_1)\bar{u}(k_1) \left( 1 + \gamma_5 \right) \gamma_\rho u(k_2) \right], \] (9)

where the top and \( W \) propagators (and their widths) are shown on the first line, while the other two lines contain the squared matrix element for (off-shell) top production, and (off-shell) \( W \) decay. Here \( M = \Gamma u(K) \), with \( \Gamma \) a combination of \( \gamma \)-matrices, and \( k_1, k_2 \) the four-momentum of a fermion entering the hard scattering.

The narrow top width approximation \( \Gamma \rightarrow 0 \) now amounts to making the replacement
\[ \frac{1}{(p^2 - m^2)^2 + (m\Gamma)^2} \rightarrow \frac{\pi}{m\Gamma} \delta \left( p^2 - m^2 \right), \] (10)
yielding an on-shell condition for the top quark momentum \( p \). Summing over spins one may now write the squared amplitude in (9) as
\[ \sum_{\text{spin}} |A|^2 = \frac{\pi}{m\Gamma} \sum_{\lambda\lambda'} \tilde{M}_\lambda \rho_{\lambda\lambda'} \tilde{M}_{\lambda'}^* \delta \left( p^2 - m^2 \right). \] (11)

The matrix \( \rho \) is the decay spin-density matrix, encoding spin correlations between production and decay, with \( \lambda, \lambda' \) labelling the top quark spin states. The above procedure to include spin correlations in the NWA can be implemented in Monte Carlo programs, even in those matched to NLO \cite{19,21}. This works in many cases very well. Other studies in this regard for \( t\bar{t} \) production can be found in Refs. \cite{22} and \cite{23}.

One should however not take the validity of the NWA for granted for all observables. Especially for those cases where there is a sizeable contribution from intermediate top quarks that are not near their mass shell this is an important issue. In these phase space regions there can moreover be appreciable contributions from subprocesses producing the same final state, but having no intermediate top quark. A recent study \cite{24} investigated the off-shell effects in \( t\)-channel single-top production, in part as a test of the NWA. Also an effective theory approach \cite{25} was compared to the exact calculation, including non-resonant diagrams and off-shell effects in the aMC@NLO \cite{26} framework using the complex mass scheme \cite{27,29}.
It was shown that indeed the NWA approximation does not always work well, in particular for observables sensitive to the $W$-$b$ invariant mass, while the effective theory approach does track the exact NLO calculation rather well.

Another NLO study \cite{30} comparing $t\bar{t}$ production plus decay in the NWA with $W^+W^-b\bar{b}$ production, the latter including also singly resonant and non-resonant contributions, found that these contributions have a significant impact on shapes of distributions, and thereby also the uncertainty of top mass measurements.

For most of the results discussed below, however, except where stated otherwise, the NWA is taken, and should be a good approximation.

### 2.4 The Higgs-top interaction and the $W$ polarization

Top interacts with the Higgs boson through the Yukawa interaction

$$y_t h\bar{t},$$

where $y_t = \sqrt{2}m_t/v$. This is a relation that can be kept at the renormalized level as well, whatever the choice of renormalization scheme for the top quark mass (about which more below). As remarked, for the top quark pole mass of about 173 GeV and with $v = 246$ GeV, $y_t = 1$ to a very good approximation.

It is interesting to note that the large Yukawa coupling $y_t$ of the top with the Higgs boson is related to the large fraction of top quarks decaying into longitudinal $W$ bosons. In fact, although it is reasonable to expect that in the decay $t \rightarrow W^+ b$, the width be proportional to the weak coupling $g^2$ and to the top mass, a direct calculation \cite{31} shows that the expression for the width reads

$$\Gamma(t \rightarrow W^+ b) \propto g^2 m_t a \left(1 + O\left(a^{-1}\right)\right),$$

with

$$a = \frac{m_t^2}{2m_W^2} = \frac{y_t^2}{g^2}.$$  

Note that the width is enhanced by a factor $a$ (about 2.3) with respect to the naive expectation. Looking at the breakdown of this result to different intermediate $W$ polarizations, we see that the decay to transversely polarized $W$ bosons is in line with the naive expectation, while the $a$ enhancement is due to the longitudinal polarization of the $W$ bosons \cite{31}. In fact, the Goldstone boson equivalence theorem \cite{32}, which states that the longitudinally polarized $W$ boson acts as a Goldstone boson (a member of the Standard Model Higgs doublet), predicts that in the limit $m_t \gg m_W$ the width of the top decaying into a longitudinal $W$ boson behaves as \cite{33}

$$\Gamma(t \rightarrow W_L^+ b) \propto g^2 m_t a.$$  

We can then write that the fraction of longitudinally polarized $W$ bosons is approximately given by

$$\mathcal{F}_L = \frac{\Gamma(t \rightarrow W_L^+ b)}{\Gamma(t \rightarrow W^+ b)} \approx \frac{a}{1 + a}.$$  

Because the ratio $a$ of the squared top and $W$ masses, or equivalently of the top Yukawa
and the gauge couplings, is about 2.3, we expect that about 70% of the $W$ bosons are
longitudinally polarized. In fact, a precise computation [34] which includes the NNLO QCD
corrections as well the leading electroweak contributions yields $F_L = 0.687(5)$. This value is
well within the experimental error bands, since an early combination of CDF and D0 Tevatron
Run II data [35] yielded $F_L = 0.682 \pm 0.057$, while CDF [36] obtained $F_L = 0.726 \pm 0.094$
using the full set of Tevatron Run II data. At the LHC, CMS has found $F_L = 0.682 \pm 0.045$
in the 7 TeV run [37], and $F_L = 0.720 \pm 0.054$ in the 8 TeV run [38].

