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Combined analysis of effective Higgs portal dark matter models

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We combine and extend the analyses of effective scalar, vector, Majorana and Dirac fermion Higgs portal models of dark matter (DM), in which DM couples to the Standard Model (SM) Higgs boson via an operator of the form $\mathcal{O}_{DM} H^\dagger H$. For the fermion models, we take an admixture of scalar $\bar{\psi}\psi$ and pseudoscalar $\bar{\psi}\gamma_5\psi$ interaction terms. For each model, we apply constraints on the parameter space based on the Planck measured DM relic density and the LHC limits on the Higgs invisible branching ratio. For the first time, we perform a consistent study of the indirect detection prospects for these models based on the WMAP7/Planck observations of the cosmic microwave background, a combined analysis of 15 dwarf spheroidal galaxies by Fermi-LAT and the upcoming Cherenkov Telescope Array (CTA). We also perform a correct treatment of the momentum-dependent direct search cross section that arises from the pseudoscalar interaction term in the fermionic DM theories. We find, in line with previous studies, that current and future direct search experiments such as LUX and XENON1T can exclude much of the parameter space, and we demonstrate that a joint observation in both indirect and direct searches is possible for high mass weakly interacting massive particles. In the case of a pure pseudoscalar interaction of a fermionic DM candidate, future gamma-ray searches are the only class of experiment capable of probing the high mass range of the theory.

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I. INTRODUCTION

Evidence for dark matter (DM) in our Universe has been accumulating for decades [1]. As none of the particles of the Standard Model (SM) adequately explains a wide range of astrophysical observations of DM, we are forced to look for viable candidates in theories that lie beyond the SM (BSM). It has been noted that a GeV-scale DM candidate can accurately reproduce the observed relic density of DM provided that it has a weak interaction cross section, leading to the so-called “weakly interacting massive particle” (WIMP) miracle [2,3].

Within the WIMP scenario, there are typically multiple complementary experimental probes that exploit the couplings between the WIMP and SM fermions/gauge bosons. In addition, there are two primary theoretical approaches for specifying the SM-WIMP couplings.

One is the top-down approach, where a WIMP is included amongst the new particle content of a BSM theory at high energies, and one can derive the WIMP interactions from the details of this new theory (e.g., supersymmetry with a neutralino WIMP). The other is the bottom-up approach in which effective field theories (EFTs) are constructed from the lowest-dimensional operators allowed in a weak-scale Lagrangian. In this approach, one assumes a DM-SM interaction Lagrangian of the form

$$\mathcal{L}_{int} \supset \Lambda^{-n} \mathcal{O}_{DM}\mathcal{O}_{SM},$$

where $\Lambda$ is the EFT cutoff scale and $\mathcal{O}_{DM}$ ($\mathcal{O}_{SM}$) are the DM (SM) operators that are singlets under the SM gauge groups. An EFT ceases to be valid when the momentum transfer in an experimental process of interest approaches the interaction mediator mass. An option that has generated much interest in the literature is that in which the DM interacts with SM particles via a Higgs portal [4–35]. This is motivated partly by the simplicity of the model in terms of the required BSM particle content, and partly by the fact that the operator $H^\dagger H$ is one of the two lowest-dimensional gauge-invariant

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operators that one can write in the SM (the other being the hypercharge field strength tensor $B_{\mu \nu}$). The recent discovery of the Higgs boson by the ATLAS [36] and CMS [37] experiments of the Large Hadron Collider gives us an extra experimental window into the possible connections between the Higgs boson and DM.

In this paper, we revisit four simple effective field theories for Higgs portal dark matter, namely those in which the DM is assumed to be a scalar, vector or Majorana/Dirac fermion. For the fermion models, one may consider parity-conserving scalar couplings ($\mathcal{O}_{\text{DM}} \propto \bar{\psi} \gamma \psi$) and/or parity-violating pseudoscalar couplings ($\mathcal{O}_{\text{DM}} \propto \bar{\psi} \gamma_5 \gamma \psi$). There are a few omissions in the literature that make a new combined study of these scenarios worthwhile. The first is that most studies have not properly summarized the impacts of current and future indirect search experiments on the parameter space of the models. In some cases (e.g., fermionic dark matter with scalar interactions) this is because the indirect signatures are velocity suppressed, while in others it has been assumed that the ability of direct detection to probe much of the parameter space renders indirect detection less important. We argue instead that it is vital to establish which regions of the model space are visible in multiple experiments, since this might ultimately present us with a much deeper understanding of the DM problem (e.g., one could imagine using a more detailed knowledge of the particle physics obtained in one experiment to better measure astrophysical factors such as the DM halo distribution, as proposed in Ref. [38]). With the approaching availability of the Cherenkov Telescope Array (CTA), it is an opportune moment to explore its expected reach. It is also the case that studies of the direct detection constraints on fermionic models have not always treated the momentum suppression of the direct detection cross section correctly in the case of a pseudoscalar coupling. We remedy this in our study by rederiving the limits from the XENON100 and LUX experiments using a dedicated code that includes an accurate treatment of the momentum-dependent scattering cross sections. However, in such cases, the results do not change significantly and the impact of the direct detection experiments remains negligible. Finally, we perform the same comprehensive study across all of the four models, and thus are able to present a consistent set of detailed results for the first time.

Our paper can be viewed as an extension of the scalar singlet DM model study presented in Ref. [39]. We leave consideration of UV completions of EFTs for future work. A global fit of the same model was presented in Ref. [40]. The scalar, vector and Majorana fermion Higgs portal models we consider have previously been studied in Ref. [41] in light of the WMAP and XENON100, as well as the Higgs invisible width and XENON1T prospects. Current LHC constraints from a CMS vector boson fusion analysis and LHC monojet and mono-Z analyses are shown to be very weak for the scalar singlet DM model in Ref. [42]. Monojet constraints on all Higgs portal models (scalar, fermion and vector) are shown to be weak in Ref. [43]. Constraints on the parameter space of the scalar model from perturbativity and electroweak vacuum stability in the early Universe were imposed first in Ref. [44] and recently in Ref. [45]. Limits on the scalar model from gamma-ray line searches in the Higgs resonance region were considered recently in Ref. [46]. The LUX limits on the scalar DM model were first considered in Ref. [47]. The same paper also evaluated the limits from antiproton data, which were shown to be important in the region of the Higgs resonance and competitive with the LUX limits at higher DM masses. For the fermion models, the corresponding antiproton limits in scalar interactions are weak due to the velocity suppression of the annihilation cross section $\sigma v_{\text{rel}}$. In all cases, the results are strongly dependent on the propagation model. A combination of the parity-conserving and parity-violating terms in fermion models were first considered in Ref. [48] and recently in Refs. [49,50] where it was noted that the parity-violating term can significantly generate a parity-conserving coupling after electroweak symmetry breaking (EWSB).

We aim to provide a complete description of the current and projected future limits on the parameter space of the effective scalar, vector, Majorana and Dirac fermion models in this paper. For each model, we apply currently available constraints from the dark matter relic density, Higgs invisible width, indirect and direct detection limits. For the indirect searches, the current limits we impose come from the WMAP 7-year observations of the cosmic microwave background (WMAP7) and a combined analysis of 15 dwarf galaxy observations by Fermi-LAT; for projected future limits, we use the Planck polarization data, projected improvements in Fermi dwarf galaxy observations and prospects for the planned CTA. Our direct detection data are taken from XENON100 (2012) and LUX (2013).

The paper is organized as follows. In Sec. II, we introduce our four Higgs portal models based on the DM spin. Section III contains a detailed description of the model constraints and their implementations. Our model results and their prospects for detection at current and/or future based experiments are presented in Sec. IV. Our conclusions are given in Sec. V. A detailed derivation of the physical mass basis required to understand the fermionic EFTs after EWSB is given in the Appendix.

II. MODELS

The DM fields are assumed to be SM gauge singlets. We consider four cases for the spin of the DM: scalar ($S$), vector ($V_\mu$), Majorana ($\chi$) and Dirac ($\psi$) fermion.
Lagrangians for the corresponding DM fields, invariant under the symmetries of the SM, are

\[ \mathcal{L}_S = \mathcal{L}_{SM} + \frac{1}{2} (\partial_\mu S)(\partial^\mu S) - \frac{1}{2} \mu_S^2 S^2 - \frac{1}{4!} \lambda_S S^4 - \frac{1}{2} \lambda_{hS} S^2 H^\dagger H, \]  

(2)

\[ \mathcal{L}_V = \mathcal{L}_{SM} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} \mu_V^2 V^\mu V^\nu - \frac{1}{4!} \lambda_V (V_{\mu} V^{\mu})^2 + \frac{1}{2} \lambda_{hV} V^\mu V^\nu H^\dagger H, \]  

(3)

\[ \mathcal{L}_X = \mathcal{L}_{SM} + \frac{1}{2} (i \partial_\mu - m_X) X - \frac{1}{2} \lambda_{hX} (\cos \theta \overline{X} + \sin \theta \overline{X} \gamma_5 X) H^\dagger H, \]  

(4)

\[ \mathcal{L}_\psi = \mathcal{L}_{SM} + \overline{\psi} (i \partial_\mu - m_\psi) \psi - \frac{\lambda_{h\psi}}{\Lambda_\psi} (\cos \theta \overline{\psi} \psi + \sin \theta \overline{\psi} \gamma_5 \psi) H^\dagger H, \]  

(5)

where \( \mathcal{L}_{SM} \) is the SM Lagrangian, \( W_{\mu\nu} \equiv \partial_\mu V_{\nu} - \partial_\nu V_{\mu} \) is the field-strength tensor of the vector field \( V_{\mu} \), and \( H \) is the SM Higgs doublet. The parameter \( \theta \) determines the type of interaction between DM and the Higgs field: \( \cos \theta = 1 \) corresponds to a pure scalar interaction and \( \cos \theta = 0 \) corresponds to a pure pseudoscalar interaction.

DM particles are required to be stable on cosmological time scales. For the models introduced above, their stability is guaranteed by imposing an assumed \( Z_2 \) symmetry: \( X \rightarrow -X \) for \( X \in (S, V_{\mu}, \chi, \psi) \). Under the \( Z_2 \) symmetry, the DM fields \( (S, V_{\mu}, X, \psi) \) are assumed to be odd while the SM fields are assumed to be even. The decay of a DM particle into SM particles is therefore forbidden. Furthermore, the imposed symmetry also prohibits linear and cubic terms in the scalar field Lagrangian as well as the kinetic mixing terms in the vector field-strength tensor \( W_{\mu\nu} \).

As an explicit bare mass term for the DM field is allowed by the \( Z_2 \) symmetry, we have included it in our Lagrangians above for completeness.

The scalar model in Eq. (2) is the simplest extension of the SM and was first introduced 30 years ago [51]. It was later studied by the authors of Refs. [52,53]. It is both theoretically and phenomenologically satisfactory as long as the \( Z_2 \) symmetry remains unbroken. It is renormalizable and hence valid up to high energy scales as long as the Landau pole is not reached. The vector model in Eq. (3) is very simple, compact and appears renormalizable due to the presence of dimension-2 and dimension-4 operators only. However, in reality, it is nonrenormalizable and violates unitarity (in a similar fashion to the four-fermion interaction of the preelectroweak theory). Therefore, it is an effective model which needs to be UV completed. UV complete models will be the subject of future work and hence are considered to be beyond the scope of this study. Simple UV completions can be found in Refs. [54,55]. The Majorana and Dirac fermion models in Eqs. (4) and (5) respectively are also nonrenormalizable, and a suggested UV completion can be found in Ref. [56].

