Chapter 6

Backmixing of the liquid phase

Abstract

In this chapter, the final step in our scale up strategy, outlined in Fig. 1.8, is presented. The link between the liquid circulation, in terms of centre-line liquid velocity, and the backmixing of the liquid phase is developed.

The liquid-phase axial dispersion coefficients have been measured for the air-water system in bubble columns of 0.174, 0.38 and 0.63 m diameter. The experiments were carried out using a transient method (the pulse-response method). The dispersion coefficient was obtained by adjusting the experimental profiles of tracer concentration with the predictions of the model. The experimental results show that the one-dimensional axial dispersion coefficient, $D_{ax,L}$, reveal strong scale dependence. Backmixing of the liquid phase increases with the diameter of the reactor and with superficial gas velocity. Once the centre-line liquid velocity is estimated, the axial dispersion coefficient for large column reactors can be easily predicted from $D_{ax,L} = 0.31V_L(0)D_T$. Comparison of the model predictions with the experimental data shows that the model is capable of describing very well the liquid mixing behaviour in bubble columns. The model can be extended also for highly viscous liquids, on basis of the results obtained in Chapter 5 - practically equal liquid velocity distributions for the two systems studied (the two liquids investigated differ in viscosity by a factor 75!).

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6.1 Introduction

In a gas liquid bubble column reactor the degree of backmixing is considerably different from one phase to another. The dispersion has the effect of reducing conversion in reactors. Dispersion will also influence reaction selectivity. Mixing of the liquid phase has to be attributed to various phenomena such as turbulent vortices, liquid entrainment in the wakes of rising bubbles, large-scale liquid circulation and radial exchange flows. All these phenomena are obviously interrelated and are primarily dependent on liquid circulation flow. The model described in this chapter simply gathers the different phenomena which contributes to mixing of the liquid phase into a dispersion coefficient. The one-dimensional description based on the one-dimensional diffusion model is considered sufficient for interpretation of bubble column performance.

When the considered systems are of gas-liquid type, dispersion can be defined as a stochastic exchange process, which breaks down gradients of intensive quantities such as concentration and temperature. As a direct effect, the concentration in the reactor is reduced. In a bubble column reactor, the gas flow generates significant backmixing in the liquid phase. After a thorough analysis it can be noticed that the backmixing of the dense phase is caused by the eddies, which derive their energy from the large, fast-rising bubbles. The maximum size of the eddies, for vessels with \( H/D_T > 1 \), is limited by the column diameter \( D_T \). The induced circulation patterns have a profound effect on mass transfer and productivity of these systems and they are especially important in eliminating concentration gradients within the vessel. The design of a bubble column reactor always requires consideration of the backmixing in the liquid phase.

![Diagram](image.png)

Fig. 6.1. Strategy for scaling up bubble column reactors.
Since the axial dispersion model characterises the backmixing by only a single parameter, its simplicity makes it the most widely used representation of the non-ideal mixing behaviour for each phase in bubble column reactors. The time variation of the liquid phase concentration of a tracer is given by Fick's law:

\[
\frac{\partial C_L}{\partial t} = D_{ax,l} \frac{\partial^2 C_L}{\partial z^2}
\]

(6.1)

where the axial dispersion coefficient, \(D_{ax,l}\), is the unique parameter which characterises the degree of backmixing during the process. The term "axial" is used in order to mark clearly the difference between the mixing in the direction of flow and the mixing in the lateral or radial direction. In bubble columns these two quantities are quite different in magnitude; the axial dispersion coefficient is significantly exceeding the radial dispersion coefficient.

Hydrodynamics of bubble columns can be described in a simplified way as follows: in the churn turbulent regime, the large bubbles tend to concentrate in the middle of the column. The liquid is drawn upwards in the centre region and when the bubbles disengage at the top, the liquid returns down the column in the wall region. The radial distribution of the liquid velocities is related with the dispersion coefficient of the liquid phase. Therefore, the complete overview starts from bubble rise velocity, bubble-bubble interactions and bubble swarm velocity, as presented in the previous chapters. An updated scale up scheme is shown in Fig. 6.1.