2.5 Top mass

The top quark property that is perhaps most central in many aspects of top physics is its
mass. We already mentioned its role in the issue of stability of the Higgs potential. From
Run I and Run II data and for an integrated luminosity of up to 9.7 fb$^{-1}$, the Tevatron
experiments [39] have measured the mass with a total uncertainty of 0.64 GeV/$c^2$, i.e. to
an accuracy of less than 0.4%. From the run at a centre-of-mass energy of 7 TeV and for an
integrated luminosity of up to 4.9 fb$^{-1}$, the LHC experiments [40] have measured the mass
with a total uncertainty of 0.95 GeV/$c^2$,

$$
\text{CDF/D0 : } 174.34 \pm 0.37(\text{stat}) \pm 0.52(\text{sys}) \text{ GeV}/c^2,
\text{ATLAS/CMS : } 173.29 \pm 0.23(\text{stat}) \pm 0.92(\text{sys}) \text{ GeV}/c^2. \quad (17)
$$

The Tevatron data from Run II at a centre-of-mass energy of 1.96 TeV and for an inte-
grated luminosity of up to 8.7 fb$^{-1}$ have been combined with the LHC data mentioned above.
The resulting worldwide combination is

$$
\text{ATLAS/CDF/CMS/D0 : } 173.34 \pm 0.27(\text{stat}) \pm 0.71(\text{sys}) \text{ GeV}/c^2, \quad (18)
$$

with a total uncertainty of 0.76 GeV/$c^2$.

CMS [41] has also provided a combination of LHC data from the run at a centre-of-mass
energy of 7 TeV and for an integrated luminosity of up to 5.1 fb$^{-1}$, with the data from the
run at a centre-of-mass energy of 8 TeV and for an integrated luminosity of up to 19.7 fb$^{-1}$,
measuring the mass with a total uncertainty of 0.49 GeV/$c^2$, i.e. to an accuracy of less than
0.3%,

$$
\text{CMS : } 172.44 \pm 0.13(\text{stat}) \pm 0.47(\text{sys}) \text{ GeV}/c^2. \quad (19)
$$

Together with an accurately measured $W$ boson mass, a precisely known top mass severely
constrains the mass range of the Higgs boson [42]. Indeed the measured Higgs boson mass
seems quite consistent given present accuracies. Therefore its precise measurement is of
considerable importance, and therefore also its careful definition. This is necessary because
for the top, being coloured and thus subject to confinement, defining the mass is indeed
subtle.

A natural definition of an elementary particle mass is based on the location of the pole of
the full propagator, i.e. the pole mass. After summing self-energy corrections the full quark
propagator reads
\[ \frac{1}{p - m_0 - \Sigma(p, m_0)} \],
where \( \Sigma \) contains \( 1/\epsilon \) UV divergences from loop integrals. Renormalization (here at one loop) now amounts to replacing the bare mass \( m_0 \) by an expression involving the renormalized mass \( m \)
\[ m_0 = m \left( 1 + \frac{\alpha_s}{\pi} \left[ \frac{1}{\epsilon} + z_{\text{finite}} \right] \right) , \]
after which the UV divergences cancel in (20). The choice of \( z_{\text{finite}} \) determines the scheme. Choosing it such that
\[ \frac{1}{p - m_0 - \Sigma(p, m_0)} = \frac{c}{p - m} \]
is the pole-mass scheme, which amounts to pretending that the particle can be free and long-lived. However, because no quark can ever propagate out to infinite times due to confinement, such a pole only exists in perturbation theory, and its location is intrinsically ambiguous by \( \mathcal{O}(\Lambda_{\text{QCD}}) \) [43–45].

Experimentally, the mass of the top quark is most often reconstructed by collecting the jets and leptons from its decay. The decay channels used are the dilepton channel - two isolated leptons with opposite charge and at least two jets [46,47]; the lepton + jets channel - an isolated lepton and at least four jets [48,49]; the all-hadronic channel [50,51]. However, soft particles originating from both within and outside these jets may affect the reconstructed mass. Moreover, various experimental methods used (e.g. track quality cuts) and corrections do not have a clean perturbation theory description. Though it is considered generally a measurement of the pole mass, the full procedure has led to some discussion about what the precise “scheme” is of the mass thus measured, and to the possibility of considering a Monte Carlo mass, which would track closely but not be quite the same as the pole mass [52].

Although the experiments in this way reconstruct the pole mass (or something close to it), theoretically it would be more desirable to have a short-distance mass, free of \( \mathcal{O}(\Lambda_{\text{QCD}}) \) ambiguities. Such is the \( \overline{\text{MS}} \) mass \( \bar{m}(\mu) \), evaluated at some scale \( \mu \), whose relation to the pole mass is known in QCD to three loops analytically [53] and four loops numerically [54]. For \( \mu \) one often takes the implicit value found when intersecting the \( \bar{m}(\mu) \) curve with the \( \bar{m}(\mu) = \mu \) axis, yielding \( \bar{m}(\bar{m}) \). The \( \overline{\text{MS}} \) mass \( \bar{m}(\mu) \) may be extracted indirectly, by comparing, for instance, the measured inclusive cross section with the theoretical one expressed in the \( \overline{\text{MS}} \) mass [55]. Of course, such an indirect measurement will depend upon the accuracy of the theoretical calculation of the inclusive cross section, and its sensitivity to the mass. A recent evaluation by D0 [56] along these lines yields \( \bar{m}(m) = 154.5 \text{ GeV/c}^2 \), if the cross section is evaluated to NLO + NNLL accuracy, [57] or \( \bar{m}(m) = 160.0 \text{ GeV/c}^2 \), if the cross section is evaluated to NNLO accuracy, [55,58] with an uncertainty in both cases of about 5 GeV/c^2.

There are other definitions of short-distance masses, inspired by the top quark pair production near threshold at a future \( e^+e^- \) collider. One is the \( 1S \) mass [59,61], which is related to the peak position in the cross section for \( e^+e^- \rightarrow t\bar{t} \), and is defined as half the perturbative mass of a fictitious toponium ground state, where the top quark is assumed to be stable.
The relation between the $1S$ mass and the pole mass is known to higher orders in QCD. As both the $1S$ mass and the $\overline{\text{MS}}$ mass are short-distance masses, the relation between them is $\mathcal{O}(\Lambda_{\text{QCD}}^2/m)$. At lepton colliders, it might be feasible to determine the $1S$ top mass with a precision of about 100 MeV (see this reference also for an overview of other methods). Another promising short-distance mass is the potential-subtracted mass, which employs the fact that the IR sensitive part of the pole, discussed in section 2.3 cancels against the IR sensitivity of the top-antitop Coulomb potential in threshold production.

Current procedures to measure the top mass are the template method, which uses distributions of top mass values obtained from the event kinematics, and compares them to distribution templates for reference top mass values, and the matrix-element method (MEM), which uses the (tree level) matrix elements to estimate the likelihood of each experimental event for kinematic configurations which come from events of a given top mass. Improvements of the methods above have been proposed. As regards the former, the template overlap method for infrared safe jet observables has been put forward, which is based on the fact that the energy flow in jets which come from the decay of highly boosted top decay products is different from the one in jets which come from the QCD background. As regards the MEM, the inclusion of QCD radiation effects and the computation of NLO weighted events have been proposed.