After EWSB, the SM Higgs doublet acquires a vacuum expectation value (VEV). In the unitary gauge, the SM Higgs doublet reduces to

\[ H = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_0 + h \end{array} \right) \]  

(6)

where \( h \) is the physical SM Higgs field and \( v_0 = 246.22 \text{ GeV} \) is the SM Higgs VEV. Consequently, the \( H^\dagger H \) term in the above Lagrangians expands to

\[ H^\dagger H = \frac{1}{2} v_0^2 + v_0 h + \frac{1}{2} h^2 \]  

(7)

which generates mass and interaction terms for the DM fields \( X \in (S, V_{\mu}, \chi, \psi) \). After substituting the expanded expression for \( H^\dagger H \) into the above Lagrangians, we find

\[ \mathcal{L}_S = \mathcal{L}_{SM} + \frac{1}{2} (\partial_\mu S)(\partial^\mu S) - \frac{1}{2} m_S^2 S^2 - \frac{1}{4!} \lambda_S S^4 - \frac{1}{2} \lambda_{hS} S^2 \left( v_0 h + \frac{1}{2} h^2 \right), \]  

(8)

\[ \mathcal{L}_V = \mathcal{L}_{SM} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_V^2 V^\mu V^\nu - \frac{1}{4!} \lambda_V (V_{\mu} V^{\mu})^2 + \frac{1}{2} \lambda_{hV} V^\mu V^\nu \left( v_0 h + \frac{1}{2} h^2 \right), \]  

(9)

\[ \mathcal{L}_X = \mathcal{L}_{SM} + \frac{1}{2} i \partial_\mu X - \frac{1}{2} \lambda_{hX} \left( \frac{\mu_X}{2 \Lambda_X} \right) v_0 \left( \cos \theta \overline{X} + \sin \theta \overline{X} \gamma_5 X \right) - \frac{1}{2} \lambda_{hX} \left( \cos \theta \overline{X} + \sin \theta \overline{X} \gamma_5 X \right) \left( v_0 h + \frac{1}{2} h^2 \right), \]  

(10)

\[ \mathcal{L}_\psi = \mathcal{L}_{SM} + \overline{\psi} i \partial_\mu \psi - \overline{\psi} \left( \frac{\mu_\psi}{2 \Lambda_\psi} \right) v_0 \left( \cos \theta \overline{\psi} \psi + \sin \theta \overline{\psi} \gamma_5 \psi \right) - \frac{\lambda_{h\psi}}{\Lambda_\psi} \left( \cos \theta \overline{\psi} \psi + \sin \theta \overline{\psi} \gamma_5 \psi \right) \left( v_0 h + \frac{1}{2} h^2 \right). \]  

(11)
The parameter \( m_\chi \) and \( m_\psi \) are the physical masses of the Majorana \((\chi)\) and Dirac \((\psi)\) DM fields respectively.

### III. Constraints

For the scalar and vector models after EWSB, the remaining free model parameters are the DM mass \( m_{S, V} \) and its coupling \( \lambda_{hS, hV} \) with the SM Higgs boson. The Majorana and Dirac fermion models in Eqs. (14) and (15) respectively yield three model parameters: \( \cos \xi, m_{\chi, \psi} \), and \( \lambda_{hS, hV} / \Lambda_{\chi, \psi} \). For simplicity, we choose to keep the parameter \( \cos \xi \) fixed when imposing fermion model constraints. More specifically, we study the cases when \( \cos \xi = 1 \) (pure scalar interaction), \( \cos \xi = 1 / \sqrt{2} \) (equally mixed scalar-pseudoscalar interaction) and \( \cos \xi = 0 \) (pure pseudoscalar interaction). Hence, the remaining free parameters in the fermion models are the DM mass \( m_{\chi, \psi} \) and its coupling \( \lambda_{hS, hV} / \Lambda_{\chi, \psi} \) with the SM Higgs boson.

As we impose identical sets of constraints on each model parameter space, we will instead refer to parameters \( m_\chi \) and \( \lambda_{hS, hV} \) where \( X \in \{ S, V, \chi, \psi \} \). In situations where a given constraint is model specific, we impose those constraints on the model parameters directly.

#### A. Relic density

Currently, the best-known value of the DM relic density (or abundance) comes from the Planck satellite measurement of the cosmic microwave background (CMB) temperature and lensing-potential power spectra [64]

\[
\Omega_{DM} h^2 = 0.1199 \pm 0.0027, \tag{16}
\]

where \( \Omega_{DM} \equiv \rho_{DM} / \rho_c \) is the fraction of the DM mass density relative to the critical density \( \rho_c = 3 H_0^2 / 8 \pi G \), and the parameter \( h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \) is the reduced Hubble constant.

In general, WIMPs in thermal equilibrium in the early Universe that annihilate purely via the s-wave have a relic density that varies inversely with the velocity-averaged annihilation cross section \( \langle \sigma v_{rel} \rangle \) through

\[
\Omega_{DM} h^2 \sim \frac{3 \times 10^{-27} \text{ cm}^3 \text{s}^{-1} \langle \sigma v_{rel} \rangle}{\langle \sigma v_{rel} \rangle}, \tag{17}
\]

Consequently, a smaller \( \langle \sigma v_{rel} \rangle \) produces an overabundance of WIMPs, whereas a larger \( \langle \sigma v_{rel} \rangle \) produces an underabundance of WIMPs in the Universe today.

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2 For our analysis, we ignore the quartic self-coupling \( \lambda_{S, V} \) since it plays no observable role in the DM phenomenology. However, it is important when the constraints from electroweak vacuum stability and model perturbativity are imposed (see, e.g., Refs. [44, 57–63].)

3 In the model Lagrangians, the scalar/vector DM coupling \( \lambda_{hS, hV} / \Lambda_{\chi, \psi} \) has mass dimension \( M^0 \) while the Majorana/Dirac fermion DM coupling \( \lambda_{hS, hV} / \Lambda_{\chi, \psi} \) has mass dimension \( M^{-1} \).
For our Higgs portal models, the model relic density is mostly determined by the Higgs-mediated s-channel annihilation into SM particles. A subdominant role is played by the annihilation into $hh$ through the direct $h^2\chi^2$ vertex as well as the Higgs-mediated t-channel annihilation of $X$. Since $\langle \sigma v \rangle_{rel}$ scales as the square of the SM Higgs-DM coupling $\lambda_{hh}$, larger (smaller) values of $\lambda_{hh}$ lead to a suppressed (enhanced) relic density $\Omega_\chi h^2$ in agreement with Eq. (17).

We use the publicly available software LanHEP 3.2.0 [65,66] to implement the scalar model in Eq. (2), vector model in Eq. (3) and the redefined, chiral-rotated Majorana and Dirac models in Eqs. (14) and (15) respectively. For the calculation of the relic density $\Omega_\chi h^2$, annihilation cross section $\sigma v_{rel}$ and the gamma-ray yields per annihilation, we use micrOMEGAs 3.6.9.2 [67,68] to automate calculations in each model. In computing the relic density, we also take into account the annihilation of DM into virtual gauge bosons.

The Planck measured relic density in Eq. (16) restricts the allowed values of the SM Higgs-DM coupling. For each of our models, we scan along the DM mass axis and for each mass, find the coupling that gives the correct relic density using Brent’s method [69]. These couplings are roots (or zeros) of the function

$$f_\Omega(m_\chi, \lambda_{hh}) = \Omega_{DM} h^2 - \Omega_\chi h^2,$$

where $\Omega_\chi h^2$ is the model relic density computed in micrOMEGAs. For our analysis, we take $\Omega_{DM} h^2 = 0.1199$ corresponding to the central value of Eq. (16).

The possibility of a multicomponent dark sector where a given model constitutes a fraction of the total DM density is a strong one. To address this possibility, we define a relic abundance parameter

$$f_{rel} \equiv \frac{\Omega_\chi}{\Omega_{DM}},$$

which quantifies the contribution of $X$ towards the total DM density in our Universe. For plotting purposes, we take $f_{rel} = 1, 0.1$ and 0.01 corresponding to $X$ relic abundances of 100%, 10% and 1% respectively.

At DM masses below $m_h/2$, we found multiple values of the coupling $\lambda_{hh}$ at fixed DM masses that were consistent with the Planck measured value. Naively, one expects a monotonic decrease in $\Omega_\chi h^2$ at larger values of $\lambda_{hh}$ due to an increased $\langle \sigma v \rangle_{rel}$, as evident in Eq. (17). However, $\langle \sigma v \rangle_{rel}$ has an additional dependence on $\lambda_{hh}$ through the full Higgs boson width $\Gamma_{h}(\sqrt{s})$ as a function of the center-of-mass energy $\sqrt{s}$. Therefore, as $\lambda_{hh}$ increases, the increasing Higgs boson width counteracts the increase in $\langle \sigma v \rangle_{rel}$ from $\lambda_{hh}$ alone and at some point outstrips it, such that $\langle \sigma v \rangle_{rel}$ starts decreasing with increasing $\lambda_{hh}$, causing a minimum relic density $\Omega_\chi h^2$ for any given DM mass. When these features appear, we always found that only one root was consistent with the Higgs invisible width constraint and this is the one presented in our results. Indeed, the same feature was also seen in the preparation of Ref. [39], but was not explicitly discussed in the final manuscript.

Furthermore, roots of Eq. (18) for $f_{rel} = 0.1, 0.01$ and DM masses below $m_h/2$ do not exist as the minimum relic density at these masses is larger than the value of the contour being drawn. Consequently, gaps in our relic density contours for $f_{rel} = 0.1, 0.01$ will appear in our plots.

For DM masses above $m_h/2$, the relic density monotonically decreases with larger couplings $\lambda_{hh}$. This results in a single root (or coupling) at each DM mass that gives the Planck measured relic density. The Higgs invisible width $\Gamma_{inv}(h \rightarrow XX)$ vanishes in this region due to the decay $h \rightarrow XX$ becoming kinematically forbidden.

### B. Higgs invisible width

When $m_\chi < m_h/2$, the decay $h \rightarrow XX$ is kinematically allowed. This contributes to the invisible width $\Gamma_{inv}(h \rightarrow XX)$ of the SM Higgs boson. The LHC constraints on $\Gamma_{inv}$ continue to improve as precise measurements of the properties of the Higgs boson are shown to be increasingly in agreement with the SM expectations.

An upper limit of 19% on the SM Higgs invisible branching ratio $BR(h \rightarrow XX)$ at 2σ C.L. is obtained in Ref. [70] through combined fits to all Higgs production and decay channels probed by ATLAS, CMS and the Tevatron. An additional projected limit of 5% on the Higgs invisible branching ratio at no more than 1σ C.L. can be further imposed if no additional Higgs decay is detected at the 14 TeV LHC run after 300 fb$^{-1}$ of luminosity data is collected [71].