Bubble column reactors are chosen as the reactor type to carry out relatively slow liquid-phase reactions (see Fig. 1.2), in which the liquid phase backmixing is a preferable feature in order to achieve temperature equalisation. Therefore, the radial distribution of liquid velocities and the residence time distributions of the liquid phase are very important design parameters.

Though there is a large variety of experimental data on liquid phase backmixing available in the literature for different column diameters and superficial gas velocities, it is difficult to compare the results of one author with those of others for the scale up task. The reasons are: (1) differences in the physical properties of the liquids used in various studies, (2) presence of impurities in the liquid phase, and (3) the fact that each published study is often restricted to one column with a diameter often smaller than 0.3 m.

### 6.2 Literature survey

We have made a comprehensive study of literature correlations for \(D_{ax,L}\) and these are summarised in Table 6.1.

The dispersion coefficient is expressed in the dimensionless form as the Peclet number, its value signifying the degree of backmixing in the column. If \(Pe = 0\) backmixing is complete and if \(Pe = \infty\) plug flow prevails.

Most of the reported empirical correlations indicate the dispersion coefficient to be dependent on the gas velocity and column diameter. A significant influence of the flow direction (i.e. co-current or counter-current) has not been pointed out. If the range of liquid velocities used in industrial operation is considered, the superficial liquid velocity \(U_L\) appears to have no influence on liquid-phase dispersion. Steady state measurements by Towell and Ackerman (1972), Deckwer et al. (1973) and Badura et al. (1974) confirm this fact. It is usually assumed that the dispersion coefficient does not depend on the column height. However, studies of Schügerl (1967) and Deckwer et al. (1973) show that dispersion coefficients may differ along the column height, decreasing from top to bottom.
Vessel internals and surface-active agents that delay the coalescence also affect the degree of axial dispersion. Konig et al. (1978) demonstrated the effects of surfactants and sparger type by experimenting with weak alcohol solutions using three different porous spargers. They indicated that the interaction of surfactants and of the sparger could be very complex. Surfactants can produce either much more or much less backmixing than surfactant-free systems, depending on the bubble size, which, in turn, depends on the sparger used.

The influence of the physical properties of the liquid on the dispersion coefficient has been investigated by several authors (Aoyama et al., 1968; Hikita and Hikukawa, 1974; Riquarts, 1981; Walter and Blanch, 1983; Bernemann, 1989) but little effect has been observed.

Some authors (Tinge and Drinkenburg, 1986; Rice and Littlefield, 1987) underlined the disastrous effect of small deviations from true verticality. The scatter in the literature data was attributed to originate in very slight and unreported (or unmeasured) deviations from verticality. They quantified this effect by the following correlation:

\[
D_{\text{ax,l}} = D_{\text{ax,l}} (1 + K_\alpha D_t \alpha)^2
\]

(6.2)

For reactor scale up purposes, the most reliable relation for vertical cylindrical vessels operated with pure liquids, without coalescence inhibitors, is considered to be the one of Baird and Rice (1975). Making use of Kolmogoroff's theory of isotropic turbulence, they proposed the following equation:

\[
D_{\text{ax,l}} = 0.35 g^{1/3} D_T^{4/3} U_G^{1/3}
\]

(6.3)

Furthermore, the authors extended the correlation to fluidised beds and liquid-liquid spray columns, which proved to be partially successful. Using an isotropic turbulence, the correlation is dimensionally consistent even though the turbulence in a bubble column is not necessarily isotropic.

The dependency of \(D_{\text{ax,l}}\) on \(D_T\) and \(U_G\) expressed by Eq. (6.3) has been confirmed by many studies and can be also transcript in dimensionless terms of \(Pe\) and \(Fr\) criteria (where the constant \(K\) has different values for different authors, as can be seen in Table. 6.1):

\[
\frac{D_{\text{ax,l}}}{D_T U_G} = K \left( \frac{g D_T}{U_G^2} \right)^{1/3}
\]

(6.4)