Alternative methods to measure the top mass are also under consideration. Mostly use proxy variables that are to a varying degree sensitive to the mass and can be accurately calculated and measured. One of them uses the leptonic final states of a $J/\psi$: one may consider the process $pp \rightarrow (t \rightarrow W^+ + b \rightarrow W^+ + J/\psi) + (\overline{t} \rightarrow W^- + \overline{b})$ and require that the $W^-$ decays hadronically, the $W^+$ decays leptonically and the $J/\psi$ decays into leptons, typically muons. Then the invariant mass distribution of the $J/\psi$ and an isolated lepton can be used to evaluate the top mass. Since no jets are involved, the measurement is not plagued by jet energy scale (JES) uncertainties, which allows for an accurate reconstruction of the $m_{J/\psi l}$ invariant mass, with a projected $\mathcal{O}(1\text{GeV})$ error on the top mass. However, the leptonic decays reduce the rate substantially, and a large integrated luminosity, of the order of tens of $\text{fb}^{-1}$, is required. The error on the top mass evaluation can be further improved by including the NLO QCD corrections to production and decay.

Some methods use the kinematic distributions of the dilepton channel to either determine the pole mass while being little sensitive to long-distance effects or to perform a simultaneous evaluation of the top-quark, $W$-boson and neutrino masses, basing it on end-point determinations in the kinematic distributions. This may be convenient in the investigation of New Physics models, where several masses in a decay chain may be unknown.

Other methods use the invariant mass $m_{t\overline{t}}$ of the $t\overline{t}$ pair, or examine $t\overline{t}$ production in association with a jet, and use the invariant mass $m_{t\overline{t}\text{jet}}$ of the $t\overline{t}$-jet system. These methods complement top mass measurements from the $t\overline{t}$ total cross section.

Finally, a novel method exploits the large top Yukawa coupling to extract the top mass from loop effects in flavour physics observables.
2.6 Spin and angular correlations

Part of the attractiveness of the top quark as a study object is its power to self-analyze its spin, through its purely left-handed SM weak decay. This is both a useful aid in signal-background separation, and itself a property worthy of detailed scrutiny, as certain new physics models could introduce right-handed couplings. The correlation between top spin and directional emission probability for its decay products is expressed through

$$\frac{d \ln \Gamma_f}{d \cos \chi_f} = \frac{1}{2} (1 + \alpha_f \cos \chi_f) ,$$  \hspace{1cm} (23)

where $|\alpha_f| \leq 1$, with 1 indicating 100% correlation. Note that in the NWA the correlation between the production process and the spin of the produced top quark is indicated in eq. (11). For the dominant decay mode

$$t \rightarrow b + W^+ (\rightarrow l^+ + \nu),$$  \hspace{1cm} (24)

at lowest order, we have $\alpha_b = -0.4$, $\alpha_\nu = -0.3$, $\alpha_W = 0.4$, $\alpha_l = 1$. QCD corrections to these values are small \cite{80,81}. The charged lepton direction (or the down-type quark in a hadronic decay of the intermediate $W$) is indeed nearly 100% correlated with the top quark spin. This is notably more than for its parent $W$ boson, a consequence of interference of two amplitudes with different intermediate $W$ polarizations.

In single-top quark production, which occurs via the charged weak interaction, the top is produced left-handed, so a correlation should be a clear feature of the production process and serve as a discriminant to suppress the background. In top quark pair production a correlation of an individual quark with a fixed direction is absent\footnote{There is a tiny correlation due $Z$-boson mediated production.}, however there is a clear correlation between the top and anti-top spins. The size of the correlation depends on the choice of reference axes $\hat{a}, \hat{b}$ \cite{82,84}. At the Tevatron the beam direction $\hat{a} = \hat{b} = \hat{p}$ is good choice, at the LHC the helicity axes $\hat{a} = \hat{b} = \hat{k}_{top}$ should give near-maximal correlation

$$\frac{d \sigma}{d \cos \theta_a d \cos \theta_b} = \frac{\sigma}{4} (1 + B_1 \cos \theta_a + B_2 \cos \theta_b - C \cos \theta_a \cos \theta_b) .$$  \hspace{1cm} (25)

Indeed, the correlation coefficient $C$ depends on the correlation axis. Thus, at LO in QCD, the values for $\{ C_{hel}, C_{beam} \}$ at the Tevatron (LHC) is $\{0.47, 0.93\} (\{0.32, -0.01\})$. NLO corrections modify these numbers somewhat \cite{85}. BSM models that influence the pair production mechanism (e.g. new resonances) could noticeably influence these correlations.

There is also the interesting possibility of azimuthal angular distributions as indicators of new physics. Thus, in the dilepton decay channel, after an invariant mass cut, $t\bar{t}$ spin correlations may be revealed through the $\Delta \phi$ distribution of leptons in the laboratory frame \cite{86}. This observable is quite robust, as the correlation remains visible even after summing over spurious neutrino momentum resolutions, and persists at NLO \cite{87}. \hfill
Other angular distributions can function as quite selective probes of new physics \[88,89\]. For instance, if a $Z'$ would polarize tops at production, the azimuthal asymmetry

$$A_\phi = \frac{\sigma(\cos \phi_l > 0) - \sigma(\cos \phi_l > 0)}{\sigma(\cos \phi_l > 0) + \sigma(\cos \phi_l > 0)},$$

(26)

where $\phi_l$ is the azimuthal angle of the lepton with respect to the beam-top plane, would be sensitive to the amount of left-handed and right-handed coupling, even more so when judicious cuts on the $p_T$ of the top are chosen. When a charged Higgs is present, such an asymmetry would help distinguish $Wt$ from $H^{-}t$ production.

3 Top production

Having discussed issues concerning top quark decays, we now turn to aspects of top quark production. In this section we discuss a number of much studied top quark production observables. For each we review the theoretical issues, and present experimental status.

3.1 Top pair production cross section

Let us first discuss the cross section measurements from the four experiments that have collected tops in large quantities. Note that besides cross sections inferred from specific final states, combinations are being made that consist of analyses with different final states, with somewhat different integrated luminosities.