For our Higgs portal models, the Higgs invisible widths are given by [9,49]

$$\Gamma_{inv}(h \rightarrow SS) = \frac{\lambda_{hh}^2 v_0^2}{32\pi m_h} \left(1 - \frac{4m_\chi^2}{m_h^2}\right)^{1/2},$$

$$\Gamma_{inv}(h \rightarrow VV) = \frac{\lambda_{hh}^2 v_0^2 m_h^3}{128\pi m_V^2} \left(1 - \frac{4m_\chi^2}{m_h^2} + \frac{12m_\chi^2}{m_h^4}\right) \times \left(1 - \frac{4m_V^2}{m_h^2}\right)^{1/2},$$

The Higgs invisible branching ratio is defined as $BR(h \rightarrow XX) \equiv \Gamma_{inv}/(\Gamma_{vis} + \Gamma_{inv})$ where $\Gamma_{vis}$ is the visible (SM) contribution to the SM Higgs width.
\[ \Gamma_{\text{inv}}(h \rightarrow \mathcal{X}) = \frac{m_h v_0^2}{16\pi} \left( \frac{\lambda_{\mathcal{X}}}{\Lambda_{\mathcal{X}}} \right)^2 \left( 1 - \frac{4m_\mathcal{X}^2}{m_h^2} \right)^{1/2} \times \left( 1 - \frac{4m_\mathcal{X}^2 \cos^2 \xi \pi}{m_h^2} \right), \quad (22) \]

\[ \Gamma_{\text{inv}}(h \rightarrow \mathcal{P}\mathcal{P}) = \frac{m_h v_0^2}{8\pi} \left( \frac{\lambda_{\mathcal{P}}}{\Lambda_{\mathcal{P}}} \right)^2 \left( 1 - \frac{4m_\mathcal{P}^2}{m_h^2} \right)^{1/2} \times \left( 1 - \frac{4m_\mathcal{P}^2 \cos^2 \xi \pi}{m_h^2} \right). \quad (23) \]

Using an upper limit of \( Y (Y = 0.19 \text{ or } 0.05 \text{ from above}) \) on \( \mathcal{B}(h \rightarrow \mathcal{X}\mathcal{X}) \), the Higgs invisible width \( \Gamma_{\text{inv}} \) can be expressed in terms of the visible contribution \( \Gamma_{\text{vis}} \) through

\[ \Gamma_{\text{inv}} \leq \frac{Y}{1 - Y} \Gamma_{\text{vis}}. \quad (24) \]

Using the Higgs invisible width expressions from Eqs. (20) to (23), an upper limit on \( \Gamma_{\text{inv}} \) in Eq. (24) provides an upper limit on the SM Higgs-DM coupling as a function of the DM mass

Scalar: \( \lambda_{h\mathcal{X}} \leq \left[ \frac{Y}{1 - Y} \frac{32\pi m_h \Gamma_{\text{vis}}}{v_0^2} \left( 1 - \frac{4m_\mathcal{X}^2}{m_h^2} \right)^{-1/2} \right]^{1/2}, \)

Vector: \( \lambda_{h\mathcal{P}} \leq \left[ \frac{Y}{1 - Y} \frac{128\pi m_h \Gamma_{\text{vis}}}{v_0^2 m_h^2} \left( 1 - \frac{4m_\mathcal{P}^2}{m_h^2} \right)^{-1} \frac{12m_\mathcal{P}^2}{m_h^2} \right]^{1/2} \times \left( 1 - \frac{4m_\mathcal{P}^2 \cos^2 \xi \pi}{m_h^2} \right)^{-1/2}, \)

Majorana: \( \lambda_{h\mathcal{X}} \leq \left[ \frac{Y}{1 - Y} \frac{16\pi m_h \Gamma_{\text{vis}}}{m_h v_0^2} \left( 1 - \frac{4m_\mathcal{X}^2}{m_h^2} \right)^{-1/2} \right] \times \left( 1 - \frac{4m_\mathcal{X}^2 \cos^2 \xi \pi}{m_h^2} \right)^{-1/2}, \)

Dirac: \( \lambda_{h\mathcal{P}} \leq \left[ \frac{Y}{1 - Y} \frac{8\pi m_h \Gamma_{\text{vis}}}{m_h v_0^2} \left( 1 - \frac{4m_\mathcal{P}^2}{m_h^2} \right)^{-1/2} \right] \times \left( 1 - \frac{4m_\mathcal{P}^2 \cos^2 \xi \pi}{m_h^2} \right)^{-1/2}, \)

where we take the visible contribution \( \Gamma_{\text{vis}} \equiv \Gamma_{\text{SM}} = 4.07 \text{ MeV for } m_h = 125 \text{ GeV} \) [72].

C. Indirect detection

Indirect detection of DM involves searches for fluxes of \( \gamma \)-rays, \( e^\pm, p^\pm \) and neutrinos produced from DM annihilations in distant astrophysical sources. As the annihilation flux \( \Phi_{\text{ann}} \) scales as the square of the DM mass density (i.e., \( \Phi_{\text{ann}} \propto \rho_{\text{DM}}^2 \)), natural places to look for DM annihilations are those with high DM content such as the dwarf spheroidal (dSph) galaxies, the Galactic Center (GC) or our Sun. Current indirect experiments searching for signs of DM annihilations include the Fermi Large Area Telescope (Fermi-LAT) [73], H.E.S.S. [74] and AMS-02 [75]. The upcoming Cherenkov Telescope Array [76] is one of the next generation of ground-based gamma-ray telescopes.

To impose parameter space constraints from indirect search experiments, we use a combined log-likelihood function that depends on the model parameters

\[ \ln \mathcal{L}_{\text{total}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}) = \ln \mathcal{L}_{\text{CMB}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}) + \ln \mathcal{L}_{\text{dSph}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}) + \ln \mathcal{L}_{\text{CTA}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}). \quad (25) \]

In general, the contribution to the total log-likelihood function in Eq. (25) comes from all three indirect searches. However, the CTA log-likelihood function enters only when we discuss projected limits. Each log-likelihood function depends on \( m_\mathcal{X} \) and on the Higgs-DM coupling \( \lambda_{h\mathcal{X}} \) via the zero-velocity annihilation cross section \( \sigma v_{\text{rel}} \), the branching ratio \( B_j \), and the SM final state \( f \) and the model relic density \( \Omega_\mathcal{X} h^2 \). The final states included in our calculations involve DM annihilations into \( W^+W^-, ZZ, hh, \mu^+\mu^- \), and \( \tau^+\tau^- \).

We scale all indirect detection signals for each combination of \( m_\mathcal{X} \) and \( \lambda_{h\mathcal{X}} \) by \( f_{\text{id}} = (\Omega_\mathcal{X}/\Omega_{\text{DM}})^2 \), thereby suppressing signals where \( X \) constitutes only a fraction of the total dark matter. In regions where the model relic density is larger than the observed DM relic density, we simply rescale in the same way, thereby increasing the expected signals. This is done for the sake of simplicity and is of no practical consequence given that the relevant region is robustly excluded by the relic density constraints.

In complete analogy with the implementation of the relic density constraints, we perform scans in each model parameter space using Brent’s method to impose the indirect search limits. The function used for the root-finding algorithm is

\[ f_{\text{id}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}) = \Delta \ln \mathcal{L}_{\text{total}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}) - \Delta \ln \mathcal{L}_{\text{C.L.}}(m_\mathcal{X}, \lambda_{h\mathcal{X}}), \quad (26) \]

where \( \Delta \ln \mathcal{L}_{\text{total}} \) is the combined delta log-likelihood, defined such that it is zero for the case of no DM signal. The term \( \Delta \ln \mathcal{L}_{\text{C.L.}} \) is the delta log-likelihood for a given C.L. For our purposes, we implement 1\( \sigma \) and 90% C.L. by taking

\[ \Delta \ln \mathcal{L}_{\text{C.L.}} = \{ \begin{array}{cl} -0.50000 & \text{for } 1\sigma \text{ C.L.}, \\ -1.352771 & \text{for } 90\% \text{ C.L.} \end{array} \}

When implementing current indirect search limits, the combined log-likelihood function is
It is useful to consider a quantity \( f_{\text{eff}} \) defined in terms of a “universal WIMP annihilation” curve \( e_W(z) \) as \( f_{\text{eff}} \approx (f \cdot e_W)/(e_W \cdot e_W) \), which has the interpretation that \( f_{\text{eff}} < 1 \) denotes the average efficiency of energy injection for the annihilation channel under consideration. With the expansion \( e_W = \sum_i e_i \) for WMAP7 [80], only the first principle component is dominant. Hence, \( f_{\text{eff}} \) for WMAP7 is given by

\[
f_{\text{eff}} = \frac{(f \cdot e_1)}{c_1(e_1 \cdot e_1)} \quad \text{(WMAP)}
\]

where numerically \( c_1 = 4.64 \). For Planck [64], the contribution from all three principle components must be included and therefore

\[
f_{\text{eff}} = \frac{1}{\sqrt{\lambda_i c_1}} \left[ \sum_i \lambda_i \left( \frac{f \cdot e_i}{e_1 \cdot e_1} \right)^2 \right]^{1/2} \quad \text{(Planck)}
\]

where \( \lambda_i \) and \( e_i \) are parameters appropriate for Planck.

We use the tabulated values of the effective efficiency \( f_{\text{eff}} \) for various SM final states and DM masses from Ref. [77]. At intermediate masses, we interpolate the values of \( f_{\text{eff}} \) in terms of \( \log_{10} m_X \). For DM annihilation into multiple channels, the total \( f_{\text{eff}} \) is a weighted sum of effective efficiency \( f_{\text{eff}, f} \) over final states \( f \) with branching ratio \( B_f \) as

\[
f_{\text{eff}} = \sum_f B_f f_{\text{eff}, f}. \quad (30)
\]

To impose CMB constraints at arbitrary C.L., we use a log-likelihood function for the annihilation cross section \( \langle \sigma v_{\text{rel}} \rangle \), assuming a DM mass \( m_X \) and a branching ratio \( B_f \) into final state \( f \). For the Planck experiment, the log-likelihood is given by [77]

\[
\ln L_{\text{Planck}}(\langle \sigma v_{\text{rel}} \rangle | m_X, B_f) = -\frac{1}{2} f_{\text{eff}}^2 \lambda_1 c_1^2 \left( \frac{\langle \sigma v_{\text{rel}} \rangle}{2 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}} \right)^2 \left( \frac{\text{GeV}}{m_X} \right)^2
\]

where \( c_1 = 4.64 \) and \( \lambda_1 = 3.16 \).\(^9\) The above equation assumes a linear response on the CMB against the deposited energy, which is not accurate for WMAP. This however can be corrected by the replacement \( 2 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \to 3.2 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \) and \( \lambda_1 = 0.279 \). Therefore, for the WMAP experiment, the log-likelihood is given by [77]

\(^9\)This is in fact a projected log-likelihood for the Planck polarization data, but it agrees very well with the actual constraints in Ref. [81].
\[ \ln \mathcal{L}_{\text{WMAP}}(\langle \sigma v_{\text{rel}} \rangle | m_X, \mathcal{B}_f) = -\frac{1}{2} \frac{c_1^2}{\lambda_1} \left( \frac{\langle \sigma v_{\text{rel}} \rangle}{3.2 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}} \right)^2 \left( \frac{\text{GeV}}{m_X} \right)^2, \] (32)

where \( c_1 = 4.64 \) and \( \lambda_1 = 0.279 \).

2. Fermi dwarfs likelihood

To place upper limits on \( \langle \sigma v_{\text{rel}} \rangle \), one must quantify how it influences the flux of gamma rays detected by the Large Area Telescope aboard the Fermi satellite [82]. The non-observation of gamma rays from the dwarf spheroidal satellite galaxies of the Milky Way (dSphs) can be used to place strong upper limits on \( \langle \sigma v_{\text{rel}} \rangle \) in various SM final states.

The differential gamma-ray flux from DM annihilation in a region \( \Delta \Omega \) towards a dwarf spheroidal galaxy is given by

\[ \frac{d\Phi_{\text{ann}}}{dE} = \kappa \frac{\langle \sigma v_{\text{rel}} \rangle}{4\pi m_X^2} \frac{dN_f}{dE} \int_{\Delta \Omega} \int_{J_{\text{ann}}} \rho_{\text{DM}}^3(r) dl, \] (33)

where \( \kappa \) equals a 1/2 for self-conjugate DM (due to the phase-space considerations) and 1/4 for non-self-conjugate DM (due to the density of DM particles and antiparticles, each being half of the total DM density \( \rho_{\text{DM}} \) in the \( J_{\text{ann}} \)-term) [83,84]. The first term \( \Phi_{\text{pp}} \) describes the particle physics aspect of DM annihilation: the velocity-averaged annihilation cross section \( \langle \sigma v_{\text{rel}} \rangle \), the differential gamma-ray spectrum \( dN_f/dE \) per annihilation\(^{10} \) and the DM mass \( m_X \). The second term \( J_{\text{ann}} \) is commonly known as the “J-factor,” and describes the astrophysical aspects of a dwarf galaxy. It is given by the line-of-sight (l.o.s.) integral of the DM distribution in a dwarf galaxy, integrated over the solid angle \( \Delta \Omega \).

