Dynamics of Gauge Fields at High Temperature

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1 Introduction

1.1 Early universe

In the 1920's it was discovered that the universe is not static, but that it expands. The observed expansion lies at the basis of standard cosmology. A very successful model for the evolution of the universe is the hot Big Bang model [19, 73], which states that the universe is not infinitely old but came into existence 10-20 billion years ago. The universe started out extremely hot and dense after which it expanded and cooled down, to the present state. During its evolution and cooling down a number of interesting events took place, which we review with increasing temperature and therefore anti-chronologically (we prefer to start from the known and go towards the unknown).

At a temperature \( T = 0.3 \text{ eV} = 3575 \text{ K} \) (we use units where Boltzmann's constant \( k_B = 1 \)), about 200,000 years after the Big Bang, electrons combined with protons and photons decoupled from the plasma. The observed cosmic microwave background radiation (CMBR) is a relic of this event. The CMBR has a thermal spectrum at a temperature of about 2.7 K. This provides a direct observation of the thermal nature of matter in the early universe.

Direct observational evidence that supports the hot Big Bang model extends back to the epoch of primordial nucleosynthesis \( t = 0.01 - 100 \text{ sec} \) after the Big Bang at temperatures of about \( T = 0.1 - 10 \text{ MeV} \). The observed light-element abundances are in agreement with what would be synthesized in a hot expanding universe. Theoretical calculations of the abundances requires one input parameter, the baryon to photon ratio. From the comparison of such calculations with observational data the baryon to photon density may be inferred [101]

\[
\frac{n_B}{n_\gamma} = (1.55 - 4.45) \times 10^{-10}, \tag{1.1}
\]

with the baryon-number density \( n_B \) and photon density \( n_\gamma \).
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From the knowledge of particle physics up to energies of about a few hundred GeV it is possible to extrapolate the model further back in time. There are at least two more interesting events that are then encountered. The deconfinement-confinement phase transition at $T \approx 150$ MeV. Before this phase transition, quarks and gluons were not bounded but moved freely in the so-called quark gluon plasma (QGP). The existence of this new state of matter may have been experimentally confirmed at CERN last year.

Another event of interest in the early universe is the electroweak phase transition at $T \approx 100$ GeV $= 10^{15}$ K, about $t = 10^{-10}$ sec after the Big Bang. After the phase transition, the particles in the standard model acquire their masses through the Higgs mechanism. Before the transition, the Higgs expectation value is zero and particles are massless. (This is rather imprecise, since the particles form a plasma and we cannot consider them as free particles; in the plasma particles acquire thermal masses.)

The electroweak phase-transition forms the border between well-known cosmology and more speculative ideas about the universe. This may be illustrated by the phase transition itself. In the minimal standard model for experimentally allowed Higgs masses there is not a phase transition but instead a cross-over. However, a standard scenario for baryogenesis requires a first-order electroweak phase-transition. In extensions of the standard model, such as the minimal supersymmetric standard model, the transition may be first-order. It is possible to severely constrain the parameters of such models by the requirement that sufficient baryons are generated. This is an example, where cosmological observations are used to constrain particle-physics theories.

Finally, the evolution of the universe before the electroweak phase transition depends on the particle model (GUT, supersymmetric extensions of the standard model,...) that is valid for these higher energies. In general, more symmetry-breaking phase-transitions may have occurred.

1.2 Some dynamical processes in the early universe

An important motivation for the study of gauge fields at high temperatures comes from electroweak baryogenesis [105,106]. This deals with the question why the baryon-photon ratio has the value (1.1). One would like to explain this value without assumptions about the initial condition. Let us sketch here a standard scenario for electroweak baryogenesis due to Cohen, Kaplan and Nelson [40,100]. As we mentioned before, this scenario requires a first-order
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electroweak phase-transition. At the phase-transition, bubbles will nucleate with in the interior the broken Higgs phase and outside the symmetric phase. These bubbles will expand and collide until the entire universe is in the broken phase.

A useful ingredient for this scenario is that baryon-violating processes in the broken phase are much slower than in the symmetric phase. For a strong enough phase transition, effectively no baryon-violating processes occur in the interior of the bubbles. These processes tend to wash out a non-zero baryon or anti-baryon-number density.

The expanding bubbles together with the baryon-number violating processes can be used to generate a resulting baryon number as follows. If one assumes particles and anti-particles scatter differently off the bubble wall there may be a net baryon-number density inside the bubble wall and an opposite net anti-baryon-number density outside the bubble wall (more precisely net number of left-handed baryons or anti-baryons). Outside the net anti-baryon density will be washed out by baryon-number violating processes. But the net baryon density inside the bubble will remain, leaving a non-zero baryon number density as the bubbles have filled out the universe.