At the Tevatron at 1.96 TeV the measured pair production cross sections, based on almost all of the collected data, are

- CDF : $7.63 \pm 0.31 \text{ (stat)} \pm 0.39 \text{ (sys)} \pm 0.15 \text{ (th)} \text{ pb}$,
- D0 : $7.56 \pm 0.20 \text{ (stat)} \pm 0.56 \text{ (sys)} \text{ pb}$,
- Tevatron combined : $7.60 \pm 0.20 \text{ (stat)} \pm 0.36 \text{ (sys)} \text{ pb}$.

(27)

The combination shown \[92\] has a measured uncertainty of about 5.4%. The best present calculation \[93\] yields $7.24 + 0.23 - 0.27 \text{pb (3.4%)}$.

The measured pair production cross sections by ATLAS and CMS at 7 TeV are

- ATLAS : $177 \pm 3 \text{ (stat)} \pm 8 \text{ (sys)} \pm 7 \text{ (lum)} \text{ pb}$,
- CMS : $166 \pm 2 \text{ (stat)} \pm 11 \text{ (sys)} \pm 8 \text{ (lum)} \text{ pb}$,
- LHC Combined : $173.3 \pm 2.3 \text{ (stat)} \pm 9.8 \text{ (sys)} \text{ pb}$.

(28)

The combined result \[94,95\] and its measured uncertainty of about 5% is to be compared to the best present calculation \[93\] which yields $172 + 6.4 - 7.5 \text{pb (5.7%)}$.

For the 8 TeV data we quote two recent results, for the ATLAS di-lepton ($e\mu$) for 20.3pb$^{-1}$, and the CMS di-lepton ($e\mu$) channel for 5.3pb$^{-1}$, respectively

- ATLAS : $242.4 \pm 1.7 \text{ (stat)} \pm 5.5 \text{ (sys)} \pm 7.5 \text{ (lum)} \text{ pb}$,
CMS : $239.0 \pm 2.6 \text{ (stat)} \pm 11.9 \text{ (sys)} \pm 6.2 \text{ (lum)} \text{ pb}$,
LHC Combined : $241.5 \pm 1.4 \text{ (stat)} \pm 5.7 \text{ (sys)} \pm 6.2 \text{ (lum)} \text{ pb}$, \hspace{1cm} (29)

with an uncertainty of about 3.5% \cite{96,97}. The best current calculation \cite{93} yields $245.8 \pm 8.8 - 10.6 \text{ pb (5.6\%)}$. Interestingly, the experimental uncertainty is now again smaller than the theoretical one, providing a challenge to theory.

First results at 13 TeV are now appearing, with both CMS \cite{98} and ATLAS results, still with large errors, in agreement with theory predictions.

For both colliders and for each collision energy the measurements are clearly in agreement with each other, and with the best theoretical calculations, which we discuss below. The remarkable agreement among different collision types and energies gives us solid confidence in the value and structure of the top quark QCD coupling.

Let us now review the status of, and main ideas behind theoretical calculations for top quark pair production. The inclusive top pair production cross section has always played a role that is both useful and instructive in perturbative QCD, because it only involves QCD couplings. It moreover features a truly large produced mass whose effects play a crucial role in both the matrix elements and the phase space measure. The NLO corrections were computed \cite{99,102} in the late 80’s. For many years these were among the most difficult one-loop calculations done. In these first calculations phase space was (partially) integrated over in analytical way; a fully differential calculation was completed shortly thereafter \cite{103}. The combination of such a fully differential calculation with parton showers, such as \texttt{MC@NLO} \cite{104,105} and \texttt{POWHEG} \cite{106,107} is now the state of art at this order in perturbation theory. These codes combine the virtues of the exclusiveness of a parton shower event generator with the accuracy of a NLO calculation.

A recent major development has been the completion of the full NNLO calculation \cite{93,108,110} for the inclusive pair production cross section. This is indeed a milestone in top quark physics, even in perturbative QCD as a whole. The result is a hadronic cross section computed with a theoretical accuracy at the few percent level, as already mentioned. The calculations require NNLO corrections to both the $q\bar{q}$ and the $gg$ channel, as well as the NLO corrections to the $qg$ channels. For both the $q\bar{q}$ and $gg$ channel, the second order corrections are composed of three classes of contributions, some computed at different times by various authors. These are (i) the two-loop corrections, (ii) the one-loop plus one real emission correction, and (iii) the double real emission contribution. The double-real emission calculations were computed earlier \cite{111,113}. The one-loop, one real emission contributions are known, since the NLO calculation for $t\bar{t} + \text{ jet}$ is available \cite{114,115}. The two-loop virtual corrections have been performed \cite{116,120}. The methods used so far are a combination of analytical and numerical ones. The latter involve solving differential equations in the kinematic invariants, which requires a highly accurate initial condition (chosen to be at high energy), and avoiding singularities in the equations. The double-real emission contribution was achieved through the use of a method called \texttt{STRIPPER} \cite{111}. The one-loop, one-real emission diagrams could be computed with well-established techniques.

The full calculation, altogether a major tour-de-force, has good perturbative convergence and very small uncertainties. Given these properties and the excellent agreement with mea-
measurements, as shown in Fig. 1, a comparison of theory and data for the inclusive cross section can be used more prosaically to infer useful knowledge about the gluon density. A first study in this direction was done [121], demonstrating the feasibility and desirability of this. The top cross section has now been included in the NNPDF3.0 global fit [122]. Electroweak corrections to top pair production have also been computed [123–125], which can be large in certain phase space regions, depending on transverse momentum. They can also impact the charge asymmetry [126]. Calculations including off-shell effects are beginning to appear as well [25, 127].

On top of the exactly calculated orders one can add arbitrarily high orders in approximately using threshold resummation. The latter also underlies some theoretical estimates of the top quark charge asymmetry, discussed in section 3.2, as well as various distributions, so let us review this method briefly here.