The main advantage of the form of the differential gamma-ray flux in Eq. (33) is that the terms describing the particle physics of DM annihilation (specifically \( \Phi_{\text{pp}} \)) and the astrophysics of a dwarf galaxy (specifically \( J_{\text{ann}} \)) separate into two independent factors. Since the prefactor \( \Phi_{\text{pp}} \) is constant for a given set of model parameters, only the \( J \)-factors have to be determined individually for each dwarf galaxy. A combined analysis based on the observations of multiple dwarf satellite galaxies is therefore straightforward.

The expected number of gamma rays produced from DM annihilations between energies \( E_1 \) and \( E_2 \) in a specific region of the sky is

\( \Phi_{\text{pp}}(E) = \kappa \frac{\langle \sigma v_{\text{rel}} \rangle}{4\pi m_X^2} \frac{dN_f(E)}{dE}. \) (35)

Since the tabulated results of Ref. [85] are given between gamma-ray energies of 500 MeV and 500 GeV, we interpolate the differential gamma-ray spectrum \( dN_f/dE \) calculated within micrOMEGAs over the energy range of interest (500 MeV and 500 GeV).\(^{12} \) The \( \ln \mathcal{L}_{\text{dSphs}}(m_X, \lambda_{hX}) \) entering Eq. (27) is then a difference of the log-likelihoods for a DM-signal and background-only hypothesis (\( \Phi_{\text{pp}} = 0 \)).

For projected indirect search limits, we assume that Fermi operates for at least 10 years in its current survey mode and is able to add as many southern dwarf galaxies in the future as there are mostly northern dwarf galaxies now. Assuming that the improvements in the Fermi-LAT reach are dominated by the statistical uncertainty (and thus the

\(^{10} \)Specifically, the differential gamma-ray spectrum per annihilation is a sum over the differential spectrum from all possible final states: \( dN_f/dE = \sum_f B_f dN_f^f/dE \), where \( B_f \) is the branching ratio into the SM final state \( f \).

\(^{12} \)https://www-glast.stanford.edu/pub_data/1048/.
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limits on $\langle \sigma v_{\text{rel}} \rangle$ scale as $\sqrt{N}$, where $N$ is the number of dwarfs), we calculate the projected Fermi sensitivities by scaling the current limits by a factor of $\sqrt{2 \times 10/6} = \sqrt{20/6} \approx 1.83$.

### 3. CTA likelihood

The CTA is a multinational project to build the next generation of ground-based gamma-ray instruments and to have sensitivity over energies from a few tens of GeV to 100 TeV. It is intended to improve the flux sensitivities of the current generation of Imaging Atmospheric Cherenkov Telescopes (IACTs) such as MAGIC [86], H.E.S.S. [87] and VERITAS [88] by an order of magnitude.

CTA will consist of several tens of telescopes of 2–3 different types, with sizes varying between 5 and 25 meters, distributed over an area of several square kilometers. The sensitivity will be a factor of 10 better than current instruments, and the field of view (FoV) will be up to $10^\circ$ in diameter. It is envisaged as a two part telescope, with southern and northern sites. CTA South is most relevant for DM searches towards the GC. Its final design is not yet fixed. Apart from the construction and maintenance questions, the relevant remaining design choices are the relative emphasis on the higher or lower energies, the angular and energy resolution and the FoV. A first detailed Monte Carlo (MC) analysis was presented in Ref. [89] where 11 different array configurations for the CTA South were discussed.

We use the array configuration known as “Array I,” which has a balanced configuration with 3 large ($\sim 24$ m aperture), 18 medium ($\sim 12$ m) and 56 small telescopes ($\sim 4–7$ m). This configuration also provides a good compromise in sensitivity between lower and higher energies. Extensive information on the effective area, background rates, and angular and energy resolution of Array I is also available. Furthermore, previous DM sensitivity studies have used a very similar array: Array E in Ref. [90] and the Paris-MVA analysis of Array I in Ref. [91]. The point-source sensitivities of Arrays E and I agree very well at energies $\lesssim 1$ TeV, whereas at higher energies Array I is more sensitive (only by a factor of less than 2).

The main performance aspects of Array I include an effective area of 100 m$^2$ at its threshold energy of 20 GeV, which then increases quickly to about $4 \times 10^5$ m$^2$ at 1 TeV and $3 \times 10^6$ m$^2$ at 10 TeV. The angular resolution in terms of the 68% containment radius is about $r_{68} \approx 0.3^\circ$ at threshold and drops to below $0.06^\circ$ at energies above 1 TeV. The energy resolution is relatively large at threshold, $\sigma(E)/E \sim 50\%$, but drops to below 10% at energies above 1 TeV.

Following the analysis of Ref. [92], we use their tabulated values of the integrated gamma-ray flux times energy (i.e., $\Phi_{\text{int}}$ in units of MeV cm$^{-2}$ s$^{-1}$) and delta log-likelihoods ($\Delta \ln L_{\text{CTA}}$) between gamma-ray energies of 25 GeV and 10 TeV. The main features of their study are

(i) Assessing the impacts of all backgrounds, including protons and electrons in cosmic rays hitting the atmosphere and diffuse astrophysical emissions. Galactic diffuse emission (GDE) substantially degrades the CTA differential sensitivity (see Fig. 4 of Ref. [92] for a comparison).

(ii) Introducing a statistical framework that accounts for the impacts of differential acceptance uncertainties from sources such as event reconstruction, Monte Carlo (MC) determination of the effective areas and the uncertainty in atmospheric conditions [93] within a FoV on DM limits from CTA. Specifically, the tabulated results correspond to a systematic uncertainty of 1%.

(iii) Using the Einasto profile [94] to calculate the J-factors for the GC. The Einasto profile best fits the DM density profiles seen in the $N$-body simulations of Milky Way type galaxies

$$\rho_{\text{DM}}(r) \propto \exp \left(-\frac{2}{\alpha} \left[\left(\frac{r}{r_s}\right)^\alpha - 1\right]\right),$$

where the profile is normalized to a local DM density of $\rho_\odot \equiv \rho_{\text{DM}}(r_\odot) = 0.4$ GeV cm$^{-3}$ by choosing $\alpha = 0.17$, $r_s = 20$ kpc and $r_\odot = 8.5$ kpc [95].

(iv) Using a slightly contracted generalized Navarro-Frenk-White (NFW) profile [96] to indicate how the limits improve with a more optimistic DM distribution. The profile is parametrized as

$$\rho_{\text{DM}}(r) \propto \frac{1}{r^\gamma (r_s + r)^{3-\gamma}},$$

where $\gamma = 1.3$ is the inner slope of the profile and $r_s = 20$ kpc is the scale radius. It is normalized in the same way as the Einasto profile (i.e., $\rho_\odot = 0.4$ GeV cm$^{-3}$).

(v) Performing a “morphological” analysis by covering the area occupied by the two regions of interest (RoIs) in the left panel of Fig. 1 and dividing it into $1^\circ \times 1^\circ$ squares, giving a total of 28 RoIs as shown in the right panel of Fig. 1. The morphological analysis allows for a proper exploitation of the shape differences between the GDE, which is concentrated along the Galactic plane, and the DM annihilation signal, which is spherically distributed around the GC. The resultant constraints are found to be more stringent by a factor of a few when compared against traditional “ring” analyses.

In summary, the tabulated results of Ref. [92] are based on a morphological analysis over the 28 RoIs, assuming $\geq 100$ h of GC observation by CTA.$^{13}$ 1% instrumental

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$^{13}$Since the limits of Ref. [92] are systematics dominated, they are more or less independent of any increase in the CTA observation time beyond 100 h. For our portal models, we derive the indirect search limits based on 100 h of CTA observation time.
D. Direct detection

Direct detection experiments aim to measure the recoil of a nucleus in a collision with a DM particle. After an elastic collision between DM of mass $m_X$ and a nucleus of mass $M$, the nucleus recoils with energy $E = \mu^2 v^2 (1 - \cos \theta)/M$, where $\mu = m_X M/(m_X + M)$ is the DM-nucleus reduced mass, $v$ is the speed of DM particle relative to the nucleus rest frame and $\theta$ is the scattering angle in the center-of-mass (COM) frame.

The differential rate per day (given in units of cpd kg$^{-1}$ keV$^{-1}$ where cpd is counts per day) from primarily spin-independent (SI) interactions is given by [98,99]

$$\frac{dR}{dE} = \frac{\rho_{\odot}}{m_X M} \int d^3 v \frac{f(v,t)}{v} \frac{d\sigma_X^{SI}}{dE},$$

where $\rho_{\odot}$ is local DM mass density and $f(v,t)$ is the time-dependent DM velocity distribution. The term $d\sigma_X^{SI}/dE$ is the momentum-dependent differential SI cross section. In the typical case where the target material contains more than one isotope, the differential rate is given by a mass-fraction weighted sum over contributions from the isotopes, each of the forms given by Eq. (38).

The momentum-dependent differential SI cross section is given by

$$\frac{d\sigma_X^{SI}}{dE} = \frac{1}{E_{\text{max}}} \sigma_0 F^2(q) \Theta(q_{\text{max}} - q),$$

where $E_{\text{max}} = q_{\text{max}}^2/2M = 2\mu^2 v^2 / M$ is the maximum energy transfer in a collision at a relative velocity $v$, $\sigma_0$ is the SI cross section in the zero-velocity limit$^{14}$ and $\Theta$ is the Heaviside step function. The term $F(q)$ is the form factor accounting for the finite size of the nucleus. When DM coherently scatters off the entire nucleus with low momentum transfer, $F^2(q) \rightarrow 1$. However, when the de Broglie wavelength of the momentum transfer $q$ becomes comparable to the size of the nucleus, DM becomes sensitive to the internal structure of the nucleus and $F^2(q) < 1$, with $F^2(q) \ll 1$ at high momentum transfers.

It is traditional to define an effective SI cross section corrected with the form factor as

$^{14}$Generally, $\sigma_{SI}$ and $\sigma_{SD}$ are used to represent this term for nuclear spin-independent (SI) and spin-dependent (SD) interactions respectively. To avoid confusion with existing literatures, we will represent the SI contribution to $\sigma_0$ as $\sigma_{SI}$ in our models.
such that the dependency on the momentum transfer \( q \) is contained entirely within the form factor \( F(q) \). The actual cross section is \( \int dq^2 d\sigma_{SI}^N(q^2, v)/dq^2 \) for a given DM-nucleus relative speed \( v \). With the chosen convention in Eq. (40), the expression for the momentum-dependent differential SI cross section in Eq. (39) simplifies to

\[
\frac{d\sigma_{SI}^N}{dE} = \frac{M}{2\mu^2 v^2} \sigma(q) \Theta(q_{\text{max}} - q).
\]  

(41)

With the form of the the momentum-dependent differential SI cross section in Eq. (41), the differential rate per day in Eq. (38) becomes

\[
\frac{dR}{dE} = \frac{1}{2m_{\chi}^2}\sigma(q)\rho_{\odot}\eta(v_{\text{min}}(E), t),
\]

(42)

where

\[
\eta(v_{\text{min}}(E), t) = \int_{v>v_{\text{min}}(E)} d^3 v \frac{f(v, t)}{v}
\]

(43)
is the mean inverse speed and

\[v_{\text{min}}(E) = \sqrt{\frac{ME}{2\mu^2}}\]

is the minimum DM speed that results in a nuclear recoil with energy \( E \). The requirement \( q < q_{\text{max}} \) in the Heaviside step function of Eq. (39) results in a lower limit \( v > v_{\text{min}}(E) \) on the integral in Eq. (43).

The main advantage of writing the recoil spectrum in the form of Eq. (42) instead of Eq. (38) is that the particle physics and astrophysics aspects separate into two distinct factors: \( \sigma(q) \) describes the particle physics aspect; while \( \rho_{\odot} \) and \( \eta(v_{\text{min}}(E), t) \) describe the astrophysical aspect.