In 1996 Degaleesan et al. developed a phenomenological model for liquid mixing, mainly based on the monitored flow circulation and turbulent eddy diffusion. The model was called "recirculation and cross flow with dispersion" (RCFD) and accounts for two mixing mechanisms. The first one is the convective recirculation, due to the large scale liquid motion. In this context, the column can be simply considered as a big recirculation cell, with liquid moving upwards in the central core zone and descending along the annular region, near the walls of the column. Besides this, fluctuated motion of the liquid elements superimposes, due to the bubble turbulence. This second mixing contribution takes into account the complex motion of the gas bubbles, and therefore the random motion of fluid elements in axial, radial and azimuthal direction caused by the wakes of fast-rising large bubbles. The upflow and downflow, characterised by averaged interstitial velocities are connected to each end by two regions, which are assumed well-mixed. The turbulent axial mixing is accounted for by an axial dispersion coefficient in each section; the radial mixing is incorporated into an exchange coefficient between the two sections (i.e. the upflow region and the downflow region). The input parameters of the model (such as local liquid velocities and void fraction) were obtained...
using Computer Automated Particle Tracking (CARPT) and Computed Tomography (CT) as experimental techniques.

This model was recently further developed and extended also for bubble columns slurry reactors by Degaleesan et al. in 1997. The two-compartment convective-diffusion model has been formulated by considering a finite volume discretisation of the two-dimensional axis-symmetric convective-diffusion model developed for bubble columns flows. The same experimental techniques were used for the data base, together with data from literature. The existing data-base was limited to air-water systems, certain column sizes and superficial gas velocities. The methodology aimed to extrapolate the data giving the following cross sectional averaged eddy diffusivities $D_\alpha$ and $D_\pi$:

$$D_\alpha = -\frac{0.00584}{D_T^{0.8}} + 0.1689(D_T U_G)^{0.3}$$(6.5) \\
$$D_\pi = -\frac{0.000879}{D_T^{0.8}} + 0.0206(D_T U_G)^{0.3}$$(6.6)

for $U_G \geq 0.05$ m/s. The authors limited the applicability of the above developed correlations only to air-water systems in churn turbulent regime, at atmospheric pressure. When the axial term is compared with experimental data from this work, they show a good agreement for low gas velocity only. This can not be attributed only to the fact that the radial contribution is also taken into account, as long as this term is much smaller, but to the currently limited data base, as is also mentioned by the authors. They advised that for higher gas velocities the equations presented above have to be verified in compliance with experimental data.