When the top quark pair is produced near threshold in hadronic collisions, logarithms whose argument represents the distance to threshold in the perturbative series become numerically large. The definition of the threshold depends on the observable. Thus, for the inclusive cross section the threshold is given by the condition \( T_1 : s - 4m^2 = 0 \). For the transverse momentum distribution we have \( T_2 : s - 4(m^2 + p_T^2) = 0 \), and for the doubly differential distribution in \( p_T \) and rapidity we can choose

\[
T_3 : s - 4(m^2 + p_T^2) \cosh y = 0 \quad \text{or} \quad T_3 : s + t + u - 2m^2 = 0 .
\] (30)

The perturbative series for any of these (differential) cross sections can be in general expressed

![Figure 1: Data and theory for the inclusive top quark pair production cross section at Tevatron and LHC as function of the collider c.m. energy, compared to recent measurements of ATLAS and CMS, compiled by the Top Physics LHC Working Group.](image)
\[ d_i \sigma(T_i) = \sum_n \sum_k \alpha_s^n c_{n,k}^i \ln^k(T_i), \]

(31)

plus non-logarithmic terms. Here \( T_i \) represents any of the threshold conditions, suitably normalized, for the observables enumerated by \( i \). Note that it is allowed to use e.g. \( T_2 \) for the inclusive cross section, by first analyzing \( d\sigma/dp_T \) and then integrating over \( p_T \), and similarly for \( T_3 \). For any complete fixed order calculation this will give the same answer, but if one only selects the logarithmic terms because the exact answer is unknown, numerical differences will occur. Such kinematic differences can then be viewed as theoretical uncertainties \[128\].

The threshold logarithms result from integration over phase space regions where the emitted gluons are soft and/or collinear to their on-shell emitter. Resummation concerns itself with carrying out the sum in Eq. (31), and the result takes the generic form

\[ d\sigma = \exp \left( L g_0(\alpha_s L) + g_1(\alpha_s L) + \alpha_s g_2(\alpha_s L) + \ldots \right) \times C(\alpha_s). \]

(32)

Including up to the function \( g_i \) in the exponent amounts to \( N^i \text{LL} \) resummation, with the coefficient \( C(\alpha_s) \) then evaluated to order \( i - 1 \). Key benefits of threshold resummation are (i) gaining all-order control of the large terms which plague fixed-order perturbation theory, to restore predictive power, and (ii) reduction of scale uncertainty. Regarding the first point, the reason these resummmable terms are large for the top quark pair inclusive cross section is that, while the hadronic cross section is Sudakov suppressed near threshold, the PDF’s are over-suppressed, which the partonic cross section must then partially compensate for. Regarding the second point, when examining the sources of scale dependence, they occur both in the PDF and in the partonic cross section now both in the exponent, which improves the cancellation \[129\].

The state-of-the-art accuracy for threshold resummation for inclusive pair production cross section at present is NNLL \[130–133\]. A consistent combination of NNLL accuracy in both threshold and Coulomb corrections has now also been achieved \[77\]. The latter are only relevant for threshold \( T_1 \) and behave as \( (\alpha_s/\beta)^n \), with \( \beta^2 = 1 - 4m^2/s \). From such all-order results, approximate NNLO results were constructed before the completion of the exact calculation. This is of particular interest for thresholds \( T_1 \) and \( T_3 \). The latter, being dependent on \( t \) and \( u \), then allows estimating threshold resummation corrections to the forward-backward asymmetry, a point we return to in section 3.2. Other approximate NNLO calculations use threshold \( T_3 \) and results based on these \[133\] are typically larger than for \( T_1 \), closer to the exact result; estimates are also made for approximate NNNLO \[134\].

As mentioned above, calculations using \( T_3 \) can assign ambiguities due to using either pair-invariant mass (PIM) or one-particle inclusive (1PI) kinematics in the precise definition of the threshold to a theoretical error \[78,128\].

The various theoretical calculations are available in a number of codes, such as HATHOR \[135\] (contains full NNLO corrections, and possibility of using a running top quark mass), TOP++ \[136\] (contains full NNLO corrections, and NNLL threshold resummation), TOP1XS \[137\] (contains NLO, approximations for NNLO, and NNLL resummation, including Coulomb corrections).
In the above, the top quarks are treated as on-shell stable particles, using the narrow-width approximation. It is interesting to include in the full description also the top quark decays, including the effects of off-shellness and spin-correlations. Thus, one considers then as final state of interest $WWbb$. For zero $b$-quark mass two groups have computed the NLO corrections to this production process \[127, 138\], establishing an interesting tool to study such effects.

### 3.2 Charge asymmetry

A different test of the QCD production mechanism of top quarks, one that has received much attention in recent years, is the charge asymmetry: the normalized difference in production rate between top and anti-top at some fixed angle or rapidity

$$A_t(y) = \frac{N_t(y) - N_{\bar{t}}(y)}{N_t(y) + N_{\bar{t}}(y)}.$$  \hspace{1cm} (33)

While electroweak production via a $Z$-boson could produce a (very small) asymmetry at LO, QCD itself produces it at $O(\alpha^3_s)$ through a term proportional to the SU(3) $d_{abc}$ symbol \[100, 102, 114, 139\]. A more precise look \[139\] shows that the asymmetry is due to an interference between C-odd and C-even terms. In top quark pair production in the $q\bar{q}$ channel this amounts to the Born diagram and the one-loop box diagram, respectively. When computing such an interference contribution, the asymmetry reveals itself in terms of the Mandelstam variables $t$ and $u$ as terms that are odd under $t \leftrightarrow u$ interchange, e.g. $t^2 - u^2$.

In $t\bar{t}$ plus 1 jet production an asymmetry can already occur at tree level (essentially, this amounts to a different cut of the same amplitude). Measurements \[140-143\] by the Tevatron experiments show substantial deviations from the Standard Model prediction for pair production, especially a deviation of more than 3 standard deviations by CDF at large invariant $t\bar{t}$ masses \[141\]. For this reason there has been considerable interest in this observable.

We discuss here the Standard Model calculations for this observable. A discussion of the many studies of specific New Physics effects on the charge or forward-backward asymmetry is beyond the scope of this review.

The effect of this interference can be understood more intuitively by the statement that the incoming quarks, via the interference, tend to repel the produced top quarks towards larger rapidity, and/or attract the produced anti-top quarks toward slightly smaller rapidities. The net effect, therefore, at the Tevatron, where the top- anti-top pairs are produced in $q\bar{q}$ annihilation, is a shift of the top quark rapidity distribution towards larger rapidity, and of the anti-top distribution towards smaller values. This clearly creates a $y$-dependent asymmetry of the type (33). Because of these shifts, this also corresponds to a forward-backward asymmetry $A_{FB}$.