For our Higgs-mediated scalar and/or pseudoscalar DM coupling, the SI cross section \( \sigma_{SI} \) takes the form

\[
\sigma_{SI} = \mu^2 \left( ZG_{SI}^p + (A - Z)G_{SI}^n \right)^2 + \frac{q^2}{4m_{\chi}^2} \mu^2 \left( Z\tilde{G}_{SI}^p + (A - Z)\tilde{G}_{SI}^n \right)^2,
\]

(44)

where \( Z \) and \( A - Z \) are the number of protons and neutrons in the nucleus respectively and \( G_{SI}^N(G_{SI}^\lambda) \) for \( N \in \{ p, n \} \) are the effective scalar (pseudoscalar) DM-nucleon couplings in analogue with the \( G_{\lambda} \)-like effective four-fermion coupling constants [100]. For the scalar and vector models, \( G_{SI}^N = 0 \), whereas in fermion models both terms contribute to \( \sigma_{SI} \). Due to the \( q^2/4m_{\chi}^2 \) factor arising from the pseudoscalar DM-nucleon interaction in fermion models [101], the corresponding detection rates are momentum suppressed.

When the effective scalar and pseudoscalar couplings are approximately equal for protons and neutrons, leading to \( \sigma_{SI,p} \approx \sigma_{SI,n} \), the SI cross section in Eq. (44) is enhanced by a factor of \( A^2 \) because the matrix elements for the cross section are a coherent sum over the individual protons and neutrons in the nucleus.

For the SI interaction, the form factor is a Fourier transform of the nucleus mass distribution. As a reasonably accurate approximation, we use the Helm form factor [102], which was first introduced as a modification to the form factor for a uniform sphere with a Gaussian function to account for the soft edge of the nucleus [103]. It is given by

\[
F(q) = 3e^{-q^2/2} \left[ \sin(qr_n) - qr_n \cos(qr_n) \right]/(qr_n^3),
\]

(45)

where \( q \approx 0.9 \) \( \text{fm} \) and \( r_n^2 = c^2 + \frac{q^2}{\pi} a^2 - 5s^2 \) is the effective nuclear radius with \( a = 0.52 \) \( \text{fm} \) and \( c = 1.23 A^{1/3} - 0.60 \) \( \text{fm} \).

We use the standard astrophysical parameters, namely the local DM density \( \rho_{\odot} = 0.4 \) \( \text{GeV} \text{ cm}^{-3} \) and the local galactic disk rotation speed \( v_{\text{esc}} = 220 \text{ km s}^{-1} \), with the same value for the most probable speed \( \pi \) of the Maxwell-Boltzmann velocity distribution. We take the Galactic escape speed as \( v_{\text{esc}} = 544 \text{ km s}^{-1} \) (see Ref. [104] for a discussion).

In the standard analysis where only a single component of DM constitutes the total DM relic density, the differential rate of detection is proportional to \( \rho_{\odot} \sigma_{SI} \) as evident in Eq. (42). To address the multicomponent dark matter scenario in our models, we rescale the limiting value of \( \sigma_{SI} \) by the fraction \( f_{\text{rel}} = \Omega_\lambda/\Omega_{DM} \) of the energy density contributed by \( X \) to the total DM density. This results in the local \( X \) energy density of \( f_{\text{rel}}\rho_{\odot} \).

1. Nonfermion models

The SI cross section in scalar and vector models is given by

Scalar: \( \sigma_{SI} = \frac{\mu^2 \lambda_{SI}^2 r_N^2 m_N^2}{\pi 4m_{\chi}^2 m_h^4} \)

(46)

= \frac{m_N^4}{4\pi(m_s + m_n)^2 \lambda_{SI}^2 f_N^2}

Vector: \( \sigma_{SI} = \frac{\mu^2 \lambda_{V}^2 f_N^2 m_N^2}{\pi 4m_{\chi}^2 m_h^4} \)

\(= \frac{m_N^4}{4\pi(m_v + m_n)^2 \lambda_{V}^2 f_N^2} \)

(47)

where \( m_N = 0.931 \text{ GeV} \) and \( f_N = 0.30 \) [39] (see also Ref. [105]). The term \( f_N \) is related to the quark matrix elements inside nucleons through

\[
f_N = \sum f_{\bar{T}q}^{(N)} = \sum_{\text{quarks}} \frac{m_q}{m_N} \langle N|\bar{q}q|N \rangle
\]

\[= \frac{2}{9} + \frac{7}{9} \sum_{q = u, d, s} f_{\bar{T}q}^{(N)},
\]

(48)
where the last equality follows from the heavy-quark expansion [106].

For the XENON1T [107–109] experiment, we apply the 90% C.L. from XENON100 [110] appropriately weighted by the relic abundance parameter $f_{\text{rel}} = \Omega_X/\Omega_{DM}$. Assuming that the sensitivity as a function of the DM mass scales relative to that of XENON100 simply by the exposure $\epsilon$, we demand for every value of $\{m_X, \lambda_{hX}|X \in S, V\}$ that

$$\sigma_{\text{eff}}(m_X, \lambda_{hX}) = \epsilon f_{\text{rel}}\sigma_{\text{SI}}(m_X, \lambda_{hX}) \leq \sigma_{\text{Xe}}$$  \hspace{1cm} (49)

where $\sigma_{\text{Xe}}$ is the 90% C.L. from XENON100. For the projected XENON1T, the expected improvement in sensitivity over XENON100 is $\epsilon = 100$.

For the LUX [111,112] experiment, we construct the log-likelihood function from the Poisson distribution [113] in the observed number of signal events $N$ by

$$\mathcal{L}_{\text{Poisson}}(s|N) = P(N|s) = \frac{(b + s)^N e^{-(b+s)}}{N!},$$  \hspace{1cm} (50)

where $b$ is the expected number of background events and

$$s = MT\int_0^\infty dE \phi(E) \frac{dR}{dE}$$ \hspace{1cm} (51)

is the expected number of signal events, $MT$ is the detector mass $\times$ exposure time and $\phi(E)$ is a global efficiency factor incorporating trigger efficiencies, energy resolution and analysis cuts.

We perform the LUX log-likelihood calculations using the LUXCalc 1.0.1 [114] package.\(^{15}\) For the LUX analysis region, we use $N = 1$ and $b = 0.64$ [115]; while for XENON100, we use $N = 2$ and $b = 1$ [110]. The efficiency curves $\phi(E)$ are generated by TPMC [116] using the NEST [117,118] model.\(^{16}\)

The LUXCalc package requires as inputs the effective SI scalar DM-nucleon coupling $G_N^\text{SI}$ and the pseudoscalar DM-nucleon coupling $G_N^\text{PS}$ to calculate the SI cross section in Eq. (44). For the scalar and vector models, the effective DM-nucleon couplings are

Scalar: \begin{equation}
G_N^\text{SI} = \frac{\lambda_{hX} f_N m_N}{2 m_s m_h^2}, \quad \tilde{G}_N^\text{SI} = 0,
\end{equation}

Vector: \begin{equation}
G_N^\text{SI} = \frac{\lambda_{hX} f_N m_N}{2 m_v m_h^2}, \quad \tilde{G}_N^\text{SI} = 0,
\end{equation}

which reproduce the SI cross sections in Eqs. (46) and (47) respectively. The pseudoscalar effective DM-nucleon couplings are zero because the interaction between DM and the nucleons proceeds through a spin-0 boson.

\(^{15}\)LUXCalc homepage: http://www.nordita.org/~savage/LUXCalc/index.html.

\(^{16}\)NEST homepage: http://nest.physics.ucdavis.edu/site/.

2. Fermion models

Due to the mediator being a spin-0 boson, the interaction between DM and nucleons are nucleon-spin independent. From the Majorana and Dirac model Lagrangians in Eqs. (14) and (15) respectively, a mixing between a pure scalar ($\cos \xi = 1$) and a pure pseudoscalar ($\cos \xi = 0$) interaction occurs.

For a pure pseudoscalar interaction, the SI cross section is suppressed by a factor of $q^2/4m_X^2$ [101], where $q$ is the momentum transfer. A direct comparison between the analytical SI cross section expressions and the limits imposed by XENON100 or LUX experiments is not accurate. We therefore use a generalized and augmented version of LUXCalc to include the $q^2/4m_X^2$ factor in the differential rate per day $dR/dE$.

For the Majorana and Dirac fermion models, the effective scalar and pseudoscalar DM-nucleon couplings required by LUXCalc are

Majorana: $G_N^\text{SI} = \frac{\lambda_{hX}}{\Lambda_y} \frac{f_N m_N \cos \xi}{m_h^2}$, \quad $\tilde{G}_N^\text{SI} = \frac{\lambda_{hX}}{\Lambda_y} \frac{f_N m_N \sin \xi}{m_h^2}$, $\tilde{G}_N = 0$.

Dirac: $G_N^\text{SI} = \frac{\lambda_{hX}}{\Lambda_y} \frac{f_N m_N \cos \xi}{m_h^2}$, \quad $\tilde{G}_N^\text{SI} = \frac{\lambda_{hX}}{\Lambda_y} \frac{f_N m_N \sin \xi}{m_h^2}$, $\tilde{G}_N = 0$.

For the XENON1T experiment, the expected improvement in sensitivity over XENON100 is $\epsilon = 100$, whereas for the LUX experiment, limits are derived with $\epsilon = 1$.

With the inclusion of the momentum-suppressed term in the differential rate per day, we perform root-finding scans in the $(m_X, \lambda_{hX}/\Lambda_X)$ plane for $X \in (\tau, \psi)$ using Brent’s method. The function for the root-finding algorithm is

$$f_{DD}(m_X, \lambda_{hX}/\Lambda_X) = \Delta \ln \mathcal{L}_{\text{Poisson}}(m_X, \lambda_{hX}/\Lambda_X)$$

for a pure DM signal ($s \neq 0$) and no DM signal ($s = 0$), whereas $\Delta \ln \mathcal{L}_{\text{90% C.L.}} = -1.352771$ is the delta loglikelihood for a 90% C.L.

IV. RESULTS

Having outlined an implementation of our model constraints from various experiments, we now present our results. As our current study is a generalization of the scalar singlet analysis in Ref. [39] to nonscalar models, we first start by presenting our scalar model results obtained using micrOMEGAs to validate their consistency.

Although QCD corrections for quark final states at low scalar masses were included in Ref. [39], they are absent in our analysis because their inclusion in either micrOMEGAs or LanHEP is rather nontrivial, given the fact that micrOMEGAs relies heavily on autogenerated LanHEP codes.
The relic density from micrOMEGAs shows a local step-function reduction of 5%-12% in a small range of couplings over the mass range $100 \lesssim m_S/\text{GeV} \lesssim 400$ and $5 \lesssim m_{X,\psi}/\text{TeV} \lesssim 180$. This feature is not reproduced in the annihilation cross section, relic densities from the micrOMEGAs routines employing the freeze-out approximation, or in our previous calculations for the scalar model [39]. Away from the feature, the full micrOMEGAs relic densities are in good agreement with the freeze-out approximation and our previous results [39]. The drop appears symptomatic of a numerical error in the full micrOMEGAs Boltzmann solver, possibly due to poor convergence properties (it is not solved by the accurate setting). As the regions where this occurs are only a few tenths of a unit wide in the log of $\lambda_{hx}/\Lambda_X$ or the log of $\lambda_{hx}/\Lambda_X$ as appropriate, we simply omit them from our limit curves, interpolating across the small resulting gap.

Another important consideration is the validity of our EFT approximation for the fermionic models, which involve dimension-5 effective operators. In general the validity of an EFT relies on the momentum exchange involving dimension-5 effective operators. In general the EFT approximation for the fermionic models, which interpolate across the small resulting gap.

In the fermion model plots, we show the regions $\lambda_{hx}/\Lambda_X > 4\pi/2m_X$ where the EFT approximation breaks down for DM annihilation. Whilst the EFT is valid for direct searches at all couplings that we show, by choosing to rescale the limits by the relic abundance parameter $f_{rel} \equiv \Omega_X/\Omega_{DM}$ which is itself subject to UV corrections, our direct search limits are indirectly affected when $\lambda_{hx}/\Lambda_X > 4\pi/2m_X$.