### Table 6.1

**Literature correlations for the axial dispersion coefficient**

<table>
<thead>
<tr>
<th>Author</th>
<th>Axial dispersion coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Reith et al. (1968)</td>
<td>$Pe_e = \frac{U_c D_\alpha}{D_{\alpha,t}} = 3 \pm 0.3$; $U_c = 2U_G + u_{sw}$</td>
</tr>
<tr>
<td>2. Ohki and Inoue (1970)</td>
<td>for $0 \leq U_G \leq 0.07$ m/s (bubbly flow regime) $D_{\alpha,t} = 75.4 D_T^{1.2} U_G^{1.2} + 17000 d_o$</td>
</tr>
<tr>
<td></td>
<td>for $U_G \geq 0.1$ m/s (coalesced bubble slug flow regime) $D_{\alpha,t} = \frac{0.14 D_T}{(1 - \varepsilon_g)^2}$</td>
</tr>
<tr>
<td>3. Kato and Nishiwaki (1972)</td>
<td>$Pe = \frac{13 Fr_G}{1 + 6.5 Fr_G^{0.8}}$; $D_{\alpha,t} = \frac{g^{0.5} D_T^{1.5} \left[ 1 + 6.5 \left( \frac{U_G}{gD_T} \right)^{0.8} \right]}{13}$</td>
</tr>
<tr>
<td>4. Towell and Ackerman (1972)</td>
<td>$D_{\alpha,t} = 1.23 D_T^{1.2} U_G^{0.5}$</td>
</tr>
<tr>
<td>Author</td>
<td>Axial dispersion coefficient</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Akita and Yoshida (1973)</td>
<td>$D_{ax,L} = U_G D_T \left( \frac{U_G}{\sqrt{gD_T}} \right)^{0.7}$</td>
</tr>
<tr>
<td>Badura (1974)</td>
<td>$D_{ax,L} = 0.692D_T^{1.4} U_G^{0.33}$</td>
</tr>
<tr>
<td>Deckwer (1974)</td>
<td>$D_{ax,L} = 0.678D_T^{1.4} U_G^{0.33}$</td>
</tr>
<tr>
<td>Hikita and Kikukawa (1974)</td>
<td>$D_{ax,L} = (0.15 + 0.69U_G^{0.77}) D_T^{2.5} \left( \frac{10^{-1}}{\eta_L} \right)^{0.12}$</td>
</tr>
<tr>
<td>Baird and Rice (1975)</td>
<td>$D_{ax,L} = 0.33 g^{1/3} D_T^{4/3} U_G^{1/3}$</td>
</tr>
<tr>
<td>Field and Davidson (1980)</td>
<td>$D_{ax,L} = 0.9 D_T^{1.3} \left[ H_d \left( U_G - \delta G V_s \right) \right]^{1/3}$</td>
</tr>
<tr>
<td>Joshi (1980)</td>
<td>$D_{ax,L} = 0.33 D_T V_c$</td>
</tr>
<tr>
<td>Miyachi et al. (1981)</td>
<td>$D_{ax,L} = U_G D_T^{2/3} \left( 0.291 + \frac{0.341}{\left( 1 - 0.54 U_G^{1/3} \right)^2} \right)$</td>
</tr>
<tr>
<td>Riquarts (1981)</td>
<td>$D_{ax,L} = 0.068 g^{3/8} D_T^{3/2} U_G^{3/5} L^{1/5}$</td>
</tr>
<tr>
<td>Zehner (1982)</td>
<td>$D_{ax,L} = 0.368 g^{1/3} D_T^{3/2} U_G^{1/3}$</td>
</tr>
<tr>
<td>Walter and Blanch (1983)</td>
<td>$D_{ax,L} = 0.606D_T V_i$; $V_i = \left( \frac{U_G g H_0}{70 + \frac{H_0}{\text{Re}^{0.25} D_T}} \right)^{1/3}$</td>
</tr>
<tr>
<td>Kawase and Moo-Young (1986)</td>
<td>$D_{ax,L} = 0.343 n^{-2/3} g^{1/3} D_T^{4/3} U_G^{1/3}$</td>
</tr>
<tr>
<td>Bernemann (1989)</td>
<td>$D_{ax,L} = 0.208 U_G^{0.4} \left( D_T + n_i d_i \right)^{3/4} A_i^{1.8} V_L^{0.12}$</td>
</tr>
<tr>
<td>Kantak (1994)</td>
<td>$D_{ax,L} = 0.632 D_T^{2/5} \frac{U_G}{\delta G}$</td>
</tr>
</tbody>
</table>
We performed further a statistical comparison, in the same manner we described before for the radial distribution of the liquid velocities and for the centre-line liquid velocity [Eq. (5.12)]. Table 6.2 shows the results of the standard deviations of literature correlations from our experimental data. The analysis takes into account \( ND = 92 \) data points, representing the complete set of data points measured in our laboratory and reported in detail later in this chapter. We are using a “preview” of our experimental results in order to “rank” the literature correlations.

**Table 6.2**

Standard deviation of literature correlations for axial dispersion coefficient from experimental data

<table>
<thead>
<tr>
<th>Reference</th>
<th>Standard deviation, ( S / [m^2/s] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reith et al. (1968)</td>
<td>0.06595</td>
</tr>
<tr>
<td>Ohki and Inoue (1970)</td>
<td>0.05776</td>
</tr>
<tr>
<td>Kato and Nishiwaki (1972)</td>
<td>0.03162</td>
</tr>
<tr>
<td>Towell and Ackerman (1972)</td>
<td>0.04032</td>
</tr>
<tr>
<td>Akita and Yoshida (1973)</td>
<td>0.03047</td>
</tr>
<tr>
<td>Badura (1974)</td>
<td>0.03280</td>
</tr>
<tr>
<td>Deckwer (1974)</td>
<td>0.03390</td>
</tr>
<tr>
<td>Hikita and Kikukawa (1974)</td>
<td>0.03187</td>
</tr>
<tr>
<td>Baird and Rice (1975)</td>
<td>0.03189</td>
</tr>
<tr>
<td>Field and Davidson (1980)</td>
<td>0.09577</td>
</tr>
<tr>
<td>Joshi (1980)</td>
<td>0.03479</td>
</tr>
<tr>
<td>Miyauchi et al. (1981)</td>
<td>0.05821</td>
</tr>
<tr>
<td>Riquarts (1981)</td>
<td>0.03225</td>
</tr>
<tr>
<td>Zehner (1982)</td>
<td>0.03529</td>
</tr>
<tr>
<td>Walter and Blanch (1983)</td>
<td>0.05105</td>
</tr>
<tr>
<td>Kawase and Moo-Young (1986)</td>
<td>0.03115</td>
</tr>
<tr>
<td>Bernemann (1989)</td>
<td>0.05186</td>
</tr>
<tr>
<td>Kantak 1994</td>
<td>0.05780</td>
</tr>
</tbody>
</table>