This intuition may also be obtained in threshold resummation from the so-called soft anomalous dimension in the $q\bar{q}$ channel, which governs subleading threshold logarithms; leading logarithms are symmetric under $t \leftrightarrow u$ interchange, and therefore cancel in the
asymmetry. The subleading contribution in the $q\bar{q}$ channel reads \cite{144}

$$
\Delta \sigma = \exp \left\{ \alpha_s L \left[ \frac{32}{6} - \frac{27}{6} \right] \ln \frac{u}{t} \right\} \sigma_{\text{Born}},
$$

(34)

where $L$ is the threshold logarithm. This expression, through $\ln(u/t)$, is indeed antisymmetric under $t \leftrightarrow u$ interchange.

Since the leading contribution to this effect for NLO pair production involves a loop diagram, the asymmetry itself is then of leading order accuracy. The impact of even higher orders is then very interesting. They have first been estimated from approximate, resummation based calculations to NLL \cite{128,145} and NNLL \cite{146,148}. For this only resummations based on threshold $T_3$, see Eq. (30), can be used, as the other two thresholds are not sensitive to the top quark rapidity. The higher order corrections so computed are small, and reasonably insensitive to scale variations. Hence, based on these approximate calculations, the discrepancy would persist, although in recent analyses by D0 \cite{149,150} it is found to be not so large.

For the $t\bar{t}$ case the electroweak corrections have been calculated \cite{126,151,152}. They are unexpectedly large, thus also diminishing the overall discrepancy. It is worth noting that, from a slightly different perspective, effects of colour reconnection in parton shower algorithms can already cause an asymmetry at what is formally leading order \cite{153}.

Very recently the exact calculations for the charge asymmetry to NNLO were completed \cite{154}. Upon taking into account the second order QCD corrections in addition to the first order EW corrections a shift of no less than 27% with respect to the NLO QCD asymmetry was found, yielding a value of $A_{FB} = 0.095 \pm 0.007$. This is now in good agreement with the most recent D0 measurement of $0.106 \pm 0.03$ \cite{155}, and only somewhat below the CDF \cite{156} value of $0.164 \pm 0.047$, which seems to settle this issue to a large extent.

Besides defining the asymmetry in terms of the top quark itself \cite{33}, one may define it also in terms of the leptons produced in top and/or anti-top decay, either in the lepton-plus-jets or the di-lepton channel. The $A_{FB}^{ll}$ asymmetry will be in general a little washed out, but leptons are relatively easy to measure. There is however still a need for unfolding due to limited acceptance. A recent compilation of theory predictions including leptonic asymmetries is available \cite{151}.

At the Tevatron, CDF and D0 have performed a set of measurements for various types of asymmetries. At the constructed top quark level the measured asymmetries exceed the theory prediction by a few standard deviations. We already mentioned the top quark level asymmetries by D0 and CDF. Recent $A_{FB}^{ll}$ measurements in the lepton-plus-jets channels corrected to the parton level are $16.4 \pm 4.7\%$ (CDF) and $12.6 \pm 6.5\%$ (D0), vs. $8.8 \pm 0.6\%$ according to the SM. An overview can be found in Ref. \cite{157}.

As noted above, the charge asymmetry is present at leading order in $t\bar{t} + \text{jet}$ production. However, here NLO corrections \cite{114,115} tend to wash out the asymmetry for this reaction. An explanation for this effect was given in Ref. \cite{115}, based on the following structure of the NLO forward-backward asymmetry for this reaction

$$
A_{FB}(t\bar{t}j) = \alpha_s^2 \frac{C}{\ln(m/p_{T,j})} + \alpha_s^4 D_{\text{hard}},
$$

(35)
The second term, appearing at NLO, cancels the first as they have opposite signs. The inverse logarithm is due to the fact that the denominator in the asymmetry has a higher power of leading soft logarithms. Also for $t\bar{t}jj$ the NLO term seems to reduce the LO contribution to the asymmetry [158].

At the LHC, the net effect of the QCD induced asymmetry is an overall broadening of the top quark rapidity distributions and a slight narrowing of the anti-top rapidity distribution. Here there is therefore no forward-backward asymmetry, but a charge asymmetry that is most pronounced at larger rapidities. One proposal [159] is e.g. to assess the asymmetry using only events with (anti)tops above a certain minimum rapidity, of about 1.5. New observables with promising sensitivity have been proposed [160, 161].

At 7 TeV, a combination of CMS and ATLAS measurements [162, 163] of the charge asymmetry yields $0.005 \pm 0.007 \text{(stat)} \pm 0.006 \text{(syst)}$, in agreement with the NLO QCD and electroweak theory [126], although also compatible with a lack of asymmetry.

3.3 Invariant mass and other distributions

Besides inclusive observables such as the cross section and charge asymmetry, differential distribution afford a more detailed look into production dynamics. For instance, a moderate enhancement in tails of distributions due to New Physics would possibly not be visible in the inclusive observables. An important distribution for both the Tevatron and the LHC is in the invariant mass $M_{tt}$. The shape of the distribution in the SM has a relatively small uncertainty. It has been computed in approximate NNLO in resummed NNLL accuracy [57]. It is sensitive to the top mass, and may thus assist in determining it. Shape deviations from the QCD predictions in this distribution (peaks, peak-dip structures) are telltales of new physics, such as resonances with various spin, parity and colour quantum numbers. A study employing the flexibility of MadGraph in a bottom-up approach was performed in Ref. [74], in which only the most generic aspects of new models are used. Given that the exact charge asymmetry calculation [154] was based on a fully differential NNLO calculation for pair production, various differential distributions will soon be available at that accuracy.

Approximate calculations based on resummation methods, discussed earlier, have already been done, e.g. for the invariant mass distribution [164], and for single particle inclusive distributions at the NNLO and beyond level [131, 165, 166]. Measurements of differential distributions in variables associated with the top quark pair have been performed, as well as of single particle inclusive distributions [167, 171].