### A. Scalar model

We perform scans in the $(m_S, \lambda_{hS})$ plane and generate contours of fixed scalar relic density ($\Omega_Sh^2$) for $f_{rel} = 1$ (black solid), 0.1 (red dashed) and 0.01 (blue dotted) as shown in Fig. 2. Values of $\lambda_{hS}$ in the grey shaded region are excluded by the relic density constraint. This is due to the fact that a lower value of $\lambda_{hS}$ gives smaller $\langle \sigma v_{rel}\rangle$. Following Eq. (17), there is therefore an overabundance of the scalar $S$ relic density.

In the region $m_S < m_h/2$, an upper limit of 19% (pink solid) at 2$\sigma$ C.L. and 5% (pink dotted) at 1$\sigma$ C.L. on the Higgs invisible branching ratio $Br(h \rightarrow SS)$ excludes coupling larger than $\log_{10} \lambda_{hS} \sim -1.75$ and $\log_{10} \lambda_{hS} \sim -2.1$ respectively. The combined constraints on the scalar relic density and the Higgs invisible width exclude lower scalar masses apart from a small triangle between 54 GeV and $m_h/2$.

Near the resonance $m_S \sim m_h/2$, the annihilation cross-section $\sigma_{v_{rel}}$ is enhanced. Consequently, the relic density contours move to lower values of $\lambda_{hS}$ to compensate for the enhancement. Above $m_S > m_h/2$, the relic density contours scale essentially linearly with $\log_{10} m_S$.

In Fig. 3, we show the combined sensitivity of indirect searches to various regions of the scalar model parameter space. For the current limits, which include the combined...
analysis of 15 dwarf galaxies by Fermi-LAT and 7-year observations of the CMB (WMAP7), we only present 1σ C.L. (brown solid). The region \( mh \approx 2 \lesssim m_S < 70 \text{ GeV} \) with \( \log_{10} \lambda h_S \in [0.5, -2.5] \) can be seen to be in tension with the current indirect searches at slightly more than 1σ C.L. The same is true for scalar masses below \( \sim 51 \text{ GeV} \); however this region is currently excluded by the Higgs invisible width constraint at more than 1σ C.L. Regions below the expected 90% C.L. curve with the Einasto (blue dashed) and contracted NFW (brown dotted) profile will be excluded. Left: A close-up of the resonantly enhanced annihilation region, \( m_S \sim m_h/2 \). Right: The full range of \( m_S \).

The combined future limits incorporate the Planck polarization data, \( \gtrsim 100 \) h GC observation by CTA and extended improvements in the Fermi-LAT data from the addition of more southern dwarf galaxies in its search. Due mainly to better exposure, future indirect DM searches will be sensitive enough to probe higher scalar DM masses, if \( S \) makes up all of the dark matter. At low scalar masses, the future DM searches are relatively insensitive to the assumed DM density profile (Einasto or a contracted NFW). This is mainly due to the fact that the Fermi log-likelihood dominates in this regime. However, at higher scalar masses, the CTA log-likelihood entering in Eq. (28) gives the dominant contribution to the total log-likelihood.

The resulting limits in the \((m_S, \lambda h_S)\) plane from the LUX (blue dashed) and the projected XENON1T (blue dotted) experiment are shown in Fig. 4. In the left panel near the resonance \( m_S \sim m_h/2 \), a small triangle will continue to evade detection at the LUX and projected XENON1T
experiments. At high scalar masses, the LUX experiment excludes scalar masses up to 120 GeV for a narrow range of $\lambda_{hS}$ values. Most of the remaining parameter space will be tested and ruled out by the XENON1T experiment for a wide range of couplings $\lambda_{hS}$. In particular, it will be able to exclude scalar masses up to 10 TeV, if $S$ makes up all of the dark matter.

**B. Vector model**

The contours of fixed vector relic density ($\Omega_v h^2$) for $f_{rel} = 1$ (black solid), 0.1 (red dashed) and 0.01 (blue dotted) are shown in Fig. 5. For $f_{rel} = 0.1$ (0.01), no values of $\lambda_{hV}$ satisfy Eq. (18) below 46 (56) GeV. The minimum relic density below these vector masses stays above the values of the contour being drawn, thereby leaving gaps that are evident in the left panel of Fig. 5.

In the region $m_V < m_h/2$, upper limits of 19% (pink solid) at 2$\sigma$ C.L. and 5% (pink dotted) at 1$\sigma$ C.L. on the Higgs invisible branching ratio $BR(h \to VV)$ exclude couplings larger than $\log_{10} \lambda_{hV} \sim -2.28$ and $\log_{10} \lambda_{hV} \sim -2.6$ respectively. The combined constraints on the vector relic density and the Higgs invisible width exclude most of the low vector masses apart from a small triangle between 56.5 GeV and $m_h/2$. Above $m_v > m_h/2$, the relic density contours scale linearly with $\log_{10} m_v$ in a similar fashion to the scalar model.

The combined sensitivity of indirect searches to various regions in the $(m_V, \lambda_{hV})$ plane is shown in Fig. 6. The current limits involve contributions from WMAP7 and the combined analysis of 15 dwarf galaxies by Fermi-LAT. The region $m_h/2 \leq m_v \leq 70$ GeV with $\log_{10} \lambda_{hV} \in [-2.5, -0.75]$ can be seen to be in tension with the current indirect searches. The same is true at vector masses below
Vector masses up to $\sim 230$ GeV are excluded by the current indirect DM searches at more than $1\sigma$ C.L., if $V$ makes up all of the dark matter. Limits from the future indirect DM searches will be able to probe parts of the parameter space that are not already excluded by either the current indirect searches or the relic density constraints. In particular, future searches based on the Einasto and contracted NFW DM profiles will exclude vector masses up to $\sim 173$ GeV and $\sim 9$ TeV respectively at more than 90% C.L., if $V$ makes up all of the dark matter.

Limits from the LUX (blue dashed) and the projected XENON1T (blue dotted) experiment in the $(m_V, \lambda_V)$ plane are shown in Fig. 7. The LUX experiment excludes vector masses up to $\sim 300$ GeV for a moderate range of $\lambda_V$ values, whereas the projected XENON1T experiment will exclude most parts of the parameter space that are not presently ruled out by the DM relic density and the Higgs invisible width constraints. In particular, it will be able to exclude vector masses up to $\sim 30$ TeV, if $V$ makes up all of the dark matter.

C. Majorana fermion model

The contours of fixed Majorana relic density in the $(m_V, \lambda_V/\Lambda_X)$ plane for $f_{\text{rel}} = 1$ (black solid), 0.1 (red dashed) and 0.01 (blue dotted) are shown in Fig. 8. Contours in each row are generated at fixed values of $\cos \xi$: $\cos \xi = 1$ for a pure scalar interaction (top row), $\cos \xi = 1/\sqrt{2}$ for an equal mix between the scalar and pseudoscalar terms (middle row), and $\cos \xi = 0$ for a pure pseudoscalar interaction (bottom row). As the interactions between the SM Higgs boson and the Majorana fermion DM change from pure scalar to pure pseudoscalar, the Majorana relic density contours move to lower values of $\lambda_V/\Lambda_X$. 
In the region \( m_\chi < m_h/2 \), most values of \( \lambda_{h}\chi / \Lambda_\chi \) for \( \cos \xi = 1, 1/\sqrt{2} \) and 0 are excluded by the combined constraints on the Majorana relic density and the Higgs invisible width apart from a small triangle that continues to evade these limits in a similar fashion to the scalar and vector models. Above Majorana masses of \( \sim 300 \) GeV, the relic density contours for \( \cos \xi = 1/\sqrt{2} \) and 0 essentially remain constant and independent of the coupling \( \lambda_{h}\chi / \Lambda_\chi \). When \( f_{\text{rel}} = 0.1 \) or 0.01, the relic density contours at higher DM masses lie in the green shaded region where \( \lambda_{h}\chi / \Lambda_\chi > 4\pi/2m_\chi \). Hence, the validity of the results in these regions cannot be guaranteed within the EFT framework.

FIG. 8. Contours of fixed Majorana relic density for \( f_{\text{rel}} = 1 \) (black solid), 0.1 (red dashed) and 0.01 (blue dotted). The grey shaded region is ruled out due to an overabundance of dark matter. The green shaded region is where the EFT approximation of the full theory breaks down for \( \lambda_{h}\chi / \Lambda_\chi > 4\pi/2m_\chi \). Left: A close-up of the resonantly enhanced annihilation region, \( m_\chi \sim m_h/2 \). Larger values of \( \lambda_{h}\chi / \Lambda_\chi \) are excluded by an upper limit of 19% (pink solid) at 2\( \sigma \) C.L. or 5% (pink dotted) at 1\( \sigma \) C.L. on \( \text{BR}(h \rightarrow \bar{\chi}\chi) \). Right: Relic density contours for the full range of \( m_\chi \).
The combined sensitivity of indirect searches to various regions in the $(m_\chi, \lambda_{h\chi}/\Lambda_\chi)$ plane is shown in Fig. 9 for $\cos \xi = 1$ (top row), $1/\sqrt{2}$ (middle row) and 0 (bottom row). In the case of $\cos \xi = 1$, the annihilation cross section $\sigma v_{\text{rel}}$ entering in the calculation of indirect detection rates receive a $v^2$ suppression, where $v \sim 10^{-3}c$ is the typical speed of dark matter in a local halo. Consequently, the resulting indirect search limits are weak and no exclusion is possible.

When interactions are pure pseudoscalar, the velocity suppression of $\sigma v_{\text{rel}}$ is lifted. Consequently, the indirect search limits are nontrivial. In the case of $\cos \xi = 1/\sqrt{2}$
and 0. Majorana masses between $m_h/2$ and 70 GeV with $\log_{10}(\lambda_{h\chi}/\Lambda_{\chi}) \in [-4.1, -2.8]$ can be seen to be in tension with the current indirect searches at more than 1σ C.L. Analogous to the indirect search limits in the scalar and vector models, a small region around $m_\chi \sim m_h/2$ will continue to evade the current and future indirect searches.

To illustrate the degree to which different indirect searches contribute to the limits, in Figs. 10–12 we show the breakdown of the current 1σ C.L., future 90% C.L. (Einasto) and projected limits from the CTA using the Einasto profile (orange dotted). Left: A close-up of the resonantly enhanced annihilation region, $m_\chi \sim m_h/2$. Right: The full range of $m_\chi$.

Projected Fermi-LAT results based on the discovery of a further 15 southern dwarf galaxies over 10 years, and projected CTA results from the GC. At low DM masses, limits from the Fermi-LAT are strongest, whereas at higher DM masses, limits from the upcoming CTA experiment are strongest. Constraints from the CMB are weak in all parts of the parameter space. Future indirect searches will be sensitive enough to exclude Majorana masses up to $\sim 170$ GeV (Einasto) and $\sim 12$ TeV (NFW, $\gamma = 1.3$) at more than 90% C.L., if $\chi$ makes up all of the dark matter.

In Fig. 13, we present limits from the LUX (blue dashed) and the projected XENON1T (blue dotted) experiment in the $(m_\chi, \lambda_{h\chi}/\Lambda_{\chi})$ plane for $\cos \xi = 1$ (top row), $1/\sqrt{2}$ (middle row) and 0 (bottom row). Although the use of...
an EFT at direct search experiments is perfectly valid, our LUX and projected XENON1T limits within the green shaded regions are still expected to be subject to unaccounted-for UV corrections. This is due to the scaling of our limits by the relic abundance parameter $f_{\text{rel}} = \Omega_\chi/\Omega_{\text{DM}}$. Consequently, values of $\lambda_{h\chi}/\lambda_\chi$ in the green shaded region for the Majorana fermion masses above $\sim 4.76$ TeV (cos $\xi = 1$), $\sim 20.3$ TeV (cos $\xi = 1/\sqrt{2}$) and $\sim 28$ TeV (cos $\xi = 0$) cannot be guaranteed to keep the EFT approximation for DM annihilations valid.