By interpreting the correlations found in the literature it can be easily concluded that, from the scale up point of view, data from larger columns are absolutely necessary. To correlate measured values for axial dispersion coefficient is not difficult, as it can be also seen from the large amount of models published so far. Hence, the important and difficult task is to achieve the capability to predict \( D_{axL} \) values for a big reactor, operated at industrial conditions. A selection of the literature models from Table 6.1 is plotted in Fig. 6.2 together with our experimental data, revealing the magnitude of the discrepancies between their predictions. The large scatter in the predictions increases with the diameter of the column.

The degree of backmixing is dictated by the strength of the liquid circulations, characterised by say \( V_L(0) \). Since \( V_L(0) \) is scale dependent, the axial dispersion coefficient \( D_{axL} \) is also expected to be scale dependent. As can be seen in Fig. 6.3 scaling up to commercial sizes leads to great uncertainty. The correlations are more or less comparable in
the range of laboratory experimented columns. Nevertheless, when we try to extrapolate the literature correlations up to industrial scale, we obtain values from 1 to 12 m$^2$/s for $D_{ax,L}$.

![Graph](image)

Fig. 6.2. Comparison of literature models (selection from Table 6.1) and measured experimental data from this work for the axial dispersion coefficient, in air-water system. Variation of $D_{ax,L}$ with superficial gas velocity for the 0.174, 0.38 and 0.63 m diameter columns.

This scatter in the predictions convinces us to start our own study regarding this variable. The experimental data were obtained in the same set-ups as used for the other variables presented in the previous chapters. The experimental details are presented further.
Fig. 6.3. Comparison of literature models (selection from Table 6.1) and measured experimental data from this work for the axial dispersion coefficient. Variation of $D_{ax,L}$ with column diameter for a superficial gas velocity of 0.23 m/s. Detail: extrapolation up to 6 m column.

6.3 Experimental

The experiments were carried out in three batch type bubble columns with internal diameters of 0.174, 0.38 and 0.63 m, respectively. The general constructional details for these three columns and the description of the gas distributors used can be found in the previous chapters of this thesis (Chapters 4 and 5). All the experiments were performed at atmospheric pressure. Air was used as the gas phase and demineralised water as liquid phase (see Table 3.4 for physical properties of the phases). The gas was introduced at the bottom of the columns. The experiments were carried out at various gas velocities, carefully adjusted and controlled by using rows of flowmeters. For each gas flow rate and each position of injection three measurements were performed. Before starting the data acquisition, for a given gas flow rate, the system was given time to achieve steady state. A typical experimental set-up is shown in Fig. 6.4 for the 0.63 m column.

For determination of the liquid-phase residence time distribution a saturated solution of NaCl was used as a tracer. The solution was injected into the batch liquid phase as a pulse just above the dispersion height. Different volumes of tracer were used, depending on gas velocity, column diameter and injection position in order to obtain the optimal signal. In the 0.63 m diameter column the tracer was injected both in the middle and near the wall. For the remaining two columns a single injection position was used, near the wall.
The transient tracer concentration was monitored continuously by means of three Metrohm immersing-type conductivity cells (Fig. 6.5) which were placed near the wall, at different locations along the height of the column, as shown in Fig. 6.6. The distances from the injection to the measuring points, $L_A$, $L_B$, $L_C$ and $L$ are given in Table 6.3.

Each of the three conductivity cells was connected to a Konsort K920 display unit and from there to the "PC-LabCard PCL-818L" data acquisition card. The data was acquired using a QBasic program.
The total measuring time for each experiment was chosen large enough in order to reach the final concentration in the column. Therefore the frequency was carefully chosen, 20 Hz for the 0.63 m column and 50 Hz for the 0.174 and 0.38 m columns. This values were able to reproduce very accurate the concentration signal even for the very sudden changes in concentration (for example the sharp peak obtained with the top conductivity cell).