In chapter 6 we will discuss some aspects of baryogenesis more in detail and suggest a different complementary scenario for baryon-number generation.

Another interesting dynamical process in the early universe is the formation of defects in symmetry breaking phase-transitions by the Kibble mechanism [70]. Topological stable configurations of gauge and Higgs fields exist as domain walls, cosmic strings and monopoles. These topological defects may affect the evolution of the universe, provide a dark matter candidate or, and may provide information over the earliest stages of the universe [73].

1.3 Classical approximation

There are a number of important processes in the early universe that involve dynamical Bose fields, such as bubble nucleation, the motion of a bubble wall, baryon-number violating processes and defect formation. These processes are difficult if not impossible to deal with perturbatively. An effective theory for the dynamics of Bose fields at high temperature is required.

An effective description of dynamical Bose fields is provided by the classical approximation [1,2,12,13,25,36,89,95,113]. Grigoriev and Rubakov
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[50] were the first to use a classical approximation to study a dynamical process (soliton anti-soliton pair production) at high temperatures. The essential observation is that the processes of interest (for instance those listed at the beginning of this section) involve Bose fields that have a spatial size large compared to the inter-particle distance $\hbar / T$. This implies that the typical momentum is small compared to the temperature. The classical theory is expected to be a good approximation at low-energy because the classical limit $\hbar \to 0$ and the low-energy limit of the Bose-Einstein distribution function $n$ yield the same result:

$$n(\omega_k) = \frac{1}{\exp(\beta \hbar \omega_k) - 1} \to \frac{1}{\beta \hbar \omega_k} \equiv n_{cl}(\omega_k), \quad \hbar \omega_k \ll T, \quad (1.2)$$

where $\omega_k = \sqrt{k^2}$ is the frequency at wave-number $k$, $\beta = 1/T$ the inverse temperature, and $n_{cl}$ the "classical" distribution function. The classical approximation has been applied to calculate non-perturbative phenomena such as the Chern-Simons diffusion rate [6,7,86,91,115] (relevant for theories of baryogenesis [105,106]) and the dynamics of the electroweak phase transition [85], as well as real-time (plasmon) properties of hot non-Abelian gauge theories [116].

However the classical approximation is not without problems. It has been well known since the work of Rayleigh, Einstein, and Jeans that in a classical description of a hot photon gas the free energy is ultraviolet divergent. For example, consider the Planck formula for the energy density for a gas of scalar bosons

$$E = \int \frac{d^3 k}{(2\pi)^3} \frac{\hbar k}{e^{\beta \hbar k} - 1} = \frac{\pi^2}{30} \frac{T^4}{\hbar^3}. \quad (1.3)$$

The classical limit (1.2) of the energy density is severely divergent

$$E_{cl} = \int \frac{d^3 k}{(2\pi)^3} T = \frac{T}{6\pi^2} \Lambda^3, \quad (1.4)$$

where we introduced a UV-cut-off $\Lambda$ on the integration. Hence, we cannot use the classical approximation for the calculation of the free energy. This is not surprising since the typical momentum of particles that contribute to the energy (1.3) is of the order of the temperature. For these momenta the classical approximation (1.2) is not expected to work anyway.

However, one might hope that for processes involving soft Bose fields the classical approximation is correct. An example of such a process is Chern-Simons number diffusion. Which is of interest for electroweak baryogenesis,
since it is related to the rate of baryon-number non-conservation \([54, 75]\). In the symmetric phase, the fields that have a typical momentum of order \(g^2 T\), with \(g\) the small gauge coupling, dominate the contribution to the rate \([9]\). In the broken phase, the typical momentum is of order \(g v\), with \(v\) the Higgs expectation value. Close enough to the electroweak phase-transition the typical momentum of the fields is small compared to the temperature. Then, in both cases, one may expect that the classical approximation should provide a good estimate for the diffusion rate. Around 1995-1996, classical lattice simulations have been used for the calculation of the rate by Ambjørn and Krasnitz \([6]\), Moore \([84]\), and Smit and Tang \([115]\). However around the same time, it was argued by Bödeker, McLerran, and Smilga \([25]\) that to really compute the Chern-Simons diffusion rate, hard thermal loop (HTL) corrections have to be included. HTL corrections were introduced, already around 1990, in the vocabulary of thermal field theory by Braaten and Pisarski \([31]\). They argued that bare perturbation theory breaks down in the calculation of soft amplitudes. To obtain a consistent expansion in the coupling \(g\) the HTL's have to be resummed.

A very relevant paper appeared in 1996, where Arnold, Son, and Yaffe \([12]\) showed that the naive classical estimate for the diffusion rate in the symmetric phase \(\Gamma_{CS} \sim (g^2 T)^4\), changes to

\[
\Gamma_{CS} \sim g^2 \hbar (g^2 T)^4, \tag{1.5}
\]

when HTL effects are taken into account. Their analysis made clear that the dynamics of non-perturbative soft gauge fields is affected by hard modes. One consequence of this is, as they argued, that the classical rate is sensitive to the cut-off \(\Lambda\).