When cos $\xi = 0$, the SI cross section is momentum suppressed by a factor of $q^2/4m_\chi^2$ and hence the expected number of signal events $(s)$ is small. In fact, the imposed direct search limits are significantly weaker than the Higgs invisible width constraint at $2\sigma$ C.L. Hence, the higher Majorana mass range with a pure pseudoscalar coupling (i.e., cos $\xi = 0$) will be inaccessible at the XENON1T experiment. This coincides with better prospects from indirect detection (bottom row in Fig. 9), making the latter class of observation the only type of experiment capable of probing the higher DM mass range should nature have chosen to have DM interact with the SM Higgs boson only by a pure pseudoscalar coupling.

When interactions are pure scalar, the momentum suppression of the SI cross section is lifted. In the case of an equal mixing between the scalar and pseudoscalar terms, although the DM-nucleon effective couplings ($G_{\text{SI}}^S, \tilde{G}_{\text{SI}}^S$) are equal, the pseudoscalar effective couplings ($\tilde{G}_{\text{SI}}^S$) carry a momentum dependence of $q^2/4m_\chi^2$ as evident in Eq. (44). Consequently, the direct search limits are strongest in the case of a pure scalar interaction and moderate for an equal mix between the scalar and pseudoscalar terms.

At Majorana masses above 70 GeV, direct search experiments will exclude large portions of the model parameter space as long as the EFT approximation remains valid for the model to make up all of the dark matter. The LUX experiment excludes Majorana masses up to $\sim 4.7$ TeV when cos $\xi = 1$ and $\sim 200$ GeV when cos $\xi = 1/\sqrt{2}$. Further exclusion will also be possible with the projected XENON1T experiment.

**D. Dirac fermion model**

The Dirac fermion model is similar to the Majorana fermion model. One aspect that separates Majorana fermion from Dirac fermion dark matter is the conventional factor of $1/2$ in front of each fermion bilinear $\bar{\chi}\chi$ in defining a Majorana fermion field $\chi$. This factor of $1/2$ accounts for the field normalization and self-conjugation. The Majorana model results carry over to the Dirac model in a relatively straightforward way after this factor of 1/2 is accounted for.

In Fig. 14, we show the contours of fixed Dirac relic density in $(m_\psi, \lambda_{h\psi}/\Lambda_\psi)$ plane for $f_{\text{rel}} = 1$ (black solid), 0.1 (red dashed) and 0.01 (blue dotted). Analogous to the Majorana model, contours in each row are generated at fixed values of cos $\xi$: cos $\xi = 1$ for a pure scalar interaction (top row), cos $\xi = 1/\sqrt{2}$ for an equal mix between the scalar and pseudoscalar terms (middle row), and cos $\xi = 0$ for a pure pseudoscalar interaction (bottom row). For $m_\psi < m_\chi/2$, most of the model parameter space except a small triangular region between $\sim 57.5$ GeV and $m_\chi/2$ is excluded by the combined constraints on the Dirac relic density and the Higgs invisible width. Similar to the Majorana model, roots of Eq. (18) for $f_{\text{rel}} = 0.1$ and 0.01 do not exist at Dirac masses below $m_\chi/2$. At higher Dirac fermion masses, the relic density contours continue to increase and ultimately become independent of the coupling $\lambda_{h\psi}/\Lambda_\psi$.

The sensitivity of indirect searches to various regions in the $(m_\psi, \lambda_{h\psi}/\Lambda_\psi)$ plane when cos $\xi = 1$ (top row), $1/\sqrt{2}$ (middle row), and $0$ (bottom row). The sensitivity is strongest for pure scalar interactions and moderate for equal mixtures between the scalar and pseudoscalar terms.
Again, in the case of \( \cos \xi = 1 \), the annihilation cross section \( \sigma v_{\text{rel}} \) is velocity suppressed. Consequently, the resulting indirect search limits are weak. In fact, the relic density constraint alone is strong enough to exclude the entire region probed by the indirect search experiments in this case.

As interactions become pure pseudoscalar, the velocity suppression of \( \sigma v_{\text{rel}} \) is lifted. Therefore, parts of the model parameter space can be excluded by the indirect
search experiments. With the strongest indirect limits in the case of a pure pseudoscalar interaction, Dirac masses between $m_\psi = 2$ and 70 GeV for $\log_{10}(\lambda_{\psi}/\Lambda_\psi)$ ∈ $[-4, -3]$ can be seen to be in tension with the current indirect search limits which incorporates the combined analysis of 15 dwarf galaxies by Fermi-LAT and 7 years of CMB observation by WMAP (WMAP7). Similarly, Dirac masses below $\sim 51.5$ GeV are in tension with the current indirect searches at more than 1σ C.L. On the other hand, future indirect searches will be able to exclude Dirac masses up to $\sim 74$ GeV for the Einasto profile and

FIG. 14. Contours of fixed Dirac relic density for $f_{\text{rel}} = 1$ (black solid), 0.1 (red dashed) and 0.01 (blue dotted). The grey shaded region is ruled out due to an overabundance of dark matter. The green shaded region is where the EFT approximation of the full theory breaks down for $\lambda_{\psi}/\Lambda_\psi > 4\pi/2m_\psi$. Left: A close-up of the resonantly enhanced annihilation region, $m_\psi \sim m_h/2$. Larger values of $\lambda_{\psi}/\Lambda_\psi$ are excluded by an upper limit of 19% (pink solid) at 2σ C.L. or 5% (pink dotted) at 1σ C.L. on $\text{BR}(h \rightarrow \bar{\psi}\psi)$. Right: Relic density contours for the full range of $m_\psi$. 
between 166 GeV and 1.3 TeV for a contracted NFW profile in the case of a pure pseudoscalar interaction, if \( \psi \) makes up all of the dark matter.

In Fig. 16, we present direct search limits in the \( (m_\psi, \lambda h_{\psi} / \Lambda_\psi) \) plane for \( \cos \xi = 1 \) (top row), \( 1 / \sqrt{2} \) (middle row) and 0 (bottom row). Similar to the Majorana fermion model, the LUX/XENON1T limits within the green shaded regions are subject to UV corrections solely due to our scaling of the limits by the relic abundance parameter \( f_{rel} = \Omega_\psi / \Omega_{DM} \). For the Dirac masses above \( \sim 3.3 \) TeV (cos \( \xi = 1 \)), \( \sim 14.2 \) TeV (cos \( \xi = 1 / \sqrt{2} \)) and \( \sim 19.7 \) TeV (cos \( \xi = 0 \)), our limits cannot be guaranteed to keep the
EFT approximation for DM annihilations valid, if $\psi$ makes up all of the dark matter.

In the case of $\cos \xi = 0$, the SI cross section is momentum suppressed by a factor of $q^2/4m_\psi^2$. Consequently, the expected event rates are small. In analogy with the Majorana fermion model when $\cos \xi = 0$, better prospects from indirect searches (bottom row in Fig. 15) make the latter class of observation the only type of experiment capable of probing the higher DM mass range should nature have chosen to have DM.
interact with the SM Higgs boson only by a pure pseudoscalar coupling.

As interactions become pure scalar ($\cos \xi = 1$), the momentum suppression of the SI cross section is lifted, which results in significant direct search rates. The LUX experiment excludes Dirac masses up to $\sim3.3$ TeV when $\cos \xi = 1$ and $\sim280$ GeV when $\cos \xi = 1/\sqrt{2}$. In contrast with the Majorana model when $\cos \xi = 1$, low Dirac fermion masses between 45 GeV and $m_h/2$ will be entirely excluded by the combined limits on the Dirac fermion relic density, the Higgs invisible width and projected XENON1T experiment. Furthermore, the projected XENON1T experiment will reach higher sensitivity in excluding TeV-scale Dirac masses, if $\psi$ makes up all of the dark matter.

V. CONCLUSIONS

In this work, we have performed an updated and combined analysis of effective scalar, vector, Majorana and Dirac fermion Higgs portal models of dark matter. For the fermion models, we investigated cases where interactions are either pure scalar, pure pseudoscalar or an equal mixture of the scalar and pseudoscalar terms. The presence of the pseudoscalar term and the requirement of a quadratic DM term corresponding to a real mass lead us to redefine the post-EWSB fermion fields through a chiral rotation.

In nonfermion models, the combined constraints on the DM relic density and the Higgs invisible width exclude most of the low mass parameter space in all models apart from a small triangular region close to the resonance $m_X \sim m_h/2$. However, in the special case of a Dirac fermion DM interacting with the SM Higgs boson via a pure scalar coupling, the entire resonance region will be excluded by the projected XENON1T experiment.

For the first time, we have performed a consistent study of the indirect detection prospects in all four effective Higgs portal models of dark matter. Using current and future gamma-ray astronomy data, we looked for viable regions of the parameter space that can be probed by the existing or future indirect searches. Below the resonance $m_X \sim m_h/2$, the indirect search limits are weaker than the combined constraints on the relic density and the Higgs invisible width. Depending on the assumed DM density profile, indirect search experiments can provide strong exclusions at higher DM masses, if the model makes up all of the dark matter. The forthcoming CTA experiment will be very useful in searching for signs of DM annihilation.

In agreement with similar studies performed elsewhere, direct search experiments will continue to provide the strongest limits on the parameter space in all models. The projected XENON1T experiment, for instance, will have the sensitivity to probe TeV-scale DM masses. Since previous studies on fermionic DM models with a pure pseudoscalar interaction have naively applied XENON100 and/or LUX limits without properly taking account of the momentum-suppressed SI cross section, we have rederived these limits by including the proper momentum dependence of the form $q^2/4m_X^2$ in our calculations. The resulting limits, although weak, are important in order to perform a consistent study. In such cases, indirect search experiments are our only hope of accessing the higher DM mass range of these models.

When both the indirect and direct search limits are available, as in the portal models we have considered in this paper, a joint observation in both channels is a very realistic possibility at higher WIMP masses. Such detection in multiple experiments would provide a far more robust discovery than a single signal alone.

For the fermion models, the EFT approximation that we have employed in this paper breaks down when DM annihilations are considered. A proper examination of the fermionic Higgs portal DM in these parameter regions requires a detailed and systematic study of possible UV completions. We plan to do this in a future paper.

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APPENDIX: CHIRAL ROTATION

1. Dirac fermion model

We start with the post-EWSB Lagrangian for the Dirac fermion DM field $\psi$,

$$\mathcal{L}_\psi = \mathcal{L}_{\text{SM}} + \overline{\psi} i\gamma^\mu D_\mu \psi$$

$$= \left[ \mu_\psi \overline{\psi} \psi + \frac{1}{2} \lambda_\psi \overline{\chi}^2 \cos \theta \overline{\psi} \psi + \sin \theta \overline{\psi} i\gamma^5 \psi \right]$$

$$\frac{1}{2} \Lambda_\psi \overline{\chi}^2 \cos \theta \overline{\psi} \psi + \sin \theta \overline{\psi} i\gamma^5 \psi \left( \nu_0 + \frac{1}{2} h^2 \right).$$

The square bracket term in the above expression contains a DM mass-type term and a pseudoscalar coupling of the DM field to the SM Higgs VEV ($\nu_0$). However, it is convenient to remove the latter term ($\alpha \sin \theta$) using a field redefinition through a chiral rotation

$$\psi \to \exp(i\gamma_5 \alpha/2) \psi, \quad \overline{\psi} \to \overline{\psi} \exp(i\gamma_5 \alpha/2).$$

where $\alpha$ is a real, space-time-independent parameter.
After expanding the exponential in terms of $\cos \alpha/2$, $\sin \alpha/2$ and using the properties of the $\gamma_5$ matrix in the Dirac-representation, the exponentials become

$$\exp(i\gamma_5\alpha/2) = \cos(\gamma_5\alpha/2) + i\sin(\gamma_5\alpha/2) = \cos(\alpha/2) + i\gamma_5\sin(\alpha/2),$$

where the property $\gamma_5^2 = \gamma_5 (I_{4\times4})$ for $n$ odd (even) is used.