3.4 Single top production

Tops can be produced singly through the weak interaction, in processes that are customarily categorized by names referring to kinematics in the Born approximation, see Fig. 2. Important aspects of single-top production are that $V_{tb}$ can be directly measured without assuming three fermion generations, and that the chiral structure of the associated vertex can be tested. The latter is the case because a single top produced in this way is highly polarized, which offers a chance to study the chirality of the coupling via spin correlations,
as discussed in eq. (11), section 2.3. Another feature is that the dominant \( t \)-channel at the LHC, when confronting measurements with a 5-flavour NLO calculation, will help determine the \( b \)-quark density. (In a 4-flavour scheme, one would demand an extra forward (b) jet). Finally, it is interesting that the different single top production processes are each sensitive to different varieties of new physics. Thus, the \( s \)-channel will be sensitive to e.g. \( W' \) resonances, the \( t \)-channel to FCNC’s. Note that the channel separation according to Fig. 2 holds to NLO, but not to all orders; at higher orders interference can take place between channels. Let us however maintain this separation, and discuss the channels separately.

### 3.4.1 \( s \) and \( t \) channel

Experimentally, both of these single top production processes turned out to be rather more difficult to separate from backgrounds than expected, as the latter are large, and similar in shape to the signals. Based on samples of up to 9.7 fb\(^{-1} \) per experiment, the Tevatron combination \[172\] of a number of CDF and D0 measurements yields an inclusive single top production cross section of

\[
\sigma = 3.30^{+0.52}_{-0.40} \text{ pb},
\]

and a measurement of \( |V_{tb}| = 1.02^{+0.06}_{-0.05} \). Furthermore, CDF and D0 have reported the Tevatron combination \[173\] of inclusive single top production in the \( s \) channel only, with a cross section of

\[
\sigma_s = 1.29^{+0.26}_{-0.24} \text{ pb}.
\]

At the LHC at a centre-of-mass energy of 7 TeV, the inclusive SM production rates of the \( s \)-channel, \( t \)-channel and \( Wt \) channel are approximately 4.6, 65 and 16 pb respectively; at 8 TeV they are 5.6, 88 and 22 pb, respectively. The \( t \)-channel yields clearly the dominant contribution. Besides interesting in its own right, the \( t \)-channel process is a background to many new physics processes involving both neutral and charged Higgs production. Based on samples of 4.6 fb\(^{-1} \) by ATLAS and 1.14 fb\(^{-1} \) by CMS of the run at 7 TeV, the \( t \)-channel cross section is \[174,175\]

\[
\text{ATLAS} : \quad \sigma_t = 68 \pm 2 \text{ (stat) \pm 8 (sys) pb}, \\
\text{CMS} : \quad \sigma_t = 67.2 \pm 3.7 \text{ (stat) \pm 4.8 (sys) pb}.
\]
Based on samples of 20.3 fb$^{-1}$ by ATLAS and 19.7 fb$^{-1}$ by CMS of the run at 8 TeV, the $t$-channel cross section is $[176, 177]$

\[
\text{ATLAS} : \quad \sigma_t = 82.6 \pm 1.2 \text{ (stat)} \pm 12.0 \text{ (sys)} \text{ pb}, \\
\text{CMS} : \quad \sigma_t = 83.6 \pm 2.3 \text{ (stat)} \pm 7.4 \text{ (sys)} \text{ pb}. \quad (39)
\]

A combination, based on partial data samples of 5.8 fb$^{-1}$ by ATLAS and 5.0 fb$^{-1}$ by CMS, yields $[178]$

\[
\text{ATLAS/CMS} : \quad \sigma_t = 85 \pm 4 \text{ (stat)} \pm 11 \text{ (sys)} \pm 3 \text{ (lumi)} \text{ pb}, \quad (40)
\]

with a total uncertainty of 12.1 pb, and in good agreement with the SM prediction. For all the measurements above, the values of $V_{tb}$ which are extracted are compatible with 1.

Based on the sample of 20.3 fb$^{-1}$ of the run at 8 TeV, ATLAS has found a first evidence of $s$-channel production $[179]$ at 3.2$\sigma$ level

\[
\text{ATLAS} : \quad \sigma_s = 4.8 \pm 1.1 \text{ (stat)}^{+2.2}_{-2.0} \text{ (sys)} \text{ pb}. \quad (41)
\]

On the theory side, the single top cross section has been computed at NLO accuracy in the QCD and electroweak corrections $[24, 25, 180–192]$, including resummations $[193–196]$ and matching NLO computations to parton showers $[197–199]$; and at NNLO accuracy in the QCD corrections $[200]$. The NLO (and NNLO) corrections are at a few percent level, and within errors the measured cross sections agree with them. Approximate NNLO $p_T$ distributions have recently appeared $[201, 202]$.

### 3.4.2 Wt associated production

A subtle and interesting issue arises in the $Wt$ mode of single top production at NLO. In the radiative corrections some diagrams contain an intermediate anti-top decaying into a $W$ and anti-down type quark, that can become resonant. From another viewpoint, these diagrams can be interpreted as LO $t\bar{t}$ on-shell production, with subsequent $\bar{t}$ decay, see Fig. 3. One is therefore faced with the issue to what extent the $Wt$ and $t\bar{t}$ can be properly defined and separated as individual processes. To this end several definitions of the $Wt$ channel have been given in the literature, each with the aim of recovering a well-behaved expansion in $\alpha_s$. This problem of interference is of course not uncommon in computations at order of at least $O(g_w^2\alpha_s^2)$. The cross section at this order has been previously presented in Refs. $[203–205]$, where only tree-level graphs were considered, and in Refs. $[187, 206, 207]$, where one-loop contributions were included as well. The rather vexing point here is that the $t\bar{t}$ process with
which the \( Wt \) interferes is an order of magnitude larger, rendering the NLO correction much too large.

In Ref. [208] this interference issue was addressed extensively in the context of event generation, in particular the MC@NLO framework (POWHEG has implemented the same method [209]). Two different procedures for subtracting the doubly-resonant contributions and thereby recovering a perturbatively well-behaved \( Wt \) cross section were defined. In “Diagram Removal (DR)” the graphs in Fig. 3 were eliminated from the calculation, while in “Diagram Subtraction (DS)” the doubly resonant contribution was removed via a judiciously constructed subtraction term. The DS procedure leads to the following expression for the cross section

\[
d\sigma^{(2)} + \sum_{\alpha\beta} \int \frac{dx_1 dx_2}{x_1 x_2 S} L_{\alpha\beta} \left( \hat{S}_{\alpha\beta} + I_{\alpha\beta} + D_{\alpha\beta} - \tilde{D}_{\alpha\beta} \right) d\phi_3, \tag{42}
\]

where \( \alpha\beta \) labels the initial state channel in which the doubly-resonant contribution occurs: \( gg \) or \( q\bar{q} \). \( \hat{S} \) is the square of the non-resonant diagrams, \( I \) their interference with \( D \), the square of graphs of Fig. 3. The subtraction term \( \tilde{D} \) requires careful construction [208]. It was shown that, with suitable cuts, the interference terms are small. From Eq. (42) one sees that the difference of DR and DS in essence consists of the interference term. A particularly suitable cut is imposing a maximum on the \( p_T \) of the second hardest \( b \)-flavoured hadron, a generalization of a proposal made in Ref. [187]. Thus defined, the \( Wt \) and \( t\bar{t} \) cross sections can be separately considered to NLO.