Later it was shown by Bödeker \([27]\), that the estimate \((1.5)\) is not entirely correct in the small coupling limit, since scattering effects give a logarithmic correction to the Chern-Simons diffusion rate

\[
\Gamma_{CS} \sim g^2 \hbar (g^2 T)^4 \log(1/g^2 \hbar). \tag{1.6}
\]

In his derivation, Bödeker started with an effective classical theory, where HTL corrections were included. From the above examples, it is clear that the classical approximation plays an important role in understanding non-perturbative processes at high temperature.

We end with some inspiring questions, that form a guideline for this thesis. Are there (non-perturbative) infrared processes independent of the cut-off? If not, what is the cut-off dependence? Can such a cut-off dependence
be removed by counterterms? Do we need to include quantum corrections into an effective classical theory? If so, how do these change the classical dynamics? What is the proper $\hbar$-expansion of the quantum theory at high temperature?

1.4 Preview

The main subject of this thesis is to improve classical field theory, that is, to include the dominant quantum corrections and to add counterterms for the Rayleigh-Jeans divergences. This will all be based on perturbation theory. Since the classical theory is intended for calculations where perturbation theory is of no use, this requires some explanation. The point is that for hard modes (modes with energy of the order of the temperature: $\hbar \omega_k \sim T$), for which the classical approximation (1.2) breaks down, perturbation theory is expected to work. This is confirmed by many explicit results, among which we mention the next-to-leading order calculations of Schulz [109] and Rebhan [104] and calculations presented in chapter 4 in this thesis. A pedagogical review of the argument that supports this viewpoint is given by Arnold in [14].

In chapter 2 we review some basic concepts and techniques of thermal field theory both for quantum and classical field theories. The tadpole resummation of Dolan and Jackiw [42], dimensional reduction [8,33,63,80,93], and classical thermal field theory [2,102] are discussed. Also for some simple quantities the classical results are compared with the quantum results. We find the expected result that the classical contributions may be identified with the contributions of the soft modes.

In chapter 3, we turn to dominant quantum corrections, the well known hard thermal loops [31]. After a diagrammatic calculation of the HTL photon self-energy in QED, a kinetic formulation of HTL’s is given, following the work of Blaizot and Iancu [20–22]. This formulation allows the HTL’s to be included in a classical statistical theory, as was shown by Iancu [56]. We will show that the classical HTL equation of motion is consistent with the classical statistical theory, provided a random noise term is added. We review some of the physics included in HTLs, with a focus on the plasmon and non-perturbative excitations in the non-Abelian plasma. In particular, we will discuss the typical time scale for non-perturbative excitations is estimated, as was found by Arnold, Son, and Yaffe [12].

In chapter 4, we shall argue that, both in SU($N$) gauge theory and in scalar field theory with a $\phi^4$ interaction term, the divergences are restricted
to one- and two-loop (sub)diagrams [4]. This implies that the proof of Aarts and Smit [1,2] that local mass counterterms render classical $\phi^4$-theory finite up to two loops, may be extended to any number of loops. It will be shown that classical one-loop diagrams that correspond to HTL’s in the quantum theory lead to linear divergences; all other one-loop diagrams are finite in the classical theory. Also we present a general argument that two-loop diagrams can at most give logarithmic divergences. This is explicitly verified for two-loop self-energy corrections in SU($N$) and scalar theories. We also use the Ward identities to show that the logarithmic divergence in the SU($N$) self-energy is transverse [16].

In chapter 5 we introduce counterterms for the linear divergences [98]. It was already expected that for linear divergences a subtraction in the plasmon frequency is sufficient to render the theory free of linear divergences at one loop [3,56]. We will confirm this and, using the results of chapter 4, conclude that also beyond one loop, linear divergences will be absent. Furthermore, we will investigate the introduction of counterterms for classical lattice theories. In a sense, as explained there, we will find that to match a classical to a quantum theory is less complicated then to match a lattice theory to a continuum one. Nevertheless, in the latter case approximate counterterms may be given by a lattice generalization of the model in [56].

In the final chapter, we turn to a different topic, namely the problem of explaining the baryon asymmetry (1.1). Usually the required CP-violation is included in a model by an effective dimension-six operator [47,111]. We study the effect of dimension-eight CP-violating operators on sphaleron transitions [99]. We will argue that in a pure gauge theory in equilibrium the distribution function of the Chern-Simons number (that is related to the baryon number) will develop an asymmetry. Also a scenario for baryogenesis is presented where this effect is utilized.