Under the chiral rotation in Eq. (A1), the term $\bar{\psi}i\partial_\mu \psi$ is invariant since

$$\bar{\psi}i\partial_\mu \psi \to \bar{\psi} \exp(i\gamma_5\alpha/2) i\gamma_\mu \partial_\mu (\exp(i\gamma_5\alpha/2)\psi)$$

$$= \bar{\psi} \exp(i\gamma_5\alpha/2) \gamma^\mu \exp(i\gamma_5\alpha/2) i\partial_\mu \psi$$

$$= \bar{\psi} \exp(i\gamma_5\alpha/2) \exp(-i\gamma_5\alpha/2) i\gamma^\mu \partial_\mu \psi \quad \{\gamma_5, \gamma^\mu\} = 0$$

$$= \bar{\psi}i\partial_\mu \psi.$$

On the contrary, $\bar{\psi} \psi$ and $\bar{\psi}i\gamma_5 \psi$ terms transform to

$$\bar{\psi} \to \bar{\psi} \exp(i\gamma_5\alpha/2) \exp(i\gamma_5\alpha/2)\psi$$

$$= \bar{\psi} \exp(i\gamma_5\alpha)\psi$$

$$= \cos \alpha \bar{\psi} \psi + \sin \alpha \bar{\psi} i\gamma_5 \psi$$

Therefore, terms in the Dirac Lagrangian transform to

$$\mu \psi \bar{\psi} \psi \to \mu \cos \alpha \bar{\psi} \psi + \mu \sin \alpha \bar{\psi} i\gamma_5 \psi$$

$$\frac{1}{2} \lambda_{hw} v_0^2 (\cos \theta \bar{\psi} \psi + \sin \theta \bar{\psi} i\gamma_5 \psi) \to \frac{1}{2} \lambda_{hw} v_0^2 (\cos \theta \cos \alpha \bar{\psi} \psi + \cos \theta \sin \alpha \bar{\psi} i\gamma_5 \psi + \sin \theta \cos \alpha \bar{\psi} i\gamma_5 \psi - \sin \theta \sin \alpha \bar{\psi} \psi)$$

$$= \frac{1}{2} \lambda_{hw} v_0^2 (\cos \xi \bar{\psi} \psi + \sin \xi \bar{\psi} i\gamma_5 \psi)$$

$$\lambda_{hw} (\cos \theta \bar{\psi} \psi + \sin \theta \bar{\psi} i\gamma_5 \psi) \to \lambda_{hw} (\cos \theta \cos \alpha \bar{\psi} \psi + \cos \theta \sin \alpha \bar{\psi} i\gamma_5 \psi + \sin \theta \cos \alpha \bar{\psi} i\gamma_5 \psi - \sin \theta \sin \alpha \bar{\psi} \psi)$$

$$= \lambda_{hw} (\cos \xi \bar{\psi} \psi + \sin \xi \bar{\psi} i\gamma_5 \psi)$$

where we define $\xi \equiv \theta + \alpha$ for future convenience and use the trigonometric identities $\cos \xi = \cos \theta \cos \alpha - \sin \theta \sin \alpha$ and $\sin \xi = \sin \theta \cos \alpha + \cos \theta \sin \alpha$. Therefore, the mass and interaction terms expand to

$$- \left[ \mu \psi \bar{\psi} \psi + \frac{1}{2} \lambda_{hw} v_0^2 (\cos \theta \bar{\psi} \psi + \sin \theta \bar{\psi} i\gamma_5 \psi) \right] \to - \left[ \mu \cos \alpha \bar{\psi} \psi + \mu \sin \alpha \bar{\psi} i\gamma_5 \psi + \frac{1}{2} \lambda_{hw} v_0^2 (\cos \xi \bar{\psi} \psi + \sin \xi \bar{\psi} i\gamma_5 \psi) \right]$$

$$= - \left[ \mu \cos \alpha + \frac{1}{2} \lambda_{hw} v_0^2 \cos \xi \right] \bar{\psi} \psi - \left[ \mu \sin \alpha + \frac{1}{2} \lambda_{hw} v_0^2 \sin \xi \right] \bar{\psi} i\gamma_5 \psi \quad (A2)$$

$$- \lambda_{hw} (\cos \theta \bar{\psi} \psi + \sin \theta \bar{\psi} i\gamma_5 \psi) \left( v_0 h + \frac{1}{2} h^2 \right) \to - \lambda_{hw} (\cos \xi \bar{\psi} \psi + \sin \xi \bar{\psi} i\gamma_5 \psi) \left( v_0 h + \frac{1}{2} h^2 \right). \quad (A3)$$

If $\alpha = \pi$, a sign change of the mass and interaction terms in the above expressions occurs. We can therefore, without any loss of generality, take $\mu > 0$ as long as we preserve the relative signs between the terms.

After the chiral rotation, we demand that the coefficient of $\bar{\psi} i\gamma_5 \psi$ in Eq. (A2) vanish in order to go to the real mass basis. This defines a proper chiral rotation and gives a field mass after EWSB in terms of the

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Lagrangian parameters. Setting the coefficient of $\bar{\psi} i\gamma_5 \psi$ to zero gives

\[
\mu_\psi \sin \alpha = -\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \xi
\]

\[= -\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 (\cos \theta \sin \alpha + \sin \theta \cos \alpha).\]

Dividing the above expression on both sides by $\cos \alpha$ and rearranging for $\tan \alpha$ gives

\[
\tan \alpha = \left(\frac{-1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta \right) \left(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta \right)^{-1}, \quad (A4)
\]
as stated for the Dirac fermion model in Sec. II. Using the expression for $\tan \alpha$, we can determine $\sin^2 \alpha$ and $\cos^2 \alpha$

\[
\cos^2 \alpha = \frac{1}{1 + \tan^2 \alpha} = \frac{(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta)^2}{(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta)^2 + (\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta)^2},
\]

\[
\sin^2 \alpha = \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \frac{(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta)^2}{(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta)^2 + (\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta)^2}.
\]

After EWSB and the field rotation, we define the mass $m_\psi$ as the coefficient of the term $-\bar{\psi} \psi$. Using Eq. (A2), $m_\psi$ is given by

\[
m_\psi = \mu_\psi \cos \alpha + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \xi
\]

\[= \mu_\psi \cos \alpha + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 (\cos \alpha \cos \theta - \sin \alpha \sin \theta),
\]

\[= \left[\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 (\cos \theta - \tan \alpha \sin \theta)\right] \cos \alpha.
\]

Making use of the expression for $\tan \alpha$, $m_\psi$ expands to

\[
m_\psi = \left[\left(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta\right)^2 + \left(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta\right)^2\right] \cos \alpha.
\]

Squaring both sides of the above equation and using the relation for $\cos^2 \alpha$ gives the following expression for the physical DM mass:

\[
m^2_\psi = \left[\left(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta\right)^2 + \left(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta\right)^2\right] \cos^2 \alpha
\]

\[= \left[\left(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta\right)^2 + \left(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta\right)^2\right] \cos^2 \alpha.
\]

The signs of $m_\psi$, $\cos \alpha$ and $\sin \alpha$ are common; hence, we choose the common sign to be “+” for $m_\psi$, $\cos \alpha = +\sqrt{\cos^2 \alpha}$ and $\sin \alpha = \sqrt{\sin^2 \alpha}$. Therefore, the physical DM mass after EWSB is

\[
m_\psi = \sqrt{(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta)^2 + \left(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta\right)^2}.
\]

With the sign convention chosen above, the expressions for $\cos \xi$ and $\sin \xi$ reduce to

\[
\cos \xi = \cos \alpha (\cos \theta - \sin \theta \tan \alpha)
\]

\[= \frac{\mu_\psi \cos \theta + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta}{\sqrt{(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta)^2 + \left(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta\right)^2}}
\]

\[= \frac{\mu_\psi}{m_\psi} \cos \theta + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} \mu_\psi \cos \theta.
\]

\[
\sin \xi = \cos \alpha (\sin \theta + \cos \theta \tan \alpha)
\]

\[= \frac{\mu_\psi \sin \theta}{\sqrt{(\mu_\psi + \frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \cos \theta)^2 + \left(\frac{1}{2} \frac{\lambda_{\psi\psi}}{\Lambda_\psi} v_0^2 \sin \theta\right)^2}}
\]

\[= \frac{\mu_\psi}{m_\psi} \sin \theta
\]

where we have used Eq. (A5) to simplify the expressions. Therefore, the final Lagrangian after EWSB and field redefinition is given by

\[
\mathcal{L}_\psi = \mathcal{L}_{SM} + \bar{\psi} i\gamma_5 \psi - m_\psi \bar{\psi} \psi
\]

\[= \frac{-\lambda_{\psi\psi}}{\Lambda_\psi} [\cos \xi \bar{\psi} \psi + \sin \xi \bar{\psi} i\gamma_5 \psi] \left(v_0 h + \frac{1}{2} h^2\right),
\]

where
\[
\cos \xi = \frac{\mu_y}{m_y} \left( \cos \theta + \frac{1}{2} \lambda_{hy} v_0^2 \cos \theta \right), \quad \sin \xi = \frac{\mu_y}{m_y} \sin \theta,
\]
\[
m_y = \sqrt{\left( \frac{\mu_y + \frac{1}{2} \lambda_{hy} v_0^2 \cos \theta}{2 \lambda_y} \right)^2 + \left( \frac{1}{2} \lambda_{hy} v_0^2 \sin \theta \right)^2}.
\]

2. Majorana fermion model

Consider the Majorana fermion DM field \( \chi \) Lagrangian after EWSB,
\[
\mathcal{L}_x = \mathcal{L}_{SM} + \frac{1}{2} \tilde{\chi} i \partial \chi - \frac{1}{2} \left[ \mu_\chi \tilde{\chi} + \frac{1}{2} \lambda_{hy} v_0^2 \left( \cos \theta \tilde{\chi} + \sin \theta \tilde{\chi} \gamma^5 \right) \right] - \frac{1}{2} \lambda_{hy} \left( \cos \theta \tilde{\chi} + \sin \theta \tilde{\chi} \gamma^5 \chi \right) \left( v_0 h + \frac{1}{2} h^2 \right).
\]

With the conventional factor of 1/2 in front of each Majorana field bilinear, the form of the Majorana model Lagrangian after EWSB and chiral rotation is analogous to that of the Dirac model. The proper chiral rotation is again given by
\[
\tan \alpha = \left( \frac{1}{2} \lambda_{hy} v_0^2 \sin \theta \right) \left( \mu_\chi + \frac{1}{2} \lambda_{hy} v_0^2 \cos \theta \right)^{-1},
\]
which leads to a real, physical DM mass of
\[
m_x = \sqrt{\left( \mu_\chi + \frac{1}{2} \lambda_{hy} v_0^2 \cos \theta \right)^2 + \left( \frac{1}{2} \lambda_{hy} v_0^2 \sin \theta \right)^2}.
\]

Finally, the Majorana fermion Lagrangian after EWSB and chiral rotation is given by
\[
\mathcal{L}_x = \mathcal{L}_{SM} + \frac{1}{2} \tilde{\chi} i \partial \chi - \frac{1}{2} m_x \tilde{\chi} \chi - \frac{1}{2} \lambda_{hy} \left[ \cos \xi \tilde{\chi} + \sin \xi \tilde{\chi} \gamma^5 \chi \right] \left( v_0 h + \frac{1}{2} h^2 \right),
\]
where
\[
\cos \xi = \frac{\mu_\chi}{m_x} \left( \cos \theta + \frac{1}{2} \lambda_{hy} v_0^2 \cos \theta \right), \quad \sin \xi = \frac{\mu_\chi}{m_x} \sin \theta,
\]
\[
m_x = \sqrt{\left( \mu_\chi + \frac{1}{2} \lambda_{hy} v_0^2 \cos \theta \right)^2 + \left( \frac{1}{2} \lambda_{hy} v_0^2 \sin \theta \right)^2}.
\]

as stated for the Majorana fermion model in Sec. II.