The constructive details about the backmixing experiments, in the three bubble columns, are specified in Table 6.3. The operating conditions used for the performed experiments are given in Table 6.4.

### Table 6.3

Constructional details about the backmixing experiments

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>Column diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_t = 0.174$ m</td>
</tr>
<tr>
<td>Liquid height (range), $H_0$ / [m]</td>
<td>1.97 - 2.55</td>
</tr>
<tr>
<td>Dispersion height, $H_d$ / [m]</td>
<td>2.89</td>
</tr>
<tr>
<td>Distance to the measuring point (see Fig. 6.6), $L_A$, $L_B$, $L_C$ / [m]</td>
<td>$L_A = 0.59$</td>
</tr>
<tr>
<td></td>
<td>$L_B = 1.39$</td>
</tr>
<tr>
<td></td>
<td>$L_C = 2.19$</td>
</tr>
</tbody>
</table>

### Table 6.4

Operating conditions used for measuring the axial dispersion coefficient

<table>
<thead>
<tr>
<th>System studied</th>
<th>Column diameter $D_t$ / [m]</th>
<th>Superficial gas velocity range, $U_G$ / [m/s]</th>
<th>Gas hold up range $e_G$ / [-]</th>
<th>Number of experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>air - water</td>
<td>0.174</td>
<td>0.049 - 0.331</td>
<td>0.11 - 0.26</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>0.38</td>
<td>0.049 - 0.331</td>
<td>0.137 - 0.276</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>0.63</td>
<td>0.049 - 0.33</td>
<td>0.12 - 0.306</td>
<td>48</td>
</tr>
<tr>
<td><strong>Total number of experiments:</strong></td>
<td></td>
<td></td>
<td></td>
<td>92</td>
</tr>
</tbody>
</table>
Data analysis

Figure 6.7 shows typical transient tracer concentrations from the 0.174 m column, operated at 0.144 m/s superficial gas velocity. These signals were fitted using the analytic solution to the diffusion equation presented in Deckwer (1992). In this way, for a given experiment, only one variable was adjusted, i.e. the axial dispersion coefficient, \( D_{ax,L} \).

\[
D_t = 0.174 \text{ m} \\
U_g = 0.144 \text{ m/s}
\]

Fig. 6.7. Normalised liquid-phase tracer concentration measured at three different locations along the height of the column in response to pulse tracer injection. The smooth curves represent the fits to the curves from fitting a diffusion model presented in Deckwer (1992).

From the literature investigation was found that there are two different measuring techniques widely used along the time. Since the stationary method is rather time consuming and expensive (Aoyama et al., 1968; Reith et al., 1968; Badura et al., 1974; Deckwer et al., 1974) it is convenient to use a non-steady-state technique, such as the pulse-response method (the mixing time determinations). The mixing time is defined as time necessary to achieve the homogeneity in the column, after all amount of tracer was completely mixed with the liquid. Siemens and Weiss (1957) were the first using the pulse method for measuring the dispersion in bubble columns. They were followed by Ohki and Inoue (1970), Kato and Nishiwaki (1970), Hikita and Kikukawa (1974), Field and Davidson (1980), to name just a few.

The partial differential equation based on the one-dimensional model has been solved analytically by Siemes and Weiss (1957). The boundary conditions for this diffusion equation [Eq. (6.1)] are:

\[
\frac{\partial C_l}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = L
\]

and the initial conditions are:

\[
C_l(z,0) = C_0 \text{ for } 0 \leq z \leq \lambda \\
C_l(z,0) = 0 \text{ for } z \geq \lambda
\]
where \( \lambda \) is the height at which tracer is injected. The solution of Eq. (6.1) under consideration of the corresponding boundary conditions gives:

\[
\frac{C_L}{C_0} = 1 + 2 \sum_{n=1}^{\infty} \left[ \cos \frac{n \pi}{L} L \right] \exp \left(- \frac{n^2 \pi^2}{L^2} D_{ax} t \right)
\]

(6.9)

where \( C_0 \) is the final concentration achieved when \( t = \infty \) and \( L \) is the distance to the measuring point \((L_A, L_B \text{ and } L_C)\).

