Games, walks and grammars: Problems I've worked on

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Chapter 1

Introduction

This dissertation is about games, walks and grammars. That's what the title says, and it's completely correct. The second part of the title is 'problems I've worked on', and that too is correct. However, those of you that are expecting deep and interesting connections between these subjects, I have to give warning: you are going to be disappointed. I worked on these problems as separate problems in separate fields, and quite frankly, it seems almost hypocritical to try to establish a connection at this point. If you wish, you may see it as a demonstration that the techniques and the manner of thinking used in different fields of mathematics are not all that different.

The first part, titled 'Blackwell Games', is an extension of the thesis I wrote as a graduate student in 1995. It is about the problem of determinacy of Blackwell games, a class of infinite games of imperfect information, where both players simultaneously select moves from a finite set, infinitely many rounds are played, and payoff is determined by a Borel measurable function f on the set of possible resulting sequences of moves. In the original thesis I gave elementary proofs of determinacy for Blackwell games whose payoff function is an indicator function of a Borel set up to complexity $G_{\delta\sigma}$. D.A. Martin later found a reduction of the problem for general Borel payoff functions to the known result of determinacy of Borel perfect information games[16]. Both results are presented here, reworked to fit into a single framework (yielding some new proofs for auxiliary results). We also consider the Axiom of Blackwell Determinacy, an analogue for Blackwell games of the Axiom of Determinacy, and give some new results regarding the consequences of this axiom.

In the second part, titled 'Random Walks', we consider recurrence in reinforced random walks, where edges in a graph are traversed with probabilities that may be different (reinforced) at second, third etc. traversals. We focus on the case where the probability for any edge only changes once, after its first traversal. Thomas Sellke showed that in the case of the once-reinforced random walk on the infinite ladder, the walk is almost surely recurrent if reinforcement is small[31].

Here, we present some general tools which allow us to obtain the same result as a special case. After considering some other interesting cases, we combine these tools with an application of nonstandard analysis to graph theory, and show that the walk on the infinite ladder is also recurrent if reinforcement is sufficiently *large*. For readers who are not familiar with nonstandard analysis, a brief overview is provided.

The third part, titled 'The EMILE Grammar Inducer', is about the EMILE program, a program that I wrote for Pieter Adriaans this past year, and that I am still working on. The program reads in a text, and without prior knowledge attempts to determine the grammatical structure of the language, outputting the results in various ways. One purpose of the program is to verify whether natural languages satisfy a condition known as *shallowness*: if this is the case, the EMILE program should work well on natural languages. Here, we first look at the basic concepts and algorithms underlying the program. Then we consider the results of this approach, both in theory and in practice. In a separate appendix, explicit pseudo-code for each of the sub-algorithms of EMILE is given.