The experimental status of this production mode at present is as follows. In the 7 TeV run, ATLAS [210] and CMS [211] have measured the \( Wt \)-channel cross section, with the results

\[
\begin{align*}
\text{ATLAS}[2.05 \text{ fb}^{-1}] : & \quad \sigma_{Wt} = 16.8 \pm 2.9 \text{ (stat) } \pm 4.9 \text{(sys) pb}, \\
\text{CMS}[4.9 \text{ fb}^{-1}] : & \quad \sigma_{Wt} = 16^{+5}_{-4} \text{ pb}.
\end{align*}
\]

In the 8 TeV run, a combination of \( Wt \)-channel measurements has been performed [212,213], based on a data set of 12.2 fb\(^{-1}\) by CMS and 20.3 fb\(^{-1}\) by ATLAS

\[
\text{ATLAS/CMS} : \quad \sigma_{Wt} = 25.0 \pm 1.4 \text{ (stat) } \pm 4.4 \text{(sys) } \pm 0.7 \text{(lumi) pb}, \tag{44}
\]

with a total uncertainty of 4.7 pb.

Within errors, the \( Wt \)-channel cross section measurements above agree with the NLO calculations [187,206,207], approximate NNLO [214] and the NLO plus parton showers discussed above [208,209,215].

One way to avoid the above difficulties in separating \( Wt \) from \( t\bar{t} \) is to consider the common final state \( WWbb \) (in the 4-flavour scheme) and not ask if there were one or two intermediate top quarks involved in producing this final state – zero intermediate top quarks is also a possibility here. In Refs. [216,217] a unified approach in the 4-flavour scheme was taken in which both \( Wt \) and \( t\bar{t} \) produce as final state \( WWbb \), and the NLO corrections were computed.
3.5 Associated top production at higher order

One can also consider processes where a top pair or a single top are produced in association with other particles. Given its relevance for the measurement of the Higgs-top Yukawa coupling, clearly the most important associated production is the production of a top pair in association with a Higgs boson, $t\bar{t}h$. This process is known to NLO accuracy in QCD, at parton level \cite{218,219} and interfaced to parton showers \cite{220,221}. Also the electroweak corrections have been computed \cite{222,223}.

The rapid evolution of computations of scattering processes with many final-state particles to NLO accuracy in QCD – the so-called NLO revolution – has left its mark on processes involving top production as well, yielding calculations that would have been hard to imagine some years ago. Accordingly, many important backgrounds to top pair production in association with a Higgs boson, with subsequent decay of the Higgs boson into a bottom-quark pair or into a pair of photons, have been computed to NLO accuracy. In particular, production of a top pair in association with a jet is known to NLO accuracy at parton level \cite{114,115}, including the top decays \cite{224}, as well as non-resonant diagrams, interferences and off-shell effects \cite{225}, and interfaced with parton showers \cite{226,228}; top pair production in association with two jets is known to NLO accuracy at parton level \cite{158,229}, and interfaced with parton showers \cite{230}; production of a top pair in association with a $b\bar{b}$ pair is known at parton level \cite{231,233} and interfaced with parton showers \cite{234,236}; production of a top pair in association with a photon is known to NLO accuracy at parton level \cite{237} and interfaced with parton showers \cite{26,238}; production of a top pair in association with two photons is known to NLO accuracy at parton level and interfaced with parton showers \cite{26,239}.

Furthermore, the production of a top pair in association with a $Z$ boson is known to NLO accuracy at parton level \cite{240,241} and interfaced with parton showers \cite{242,243}, which is relevant to measure the $t\bar{t}Z$ coupling \cite{242,243}; the production of a top pair in association with a $W$ boson is known to NLO accuracy \cite{246,248} and interfaced with parton showers \cite{248}, which can be used as a tool to examine the top-quark charge asymmetry. Also the electroweak corrections to the production of a top pair in association with a $W/Z$ boson have been computed \cite{223}. The production of a top pair in association with two vector bosons, be it either $W$, $Z$ bosons or photons, is known to NLO accuracy at parton level, and interfaced with parton showers \cite{26,249}; as well as the production of a top pair in association with a vector boson and a jet is known to NLO accuracy at parton level, and interfaced with parton showers \cite{26}. Finally, the production of two top pairs is known to NLO accuracy at parton level \cite{249,250}, which can be used as a benchmark process to test New Physics signals.

In addition, single top production in association with a $W$ boson is known to NLO accuracy at parton level \cite{208,217}, and interfaced to parton showers \cite{208}, which is relevant for the $Wt$ mode of single top production, see Sect. \ref{sec:single_top_production}. Single top production in association with a $Z$ boson is known to NLO accuracy at parton level \cite{251}, which is a background to flavour changing neutral current decays of the top in $t\bar{t}$ production. Various single top production processes in association with a $b$ quark and a $Z$ boson, a photon or a jet are available to NLO accuracy in Ref. \cite{26}.
4 Conclusions

Top quark physics is at present at a pivotal point, in the early days of Run II of the LHC. Rather accurate studies of top quark observables from Tevatron and LHC Run I data have been done, but the bulk of (higher energy) data is still to be collected. Also in top physics the Standard Model has withstood tests so far, but many highly detailed and varied tests by the LHC experiments will follow.

Top’s attractiveness as a study object has by no means diminished. On the contrary, new observables are being enlisted for this enterprise. The characteristics of production and decay, in association with other particles, can be very revealing. The examination of many (multi-)differential distributions, a full accounting of spin and off-shellness, and especially its interaction with the Higgs boson are all still to come.

As we have reviewed here, the theoretical tools for top physics studies are of high quality, and still keep improving with remarkable pace. We are therefore confident that the top quark will remain in the focus of attention for a good many more years.

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