The axial dispersion coefficient \( D_{ax,L} \) was obtained by adjusting the experimental profiles with the solution of the model, Eq. (6.9). A number of \( n = 20 \) terms were found to be sufficient. As it can be seen, the evaluation of \( D_{ax,L} \) from the curves produced by the pulse method is fast and reasonable good.

### 6.4 Results and discussion

The last step in the scale up strategy outlined in this thesis is the liquid-phase backmixing. When the bubbles escape at the top of the column, the liquid returns down the sides of the column (in the wall region). The downflowing liquid generates backmixing. The radial distribution of the liquid velocities is related with the axial dispersion coefficient of the liquid phase. Our experimentally measured axial dispersion coefficients for the liquid phase show a strong function of the column diameter; see Fig. 6.8. Once again, the limited laboratory conditions confine us in a relatively small area far from the real industrial dimensions.

![Fig. 6.8](image_url)

**Fig. 6.8.** Axial dispersion coefficient of the liquid phase measured in three columns, as a function of superficial gas velocity.

The axial dispersion coefficients can be predicted in terms of liquid circulation velocity, as a characteristic velocity and column diameter, as a characteristic length. This description as a function of the centre-line liquid velocity is possible only because the liquid profiles were successfully normalised, as shown in the previous chapter (Fig. 5.17). The model shown in
Fig. 6.9 was obtained by making use of the dimensionless analysis. Our suggestion is thus to use the following correlation for estimation of $D_{ax,L}$:

$$D_{ax,L} = 0.31 V_L(0) D_T$$  \hfill (6.10)

once the centre-line liquid velocities are estimated either from using Eulerian simulations or say from the Riquarts (1981) correlation. The form of Eq. (6.10) was first suggested by Field and Davidson (1980) and Joshi (1980) and can be derived by considering dispersion in single phase turbulent flow. The constant 0.31 was found taking into account the whole experimented range of superficial gas velocities. However, the constant, 0.31, in the presented model does not give a different picture than some of the other published models. For instance, this constant is about 0.32 in the model of Riquarts (1981) and about 0.36 in the model of Joshi (1980). The standard deviation of Eq. (6.10) from our experimental data set is 0.02 m$^2$/s, which is significantly lower than those of the other literature correlations (see Table 6.2).

Since we have established that $V_L(0)$ is not significantly influenced by the liquid viscosity (see Chapter 5), we expect Eq. (6.10) to be valid for all systems. Following the conclusion reached in Chapter 5, we recommend the use of the Riquarts correlation, with water properties, for estimation of the centre-line liquid velocity $V_L(0)$ for use in Eq. (6.10). This estimation procedure works very well not only for our own experimental data but also for data culled from the literature; see Fig. 6.10. Equation (6.10) presents a reliable and accurate method of predicting the axial dispersion coefficients in case of large diameter columns.
Fig. 6.10. Comparison between the measured axial dispersion coefficient $D_{axL}$ (our data and from the literature) and the predictions of the model given by Eq. (6.10). The $V_L(0)$ is predicted using the Riquarts (1981) correlation.
6.5 Conclusions

The main results presented in this chapter are:

- The column diameter significantly influences the backmixing of the liquid phase. The axial dispersion coefficient of the liquid phase increases with the diameter of the reactor and with superficial gas velocity.

- The measurements on the axial dispersion coefficient in the liquid phase show that this parameter can simply be predicted as being proportional to the product of the centre-line liquid velocity, $V_L(0)$, and the column diameter, $D_T$ [Eq. (6.10)]. One should keep in mind that the centre-line liquid velocity is also a function of column diameter! As already concluded in Chapter 5, the Riquarts (1981) correlation is considered to be suitable for estimation of $V_L(0)$ and extrapolation to commercial scale reactors [Eq. (5.42)].

- We assume the liquid properties to have negligible influence on the liquid-phase axial dispersion coefficient. This hypothesis is indirectly obtained from the virtually equal results obtained in Chapter 5 for the liquid velocity distributions studied in two liquids, water and Tellus oil, which differ in viscosity by a factor 75.